Vehicle Dynamics 4L150

Eindhoven University of Technology
Mechanical Engineering - Dynamics & Control

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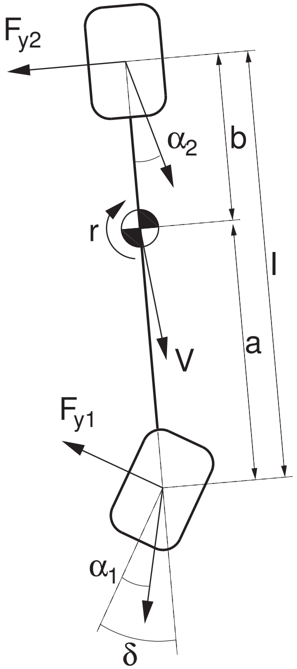
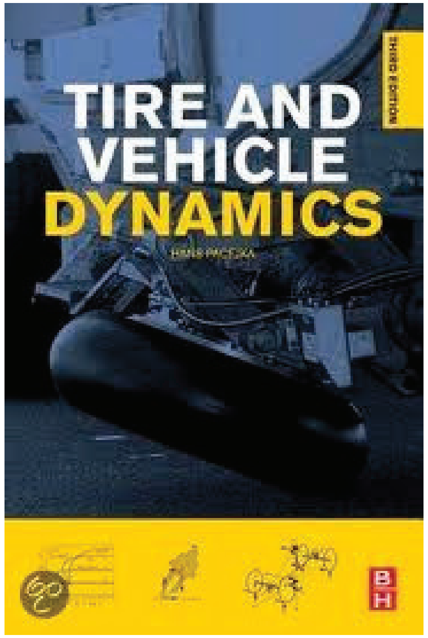
lecture notes 2012



These lecture notes are based on the book:

Tire and Vehicle Dynamics
Hans Pacejka
Third edition 2012
Butterworth-Heinemann
ISBN 978-0-08-097016-5



Contents

1. Introducing Vehicle Dynamics

contents:

- introduction to vehicle dynamics
- equations of motion
- forces acting on the vehicle
- tyre behaviour basics

Vehicle dynamics

study of vehicle behaviour (motions, vibrations) for various driving conditions

subjects:
- “ride”, e.g. comfort on uneven roads
- “handling”, e.g. response to steering inputs
- (dynamic) stability, e.g. vehicle roll-over

relevant vehicle components:
- suspension
- steering system
- braking system
- tyres

these systems cannot be designed without taking the full vehicle behaviour into consideration.

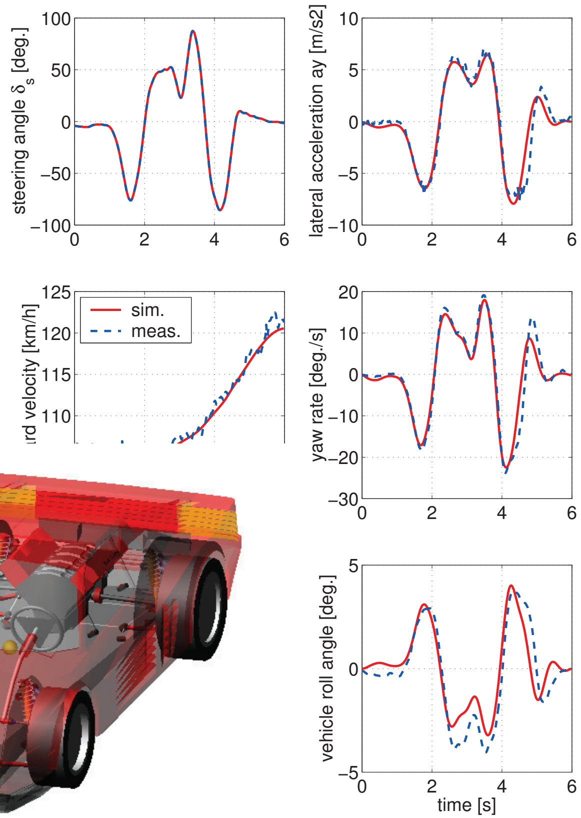
revolution over the past decades:

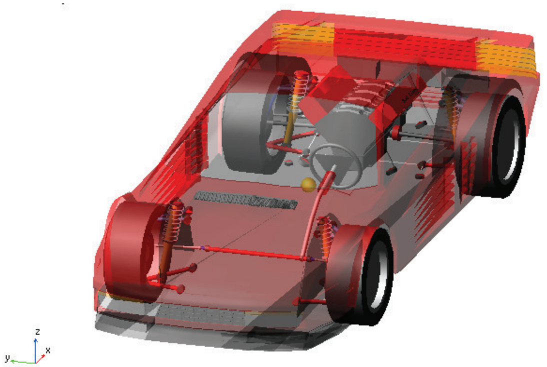
increase in computing power!

consequences:
- shift towards “virtual prototyping”: computer simulations are becoming an integral part of the design process.
- introduction of very sophisticated control systems to improve vehicle behaviour

"virtual prototyping":

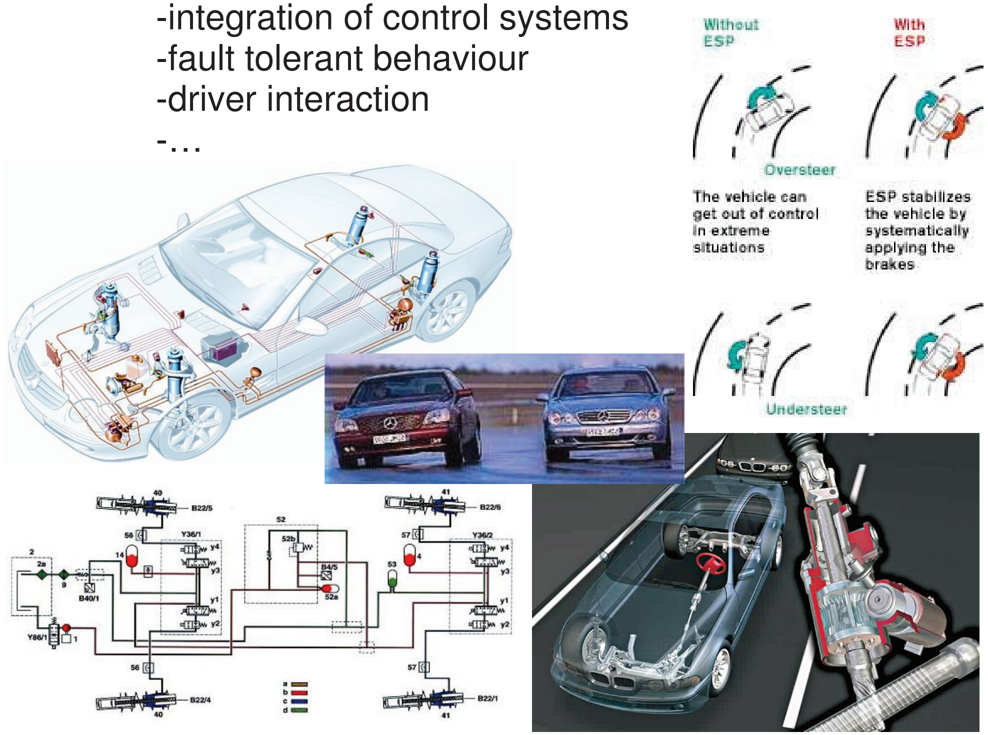
- less prototypes in hardware required, accelerated design process
- requires accurate modelling of vehicle (components); ever increasing demands… improves understanding
- may require detailed knowledge of driver perception, behaviour
- hardware testing will <u>always</u> be required!





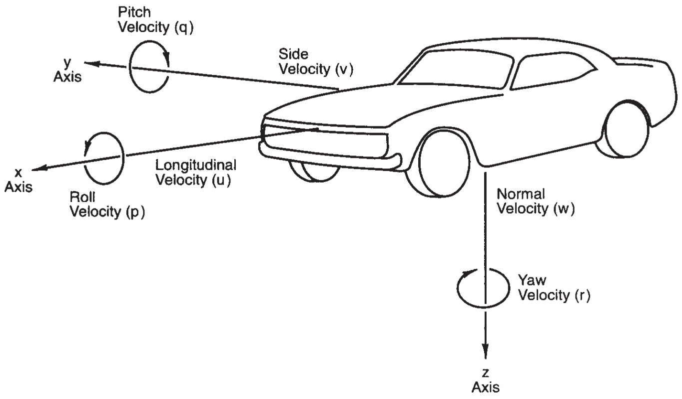
control systems:

- introduction of many new systems to improve vehicle dynamics behaviour:
  - ABS: anti-lock braking system
  - ESP: handling stability improvement
  - EDC: adjustable shock absorbers
  - AFS: active front steering
  - ACC: adaptive cruise control
  - …
- number of actuators: +
  number of sensors: ++
  controller complexity: +++++
- challenges ahead:
  -integration of control systems
  -fault tolerant behaviour
  -driver interaction
  -…



Vehicle motions

SAE sign convention



vehicle x,y,z axis system

- z: normal to the road
- x: pointing forward, through plane of symmetry
- x,y: parallel to the road

translation

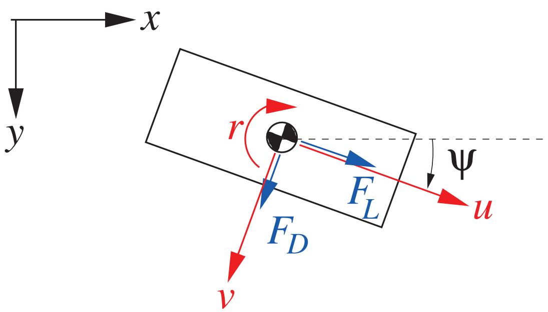
- x-direction: longitudinal velocity u
- y-direction: lateral or side velocity v
- z-direction: vertical or normal velocity w

rotation

- x-direction: roll velocity p
- y-direction: pitch velocity q
- z-direction: yaw velocity r

Equations of motion (moving axis system)

2D case



$$\dot{x} = u\cos\psi - v\sin\psi$$
$$\dot{y} = u\sin\psi + v\cos\psi$$

$$\ddot{x} = \dot{u}\cos\psi - \dot{v}\sin\psi + (-u\sin\psi - v\cos\psi)\dot{\psi}$$
$$\ddot{y} = \dot{u}\sin\psi + \dot{v}\cos\psi + (u\cos\psi - v\sin\psi)\dot{\psi}$$

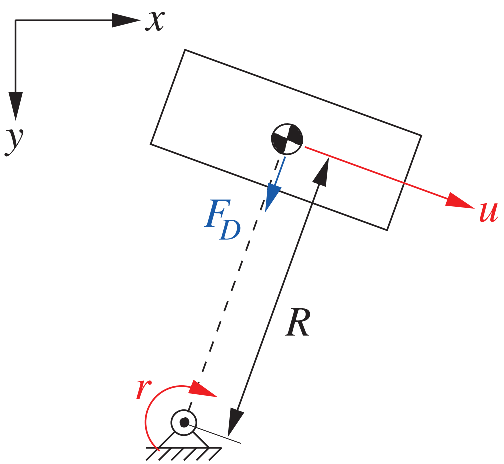
$$m\ddot{x} = F_L\cos\psi - F_D\sin\psi$$
$$m\ddot{y} = F_L\sin\psi + F_D\cos\psi$$

$$m\ddot{x}\cos\psi + m\ddot{y}\sin\psi = F_L$$
$$-m\ddot{x}\sin\psi + m\ddot{y}\cos\psi = F_D$$

substitution of accelerations (note: $r = \dot{\psi}$)

$$m(\dot{u} - vr) = F_L$$
$$m(\dot{v} + ur) = F_D$$
$$I_{zz}\dot{r} = M_z$$

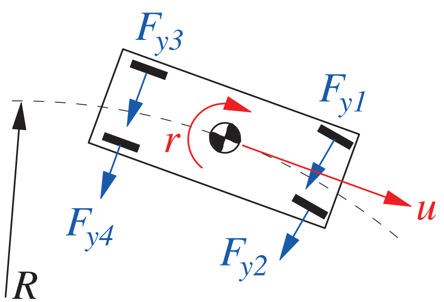
application: rotating mass



angular velocity r and radius R are constant

so $\dot{v} = 0$ and $u = rR \quad \Rightarrow \quad F_D = \frac{mu^2}{R}$

very similar: steady state cornering of a vehicle (constant forward velocity u and corner radius R)



tyres will have to generate lateral forces (F_y) to keep the vehicle on the circular track

generic 3D case:

Newton-Euler equations:

$$m(\dot{\mathbf{v}} + \boldsymbol{\omega} \times \mathbf{v}) = \mathbf{F}$$

$$\mathbf{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega} = \mathbf{M}$$

where:

$$\mathbf{v} = [u \quad v \quad w]^T$$

$$\boldsymbol{\omega} = [p \quad q \quad r]^T$$

$$\mathbf{J} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ & I_{yy} & I_{yz} \\ sym. & & I_{zz} \end{bmatrix}$$

$\mathbf{F}$ forces on the centre of gravity in the body fixed axis system

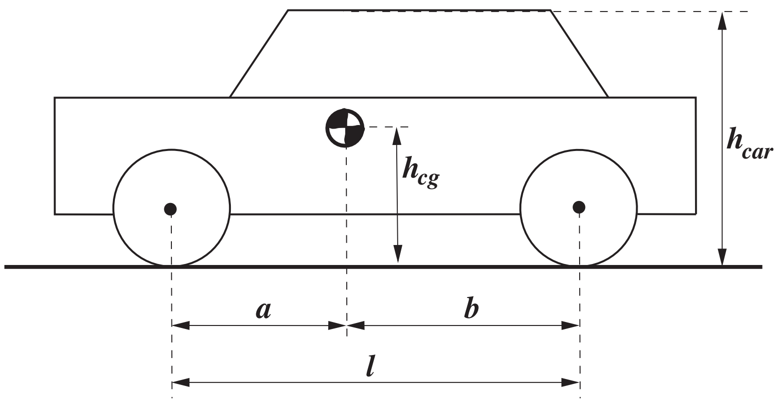
$\mathbf{M}$ moments in the body fixed axis system

note:

- inertia tensor $\mathbf{J}$ is constant
- symmetry about the x-axis: $I_{xy}, I_{yz} = 0$

Centre of gravity

centre of gravity of the complete vehicle



some rules of thumb:

(NHTSA database of cars, vans, SUV, pickup trucks; SAE paper 1999-01-1336)

- longitudinal position

  empty vehicle, driver only: $0.35 < \dfrac{a}{l} < 0.48$

  fully loaded vehicle: $0.45 < \dfrac{a}{l} < 0.57$

  (CG moves rearward when loading the vehicle)

- lateral position
  (close to) plane of symmetry of vehicle

- vertical position

  empty vehicle, driver only: $0.35 < \dfrac{h_{cg}}{h_{car}} < 0.43$

Moments of inertia

full scale pendulum tests can be done to determine these properties

some rules of thumb:

(NHTSA database of cars, vans, SUV, pickup trucks; SAE paper 1999-01-1336)

$$0.14 < \frac{I_{xx}}{mw^2} < 0.19$$

$$0.20 < \frac{I_{yy}}{ml^2} < 0.25$$

$$0.22 < \frac{I_{zz}}{ml^2} < 0.26$$

$$-0.01 < \frac{I_{xz}}{mwl} < 0.03$$ (negative values for pick up trucks only)

note:

- empty vehicle, driver only
- vehicle mass m, wheelbase l, track width w

Forces and moments acting on the vehicle:

- aerodynamics
- gravity
- tyre forces

aerodynamic forces/moments

for a normal passenger car aerodynamic forces are small compared to the tyre forces occurring during extreme vehicle manoeuvres

example:

-vehicle mass $m = 1200$ kg,
-air density $\rho = 1.226$ kg/m^3
-drag coefficient $c_D = 0.4$
-frontal area $A = 2$ m^2

- rolling resistance: 1% $\Rightarrow$ 0.1 kN
- emergency braking: $\mu \approx 0.9$ $\Rightarrow$ 11 kN
- aerodynamic drag: $F_d = \frac{1}{2} \rho c_D A V^2$

100 km/h $\Rightarrow$ 0.4 kN
200 km/h $\Rightarrow$ 1.5 kN

it is fairly common to neglect aerodynamic forces in a vehicle handling analysis (except for special condition, e.g. cross wind gust)

note:

- aerodynamic forces may affect the vertical force on front- and rear tyres (e.g. “lift”)
- roll, pitch and yaw damping due to aerodynamic moments are very small

for racing cars the situation is clearly different:

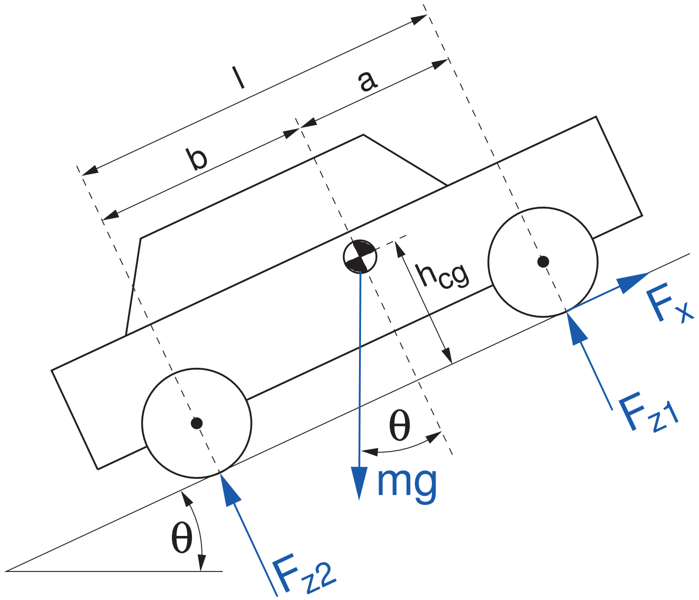
- the aim is to increase the vertical force on the tyres. In this way the potential for transferring longitudinal and lateral forces by the tyre is also increased.

e.g. modern F-1 racing car: at top speed the vertical load on the tyres is *four* times as high compared to a vehicle standing still.

gravity
for a normal road car gravity determines to a large extend the average vertical force on the tyres

some special cases:

- driving up (or down) a hill



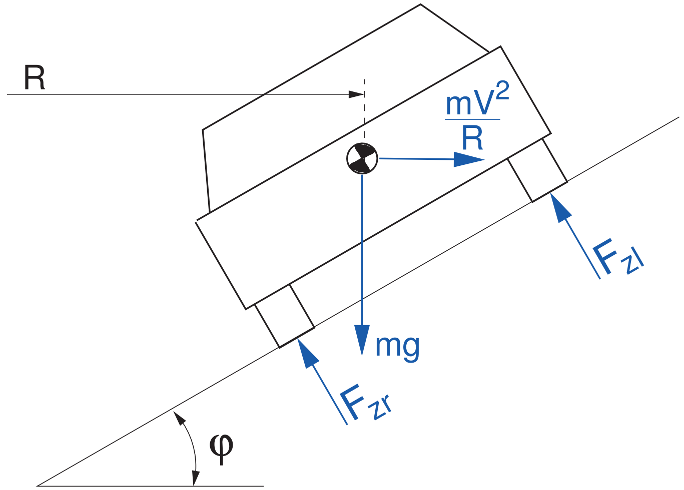
front wheel driven car:

$$F_{z1} + F_{z2} = mg\cos\theta$$

$$F_x = mg\sin\theta$$

$$F_{z1} = \frac{(bmg\cos\theta - h_{cg}mg\sin\theta)}{l}$$

- banked corners



for a banked corner a neutral speed V_n exists where no steering is required and the tyres don't produce lateral forces

the neutral speed can be calculated as:

$$\tan\varphi = \frac{mV_n^2}{R}\frac{1}{mg} \quad \Rightarrow \quad V_n = \sqrt{Rg\tan\varphi}$$

note:

- neutral speeds depends on the corner radius and banking angle
- the vertical tyre force increases
- for velocities above or below the neutral speed steering is required

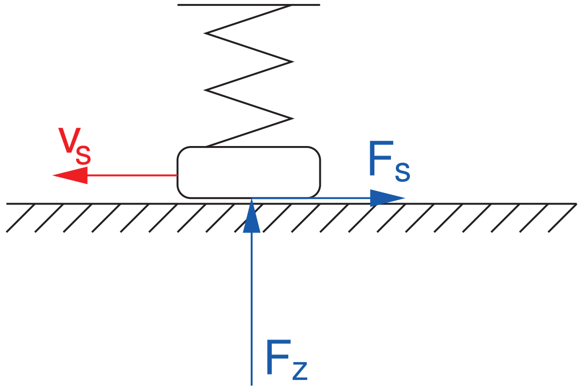
example:

- test track ATP-Papenburg (Germany)
  max. banking $\varphi = 50$ deg.
  corner radius $R = 500$ m
  neutral speed $V_n = 250$ km/h

Tyre forces/moments

simplified view of the (rolling) tyre:

- friction element which generates a shear force due to a relative sliding velocity with respect to the ground



the shear force is limited by the friction coefficient and normal load: $F_s \leq \mu F_z$

typical values for the friction coefficient μ :
(dry, clean conditions)

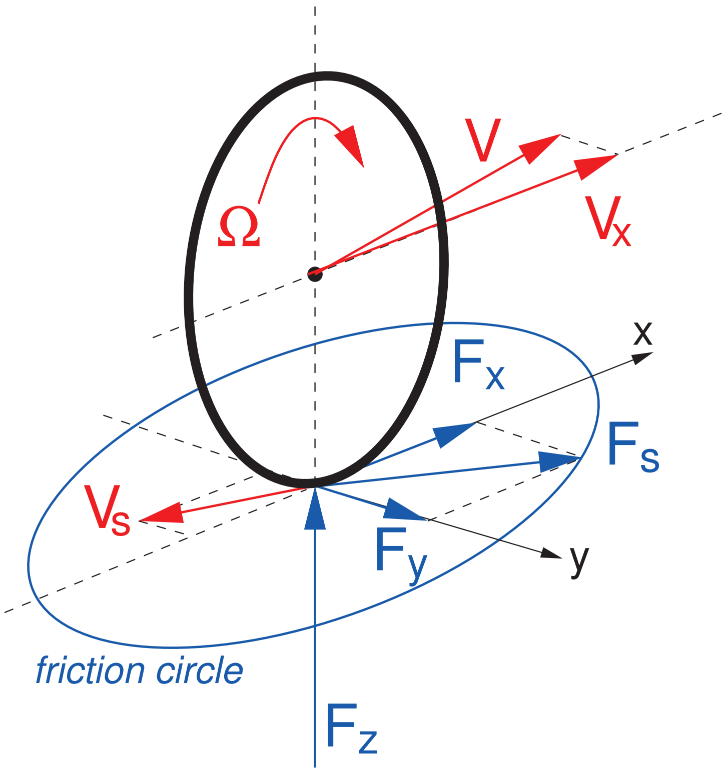
- truck, aircraft tyre: 0.6 - 0.8
- passenger car tyre: 0.9 - 1.2
- motorcycle tyre: 1.0 - 1.4
- racing tyre: 1.5 - 2.0

wet roads: speed dependent reduction
snow/ice: 0.05 – 0.2

sign convention for tyre forces:

- z: normal to the road
- x: pointing forward, through plane of symmetry
- x,y: parallel to the road

(note the analogy with the vehicle axis system)



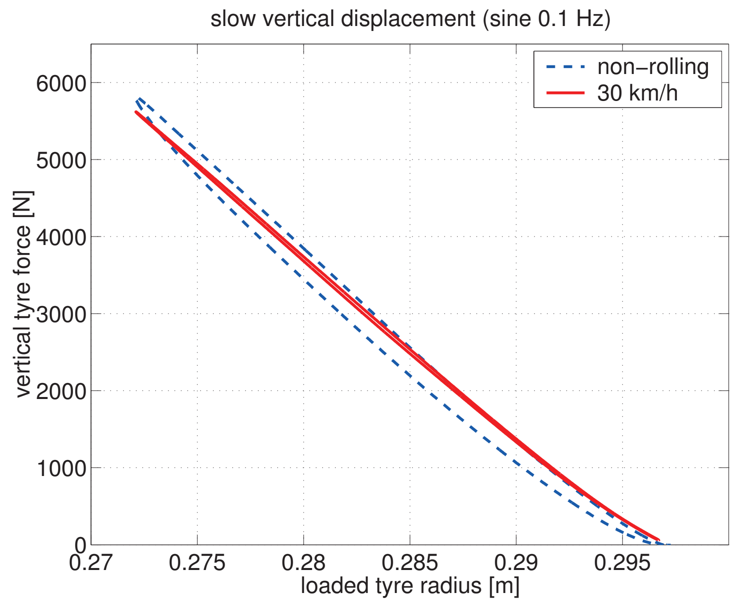
F_x longitudinal force (driving, braking)

F_y lateral force (steering)

F_z vertical or normal force (tyre compression)

vertical tyre behaviour

- tyre is almost a linear spring
- vertical tyre force F_z cannot become negative
- for a rolling tyre the amount of damping is rather small (and may be neglected)
- a non-rolling tyre has a fair amount of hysteresis



order of magnitude for a passenger car tyre:

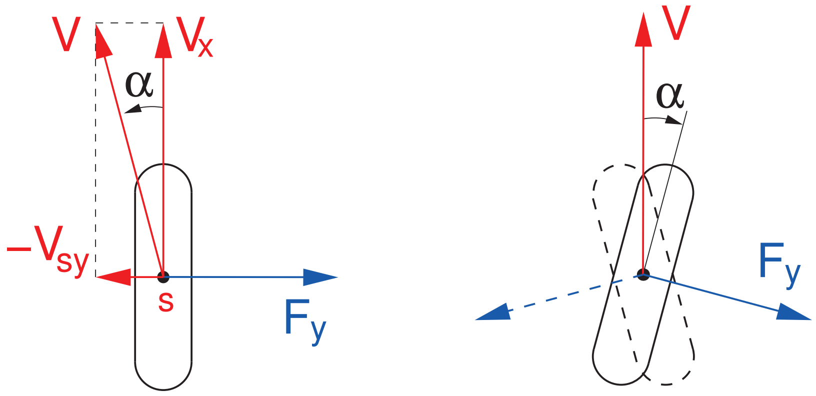
- spring stiffness 200000 N/m (200 N/mm)
- damping 50 Ns/m (rolling tyre)

tyre forces and moments generated under various slip conditions…

… will be discussed in detail!

for the moment:

