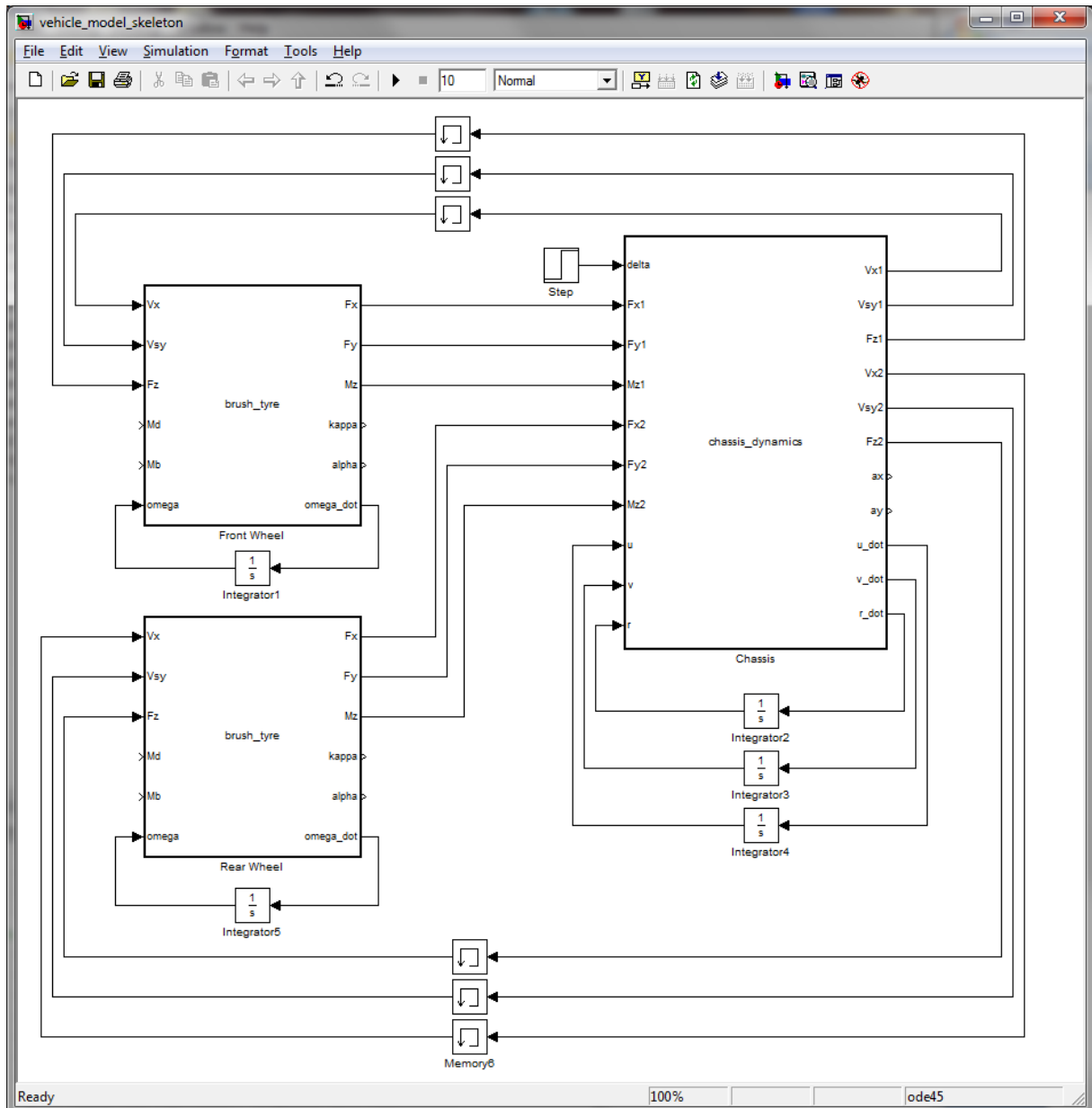


## Exercise 4: Non-linear bicycle model (draft)

In this exercise a non-linear bicycle model will be developed using MATLAB/Simulink. The brush model will be used to represent the tyres (you can reuse some of the results of exercise 3).

A Simulink model “vehicle\_model\_skeleton.mdl” is provided as a starting point. The first task is to enter the right equations in the various Embedded MATLAB function blocks (“Front Wheel”, “Rear Wheel” and “Chassis”). Note that “Front Wheel” and “Rear Wheel” are identical.



