Exercise 2: Handling dynamics using the single track model

In this exercise the theory of lectures 2 and 3 will be applied.

For people being less familiar with MATLAB, an interactive course can be found online: http://www.imc.tue.nl/. In this exercise the following MATLAB commands can be used:

<u>Creation of a "sys" object</u>

```
sys = ss(A,B,C,D)       % creates object "sys" of the continuous-time state-space model:
                        % dx/dt = Ax(t) + Bu(t)
                        % y(t) = Cx(t) + Du(t)
```

<u>Eigenvalues, eigenfrequencies,damping</u>

```
lambda = eig(A);                        % calculate the eigenvalues of matrix A
freq = imag(lambda)/(2*pi);             % calculate damped eigenfrequencies
dr = -real(lambda)/(norm(lambda))*100;  % critical damping ratio in percent
```

<u>Transfer function</u>

```
freq=[0.1:0.1:5];                       % frequency vector in Hz (0.1 to 5 Hz)
w= 2*pi*freq;                           % create vector w in rad/s
[mag,phase] = bode(sys,w);              % transfer functions of "sys" at frequencies "w"
```

"mag" gives the magnitude, "phase" the phase angle in degrees. Note that "mag" and "phase" have three dimensions, "mag(2,1,:)" represents the magnitude of the second output to the first input of the object "sys" for all frequencies specified in the vector "w". The MATLAB "plot" command does not accept a signal with three dimensions, use the "squeeze" command to remove the singleton dimensions, example:

```
plot( freq, squeeze(mag(2,1,:) ); % plot magnitude of the transfer function, input 1 - output 2
```

To plot the transfer function on a log-log scale, use "loglog" instead of "plot". "semilogx" gives a logarithmic x-axis.

As an alternative to "bode" it is also possible to use "freqresp".

```
H = freqresp(sys,w);          % complex values of the transfer function H.
```

Just as with the "bode" command H has three dimensions: output, input, frequency.

```
mag21 = norm(H(2,1,:);    % magnitude of the transfer function, input 1 - output 2
phase21 = angle(H(2,1,:); % phase angle (radians!) of the transfer function, input 1 - output 2
```

<u>Step response</u>

```
time=[0:0.01:5];              % time vector (0 to 5 sec.)
y = step(sys,time);           % calculate step response
```

The vector "y" has three dimensions: time, output, input

```
plot(time, squeeze(y(:,2,1) );  % plot time history of output 2 to a step on the input  1
```

<u>Response to an arbitrary input signal</u>

```
time=[0:0.01:5];              % time vector (0 to 5 sec.)
u = 0.1*sin(2*pi*time);       % input signal with a frequency of 1Hz with amplitude of 0.1
y = lsim(sys,u,time);         % perform the linear simulation
```

In case of multiple inputs, "u" should have as many columns as there are input to the system!

Tool development

In the exercise we need to make similar calculations as presented in lecture 2. Before analysing a new vehicle with different parameters, it is a good approach to see if the graphs presented in lecture 2 can be reproduced. In particular the focus will be on calculating the eigenvalues, step response and transfer functions using the state space representation of the single track model.

Note that:

- The equations are given on page 44
- The model parameters are given on page 46
- The eigenvalues (in terms frequency and damping) as a function of forward velocity are given on page 46
- Step responses for different velocities are given on page 47.
- Transfer functions are given on page 50 and 51.
- Degrees may be used in the graphs, whereas the equations on page 44 are in radians. Do the conversion from radians to degrees when creating the plots.

In order to calculate the step response of the side slip angles α_1 and α_2 the matrices $\boldsymbol{C}$ and $\boldsymbol{D}$ have to be modified (how?).

Vehicle measurements

A vehicle has undergone a similar test programme as shown in lecture 3, for a fully loaded condition (driver + passengers and cargo). The data is stored in a number of *.mat files and graphs are available when running the MATLAB-file “plot_measurements.m”. In the *.mat files SI units are used: N, kg, m, rad, s. So angles are in radians and the velocity is expressed in m/s. When creating the plots the conversion to km/h and degrees is made. Note that the steering wheel angle δ_s and not the steer angle of the front wheel δ is measured, these angles are related by the steering ratio i_s, $\delta_s = i_s\delta$ and $i_s = 17$. The wheel base of the vehicle: $l = 2.51$ m.