- freely rolling wheel (no braking/driving)
- constant vertical force
- side slip only, no inclination angle



the lateral force F_y is a non-linear function of the slip angle (drift angle) α

$$\tan\alpha = -\frac{V_{sy}}{V_x}$$

V_{sy} : sliding velocity of tyre w.r.t. the road

F_y : lateral force acting from the road on the tyre

tyre deformation while cornering…

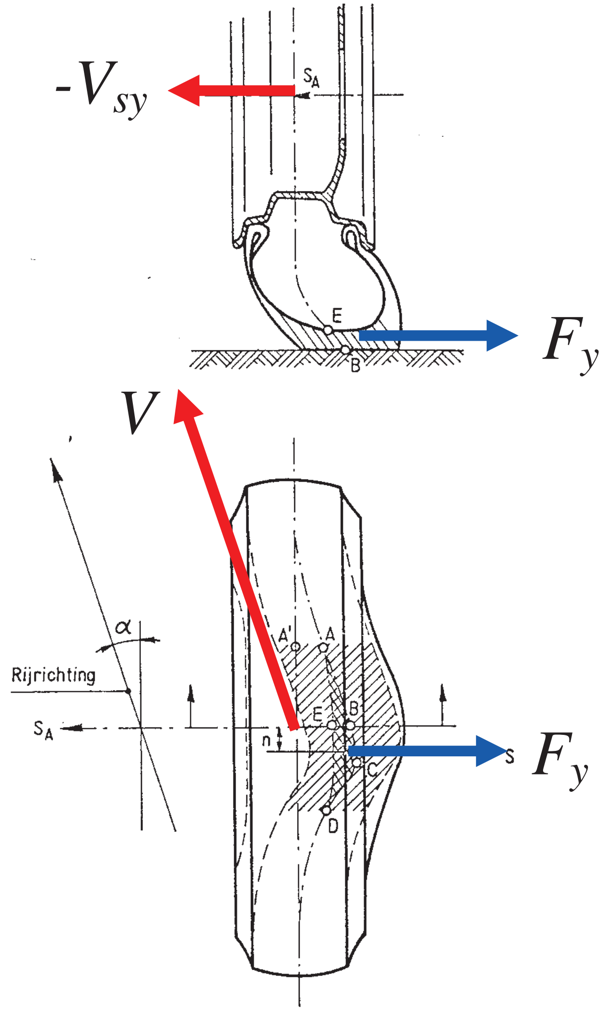
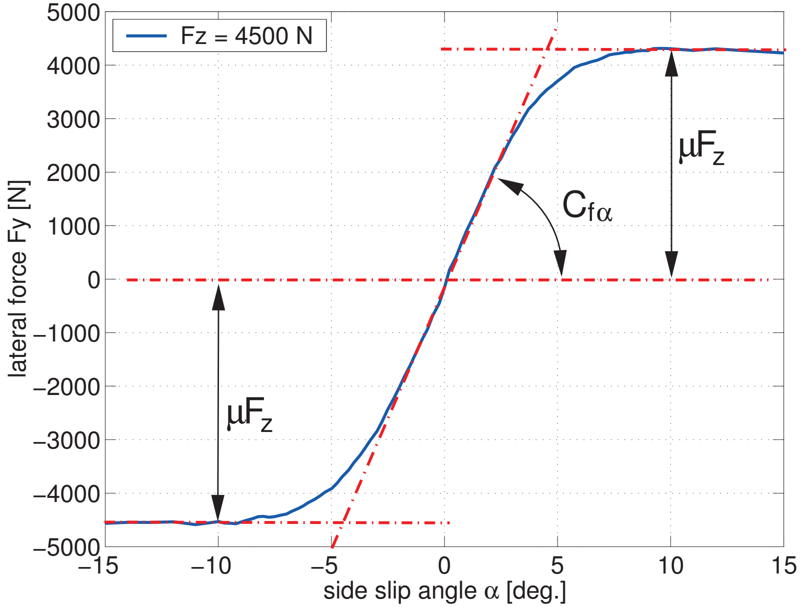


Fig. 1. Vervorming van de band.

measured lateral tyre force characteristic:



<u>linearised characteristics</u>
for small side slip angles a linear relation applies:

$$F_y = C_{f\alpha}\alpha \approx -C_{f\alpha}\frac{V_{sy}}{V_x} = -\frac{C_{f\alpha}}{V_x}\cdot V_{sy}$$

$C_{f\alpha}$: cornering stiffness

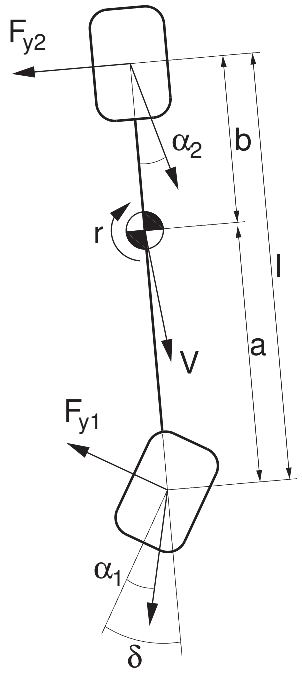
- force proportional to velocity ("damper")
- apparent "damping constant" decreases as function of forward velocity

Next time:
- the "bicycle" model

2. Single track vehicle model

analysis of cornering behaviour using the “bicycle model” or “single track vehicle model”

- equations of motion
- steady-state cornering
- dynamics



Equations of motion

assumptions:

- left, right tyre and axle characteristics can be lumped into a single, equivalent “tyre”
- no body roll
- centre point steering
- constant forward velocity *u* (≈ V)
- no aerodynamic forces
- no slopes, level road surface

steering angle of the front wheel δ

centre of gravity has two degrees of freedom:

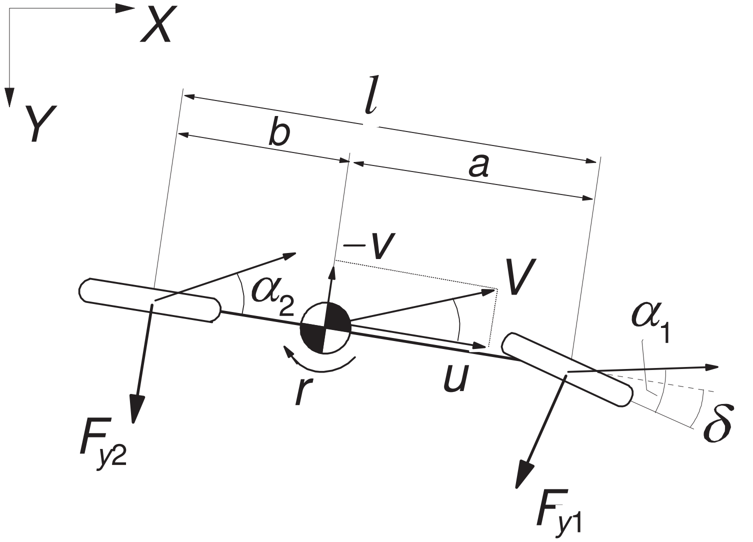
- lateral velocity v
- yaw velocity r

vehicle mass	m
vehicle yaw moment of inertia	I
distances to C.G.	a and b
wheelbase	$l(=a+b)$

small angles δ, α_1, α_2

$$\Rightarrow \sin(x) = x \text{ and } \cos(x) = 1$$

Note: all equations in this lecture refer to <u>linear</u> vehicle behaviour



equations of motion: (see also page 10)

$$m(\dot{v}+ur)=F_{y1}+F_{y2}$$

$$I\dot{r}=aF_{y1}-bF_{y2}$$

tyre side slip angles:

$$\alpha_1=\delta-\frac{1}{u}(v+ar),\ \ \alpha_2=-\frac{1}{u}(v-br)$$

linear cornering characteristics: (see page 25)

$$F_{y1}=C_1\alpha_1,\ \ F_{y2}=C_2\alpha_2$$

$C_{1,2}$: cornering stiffness (units: N/rad. or N/deg.)

vehicle side slip angle: $\beta=-\dfrac{v}{u}$

after substitution:

$$m\dot{v}+\frac{1}{u}(C_1+C_2)v+\left\{mu+\frac{1}{u}(aC_1-bC_2)\right\}r=C_1\delta$$

$$I\dot{r}+\frac{1}{u}\left(a^2C_1+b^2C_2\right)r+\frac{1}{u}(aC_1-bC_2)v=aC_1\delta$$

and after elimination of v:

$$mIu\ddot{r}+\left\{I(C_1+C_2)+m\left(a^2C_1+b^2C_2\right)\right\}\dot{r}+$$

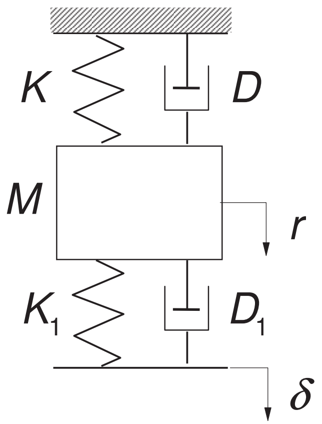
$$\frac{1}{u}\left\{C_1C_2l^2-mu^2(aC_1-bC_2)\right\}r=muaC_1\dot{\delta}+C_1C_2l\delta$$

equivalent system:

$$M\ddot{r}+D\dot{r}+K\,r=D_1\dot{\delta}+K_1\delta$$

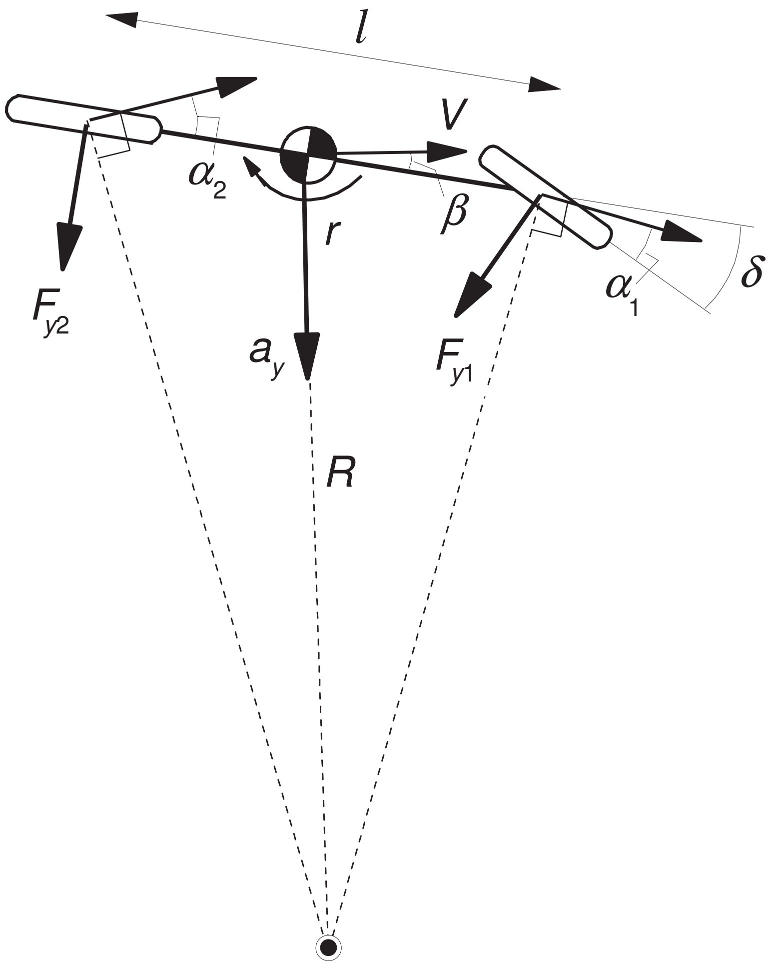
note:

- r equals yaw <u>velocity</u>
- K may become negative!



Steady-state cornering

- vehicle drives in a circle with <u>fixed</u> radius R
- constant steering angle δ



steady-state cornering: $\ddot{r}=0, \dot{r}=0, \dot{\delta}=0$

differential equation reduces to:

$$\frac{1}{u}\left\{C_1C_2l^2 - mu^2\left(aC_1 - bC_2\right)\right\} r = C_1C_2l\delta$$

furthermore:

$$R = \frac{V}{r} \approx \frac{u}{r}$$ (assumption that β is small)

the required steering angle for steady-state driving of a circle with radius R:

$$\delta = \frac{1}{R}\left(l - mV^2\frac{aC_1 - bC_2}{lC_1C_2}\right)$$

or

$$\delta = \underbrace{\frac{l}{R}}_{1} \underbrace{- \frac{mV^2}{Rl}\left(\frac{a}{C_2} - \frac{b}{C_1}\right)}_{2}$$

required steering angle has two contributions:
1. “kinematic” part (ackerman steer)
2. speed (or lateral acceleration) dependent part

lateral acceleration $a_y = \dfrac{V^2}{R}$

so:

$$\delta = \frac{l}{R} + \frac{a_y}{g}\left(\frac{mg}{l}\left(\frac{b}{C_1} - \frac{a}{C_2}\right)\right) = \frac{l}{R} + \frac{a_y}{g}\eta$$

we have now introduced the
understeer coefficient or understeer gradient η

$$\eta = \frac{mg}{l}\left(\frac{b}{C_1} - \frac{a}{C_2}\right)$$

other ways of expressing η

- using vertical equilibrium:

  $F_{z1,static} = \dfrac{b}{l}mg$, $F_{z2,static} = \dfrac{a}{l}mg$

  so: $\eta = \dfrac{F_{z1,static}}{C_1} - \dfrac{F_{z2,static}}{C_2}$

- using expressions for α_1, α_2 (page 29) or geometry (page 31):

  $\dfrac{l}{R} = \delta - \alpha_1 + \alpha_2$

  so: $\alpha_1 - \alpha_2 = \dfrac{a_y}{g}\eta$

steady-state cornering:

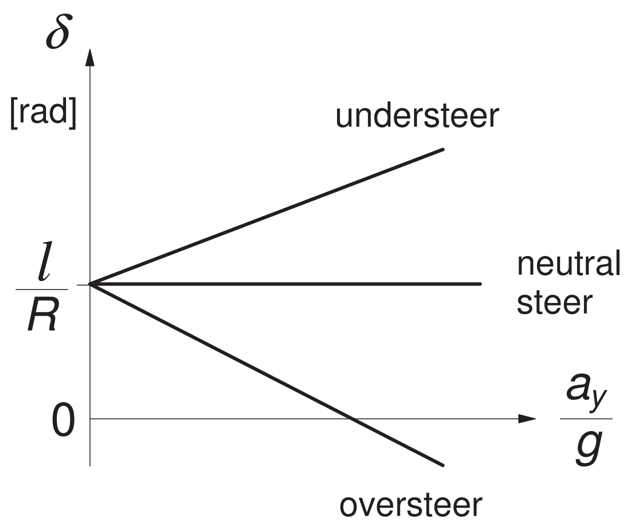
$\delta = \dfrac{l}{R} + \dfrac{a_y}{g}\eta$ or $\delta = \dfrac{l}{R}\left(1 + \dfrac{\eta}{gl}V^2\right)$

meaning of the understeer coefficient η:

- $\eta = 0$ “neutral steer” $(\alpha_1 = \alpha_2)$
- $\eta > 0$ “understeer” $(\alpha_1 > \alpha_2)$
- $\eta < 0$ “oversteer” $(\alpha_1 < \alpha_2)$

maintain a constant radius R while increasing forward speed V, then the steering angle:

- can *remain the same* for a neutral vehicle
- has to *increase* for an understeered vehicle
- has to *decrease* for an oversteered vehicle



oversteered vehicle has a critical velocity V_{crit} where the required steering angle δ equals zero

$$V_{crit} = \sqrt{\frac{gl}{-\eta}}$$

beyond this velocity the system is unstable
- in the equivalent system: $K_1 + K_2 < 0$
- will be shown when calculating eigenvalues

we may also define for an understeered vehicle the characteristic velocity V_{char}.

at V_{char} twice the steering input is required to maintain the same radius R compared to very low speeds

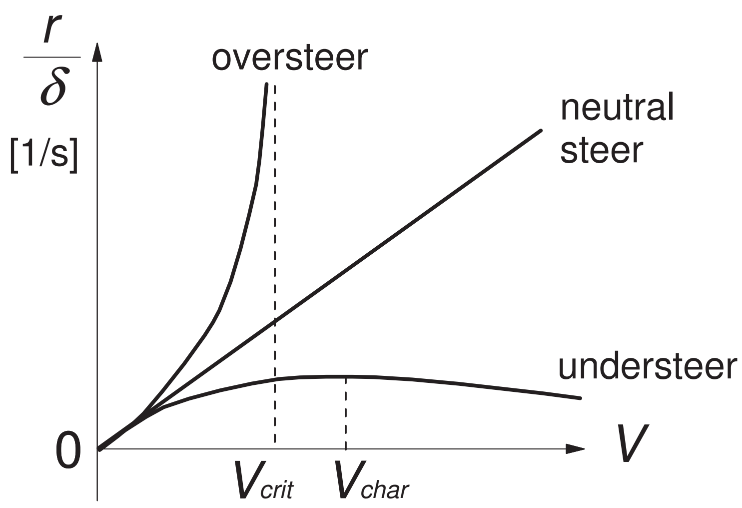
$$V_{char} = \sqrt{\frac{gl}{\eta}}$$

at this velocity the steady-state yaw velocity gain reaches its maximum for the understeered vehicle

steady-state yaw velocity gain: (see page 32)

$$\frac{r}{\delta} = \frac{C_1 C_2 l V}{C_1 C_2 l^2 - mV^2 (aC_1 - bC_2)}$$

or $\dfrac{r}{\delta} = \dfrac{V/l}{1 + \dfrac{\eta}{gl} V^2}$



similarly: steady-state lateral acceleration gain

$$\frac{a_y}{\delta} = \frac{V^2/l}{1 + \dfrac{\eta}{gl} V^2}$$

vehicle side slip angle β

$$\beta = -\frac{v}{u} \approx -\frac{v}{V}$$

$$\alpha_2 = -\frac{1}{u}(v - br) = \beta + \frac{br}{u}$$

circular driving with fixed radius R $(rR = u)$:

$$\alpha_2 = \beta + \frac{b}{R}$$

$$\beta = -\frac{b}{R} + \alpha_2$$

furthermore we may write:

$$\alpha_2 = \frac{F_{y2}}{C_2} = \frac{1}{C_2}\frac{mV^2}{R}\frac{a}{l}$$

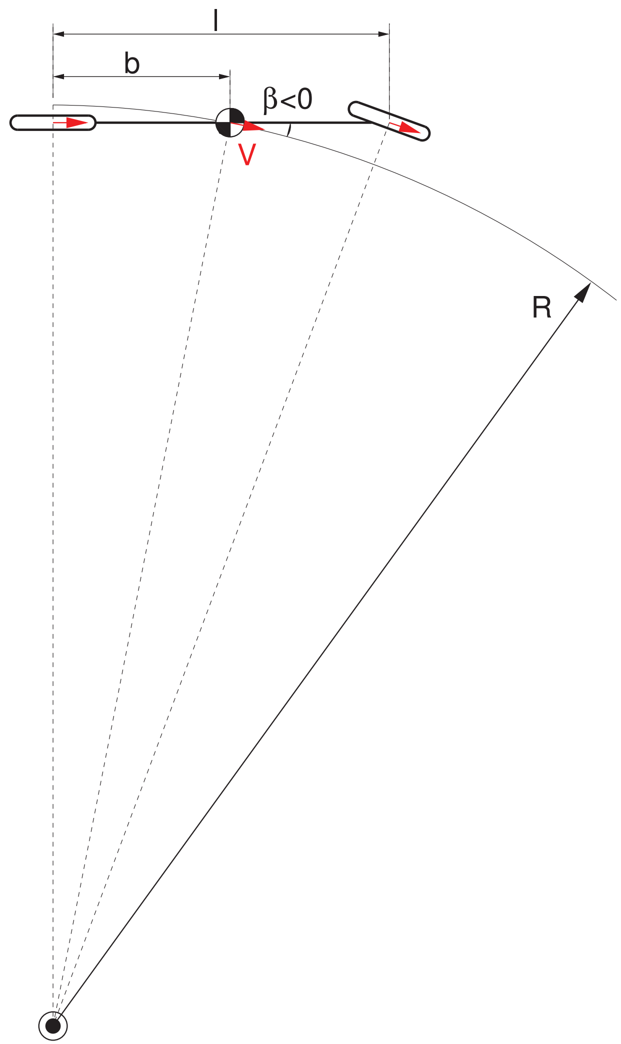
thus:

$$\beta = -\frac{b}{R} + \frac{am}{C_2 l}\frac{V^2}{R}$$

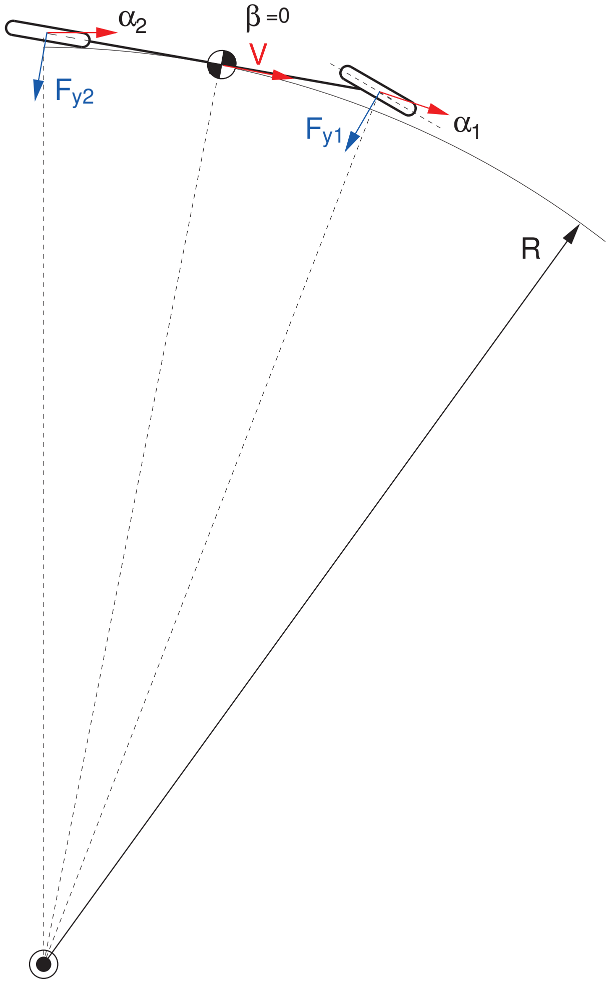
note: the vehicle side slip angle β will change sign with increasing forward velocity V …

example: neutral vehicle (δ fixed), increasing V

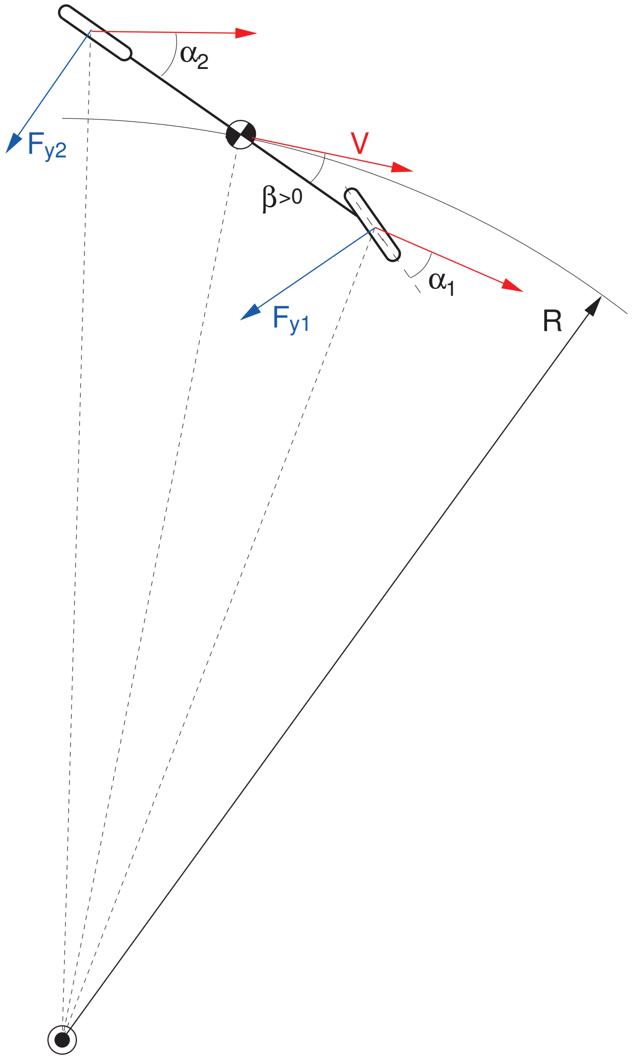
V very low: $\beta \approx -\frac{b}{R}$, $\delta \approx \frac{l}{R}$



increasing V : at some point β will become zero



increasing V further: β has same sign as α



Summarising…

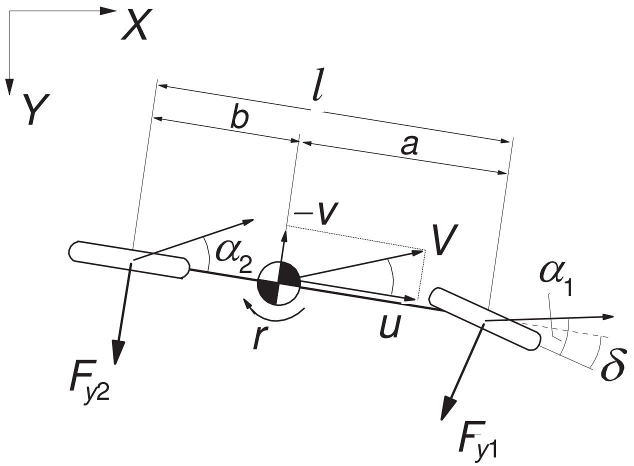
the tyres need to develop lateral forces to keep the vehicle on a fixed radius R at a certain (non-zero) forward velocity

the lateral tyre forces at the front and rear tyre are a result of the tyre side slip angles α_1, α_2.

$$F_{y1} = C_1\alpha_1, \;\; F_{y2} = C_2\alpha_2$$

for steady-state cornering we need to have both force and moment equilibrium

- $F_{y1} + F_{y2} = \dfrac{mV^2}{R}$
- $aF_{y1} - bF_{y2} = 0$



if we increase the forward velocity V and want to maintain the same corner radius R the lateral tyre forces F_{y1} and F_{y2} have to increase and therefore also the side slip angles will increase.

the side slip angle of the rear tyre can only increase if the whole vehicle is oriented more “nose inwards” with respect to the corner

if we don’t change the steering angle δ, the front tyre will have the same increase in side slip angle as the rear tyre (see page 38 to 40).

what about the moment equilibrium then?