First make sure that you can execute a simulation with “vehicle\_model\_skeleton.mdl”. To be able to do this, a compiler needs to be installed, as described in the document “VD2012\_information\_v1.pdf”

A description of the in and outputs of each block:

$[F_x, F_y, M_z, \kappa, \alpha, \omega_{\dot{}}] = \text{brush\_tyre}(V_x, V_{sy}, F_z, M_d, M_b, \omega)$

*inputs*

$V_x$	longitudinal velocity at the wheel centre, in the plane of symmetry of the wheel
$V_{sy}$	lateral velocity of the wheel, perpendicular to the wheel plane
$F_z$	vertical force
$M_d$	drive moment applied to the wheel (see lecture notes, page 181/209)
$M_b$	brake moment applied to the wheel (see lecture notes, page 181/209)
$\omega$	angular velocity of the wheel

*outputs*

$F_x$	longitudinal force
$F_y$	lateral force
$M_z$	self aligning moment
$\kappa$	longitudinal slip
$\alpha$	side slip angle
$\omega_{\dot{}}$	wheel angular acceleration (see lecture notes, page 181/209).

$[V_{x1}, V_{sy1}, F_{z1}, V_{x2}, V_{sy2}, F_{z2}, a_x, a_y, u_{\dot{}}, v_{\dot{}}, r_{\dot{}}] = \text{chassis\_dynamics}(\delta, F_{x1}, F_{y1}, M_{z1}, F_{x2}, F_{y2}, M_{z2}, u, v, r)$

*inputs*

$\delta$	front wheel steer angle
$F_{x1}$	longitudinal force of front tyre (in the wheel frame)
$F_{y1}$	lateral force of front tyre (in the wheel frame)
$M_{z1}$	self aligning moment of front tyre
$F_{x2}$	longitudinal force of rear tyre (in the wheel frame)
$F_{y2}$	lateral force of rear tyre (in the wheel frame)
$M_{z2}$	self aligning moment of rear tyre
$u$	longitudinal velocity of the centre of gravity (see lecture notes, page 10)
$v$	lateral velocity of the centre of gravity
$r$	yaw velocity

*outputs*

$V_{x1}$	longitudinal velocity of the front wheel (in the wheel plane of symmetry)
$V_{sy1}$	lateral velocity of the front wheel (perpendicular to the wheel plane)
$F_{z1}$	vertical tyre force front tyre
$V_{x2}$	longitudinal velocity of the rear wheel (in the wheel plane of symmetry)
$V_{sy2}$	lateral velocity of the rear wheel (perpendicular to the wheel plane)
$F_{z2}$	vertical tyre force rear tyre
$A_x$	longitudinal acceleration
$A_y$	lateral acceleration
$u_{\dot{}}$	time derivative of the longitudinal velocity of the centre of gravity
$v_{\dot{}}$	time derivative lateral velocity of the centre of gravity
$r_{\dot{}}$	time derivative of the yaw velocity

Both blocks are still empty and the first task is to enter the right set of equations.

To calculate the side slip angle  $\alpha$  you may want to use the *atan2* –function instead of the *atan* function. When  $V_x$  is zero, *atan*( $-V_{sy}/V_x$ ) may lead to incorrect results, whereas *atan2*( $-V_{sy}, V_x$ ) will return  $\pi/2$  (90 degrees).

Parameters of the vehicle model:

$m$	vehicle mass	1500 kg
$I_{zz}$	vehicle yaw moment of inertia	2700 kgm <sup>2</sup>
$l$	wheel base	3.0 m
$a$	distance between front tyre and centre of gravity	1.5 m
$h$	vehicle centre of gravity height above the road	0.6 m
$r_l$	loaded tyre radius (assumed to be constant)	0.3 m
$I_p$	polar moment of inertia of a wheel	2 kgm <sup>2</sup>

Tyre model parameter (brush model exercise 3)

$r_f$	free tyre radius	0.3 m
$c_z$	tyre vertical stiffness	250000 N/m
$c_p$	tread element stiffness	$9 \cdot 10^6$ N/m <sup>2</sup>
$\mu$	friction coefficient	1.2

Initial conditions:

$u$	vehicle longitudinal velocity	20 m/s
$v$	vehicle lateral velocity	0 m/s
$r$	vehicle yaw velocity	0 rad/s
$\Omega$	wheel angular velocity	zero longitudinal slip

Simulations to be executed:

- 1) Step steer at  $t=1$  sec.  
small steering input of 0.02 rad (1.15 deg.)  
large steering input of 0.1 rad. (5.73 deg.)
- 2) Introduce a braking system which applies 75% of the brake torque to the front wheel and 25% to the rear wheel.  
start the simulation with the small step steer input of 0.02 rad at  $t=1$  sec. Apply a step in the brake torque of 1500 Nm at  $t=5$  sec.
- 3) Introduce rear wheel drive, by applying a driving torque at the back wheel  
start the simulation with the small step steer input of 0.02 rad at  $t=1$  sec.  
-Apply a step in the rear wheel drive torque of 1000 Nm at  $t=5$  sec.  
-Apply a step in the rear wheel drive torque of 3000 Nm at  $t=5$  sec.
- 4) Introduce front wheel drive, by applying a driving torque at the front wheel  
start the simulation with the small step steer input of 0.02 rad at  $t=1$  sec.  
-Apply a step in the front wheel drive torque of 1000 Nm at  $t=5$  sec.  
-Apply a step in the front wheel drive torque of 3000 Nm at  $t=5$  sec.