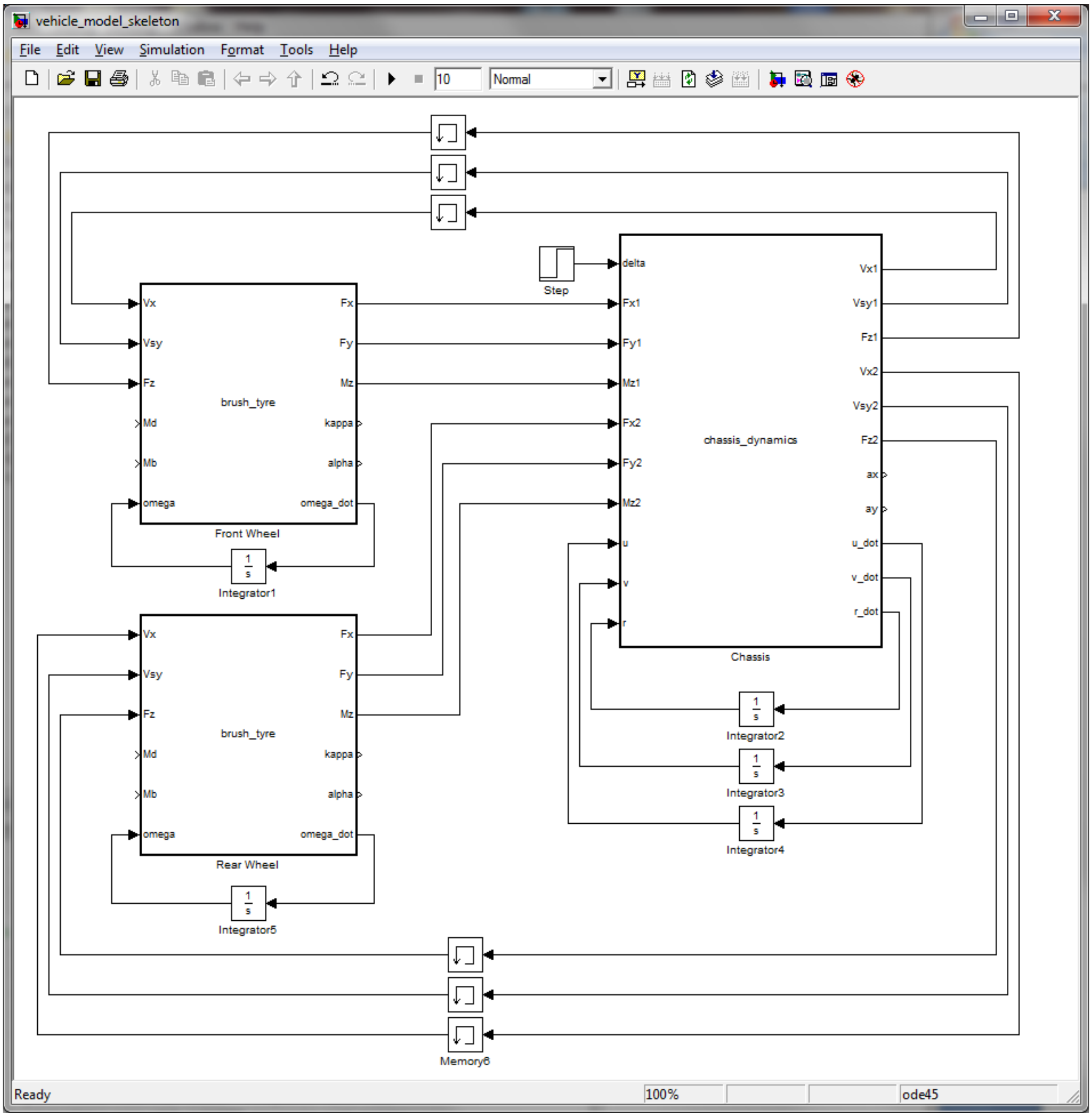
Exercise 4: Non-linear bicycle model

In this exercise a non-linear bicycle model will be developed using MATLAB/Simulink. The brush model will be used to represent the tyres (you can reuse some of the results of exercise 3).

A Simulink model “vehicle_model_skeleton.mdl” is provided as a starting point. The first task is to enter the right equations in the various Embedded MATLAB function blocks (“Front Wheel”, “Rear Wheel” and “Chassis”). Note that “Front Wheel” and “Rear Wheel” are identical.



First make sure that you can execute a simulation with “vehicle_model_skeleton.mdl”. To be able to do this, a compiler needs to be installed, as described in the document “VD2012_information_v1.pdf”. If the “mex” command still gives errors, typically on 64 bit windows machines, then it may be necessary to install an additional patch KB2519277 (see: http://www.microsoft.com/en-us/download/details.aspx?id=4422)

A description of the in and outputs of each block:

[Fx,Fy,Mz,kappa,alpha,omega_dot]=brush_tyre(Vx,Vsy,Fz,Md,Mb,omega)

inputs

Vx	longitudinal velocity at the wheel centre, in the plane of symmetry of the wheel
Vsy	lateral velocity of the wheel, perpendicular to the wheel plane
Fz	vertical force
Md	drive moment applied to the wheel (see lecture notes, page 209)
Mb	brake moment applied to the wheel (see lecture notes, page 209)
omega	angular velocity of the wheel

outputs

Fx	longitudinal force
Fy	lateral force
Mz	self aligning moment
kappa	longitudinal slip
alpha	side slip angle
omega_dot	wheel angular acceleration (see lecture notes, page 181/209).