- if the increase in lateral force for the front tyre is too small, the driver has to increase the steering angle δ to maintain moment equilibrium: the vehicle has understeer

- if the increase in lateral force for the front tyre is too big, the driver has to decrease the steering angle δ to maintain moment equilibrium: the vehicle has oversteer

- no additional steering action required: the vehicle has neutral steer an almost trivial example:

suppose $a = b$

…then the vehicle has neutral steer if the front and rear tyre cornering stiffness are equal $(C_1 = C_2)$

…then the vehicle has oversteer if the front cornering stiffness is higher than the rear tyre cornering stiffness $(C_1 > C_2)$

…then the vehicle has understeer if the front cornering stiffness is lower than the rear tyre cornering stiffness $(C_1 < C_2)$

Dynamics

State-space description:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$
$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$$

with:

$$\mathbf{x} = \begin{pmatrix} v \\ r \end{pmatrix}, \ \mathbf{u} = \delta, \ \mathbf{y} = \begin{pmatrix} a_y \\ r \\ \beta \end{pmatrix} = \begin{pmatrix} \dot{v} + ur \\ r \\ -v/u \end{pmatrix}$$

and

$$\mathbf{A} = -\begin{pmatrix} \dfrac{C_1 + C_2}{mu} & u + \dfrac{aC_1 - bC_2}{mu} \\ \dfrac{aC_1 - bC_2}{Iu} & \dfrac{a^2C_1 + b^2C_2}{Iu} \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} \dfrac{C_1}{m} \\ \dfrac{aC_1}{I} \end{pmatrix}$$

$$\mathbf{C} = -\begin{pmatrix} \dfrac{C_1 + C_2}{mu} & \dfrac{aC_1 - bC_2}{mu} \\ 0 & -1 \\ 1/u & 0 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} \dfrac{C_1}{m} \\ 0 \\ 0 \end{pmatrix}$$

system stability

eigenvalues of matrix $\mathbf{A}$

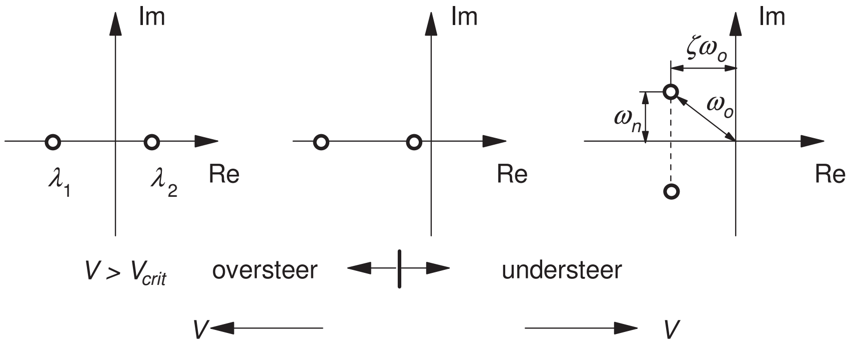
- neutral and oversteer vehicle: real eigenvalues
- oversteer vehicle has a positive real eigenvalue if $V > V_{crit}$, so unstable
- understeer vehicle has complex conjugate eigenvalues, damping ratio decreases with forward velocity.

natural frequency:

$$\omega_n^2 \approx \left(\frac{C_1 + C_2}{m}\right)^2 \frac{\eta}{gl}$$

damping ratio:

$$\zeta \approx \frac{1}{\sqrt{1 + \frac{\eta}{gl} V^2}}$$

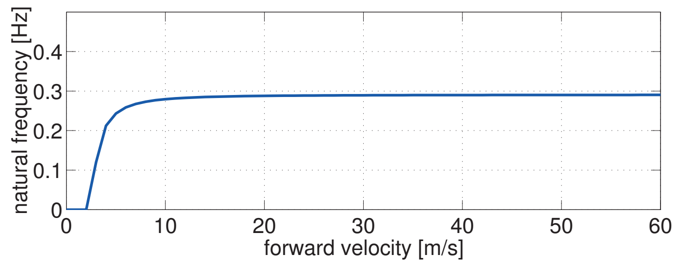


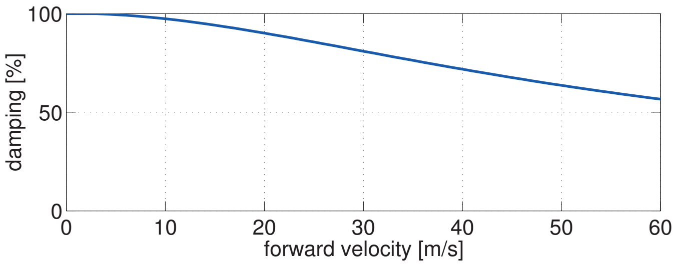
numerical example

m=1600 kg, I = 3600 kgm^2
l= 3 m, a= 1.4 m
$C_1 = C_2$=60000 N/rad
$\Rightarrow$ understeer coefficient η = 0.0174 rad

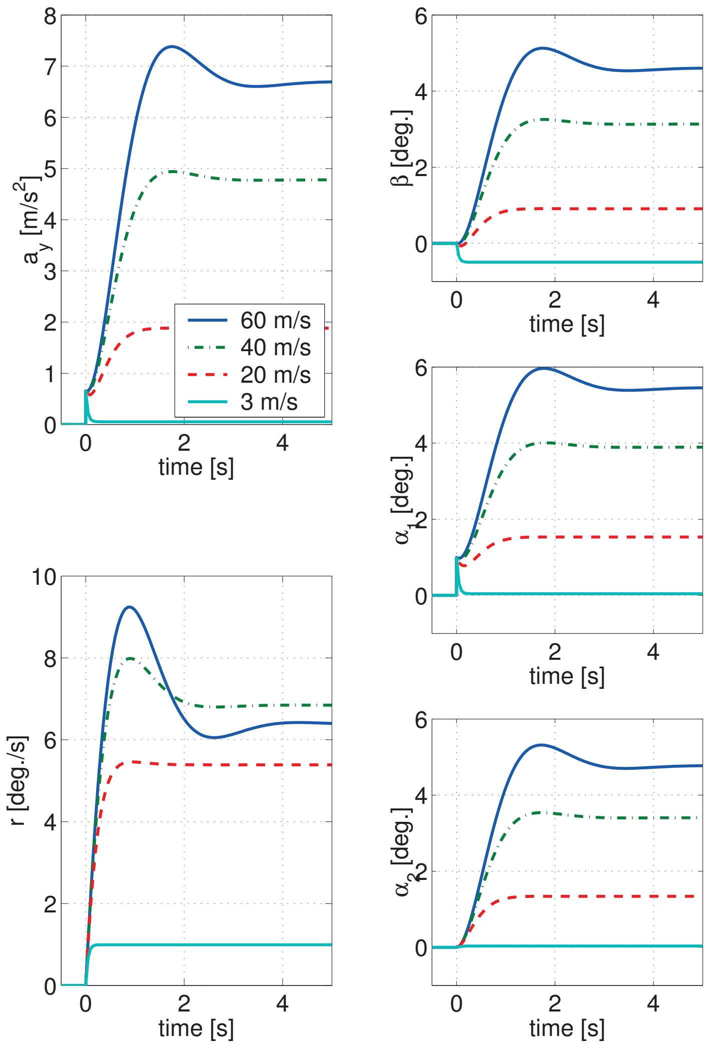
complex eigenvalue $\lambda = a \pm ib$

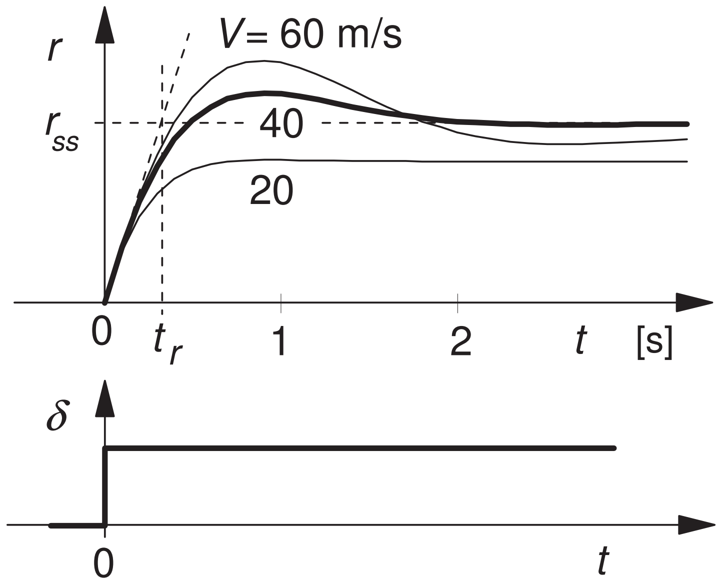
- frequency in Hz $\quad f = \dfrac{b}{2\pi}$
- damping ratio in % $\quad \zeta = -\dfrac{a}{|\lambda|} \cdot 100\%$





step response 1 deg. steer angle





yaw rate reponse rise time:

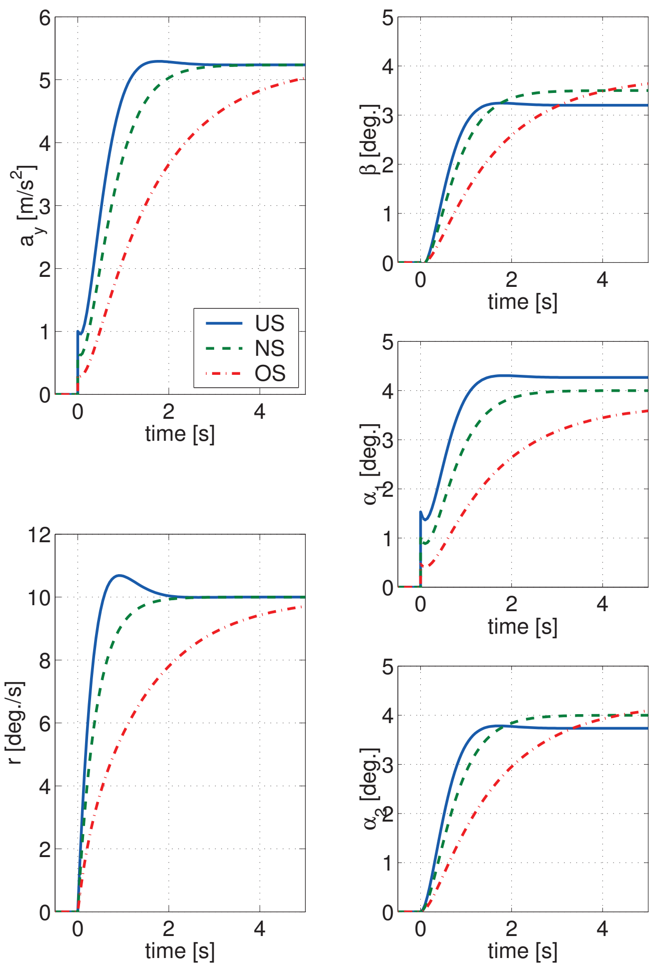
$$t_r = \frac{IV}{aC_1 l\left(1+\frac{\eta}{gl}V^2\right)}$$

note:

- for understeered vehicle:
  response time t_r reaches a maximum at V_{char}
- understeered vehicle has smaller response time t_r compared to oversteered vehicle

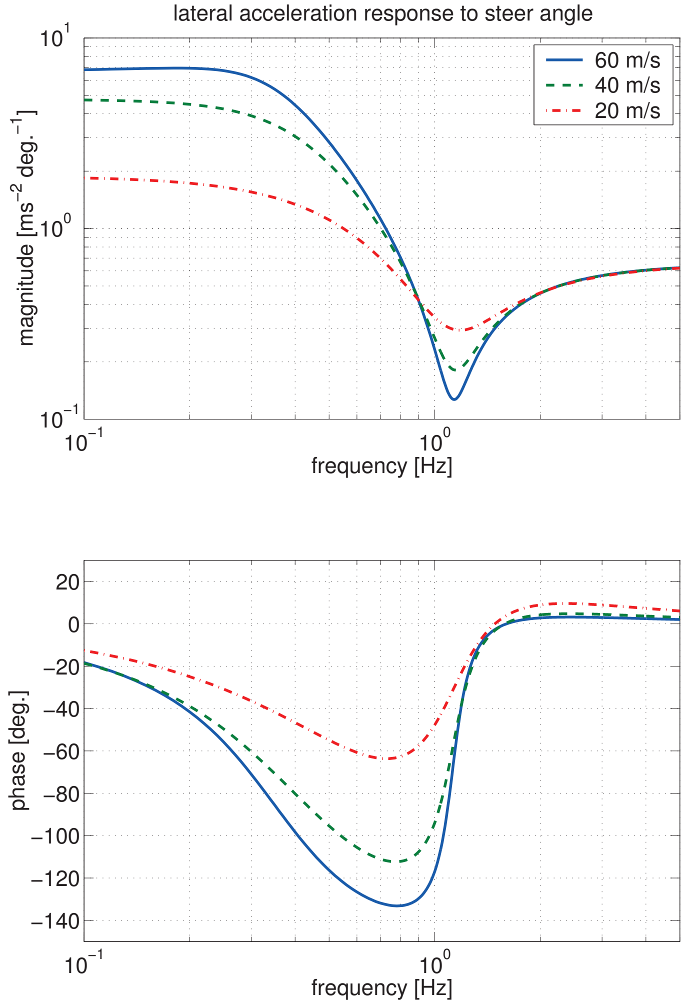
step steer response V =30 m/s

- δ adapted obtain yaw rate of 10 deg./s
- ns: neutral, os: oversteer, us: understeer

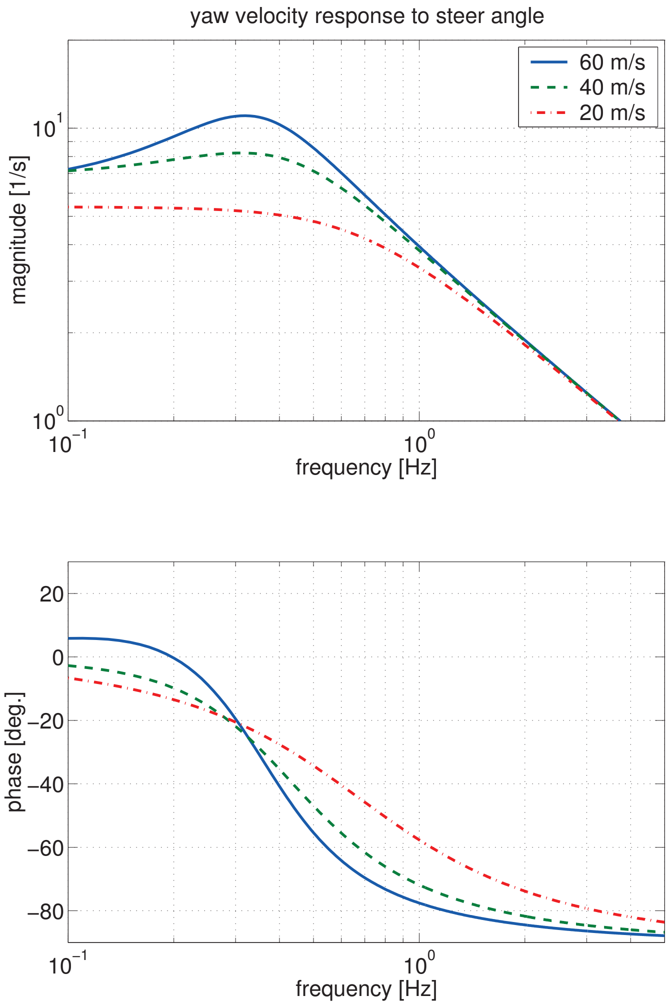


transfer functions with respect to steer angle

- lateral acceleration a_y



- yaw velocity r



Book Pacejka

- pages 22 to 35, section 1.3.2

Next time...

- validation using experimental data
- model enhancements

3. Validation of the single track vehicle model

analysis of cornering behaviour using the "single track vehicle model"

- comparison with vehicle tests
- tyre relaxation effects
- extension to non-linear behaviour (handling diagram)

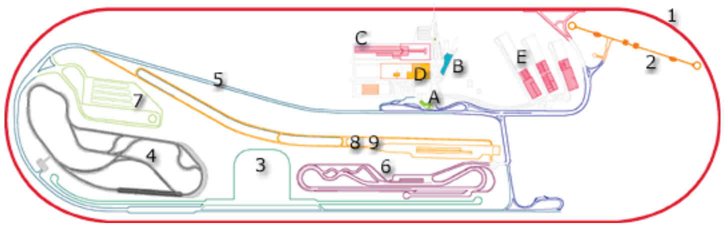
VEHICLE DYNAMICS 4L150 20121112

Vehicle tests

instrumented vehicle, measurement of:

- steer angle, steer torque
- brake pedal force
- forward, lateral and yaw velocity
- longitudinal, lateral acceleration
- roll angle
- travelled distance
- …

test track (proving ground)



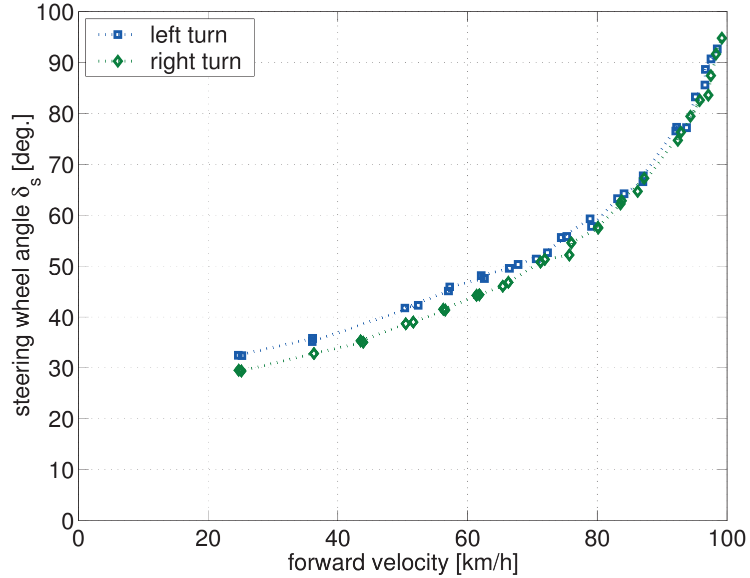
example: IDIADA, Spain

"dynamic platform" (3)
- dimensions 250x250 m
- completely level surface, gradient 0%
- marked circles (range R=10 - 120 m)

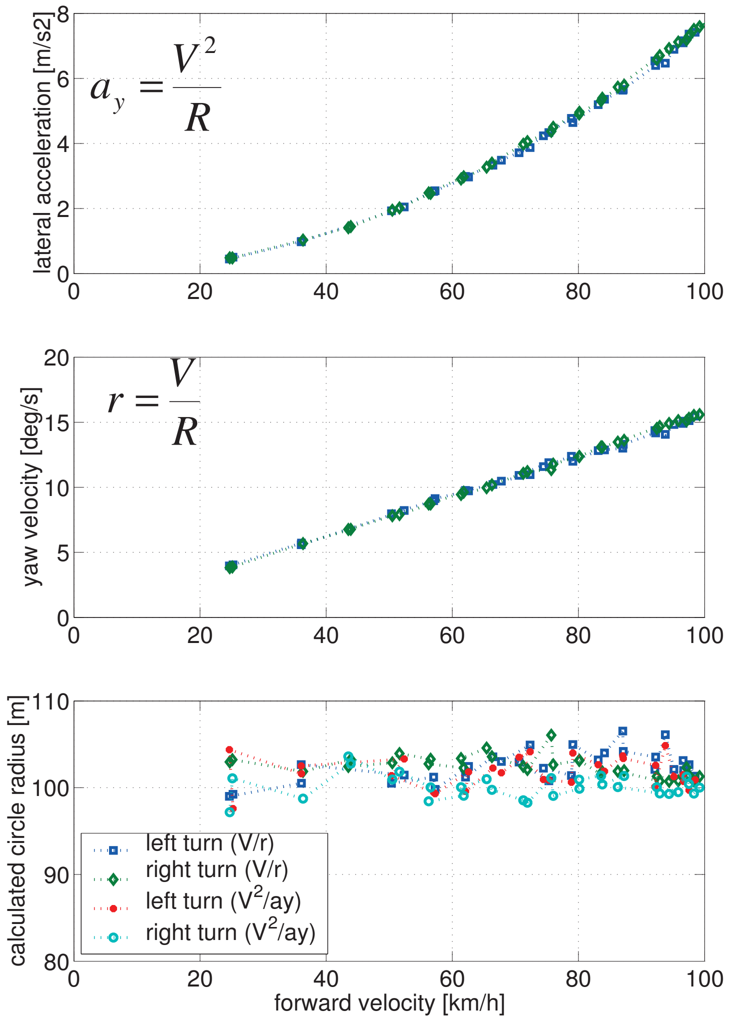
steady-state circular test
- fixed radius R (in example shown: 100 m)
- different constant forward velocities V
- steering angle adjusted to maintain radius R
- steady-state conditions

standardised in ISO 4138
left and right turn

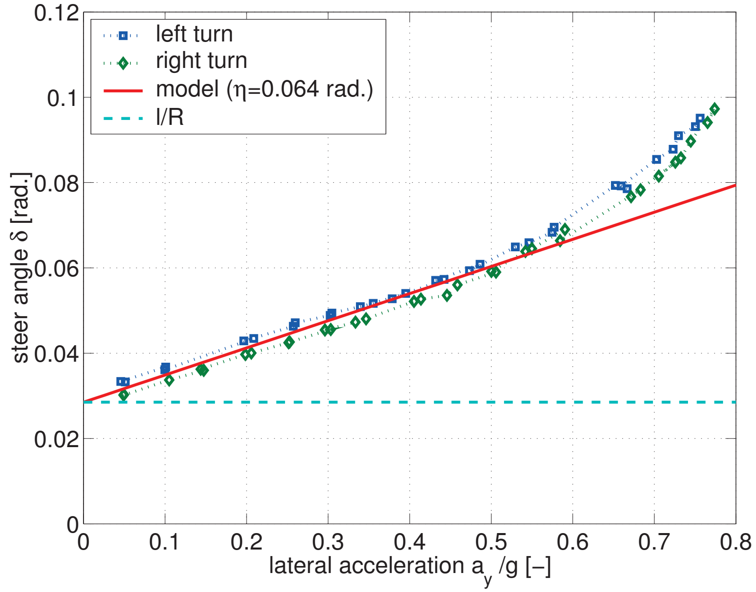
note:
data for the left hand turn is mirrored for easy comparison with the right turn (symmetry check)



checking radius R



front wheel steer angle δ vs. lateral acceleration



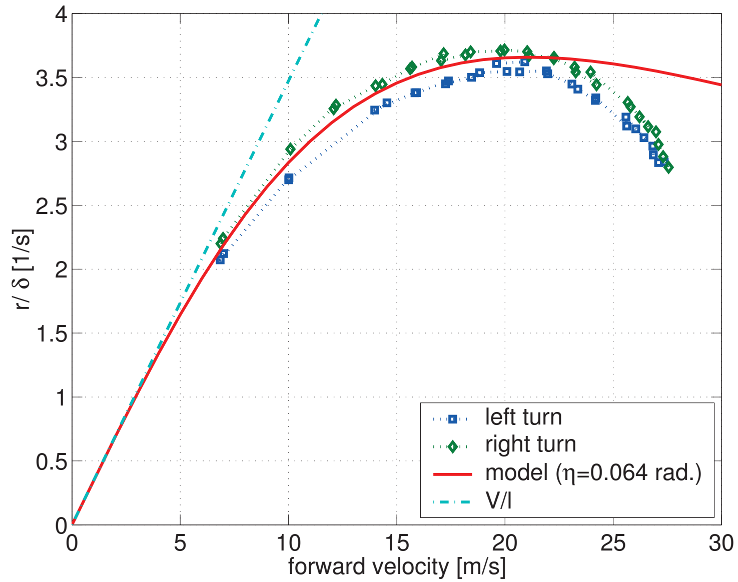
conclusions:

- understeered vehicle
- linear behaviour up to 0.4 - 0.5 g, differences high higher lateral accelerations due to non-linear tyre behaviour

model parameters:

- based on measurements (e.g. m, l, a, b)
- “tuned” to match tests (e.g. I, C_1, C_2)
- steering ratio $i_s = \dfrac{\delta_{steering\ wheel}}{\delta_{front\ wheel}}$ (typically 15 to 20)

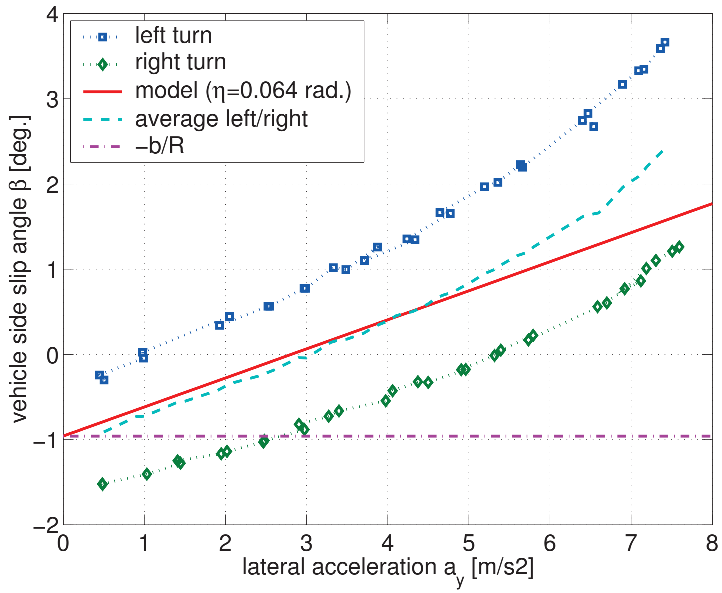
steady state yaw velocity gain $\frac{r}{\delta}$



confirms understeer behaviour

- $V_{char} \approx 20$ m/s (=72 km/h)
- deviation gets bigger for higher forward velocities (in this test: 20 m/s $\Rightarrow a_y \approx 4$ m/s^2)

vehicle side slip angle β



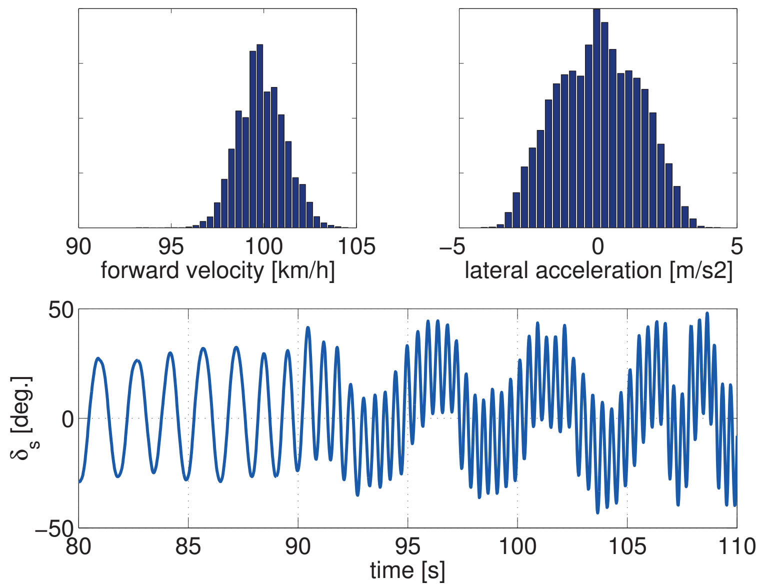
note:

- up to 4-5 m/s^2 response fairly linear
- deviation gets bigger for higher lateral acceleration levels
- relatively big difference between left and right turn. cause???