When the handling tests were executed the vehicle was put on scales, with the following results:front axle 809.5 kg and rear axle 840.0 kg

a) Determine the location of the centre of gravity based on loads on both axles. Calculate the distances a and b.

<u>Steady state cornering</u>

b) Provided with this exercise is the measurement file “steady_state_circular_loaded.mat”. Make a plot of the corner radius R for the different tests and determine an average value (various possibilities exist to check this, see VD lecture notes page 57).

c) For the steady state circular test we have information on the steering wheel angle δ_s and vehicle side slip angle β as a function of the lateral acceleration a_y. Determine the cornering stiffness C_1 and C_2 so that the single track vehicle model agrees with the measurement results in the linear range (0 to 4 m/s^2). Create plots in which the model is compared with the measurements. Hint: modify the plot_measurements file and add the results of the linear bicycle model to the existing graphs.

d) Calculate the understeer gradient η. Is the vehicle understeered, oversteered or neutral steer?

e) Make a plot of the yaw velocity gain for the velocity range 0 to 100 km/h, both experiment and model in one plot. An example is shown on the VD lecture notes 59.

Random steering

f) The file “plot_measurements.m” also displays the transfer functions of the vehicle. The numerical data can be found in the file “pseudo_random_loaded.mat”. The transfer functions of the vehicle were measured at an average forward velocity of 100 km/h. Create a state-space model of the linear bicycle model (see also VD lecture notes page 44). Calculate the transfer functions between the steering angle δ_s and lateral acceleration a_y and yaw velocity r. Use the values of the cornering stiffness C_1 and C_2 obtained under c) and estimate the yaw moment of inertia I_{zz} using the rules of thumb given on VD sheets page 14. Again plot both the measured transfer function and calculated transfer function in a single plot (magnitude and phase). You may tune the value of yaw moment of inertia I_{zz} to get the best match. Hint: again the easiest way to create this plot is to modify the plot_measurements file and add the transfer function of the model.

g) As was shown in lecture 3, adding the relaxation length of the tyre improves the accuracy of the model, see VD sheets page 65-81. Extend the state space description of the bicycle model, as given on page 44, so that it includes the tyre relaxation behaviour. Give the $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$ matrix.

h) Similar to question f) compare the measured transfer functions with the model results, now including the tyre relaxation lengths σ_1 and σ_2. The relaxation length is dependent on the vertical load on the tyres, but the lateral stiffness of the tyres is (almost) constant (stiffness k in VD lecture notes page 65). Calculate the relaxation for the front and rear tyre by dividing the cornering stiffness through the tyre lateral stiffness, so:

$$\sigma_1 = \frac{C_1}{k}, \sigma_2 = \frac{C_2}{k}$$

Tune the magnitude of the tyre lateral stiffness k (and possibly the vehicle yaw moment of inertia I_{zz}) to get the best match with the measurements for both the loaded and unloaded case. Provide the plots of the transfer functions, has the model become more accurate?

Eigenfrequency analysis

i) By analysing the eigenvalues of the matrix $\boldsymbol{A}$, we can determine the frequency and damping ratio of the system (see VD lecture notes page 46). Since the system dynamics of the vehicle are velocity dependent, this analysis has to be performed at different forward velocities. Using the results from f) and h) calculate the frequencies and damping ratios at 50, 100 and 150 km/h for the model with and without relaxation lengths. Discuss the results.

J-turn

j) With the same vehicle a J-turn manoeuvre has been performed, the measurement data is given in the file “Jturn_loaded.mat”. Use the measured steering input as input for the simulation model and calculate the lateral acceleration and yaw velocity. The forward velocity may assumed to be constant and equals 100 km/h. Plot the simulation model results on top of the measurement results (see also VD sheets page 78). Compare the model with and without relaxation length, do you see an improvement?

Double lane change

k) Do the same as j) but now for the double lane change manoeuvre (file “lane_change_loaded.mat”). Note that during the experiment the forward velocity is not constant and does not equal 100 km/h, use the average value. Compare the accuracy of the simulation model for questions j) and k). What is the reason for the difference in accuracy?