[Vx1,Vsy1,Fz1,Vx2,Vsy2,Fz2,ax,ay,u_dot,v_dot,r_dot] =
chassis_dynamics(delta,Fx1,Fy1,Mz1,Fx2,Fy2,Mz2,u,v,r)

inputs

delta	front wheel steer angle
Fx1	longitudinal force of front tyre (in the wheel frame)
Fy1	lateral force of front tyre (in the wheel frame)
Mz1	self aligning moment of front tyre
Fx2	longitudinal force of rear tyre (in the wheel frame)
Fy2	lateral force of rear tyre (in the wheel frame)
Mz2	self aligning moment of rear tyre
u	longitudinal velocity of the centre of gravity (see lecture notes, page 10)
v	lateral velocity of the centre of gravity
r	yaw velocity

outputs

Vx1	longitudinal velocity of the front wheel (in the wheel plane of symmetry)
Vsy1	lateral velocity of the front wheel (perpendicular to the wheel plane)
Fz1	vertical tyre force front tyre
Vx2	longitudinal velocity of the rear wheel (in the wheel plane of symmetry)
Vsy2	lateral velocity of the rear wheel (perpendicular to the wheel plane)
Fz2	vertical tyre force rear tyre
ax	longitudinal acceleration
ay	lateral acceleration
u_dot	time derivative of the longitudinal velocity of the centre of gravity
v_dot	time derivative lateral velocity of the centre of gravity
r_dot	time derivative of the yaw velocity

Both blocks are still empty and the first task is to enter the right set of equations.

In this exercise we will **not** linearise the equations of motion as done previously in lecture 2. As a reference the equations presented for the two track vehicle model may serve as a reference. These are given page 207 to 211 of the lecture notes, note that the roll angle φ is absent in the model to be developed in this exercise.

Also note that the load transfer due to longitudinal acceleration or deceleration should be taken into account, i.e. the vertical tyre forces F_{z1} and F_{z2} change as a function of a_x.

The brush tyre model of exercise 3 can be used again. You may consider changing the input of the brush model. As indicated (implicitly) on page 140 of the lecture notes:

$$\sigma_x = -\frac{V_{sx}}{V_r} \text{ and } \sigma_y = -\frac{V_{sy}}{V_r}$$

So the sliding velocities at the tyre-road contact point can be used directly to calculate the slip forces. Still the side slip angle α and longitudinal slip κ should be provided as output, as we are familiar with them and they give a good indication of the state of the tyre. To calculate the side slip angle α you may want to use the *atan2* –function instead of the *atan* function. When *Vx* is zero, *atan(-Vsy/Vx)* may lead to incorrect results, whereas *atan2(-Vsy, Vx)* will return pi/2 (90 degrees).

For the tyre radius we simplify things by assuming that there is no difference between the free tyre radius, the loaded tyre radius and effective rolling radius. So $r_f = r_l = r_e$. Only for the determination of the contact length a inside the brush model a tyre deflection ρ has to be calculated.

Parameters of the vehicle model:

m	vehicle mass	1500 kg
I_{zz}	vehicle yaw moment of inertia	2700 kgm^2
l	wheel base	3.0 m
a	distance between front tyre and centre of gravity	1.5 m
h	vehicle centre of gravity height above the road	0.6 m
r_l	loaded tyre radius (assumed to be constant)	0.3 m
I_p	polar moment of inertia of a wheel	2 kgm^2

Tyre model parameter (brush model exercise 3)

r_f	free tyre radius	0.3 m
c_z	tyre vertical stiffness	250000 N/m
c_p	tread element stiffness	9•10^6 N/m^2
μ	friction coefficient	1.2

Initial conditions:

u	vehicle longitudinal velocity	20 m/s
v	vehicle lateral velocity	0 m/s
r	vehicle yaw velocity	0 rad/s
Ω	wheel angular velocity	zero longitudinal slip

Please make sure that all memory blocks and integrators in the model have the correct initial conditions! The memory blocks are there to prevent algebraic loops (simply said: “a chicken and egg problem”). This includes proving an initial value for the vertical force of the tyre. You may try to omit the memory blocks, but the MATLAB will issue an error message, suggesting to “break the algebraic loop”. As the user of memory blocks can make the results dependent on the time step of the integrator, the maximum time step is set to 0.001 sec. (see menu: Simulation – Configuration parameters - Max Step Size).

a) Include a listing of both the brush_tyre and chassis_dynamics functions in the report. Whenever necessary, include comments in the code to make it more clear and readable.

The following simulations need to be executed and discussed in the report:

b) Step steer
Step steer at t=1 sec.
small steering input of 0.02 rad (1.15 deg.)
large steering input of 0.1 rad. (5.73 deg.)

c) Braking
Introduce a braking system which applies 75% of the brake torque to the front wheel and 25% to the rear wheel. start the simulation with the small step steer input of 0.02 rad at t=1 sec. Apply a step in the brake torque of 1500 Nm at t=5 sec.

d) Rear wheel drive
Introduce rear wheel drive, by applying a driving torque at the back wheel
start the simulation with the small step steer input of 0.02 rad at t=1 sec.
-Apply a step in the rear wheel drive torque of 1000 Nm at t=5 sec.
-Apply a step in the rear wheel drive torque of 3000 Nm at t=5 sec.

e) Front wheel drive
Introduce front wheel drive, by applying a driving torque at the front wheel
start the simulation with the small step steer input of 0.02 rad at t=1 sec.
-Apply a step in the front wheel drive torque of 1000 Nm at t=5 sec.
-Apply a step in the front wheel drive torque of 3000 Nm at t=5 sec.

For each of these simulations include plots of at least:

- Front and rear side slip angle
- Front and rear longitudinal slip
- Longitudinal and lateral acceleration
- Vehicle yaw rate

Briefly discuss and explain the results and include additional graphs whenever they are needed to support your observations.