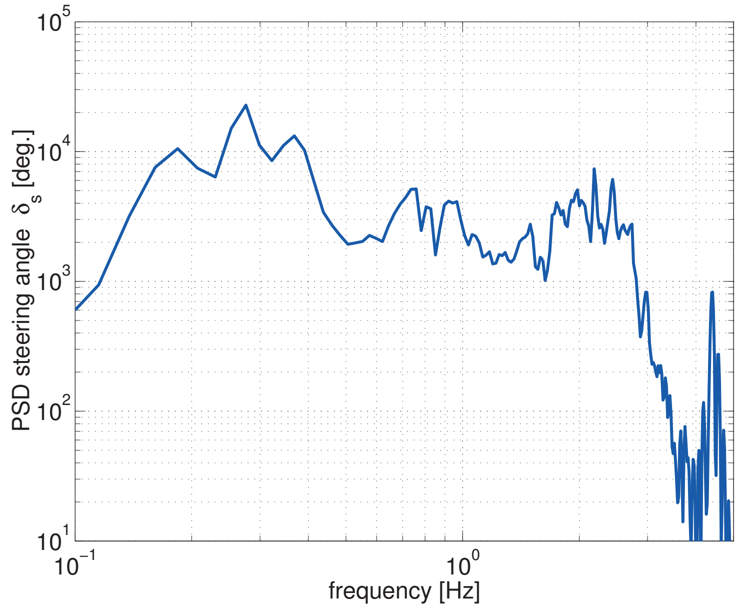
<u>random steering input test</u>

- constant forward velocity
- “pseudo” random steer input:
  –experienced test driver
  –steering robot
- accelerations within “linear” range (< 4 m/s^2)
- measured in sequences (total time: > 15 min.)

standardised in ISO 7401 and ISO/TR 8726



power spectral density of the steering input

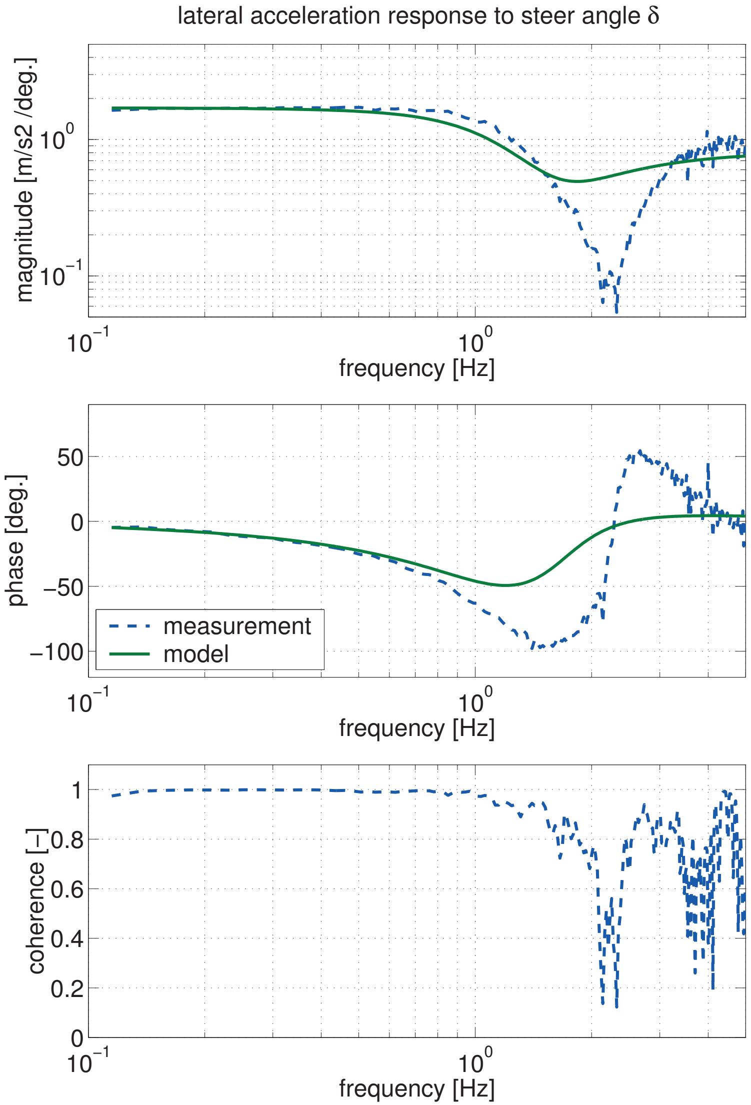


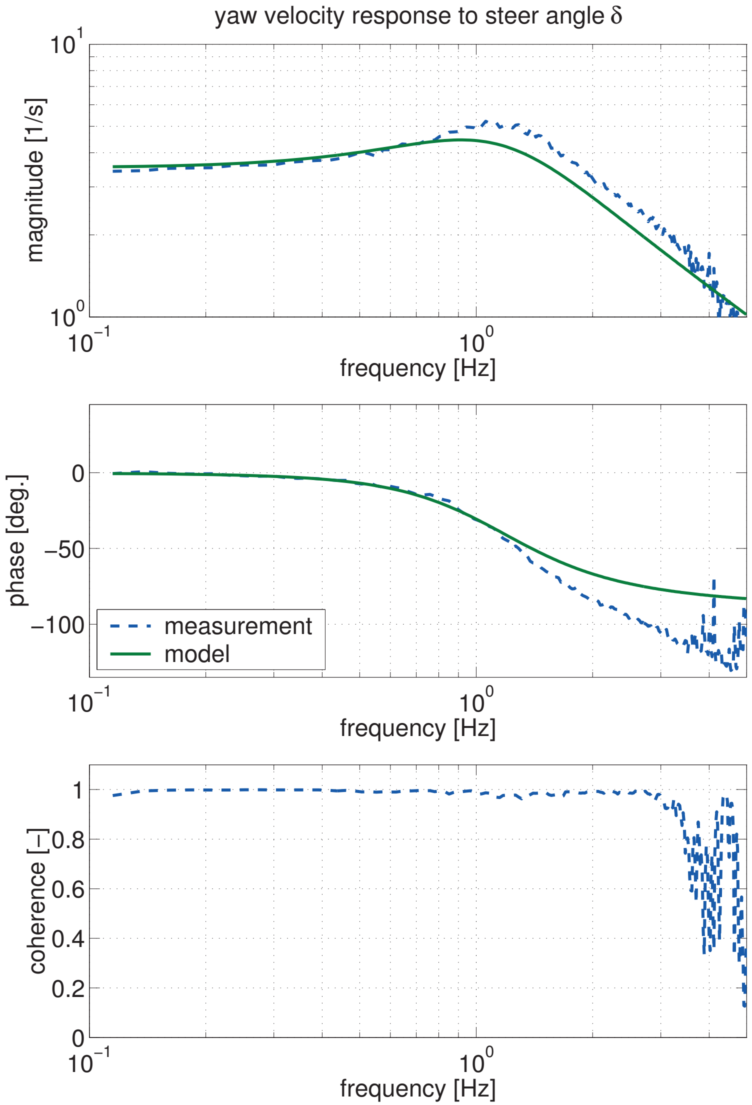
aim of the test: determination of transfer functions

model comparison with single track model

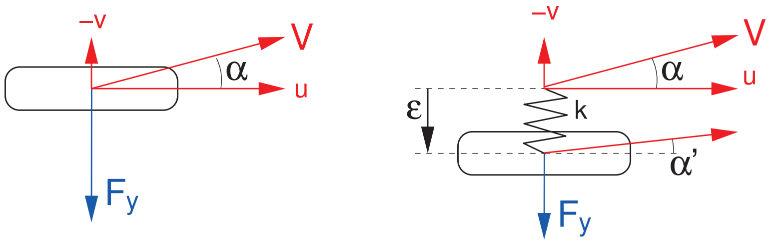
note

- vehicle model still very simple: does not include e.g. body roll, which is an important source of differences in lateral acceleration
- vehicle model uses constant forward velocity of 100 km/h


lateral acceleration response to steer angle δ
magnitude [m/s2 /deg.]
10^0
10^{-1}
frequency [Hz]
phase [deg.]
50
0
−50
−100
measurement
model
coherence [−]
1
0.8
0.6
0.4
0.2
0


yaw velocity response to steer angle δ
magnitude [1/s]
10^1
10^0
frequency [Hz]
phase [deg.]
0
−50
−100
measurement
model
coherence [−]
1
0.8
0.6
0.4
0.2
0

Tyre relaxation effects



- existing tyre model:

$$F_y = C\alpha \qquad \text{side slip angle } \alpha = -\frac{v}{u}$$

- dynamic tyre model

$$F_y = C\alpha' = k\varepsilon \quad \Rightarrow \quad C\dot{\alpha}' = k\dot{\varepsilon}$$

dynamic side slip angle $\alpha' = -\dfrac{v+\dot{\varepsilon}}{u}$

$$\frac{C}{k}\frac{1}{u}\dot{\alpha}' + \alpha' = -\frac{v}{u} = \alpha$$

introducing the relaxation length $\sigma \left(= C/k\right)$ and $V \approx u$

$$\frac{\sigma}{V}\dot{\alpha}' + \alpha' = \alpha \qquad \text{and} \qquad F_y = C\alpha'$$

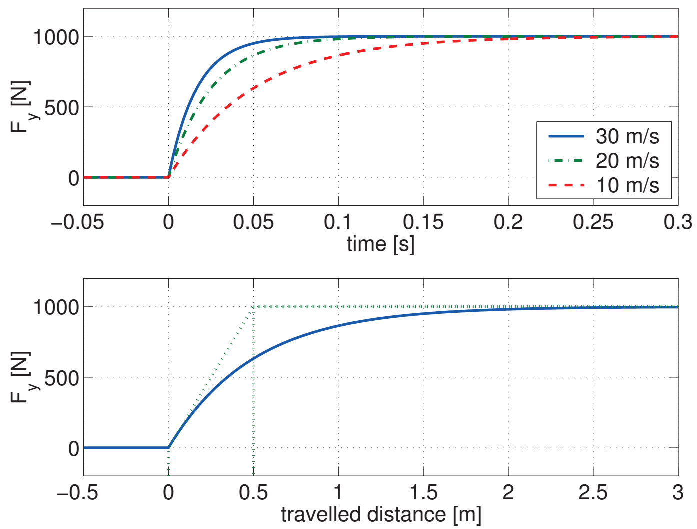
first order dynamics between lateral force and side slip angle input, transfer function:

$$H_{F_y,\alpha}(s) = \frac{C}{\frac{\sigma}{V}s+1} \qquad \text{time constant: } \frac{\sigma}{V}$$

relaxation length σ does not depend on forward velocity V:

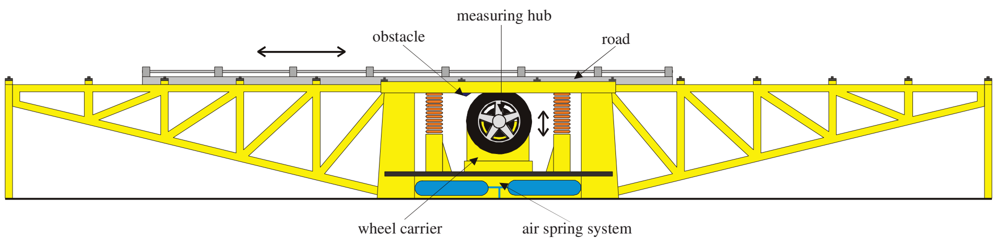
- response time reduces when increasing V
- travelled distance required to build up the lateral force remains the same

step response (α=1 deg, C=1 kN/deg, σ=0.5 m)

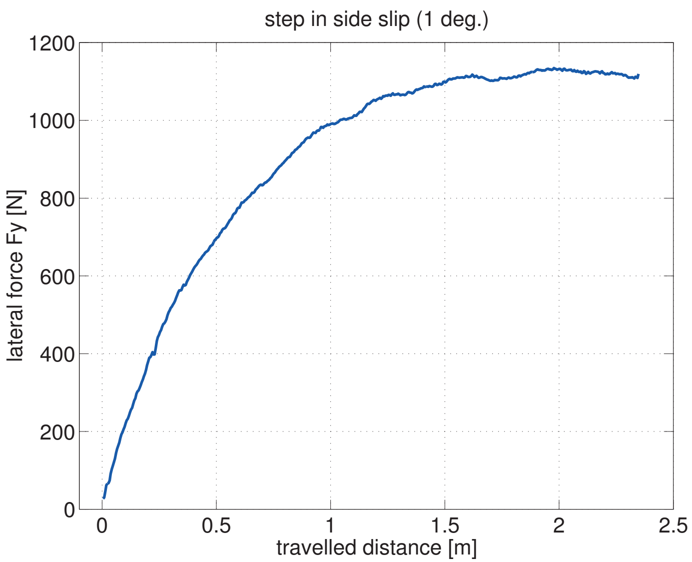


tyre relaxation length measurement

flat plank tyre tester
- fixed steering angle (e.g. 1 deg.)
- velocity 0.05 m/s



experiment



vehicle model including relaxation effects

equations of motion:

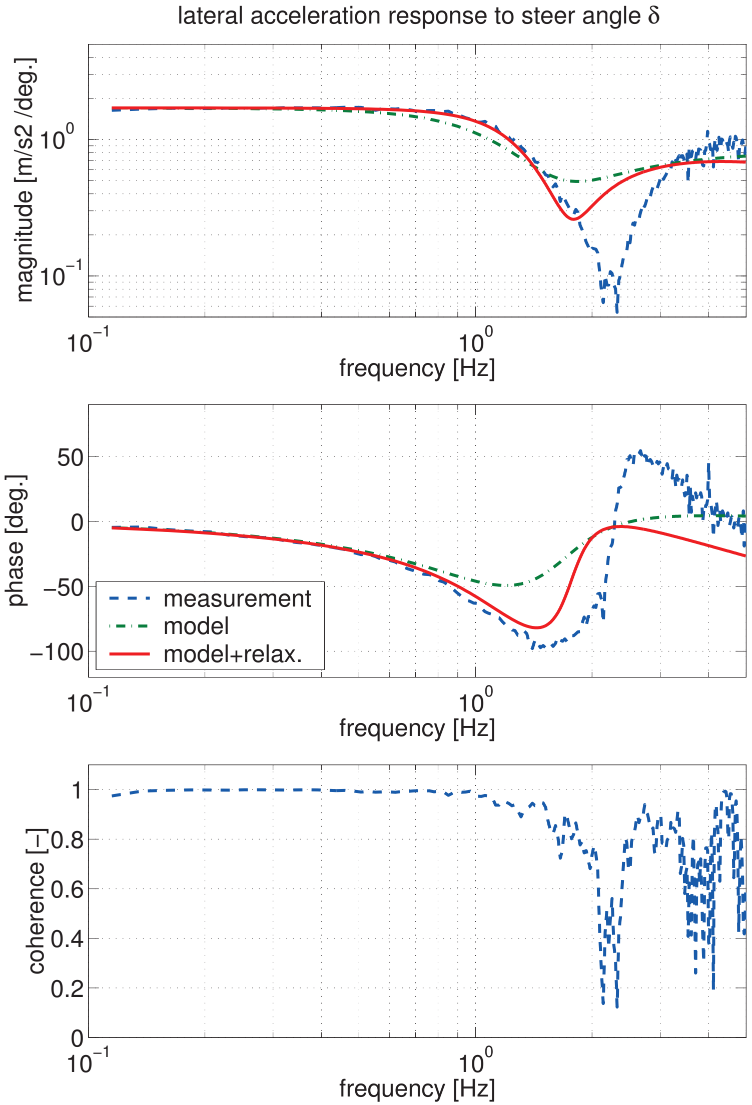
$$m(\dot{v}+ur)=C_1\alpha_1'+C_2\alpha_2'$$

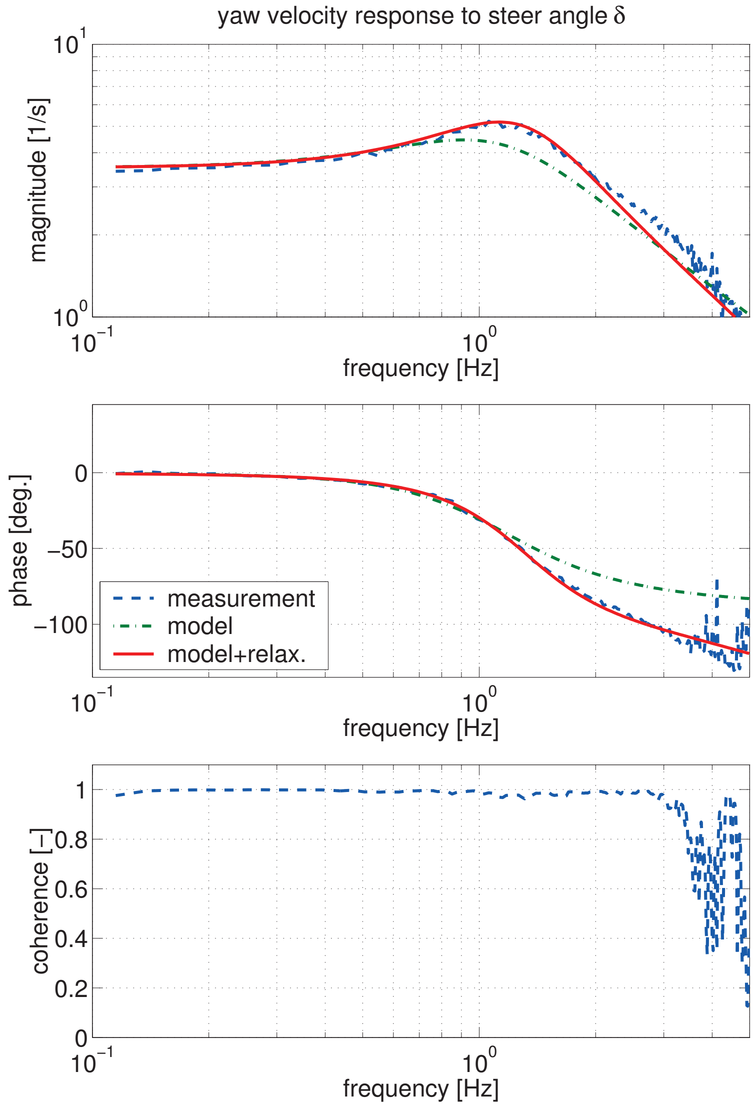
$$I\dot{r}=aC_1\alpha_1'-bC_2\alpha_2'$$

$$\alpha_1=\delta-\frac{1}{u}(v+ar) \qquad \alpha_2=-\frac{1}{u}(v-br)$$

$$\frac{\sigma_1}{u}\dot{\alpha}_1'+\alpha_1'=\alpha_1 \qquad \frac{\sigma_2}{u}\dot{\alpha}_2'+\alpha_2'=\alpha_2$$

parameters:
- m=1971.8 kg
- l= 2.88 m
- a=1.1907 m (based on vehicle weight distr.)
- $b=l-a$ = 1.6893 m
- I=3550 kgm^2
- C_1=93000 N/rad (≈1600 N/deg)
- C_2=137000 N/rad (≈2400 N/deg)
- σ_1=0.57 m
- σ_2=0.97 m
- i_s=17.0


lateral acceleration response to steer angle δ
magnitude [m/s2 /deg.]
phase [deg.]
coherence [–]
frequency [Hz]
measurement
model
model+relax.


yaw velocity response to steer angle δ
magnitude [1/s]
phase [deg.]
coherence [–]
frequency [Hz]
measurement
model
model+relax.

Handling diagram

non-linear, steady state behaviour for large values of lateral acceleration a_y

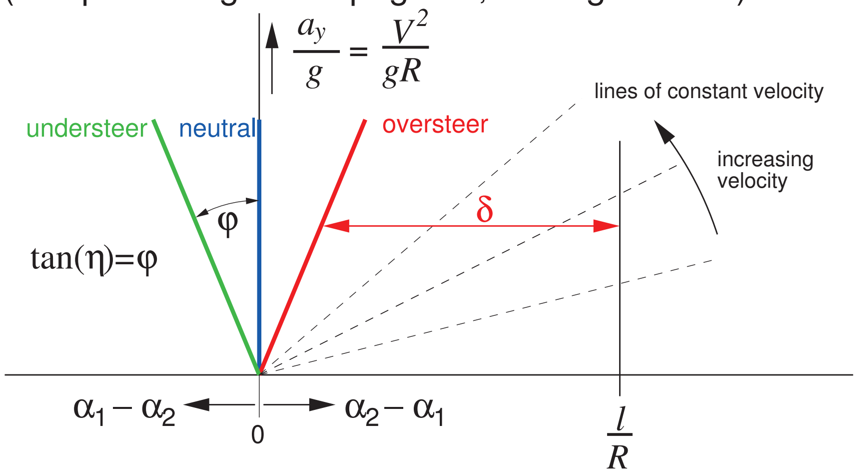
the following relations have been derived (page 33):

$$\frac{l}{R} = \delta - \alpha_1 + \alpha_2 \qquad \text{(based on geometry)}$$

$$\alpha_1 - \alpha_2 = \frac{a_y}{g}\eta \qquad \text{(understeer coefficient } \eta\text{)}$$

so: $\alpha_1 - \alpha_2 = \dfrac{a_y}{g}\eta = \delta - \dfrac{l}{R}$

different graphical representation:
(compare to figure on page 34, 90 deg. rotated)



equilibrium:

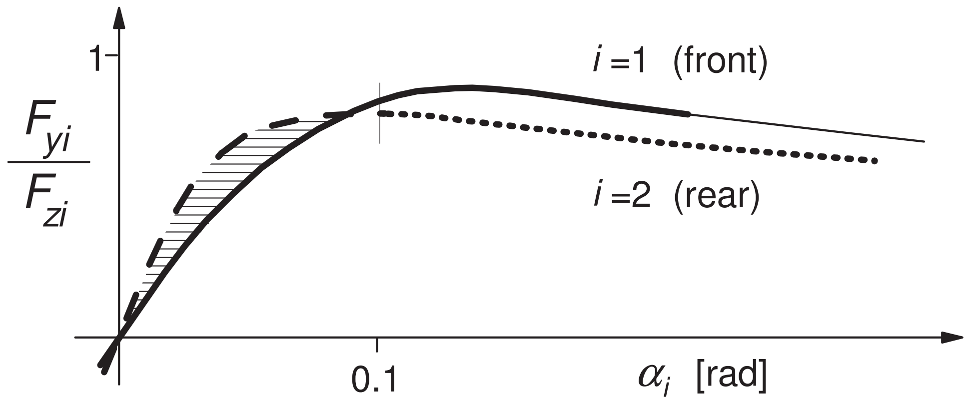
$$F_{y1} + F_{y2} = ma_y \qquad \text{(lateral)}$$
$$F_{y1}a = F_{y2}b \qquad \text{(yaw)}$$
$$F_{z1}a = F_{z2}b \qquad \text{(pitch)}$$

then

$$\frac{a_y}{g} = \frac{F_{y1} + F_{y2}}{mg} = \frac{F_{y1} + F_{y2}}{F_{z1} + F_{z2}} = \frac{F_{y1}}{F_{z1}} = \frac{F_{y2}}{F_{z2}}$$

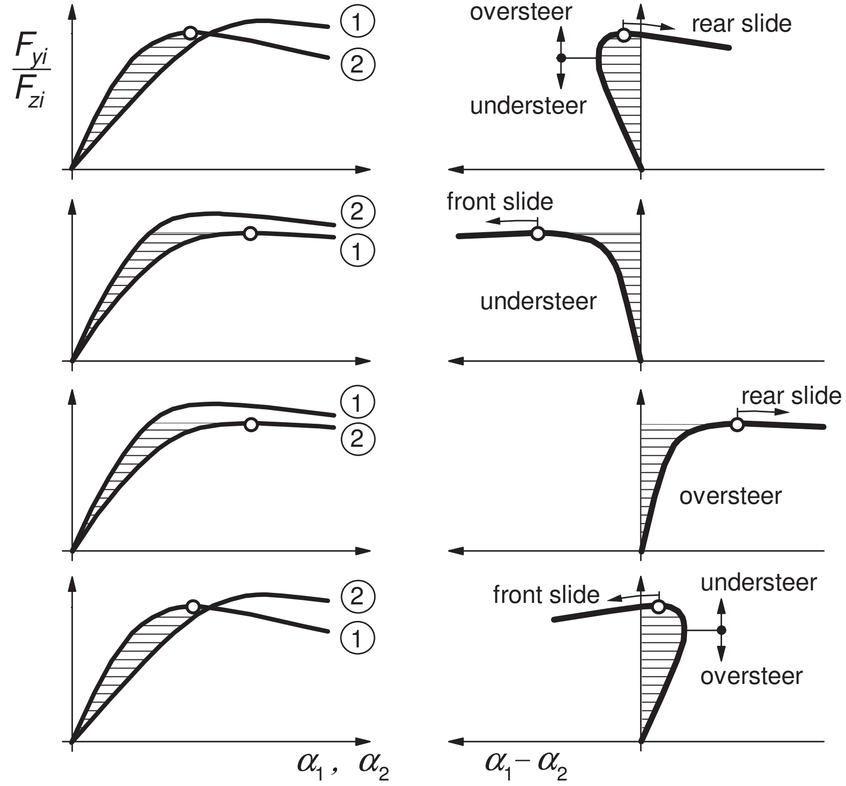
lateral tyre force F_y is a non-linear function of the side slip angle α

<u>normalised tyre characteristics</u>



by subtracting the characteristics horizontally the handling diagram can be obtained!

essentially 4 possibilities exist:
(note: 1=front tyre, 2=rear tyre)



definition of oversteer/understeer is revised:

- understeer if: $\left(\dfrac{\partial \delta}{\partial V}\right)_R > 0$
- oversteer if: $\left(\dfrac{\partial \delta}{\partial V}\right)_R < 0$

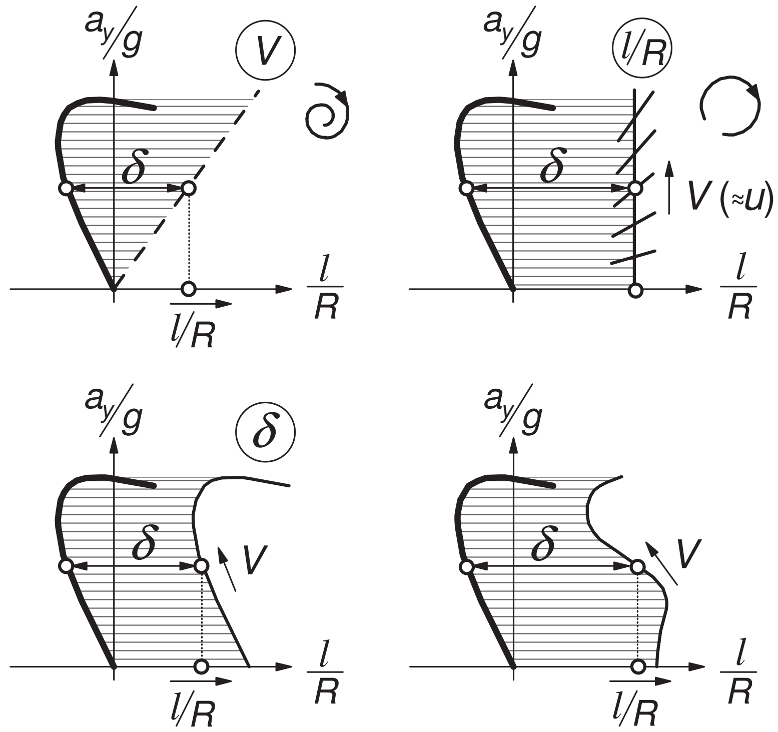
interpretation of the handling diagram

different types of steady state manoeuvres:

- constant velocity V, increasing steering δ
- constant radius R, increasing velocity V
- constant steering angle δ, increasing V
- increasing V, δ and R not fixed

note: steady-state, so either:

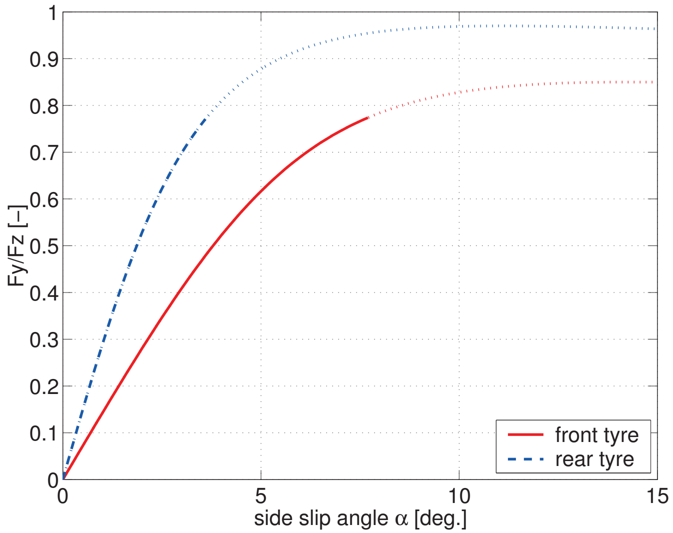
- very slow changes of V, δ
- separate tests



cross-check with vehicle tests

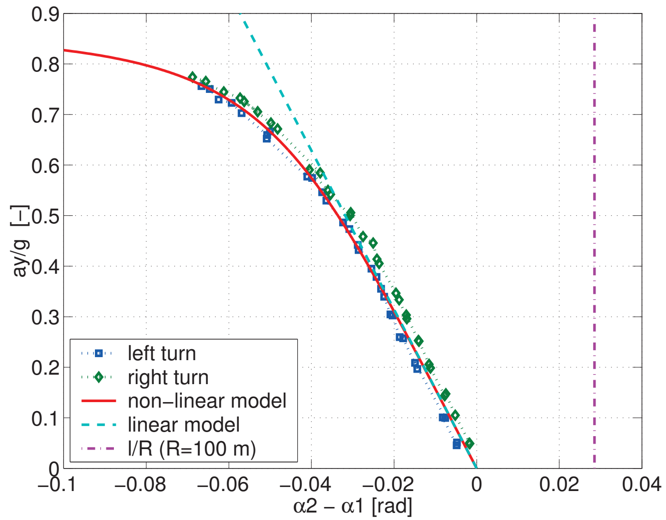
estimate for normalised tyre characteristics

- linear part: already known
- non-linear part: chosen to get a good math with vehicle tests
- dotted part: not encountered during tests, educated guess…

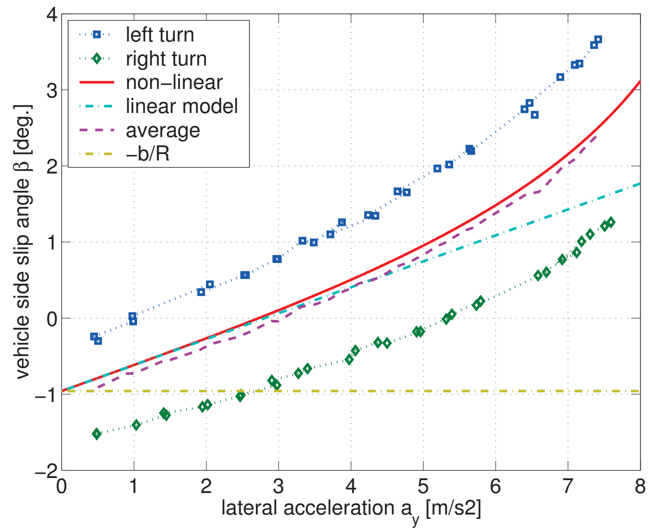


note:
simplified discussion: “tyre” includes effects of the suspension design: although front and rear tyres may be same, the normalised front and rear tyre characteristics will be different.

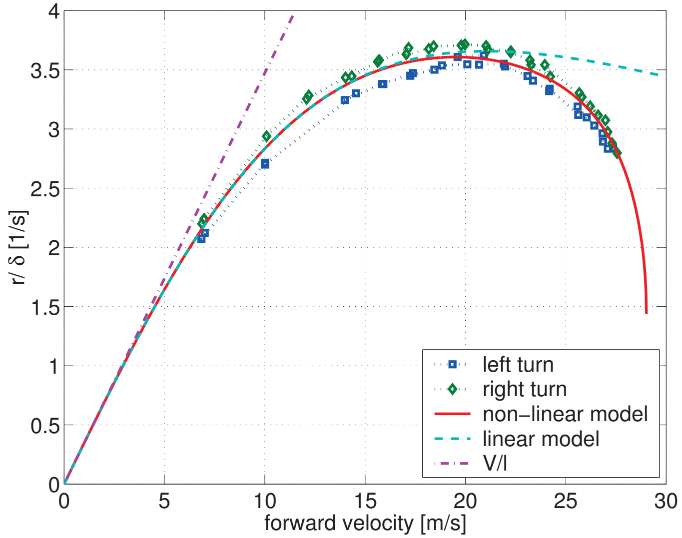
- handling diagram



- vehicle side slip angle



- yaw velocity gain

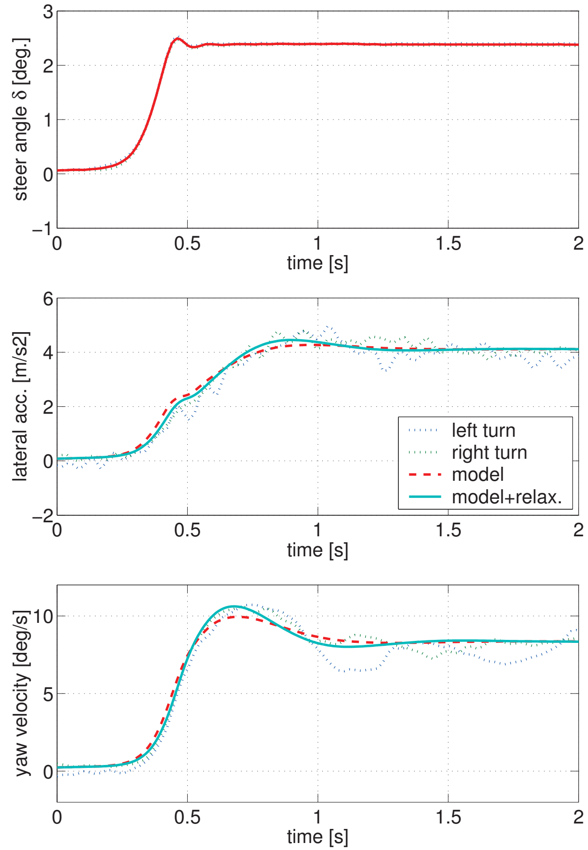


additional validation of the vehicle model:

- J-turn
- severe lane change

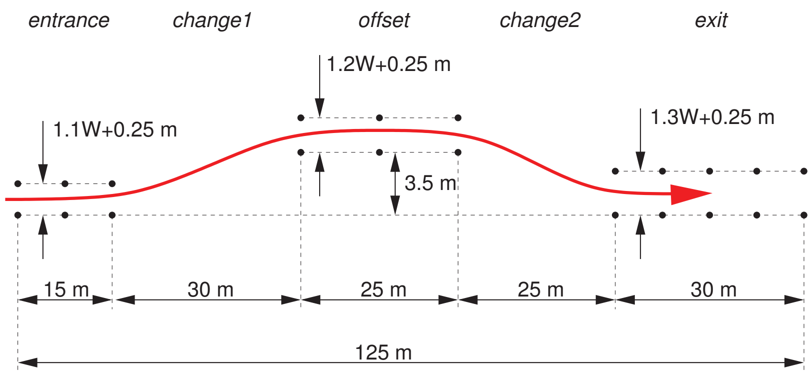
lateral transient response or “J-turn”

- constant forward velocity (example 100 km/h)
- “step” steer input
- standardised in ISO 7401



<u>severe lane change test</u>

- obstacle avoidance
- find maximum velocity where test driver is capable to complete the course, without touching the cones
- high lateral accelerations, limit handling
- standardised in ISO/TR 3888

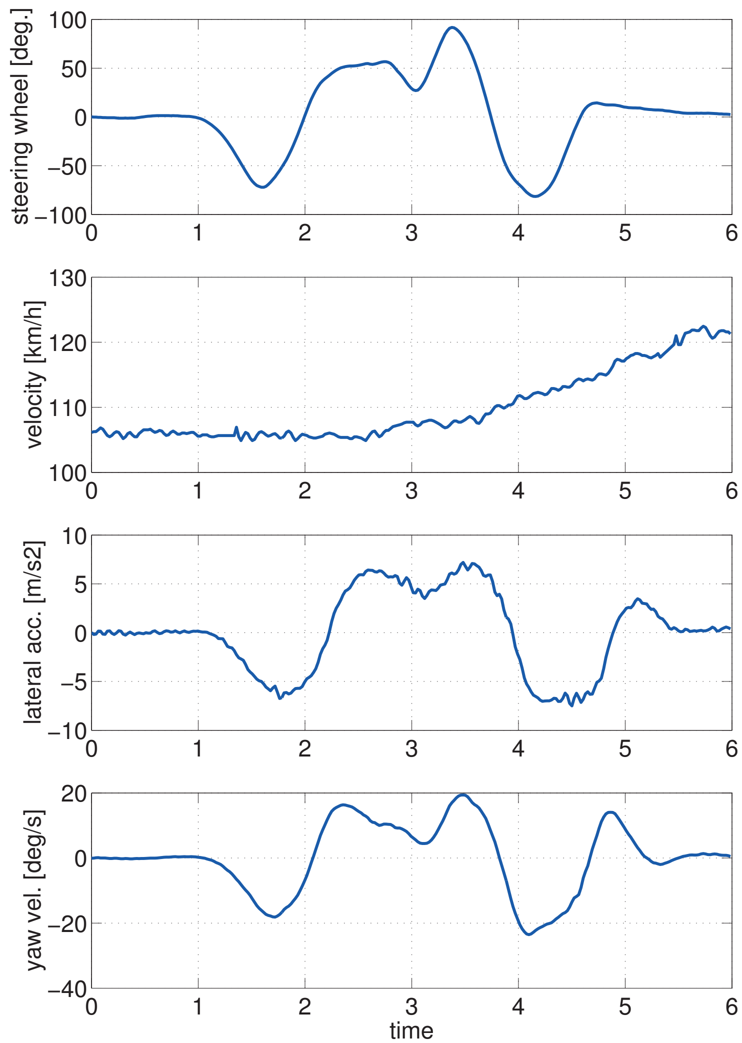


w: width of vehicle

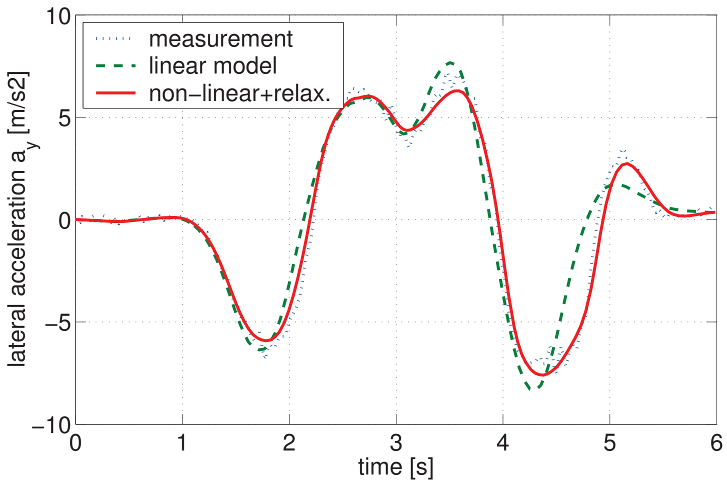
simulation model:

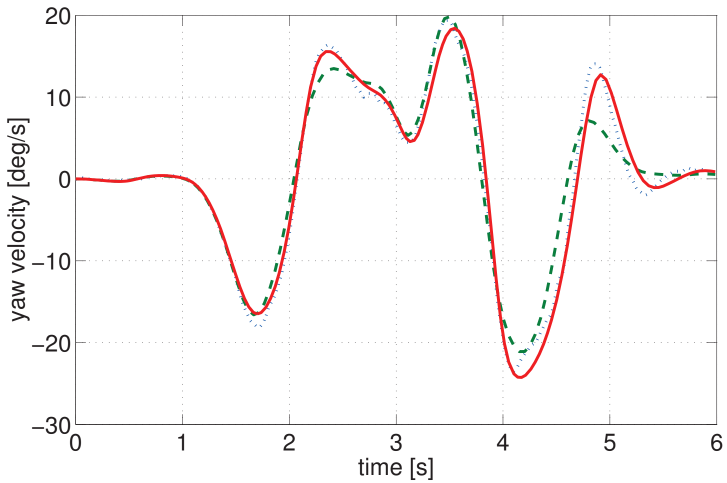
- use measured steering angle and vehicle forward velocity as input
- model parameters from steady state circular test and random steer (no additional tuning)
- compare against measurements:
  -linear model without relaxation effects
  -model with non-linear tyres and relaxation

test results…



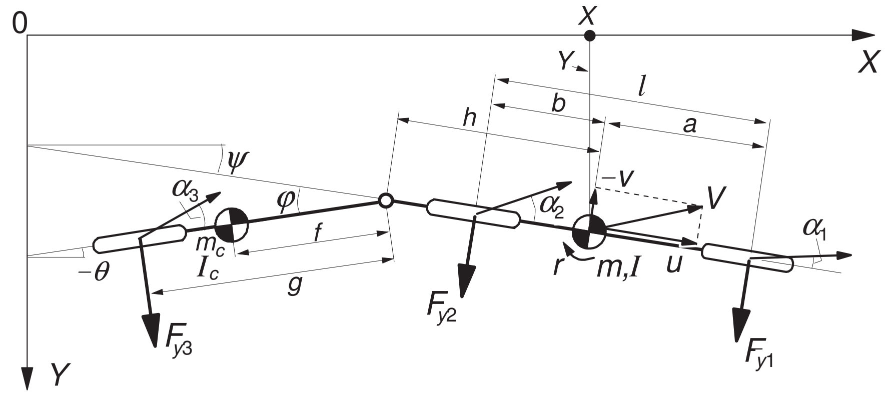
comparison with simulation model



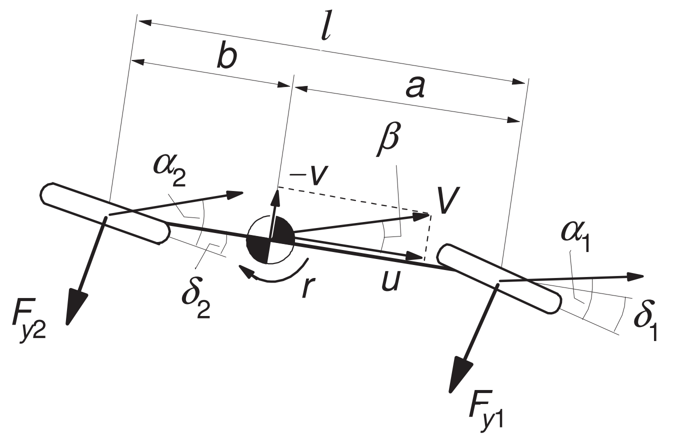


applications...

- car-trailer combination stability



- four wheel steering



Book Pacejka

- pages 35 to 39, chapter 8.1

Next time…

- tyre slip definitions
- tyre force and moment testing

4. Assessment of tyre characteristics

rolling tyre

- input quantities (e.g. slip, inclination angle,…)
- forces and moments

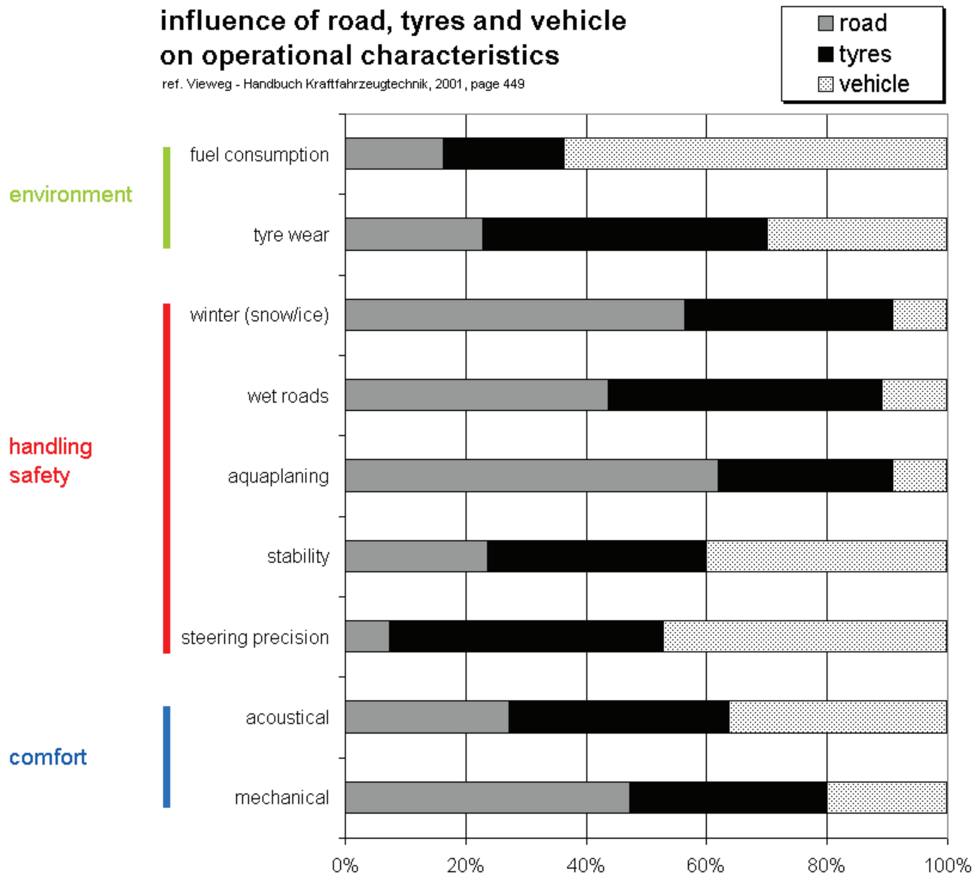
overview of results obtained from experiments

“...meten is weten!”

Tyres…

- connection between vehicle and road
- large forces are transmitted through a relatively small contact area

affects many different aspects of the vehicle behaviour



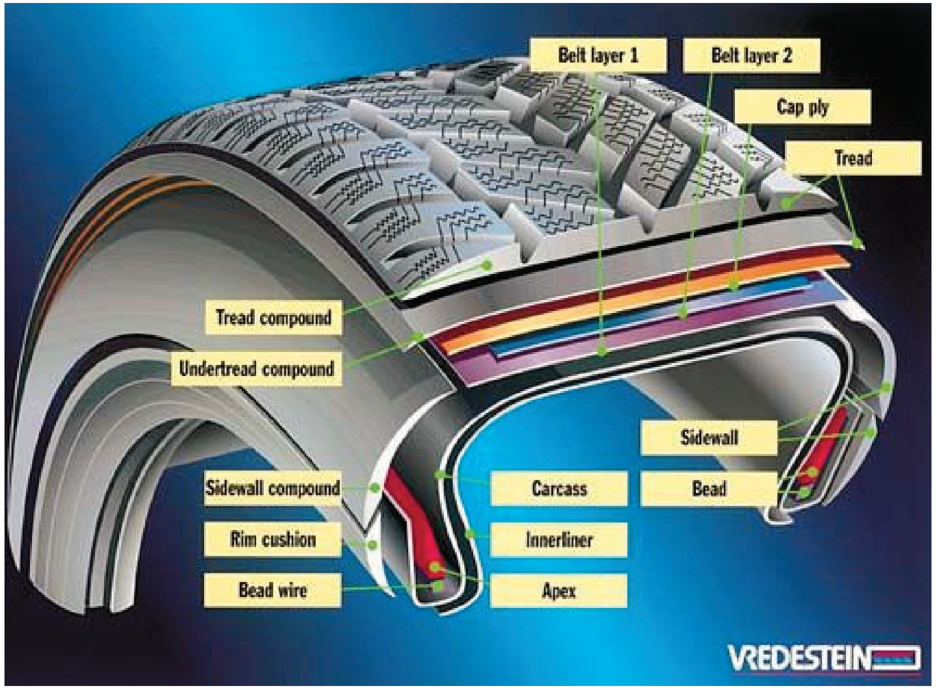
tyre construction

two carcass construction principles

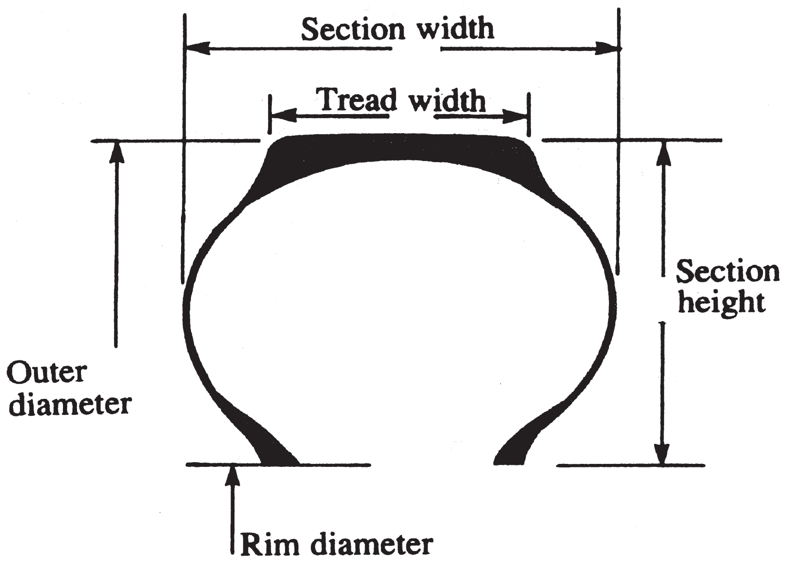
- bias ply tyre (cross ply tyre)
- radial tyre

passenger car tyres and truck tyres will (nearly) always have a radial construction.

bias ply tyres are still used for high loading/low speed applications.



tyre dimensions



identification code: 185/65R14 86H

- section width: 185 mm
- aspect ratio: 65%
  (aspect ratio=section height/width*100%)
- R = radial construction
- rim diameter: 14 inch
- load index: 86 (530 kg)
- speed symbol: H (210 km/h)

Force and Moment testing

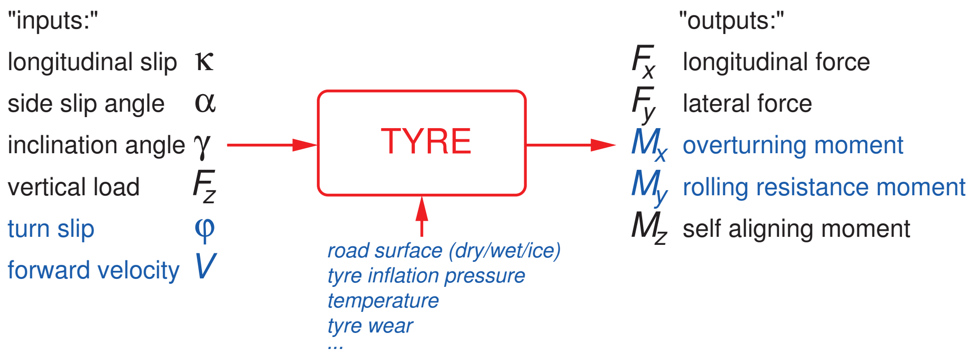
aim: determination of the forces and moments generated by the rolling tyre under various slip conditions

example results:

- cornering stiffness $C_{F\alpha}$
- full non-linear behaviour $F_y = f(\alpha, Fz, ...)$

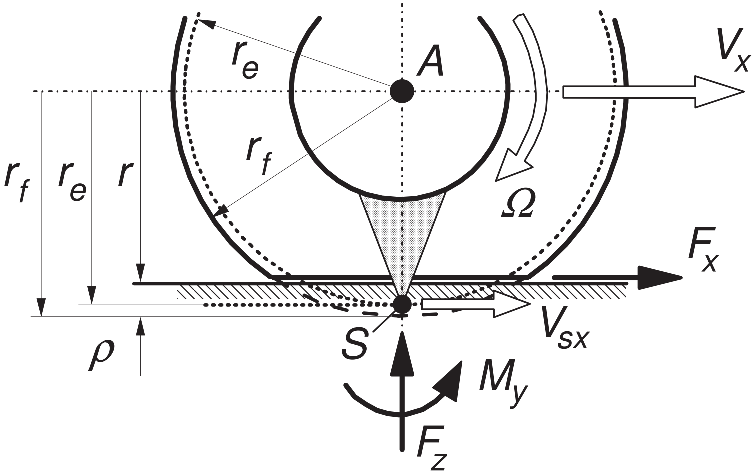
note: *steady state conditions, no dynamics! (e.g. relaxation behaviour)*

representation of the tyre:



we will focus on the relations between the inputs $\kappa, \alpha, \gamma, F_z$ and outputs F_x, F_y, M_z.

input/output definitions
in-plane behaviour



nomenclature:

- free tyre radius r_f
- effective rolling radius r_e
- loaded radius r, tyre deflection ρ
- forward velocity V_x
- wheel angular velocity Ω
- longitudinal slip speed V_{sx}
- longitudinal force F_x
- vertical force F_z
- rolling resistance moment M_y

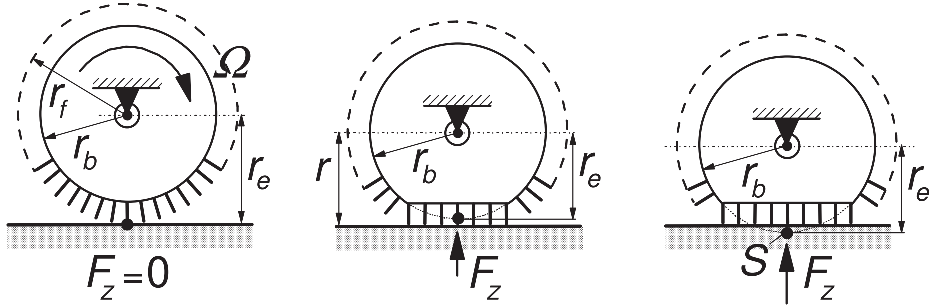
"S" is the pole of the free rolling tyre

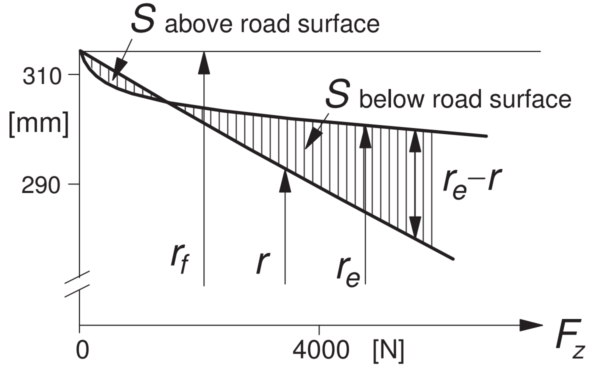
the effective rolling radius relates the angular velocity Ω with the forward velocity V_x for a freely rolling wheel (no braking/no side slip).

$$r_e = \frac{V_x}{\Omega}$$

the effective rolling radius:

- depends on the vertical load F_z
- determines the location of point S





longitudinal slip or slip ratio κ:

$$\kappa = -\frac{V_{sx}}{V_x} = -\frac{V_x - \Omega r_e}{V_x}$$

so by definition:
- for a freely rolling wheel: $\kappa = 0$
- for a fully locked wheel: $\kappa = -1$

note:
- sometimes κ is expressed as a percentage (-100% fully locked wheel)
- in the available literature sometimes a different definition for the driving side ($\kappa > 0$) may be found
- how to handle vehicle standing still? ($V_x = 0$)

for small values of longitudinal slip the tyre behaviour is linear:

$$F_x = C_{F\kappa}\kappa$$

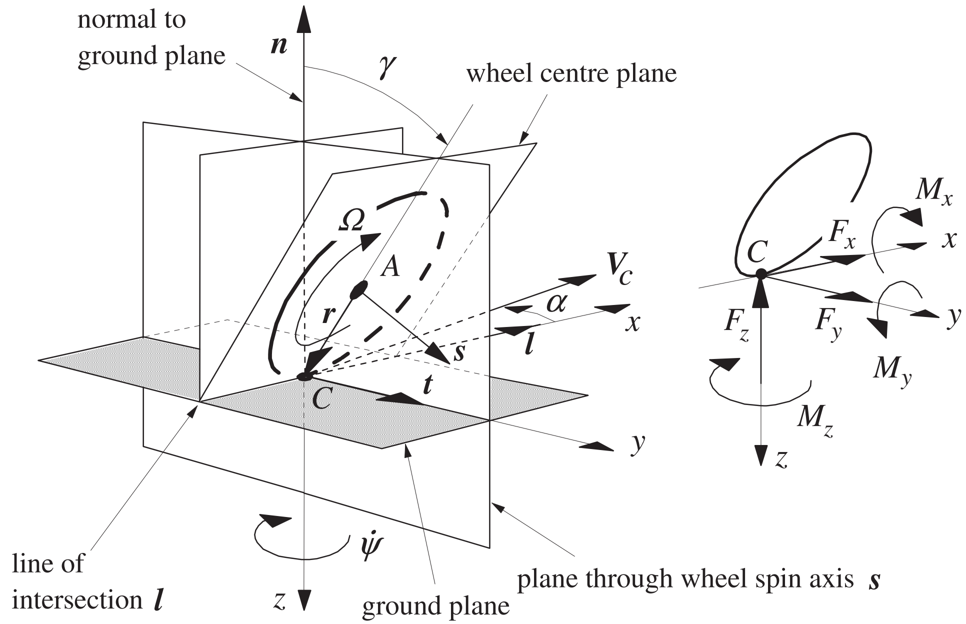
where $C_{F\kappa}$ is the longitudinal slip stiffness

<u>out-of-plane behaviour</u>

wheel+tyre considered as a disk

definition of the contact centre C:
point of intersection of three planes
- ground plane
- wheel centre plane (through plane of symmetry of the tyre)
- plane though wheel spin axis and normal to the road

distance A to C equals the loaded radius



side slip angle (or drift angle) α:

$$\tan\alpha = -\frac{V_{cy}}{V_x}$$

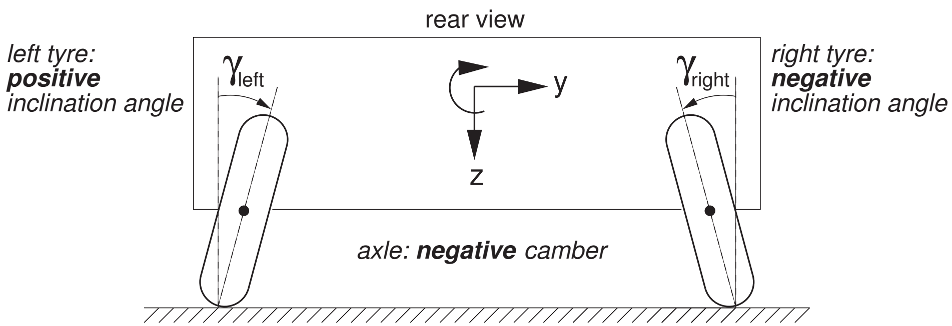
where V_{cy} is the lateral component of the velocity in point C.

the inclination angle (or camber angle) γ is the angle between the normal to road and wheel centre plane

notes:

- as pointed out in the book of Pacejka slightly different definitions may be used for α, but on a level road without rapid inclination angle changes the differences are negligible

- strictly speaking camber is only defined in the context of an axle, but is often (ab)used to define the inclination angle of a tyre…



nomenclature of the forces/moments:

- F_y : lateral force (side force)
- M_x: overturning moment
- M_z: self aligning moment/torque

linear tyre behaviour, steady state:
(small values of side slip and camber)

$$F_y = C_{F\alpha}\alpha + C_{F\gamma}\gamma$$

$$M_z = -C_{M\alpha}\alpha + C_{M\gamma}\gamma$$

where

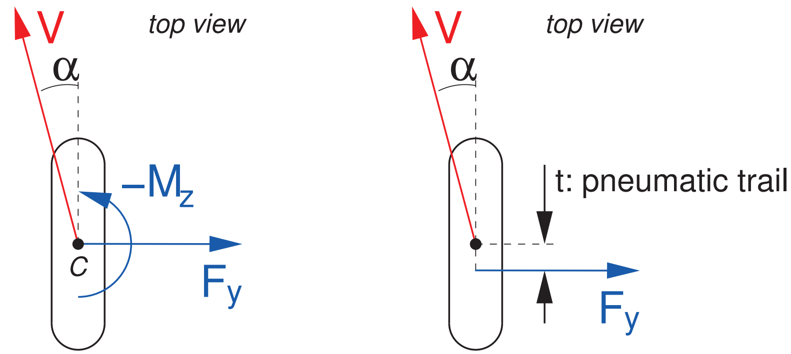
- $C_{F\alpha}$: cornering stiffness
- $C_{M\alpha}$: self aligning stiffness
- $C_{F\gamma}$: camber stiffness
- $C_{M\gamma}$: camber torque stiffness

for a “normal” tyre all these stiffnesses are positive!

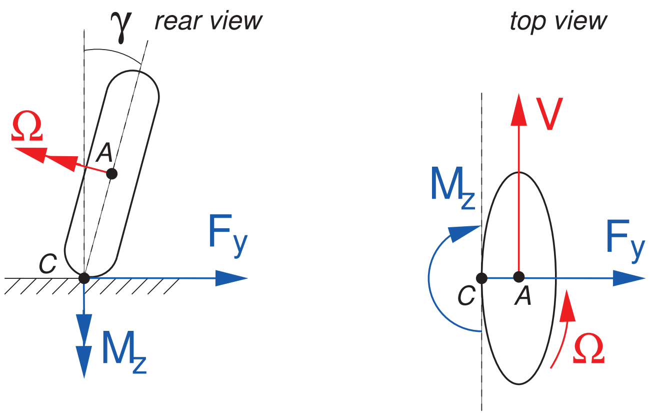
pneumatic trail: $t = -\frac{M_z}{F_y}$

or in the linear case (and $\gamma = 0$): $t = \frac{C_{M\alpha}}{C_{F\alpha}}$

side slip

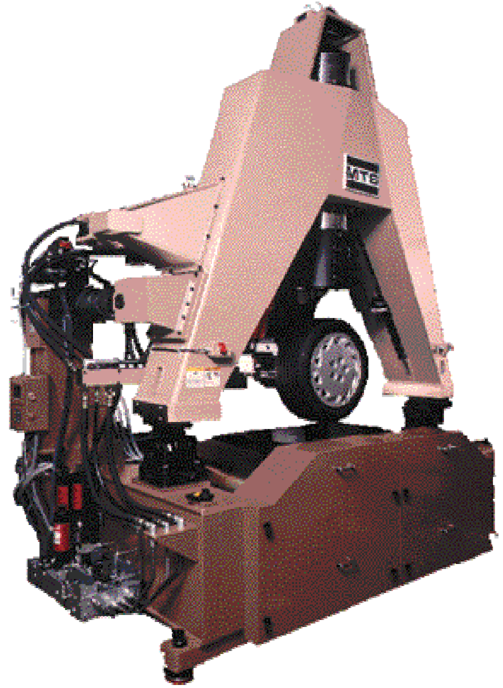


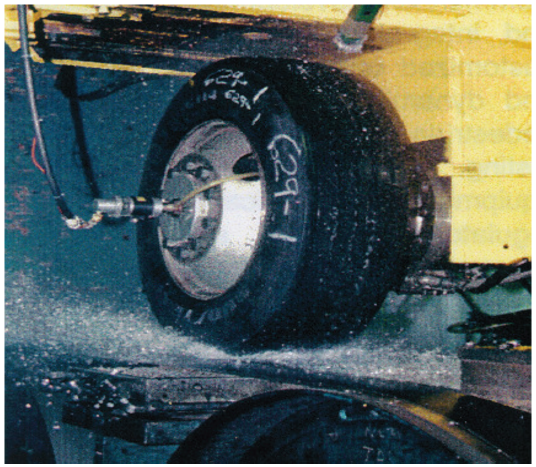
inclination angle (camber)



test facilities for force and moment testing:

- flat track machine
- drum
- tyre test trailer





traditional test programme:

- vary α or κ and fix the remaining inputs

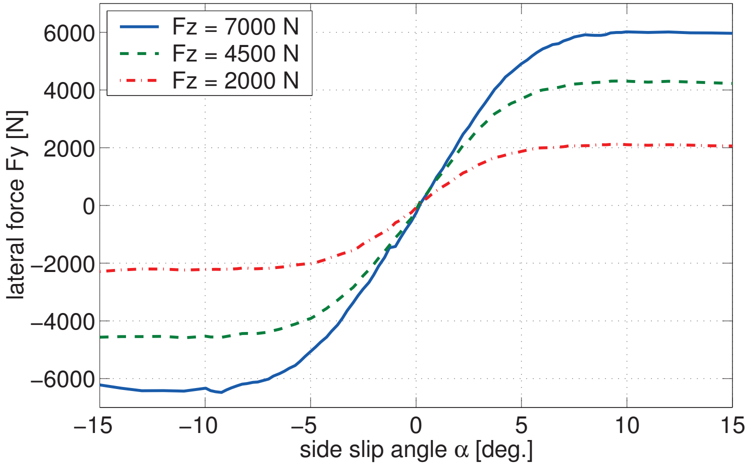
example of a test programme:

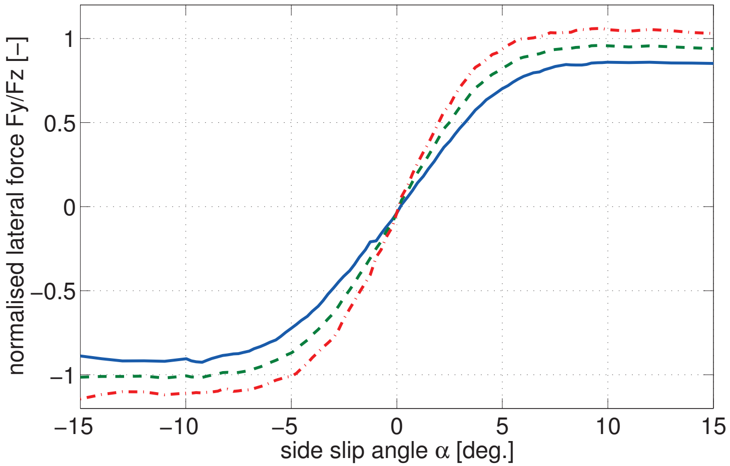
	alpha [deg.]	inclination [deg.]	Fz [kN]
"pure" slip characteristics			
alpha sweep	sweep	-5/0/5	2.0/4.5/7.0
kappa sweep	0	-5/0/5	2.0/4.5/7.0
"combined" slip characteristics			
kappa sweep	-9/-5/-2/2	0	2.0/4.5/7.0

note:

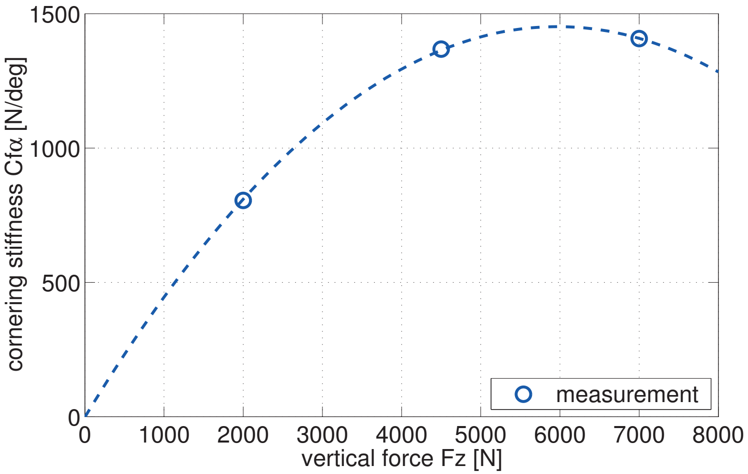
alpha sweep: -15 to 15 deg. (free rolling tyre: kappa=0)
kappa sweep: braking only up to wheel lock (kappa: 0 to -1)

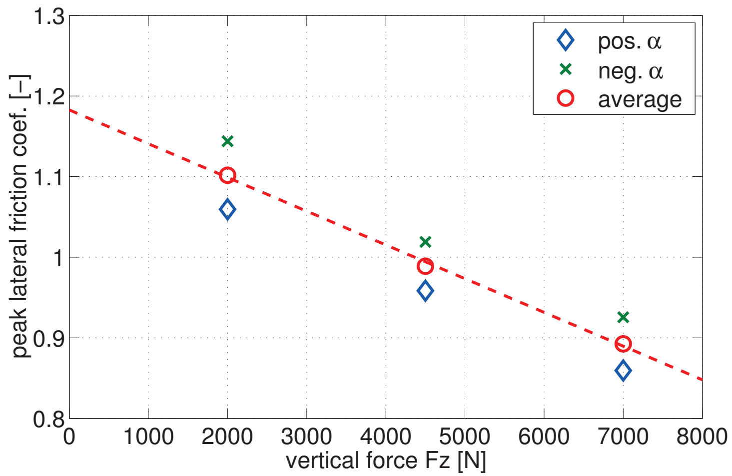
measured F_y verus α ($\gamma = 0$, free rolling)



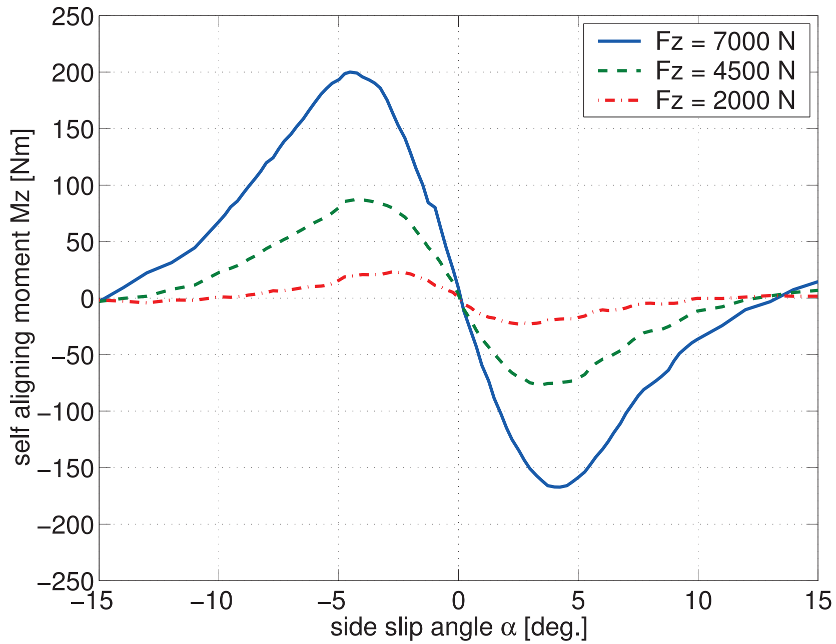


cornering stiffness and peak friction coefficient

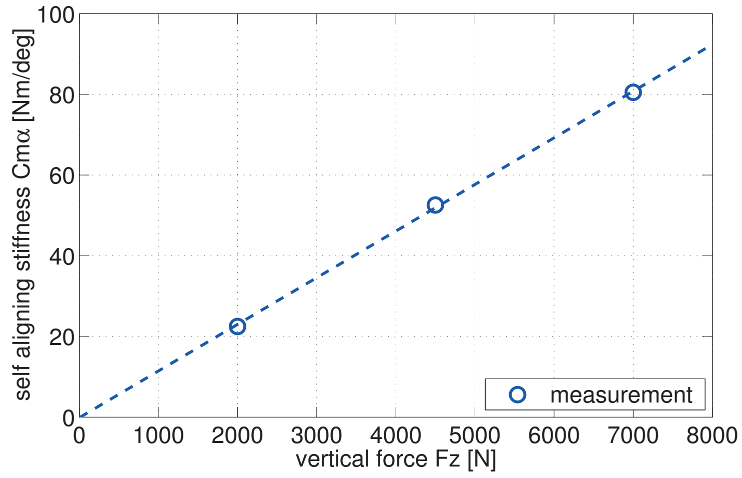


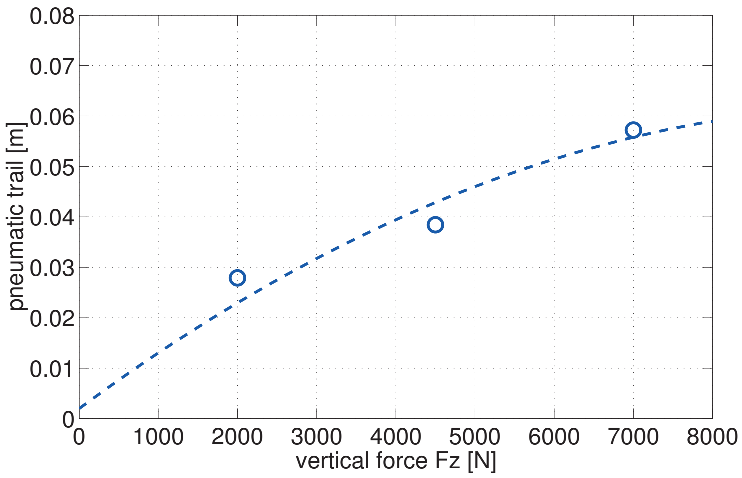


measured M_z verus α ($\gamma = 0$, free rolling)

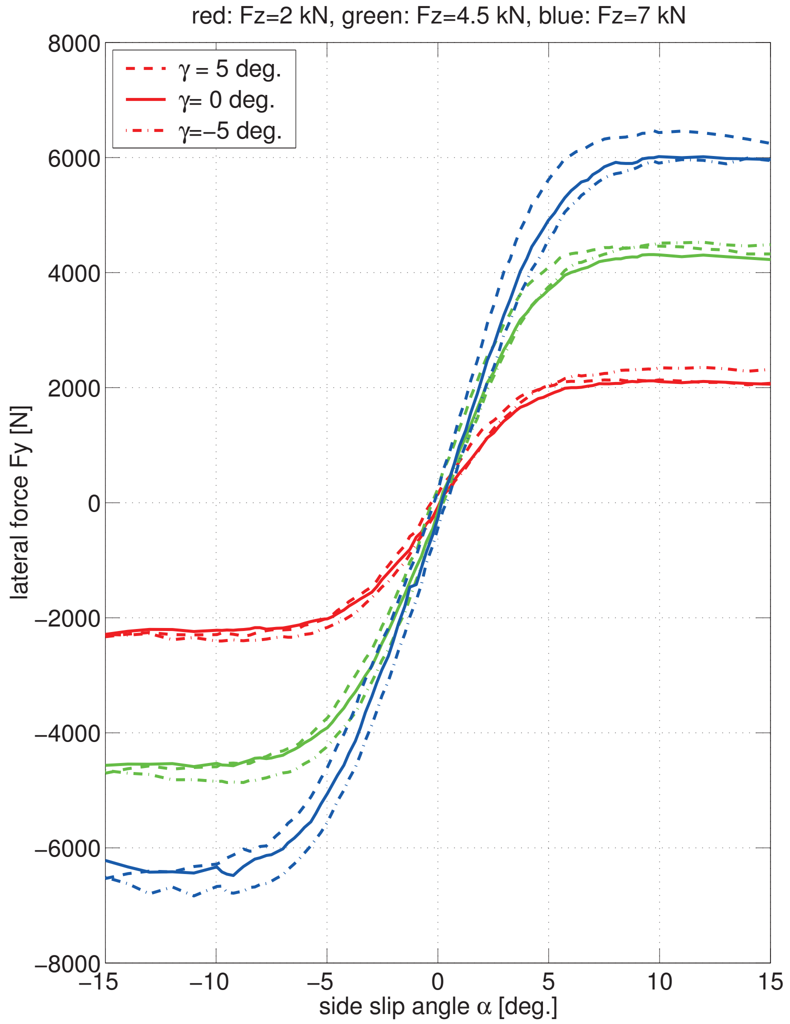


self-aligning stiffness and pneumatic trail

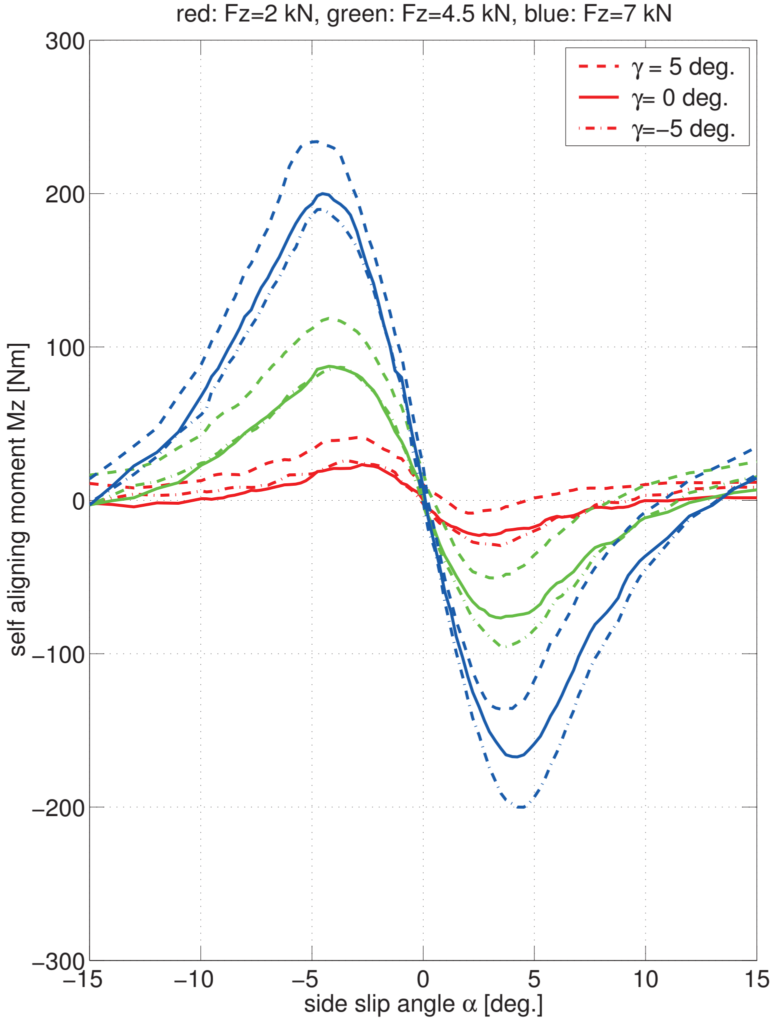




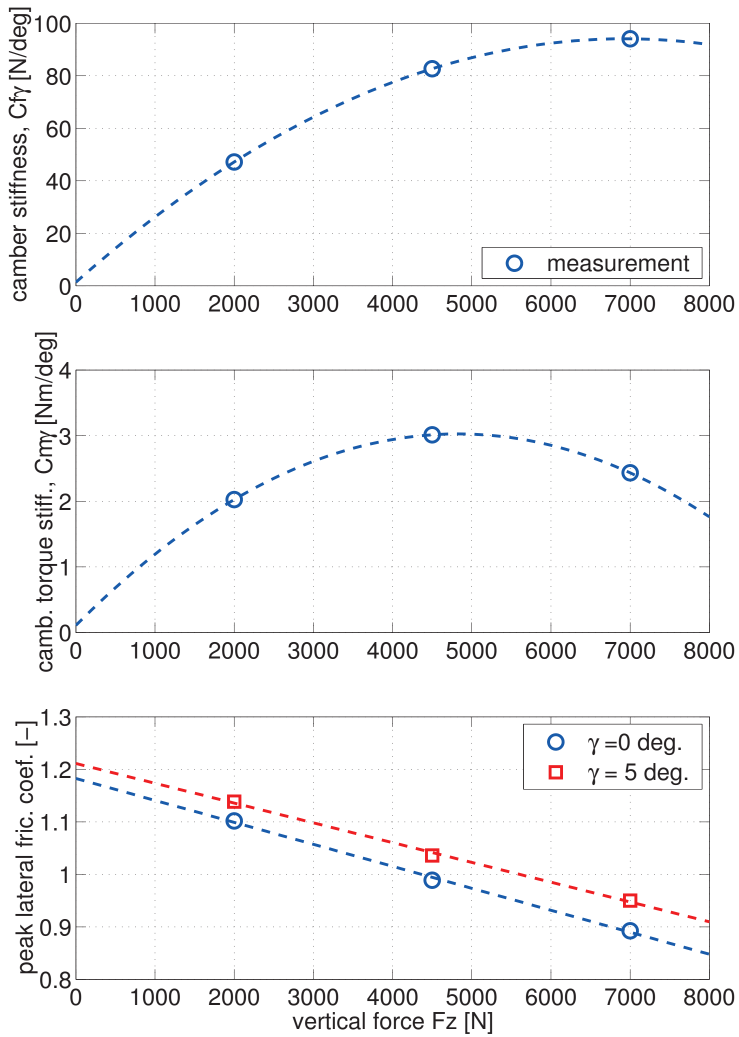
inclination angle γ; measured F_y versus α



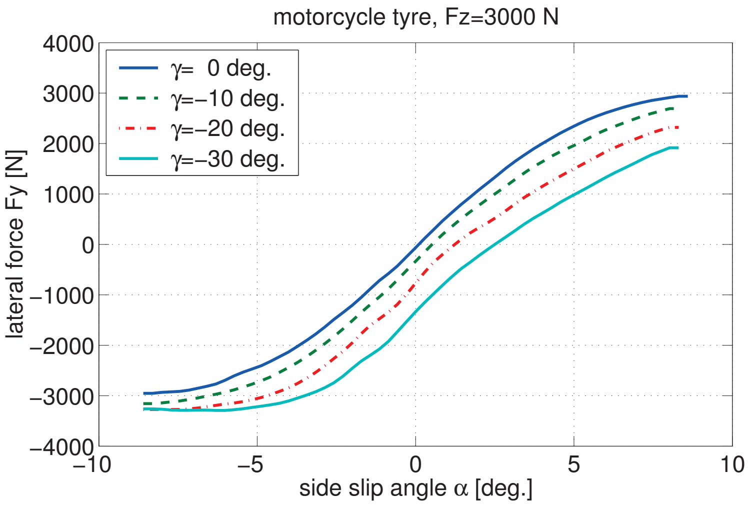
inclination angle γ; measured M_z versus α

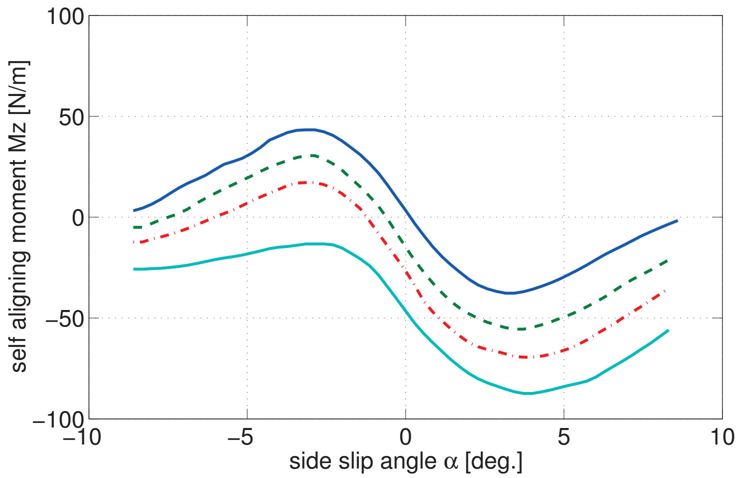


camber (torque) stiffness and peak friction



more extreme camber angles: motorcycle tyre





when looking carefully: F_y and M_z are not exactly zero when $\alpha = 0$ and $\gamma = 0$.

this is not (necessarily…) a measurement error!

can be caused by non-symmetry of the tyre construction, two effects:

- <u>plysteer</u>
  determined by construction and build-up of the carcass layers (design)

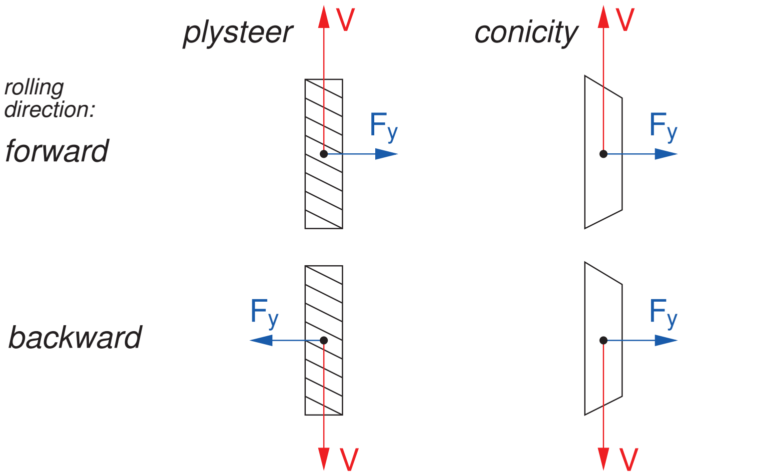
  plysteer force changes sign when going form forward to backward driving

  plysteer: may be interpreted as a “pseudo” side slip angle α_{ply}

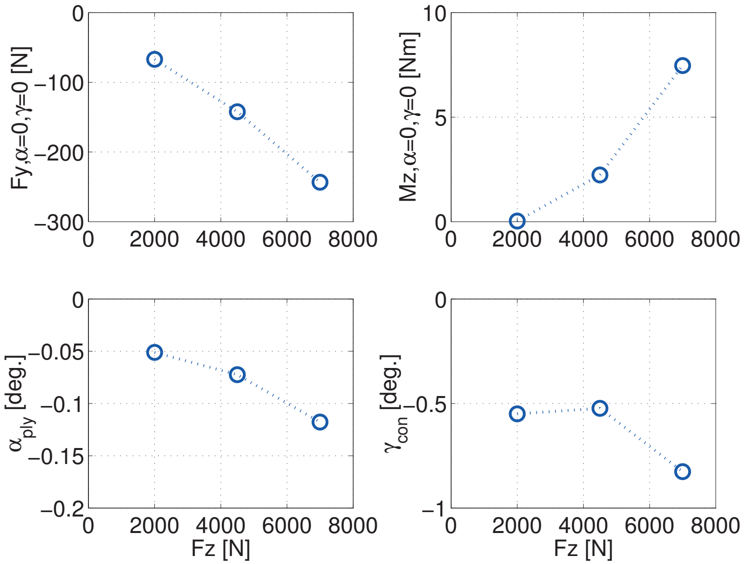
- <u>conicity</u>
  determined by the shape of the tyre/carcass (production tolerances)

  conicity force does not change sign when going from forward to backward driving

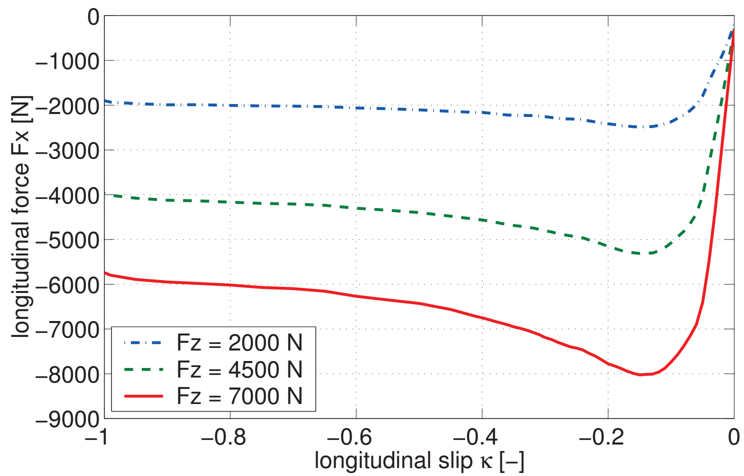
  conicity may be interpreted as a “pseudo” inclination angle γ_{con}

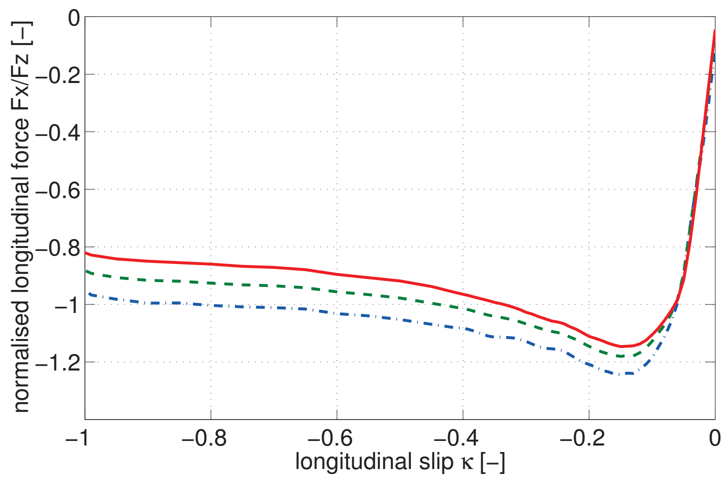


$$F_y(\alpha_{ply}, \gamma_{con}) = 0 \text{ and } M_z(\alpha_{ply}, \gamma_{con}) = 0$$

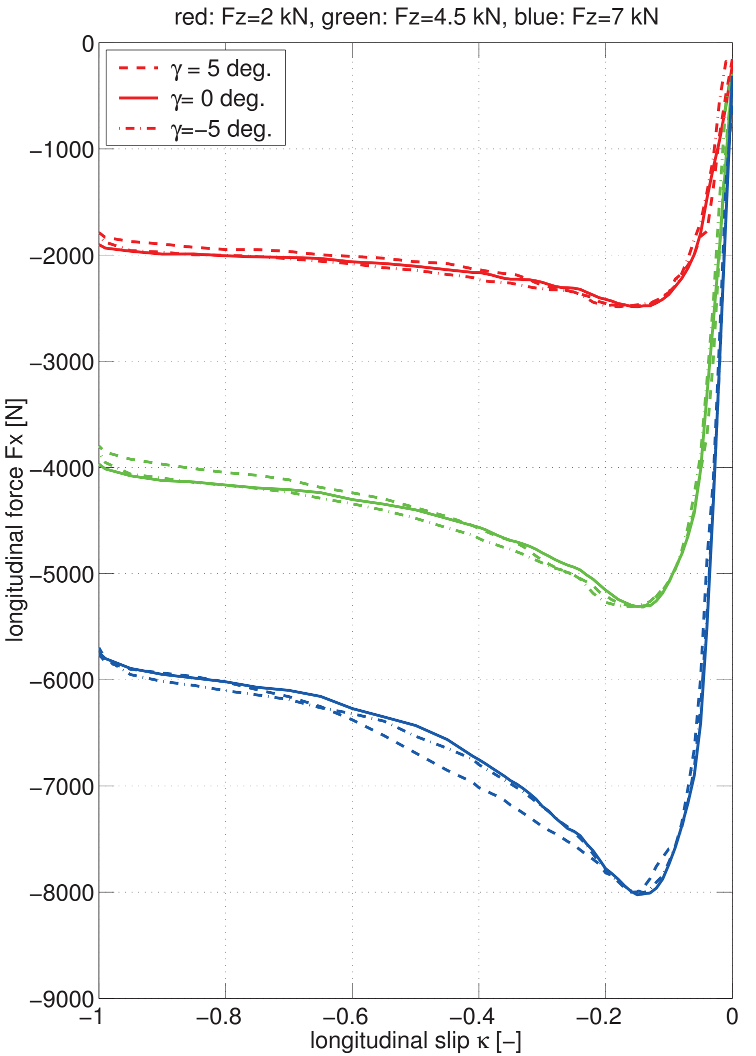


braking in a straight line ($\alpha = 0$ and $\gamma = 0$)

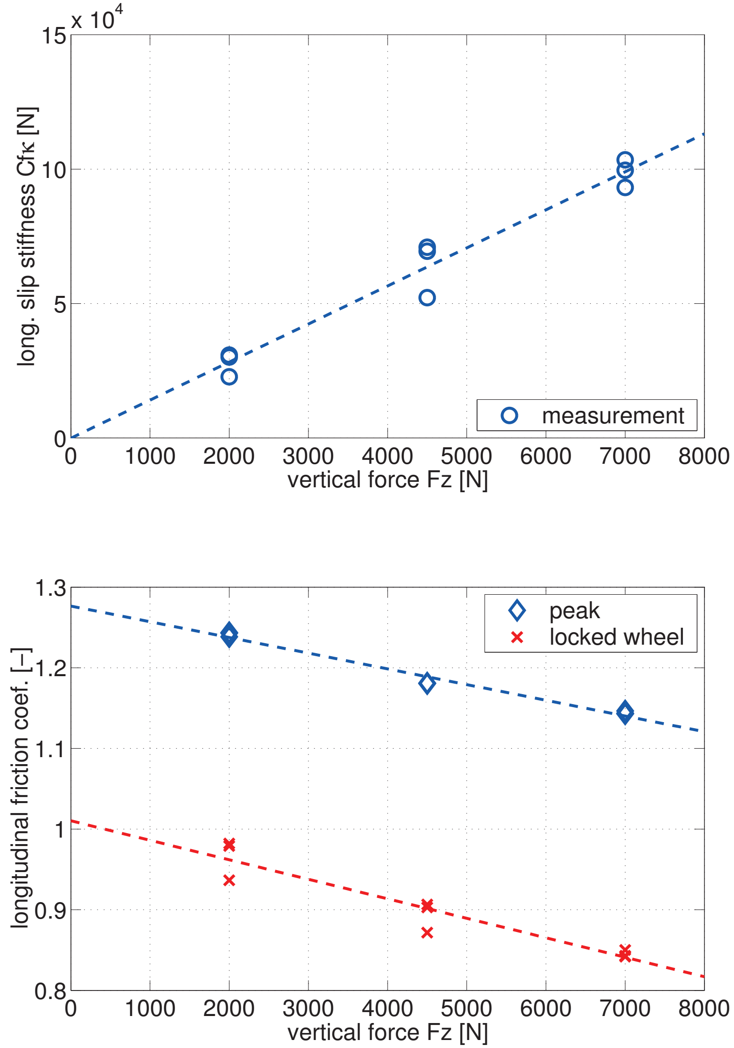




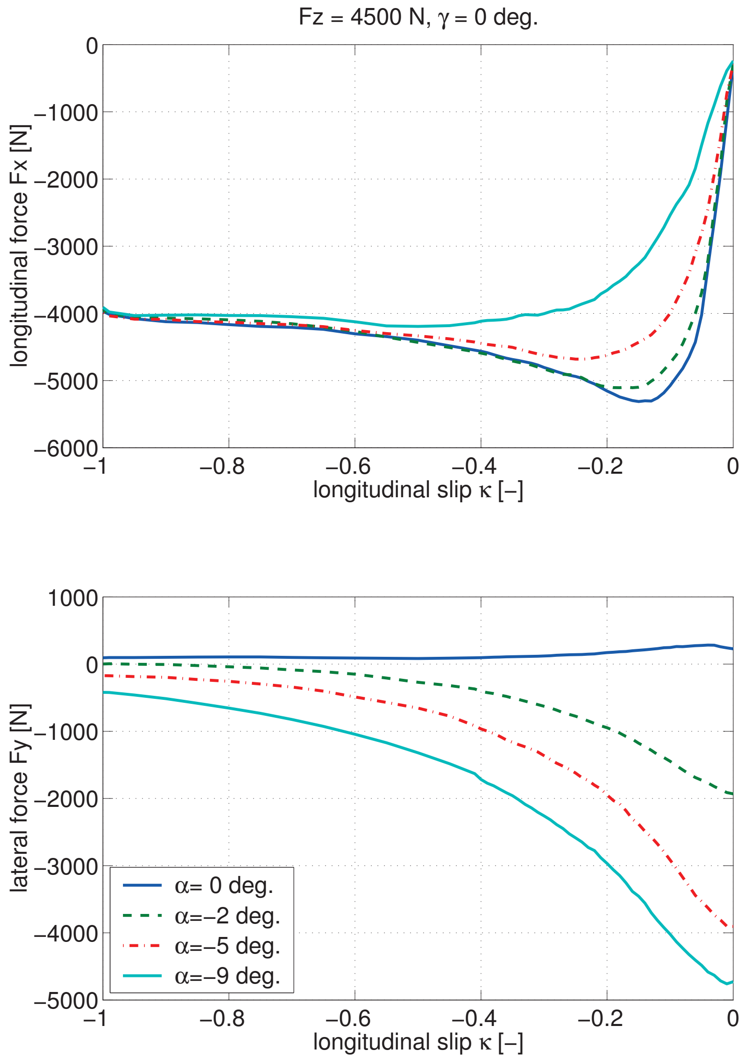
inclination angle effects are small…



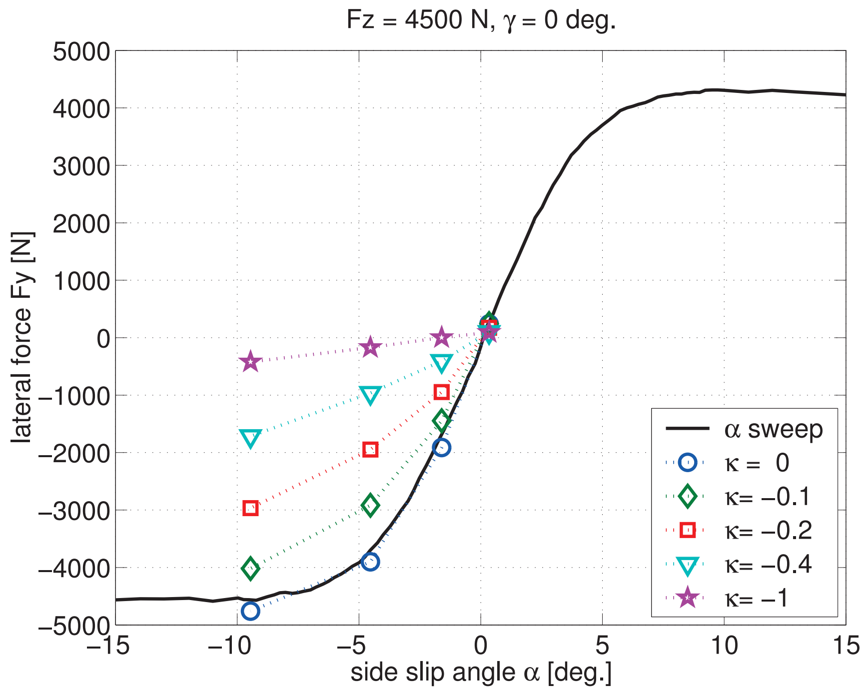
slip stiffness and friction coefficient



braking with fixed steering angle ($\gamma = 0$)



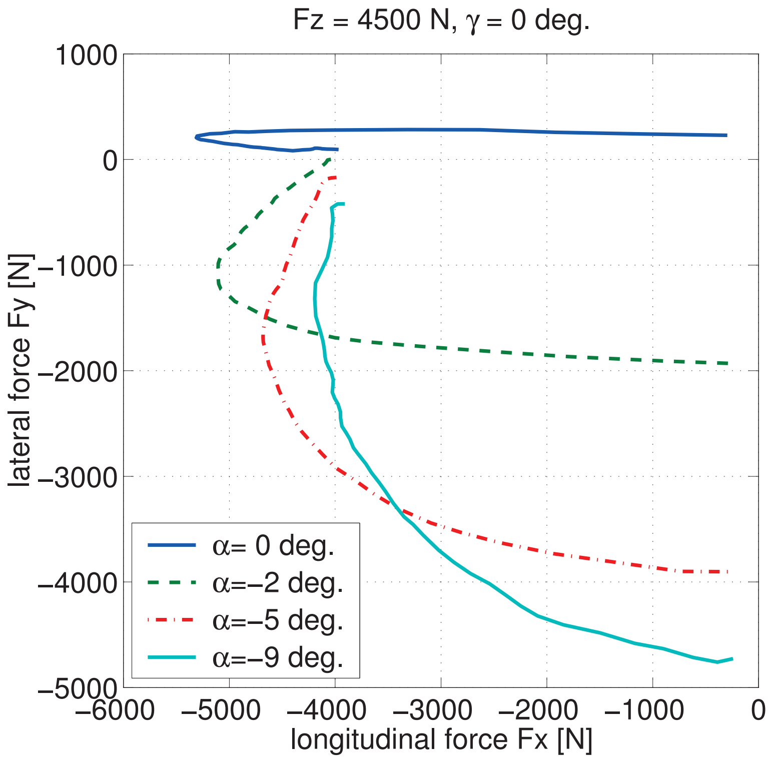
lateral force characteristic under braking



note the mutual interactions:

- introduction of a side slip angle results in a reduction of the longitudinal slip stiffness and peak longitudinal force
- introduction of longitudinal slip results in a reduction of the cornering stiffness and peak lateral force

longitudinal force versus lateral force



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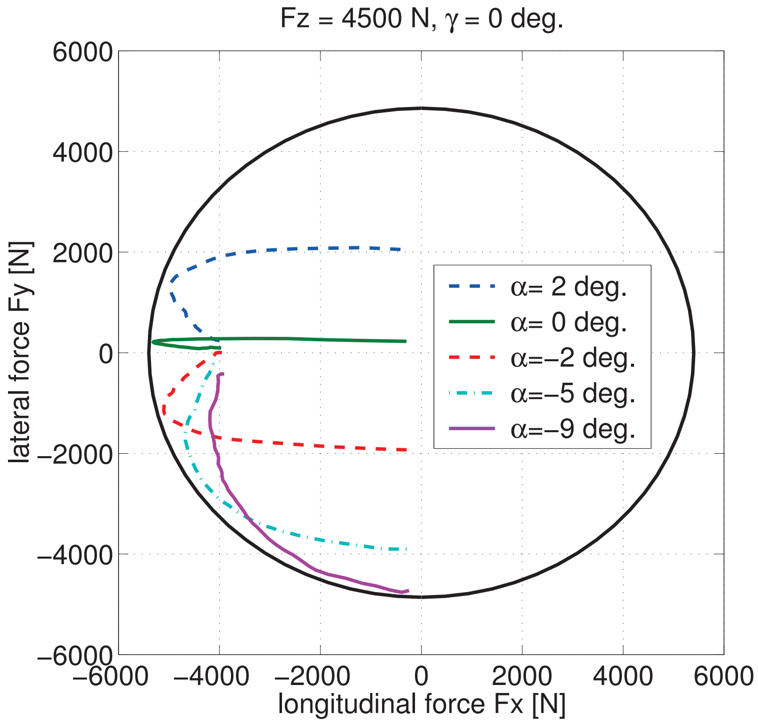
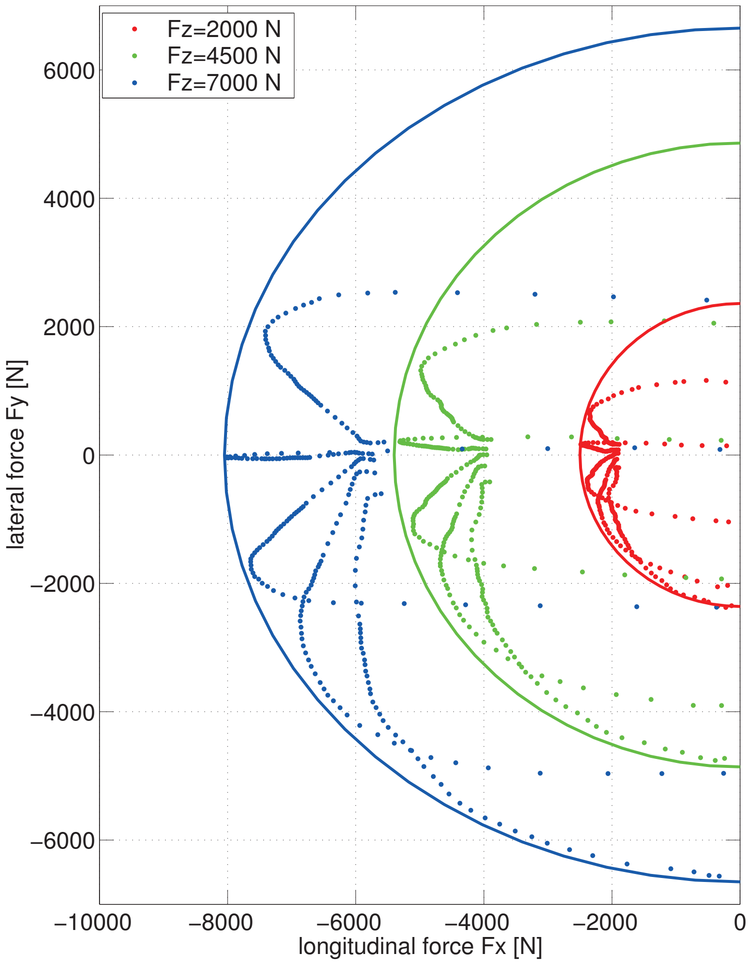
friction ellipse represents boundary:

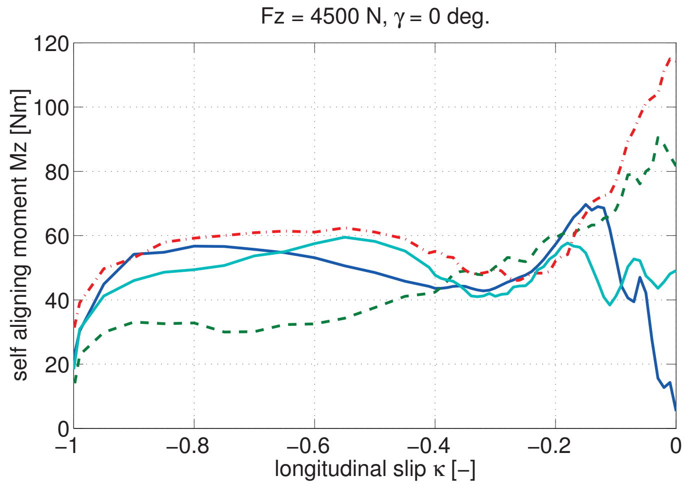


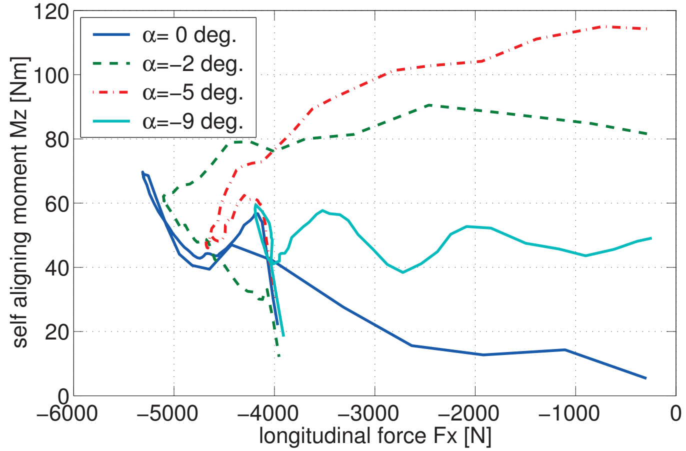
measurements at different vertical loads



self-aligning moment under braking:

- rather difficult to be measured accurately…





Sign conventions

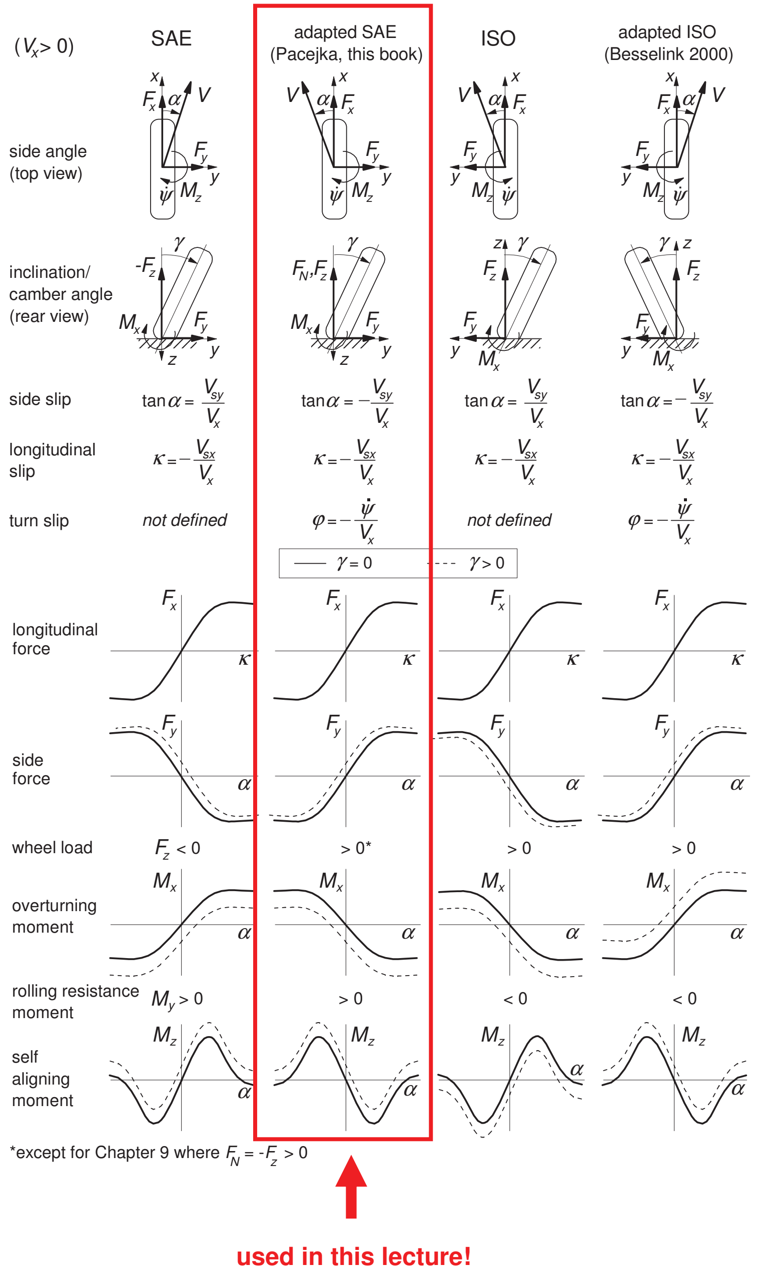
- may seem trivial, but can be the source of a lot of confusion and errors!

- SAE and ISO have standardised tyre axis systems and slip definitions

- in the SAE axis system a tyre deflection results in a negative vertical force F_z, which is not very intuitive. The sign of the negative SAE vertical force is often reversed. This results in a non-RHS axis system for the forces, which again isn't very nice.

- Pacejka reversed the sign of the slip angle with respect to the SAE definitions to enhance the similarity between the longitudinal and lateral slip characteristics

- in the axis system proposed by Besselink:
  -the vertical force F_z is positive
  -rolling resistance M_y is negative
  -positive camber: upward shift of <u>all</u> curves
  -most of the slopes near the origin are positive
  -we have the desired similarity



Book Pacejka

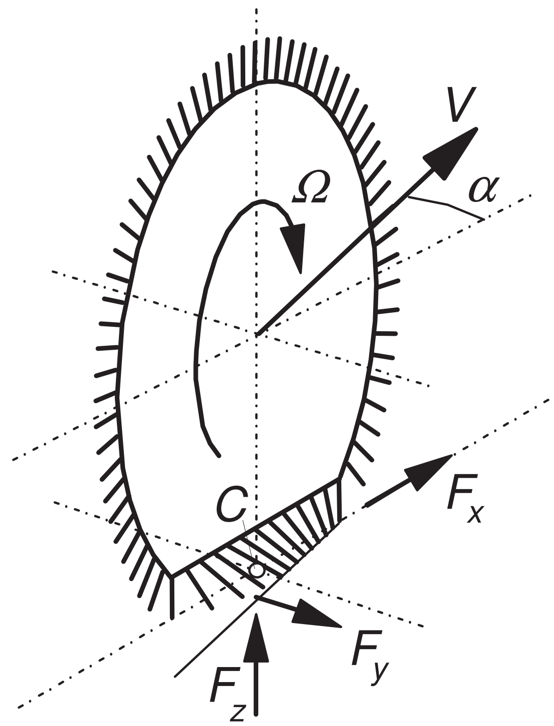
- chapter 1.1 and 1.2.1 (pages 2 to 7)
- chapter 2.1 and 2.2 (pages 59 to 68)
- appendix 1 (page 609)

Next time…

- the brush tyre model

5. The brush tyre model

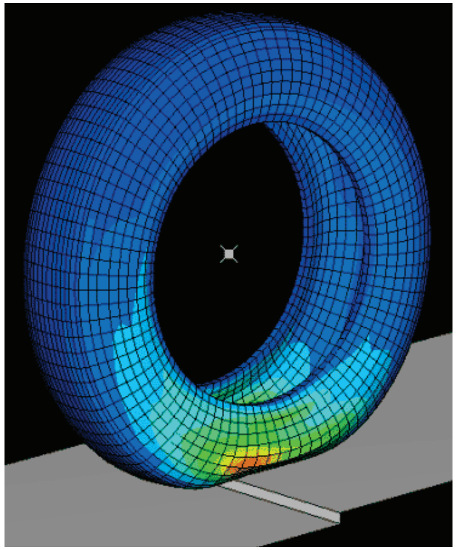
- tyre modelling: some general remarks
- the brush tyre model
  - –pure lateral slip
  - –pure longitudinal slip
  - –combined slip



Tyre modeling in general

from the perspective of a tyre manufacturer:
calculation of:

- stresses
- mechanical properties e.g. contact pressure, vertical stiffness, cornering stiffness, rolling resistance, plysteer,…
- noise
- wear
- …



from the perspective of a car manufacturer
calculation of:

- handling behaviour
- ride comfort
- load/fatigue spectra (incl. abuse…)
- space requirements
- …

limiting the discussion to forces and moments generated by a rolling tyre.

two “schools” can be distinguished:

- <u>physical tyre models</u>
  detailed modelling of the tyre structure: rubber, carcass, etc.

  insight in tyre design – tyre behaviour

  examples:
  –stretched string model
  –brush model
  –FEM models

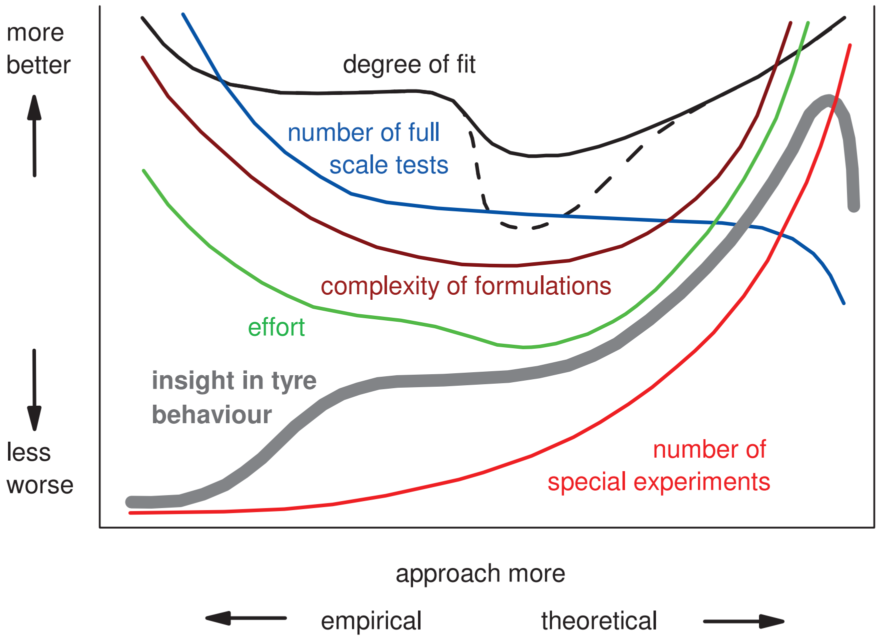
- <u>(semi-)empirical tyre models</u>
  use a mathematical formulation which can represent the measurements.

  quick and accurate

  examples:
  –similarity method
  –Magic Formula

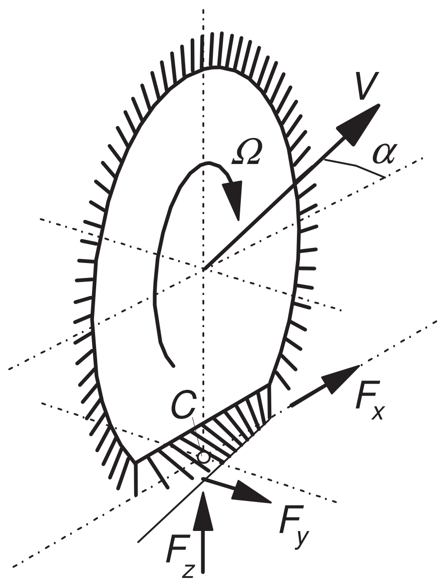
comparison of different approaches:

from experimental data only	using similarity method	through simple physical model	through complex physical model
fitting full scale tyre test data by regression techniques	distorting, rescaling and combining basic characteristics	using simple mechanical representation, possibly closed form solution	describing tyre in greater detail, computer simulation, finite element method
Magic Formula		***brush model***	



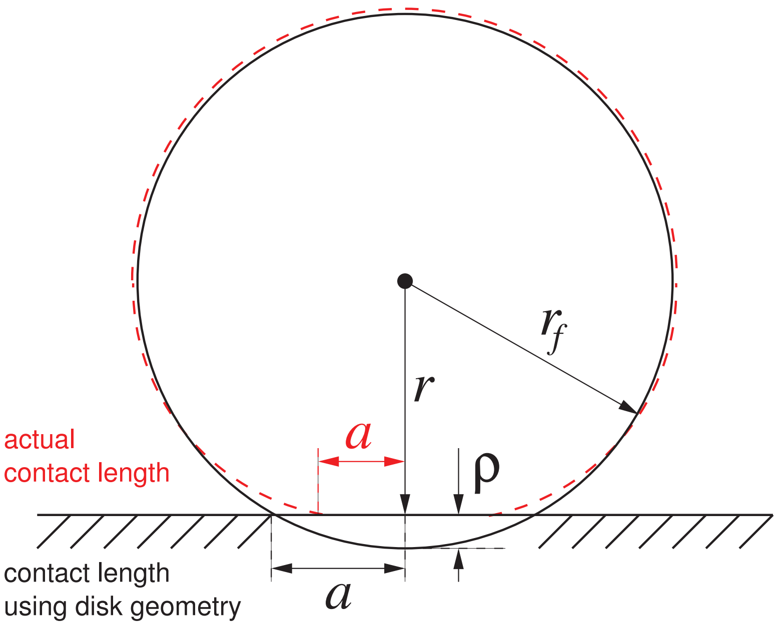
Brush model (dutch: borstelmodel)

- single row of bristles
- bristles are compliant in the fore/aft and lateral direction (representing the combined stiffness of carcass, belt and tread elements)
- distance where bristles are in contact with the road: $2a$ (the contact length)
- bristles are undeformed when not in contact with the road
- parabolic pressure distribution of the vertical force
- constant friction coefficient μ between the tip of a bristle and the road



contact length

contact length based on rigid disk penetrating the ground

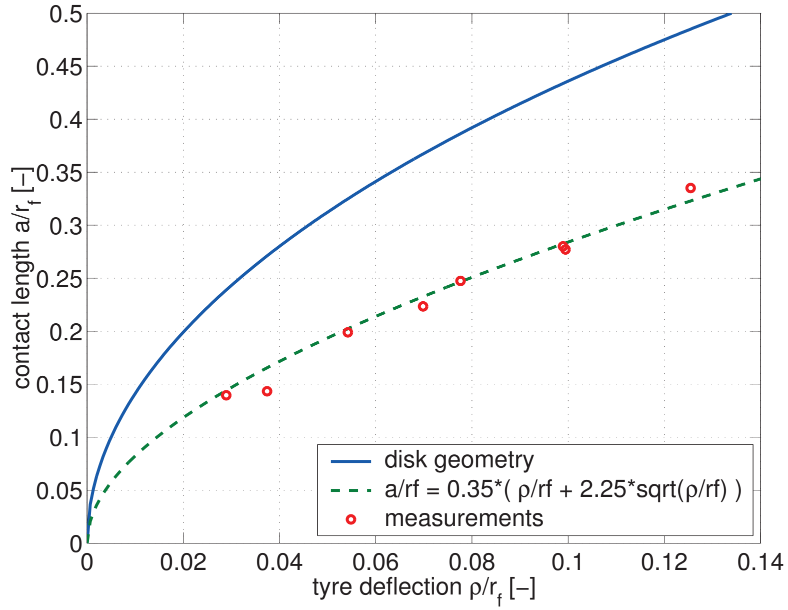


from disk geometry:

$$a^2 = r_f^2 - r^2 \text{ and } \rho = r_f - r$$

so $a = r_f \sqrt{\dfrac{2\rho}{r_f} - \left(\dfrac{\rho}{r_f}\right)^2}$

cross check with measurements on a real tyre...

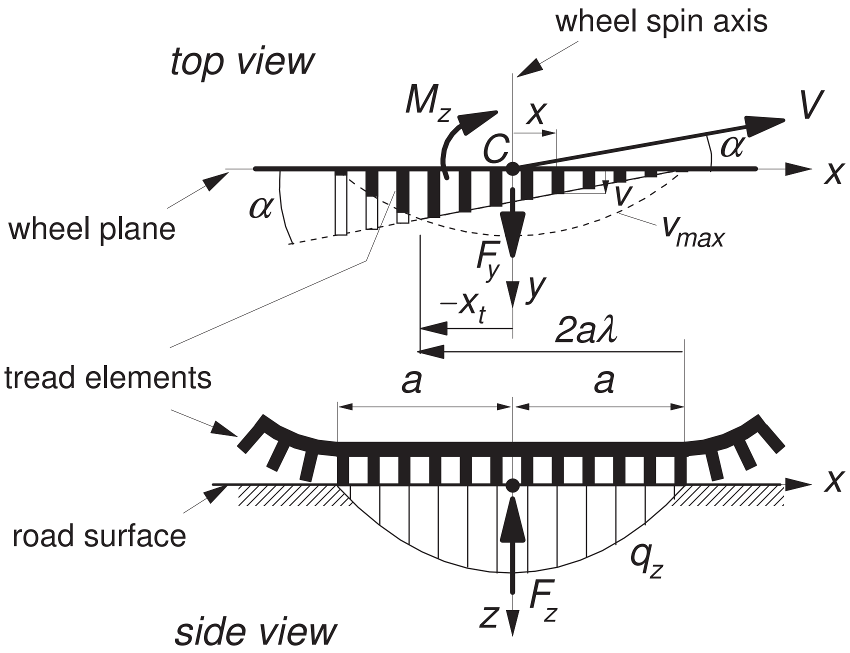


empirical formula:

$$a = 0.35 r_f \left(\frac{\rho}{r_f} + 2.25 \sqrt{\frac{\rho}{r_f}} \right)$$

note:
a equals half of the contact length!

Pure side slip



note: stationary conditions!

parabolic pressure distribution:

$$q_z = \frac{3F_z}{4a}\left\{1-\left(\frac{x}{a}\right)^2\right\}$$

obviously: $F_z = \int_{-a}^{a} q_z dx$

lateral bristle deflection (adhesion region):

$$v = (a-x)\tan\alpha$$

lateral force per unit of length:

$$q_y = c_{py} v$$

where c_{py} equals the lateral stiffness of the bristles per unit of length.

(note: v equals deflection, not velocity!)

the lateral force per unit of length q_y will be limited by the friction coefficient μ between tyre and road

two possible situations:

- $|q_y| < \mu q_z$ adhesion
- $|q_y| = \mu q_z$ sliding

in the sliding region the lateral bristle deflection becomes:

$$v = \frac{\mu}{c_{py}} q_z$$

so in the contact area we may distinguish:
- region of adhesion (at the leading edge)
- region of sliding (at the trailing edge)

the transition point is described by the factor λ, the transition from adhesion to sliding is located at a distance $2a\lambda$ from the leading edge (see page 127)

extremes:

$\lambda = 1$: full adhesion
(zero or very small side slip angles α)

$\lambda = 0$: total sliding
(large side slip angles α)

the lateral tyre force and self-aligning moment can be obtained by integrating the lateral force per unit of length over the full contact length:

$$F_y = \int_{-a}^{a} q_y dx$$

$$M_z = \int_{-a}^{a} q_y x dx$$

linear characteristics

full adhesion $\alpha \to 0$

lateral force:

$$F_y = \int_{-a}^{a} q_y dx = c_{py} \int_{-a}^{a} v dx = c_{py} \int_{-a}^{a} (a - x) \tan(\alpha) dx$$

$$F_y = 2 c_{py} a^2 \alpha$$

so the cornering stiffness becomes:

$$C_{F\alpha} = \left(\frac{\partial F_y}{\partial \alpha} \right)_{\alpha=0} = 2 c_{py} a^2$$

self-aligning moment:

$$M_z = \int_{-a}^{a} q_y x dx = c_{py} \int_{-a}^{a} v x dx = c_{py} \int_{-a}^{a} (a - x) \tan(\alpha) x dx$$

$$M_z = -\frac{2}{3} c_{py} a^3 \alpha$$

so the self aligning stiffness becomes:

$$C_{M\alpha} = -\left(\frac{\partial M_z}{\partial \alpha} \right)_{\alpha=0} = \frac{2}{3} c_{py} a^3$$

<u>non-linear characteristics</u>

transition from adhesion to sliding at $x = x_t$ where
$q_y = q_{y,\max}$

$$\left|q_y\right| = c_{py}(a - x_t)\left|\tan\alpha\right| \qquad \text{adhesion}$$

$$\left|q_{y,\max}\right| = \mu \frac{3F_z}{4a}\left\{1 - \left(\frac{x_t}{a}\right)^2\right\} \qquad \text{sliding}$$

then we obtain for the transition point:

$$x_t = \frac{4c_{py}a^3\left|\tan\alpha\right|}{3\mu F_z} - a$$

the relation between x_t and λ

$$\lambda = \frac{1}{2}\left(1 - \frac{x_t}{a}\right)$$

so for the adhesion parameter λ:

$$\lambda = \frac{a - x_t}{2a} = 1 - \frac{2c_{py}a^2\left|\tan\alpha\right|}{3\mu F_z} = 1 - \theta_y\left|\tan\alpha\right|$$

where:

$$\theta_y = \frac{2c_{py}a^2}{3\mu F_z}$$

full sliding: $\lambda = 0$

corresponding side slip angle:

$$\tan\alpha_{sl} = \frac{1}{\theta_y}$$

if $\left|\alpha\right| < \alpha_{sl}$ the following equations hold:

lateral force:

$$F_y = \mu \frac{3F_z}{4a}\int_{-a}^{x_t}\left\{1 - \left(\frac{x}{a}\right)^2\right\}dx + c_{py}\left|\tan\alpha\right|\int_{x_t}^{a}(a - x)dx$$

self-aligning moment:

$$M_z = \mu \frac{3F_z}{4a}\int_{-a}^{x_t}\left\{1 - \left(\frac{x}{a}\right)^2\right\}xdx + c_{py}\left|\tan\alpha\right|\int_{x_t}^{a}(a - x)xdx$$

if $\left|\alpha\right| > \alpha_{sl}$ then

lateral force: $F_y = \mu Fz$

self-aligning moment: $M_z = 0$

solving the integrals:

lateral force:

$$F_y = \mu F_z (1-\lambda^3)\,\mathrm{sgn}\,\alpha$$

or

$$F_y = 3\mu F_z \theta_y \sigma_y \left\{1 - |\theta_y \sigma_y| + \frac{1}{3}(\theta_y \sigma_y)^2\right\}$$

where $\sigma_y = \tan\alpha$

self aligning moment:

$$M_z = -\mu F_z \lambda^3 a (1-\lambda)\,\mathrm{sgn}\,\alpha$$

or

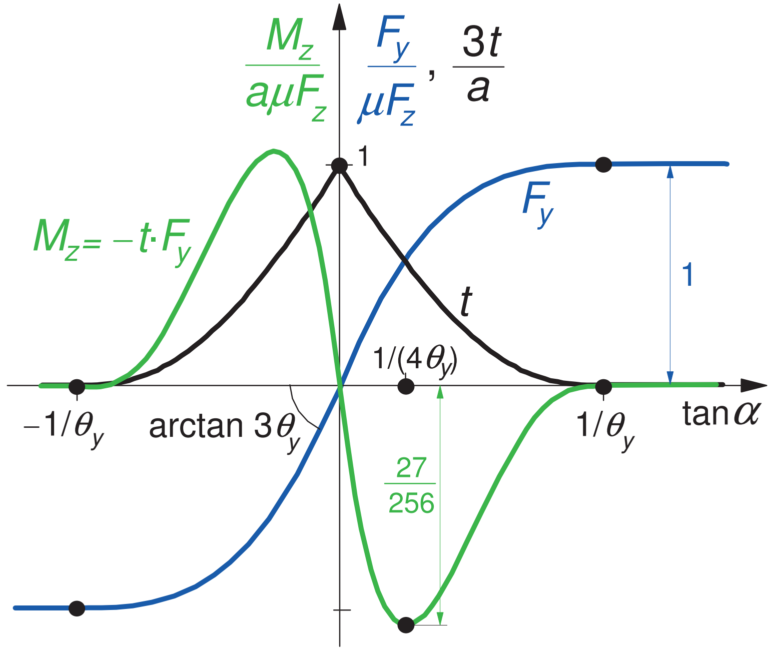
$$M_z = -\mu F_z a \theta_y \sigma_y \left(1 - 3|\theta_y \sigma_y| + 3(\theta_y \sigma_y)^2 - |\theta_y \sigma_y|^3\right)$$

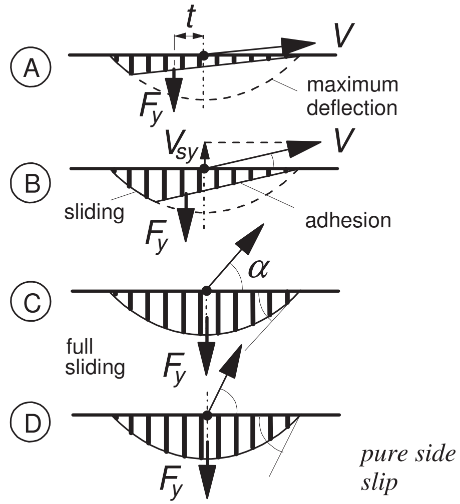
pneumatic trail:

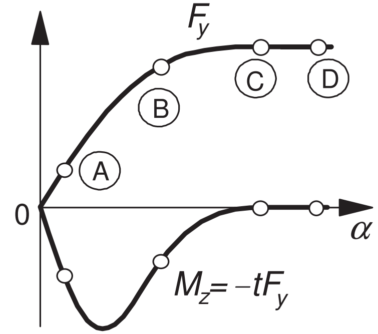
$$t = -\frac{M_z}{F_y}$$

$$t = \frac{1}{3} a \left(\frac{1 - 3|\theta_y \sigma_y| + 3(\theta_y \sigma_y)^2 - |\theta_y \sigma_y|^3}{1 - |\theta_y \sigma_y| + \frac{1}{3}(\theta_y \sigma_y)^2}\right)$$

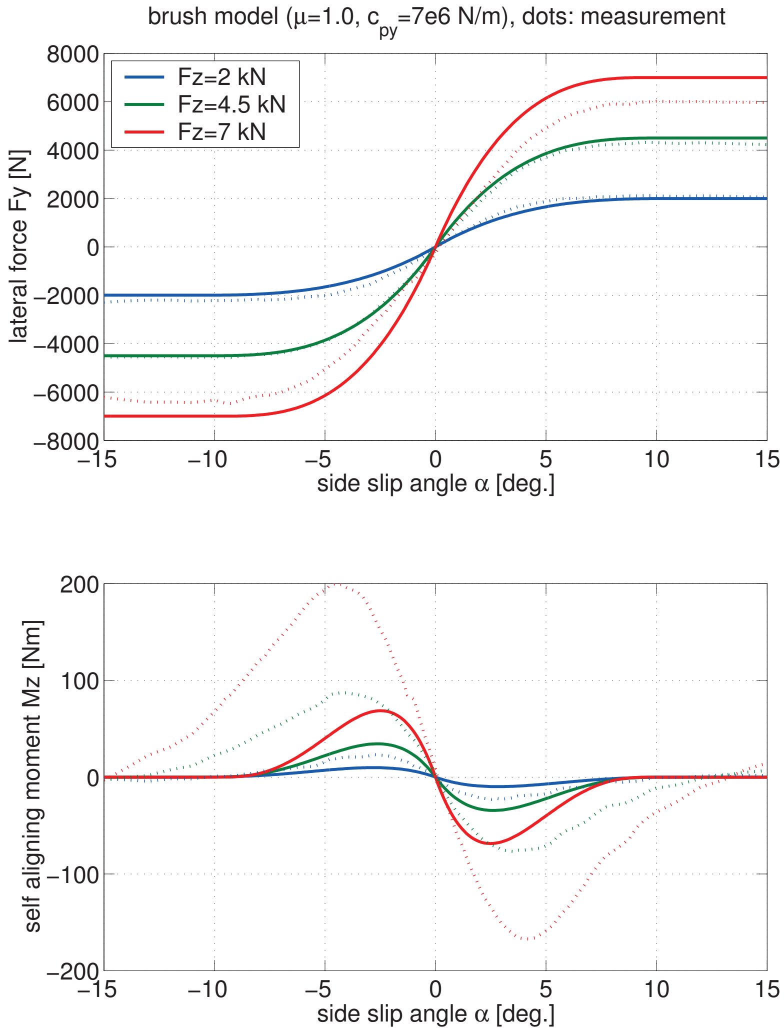
steady-state characteristics brush model:





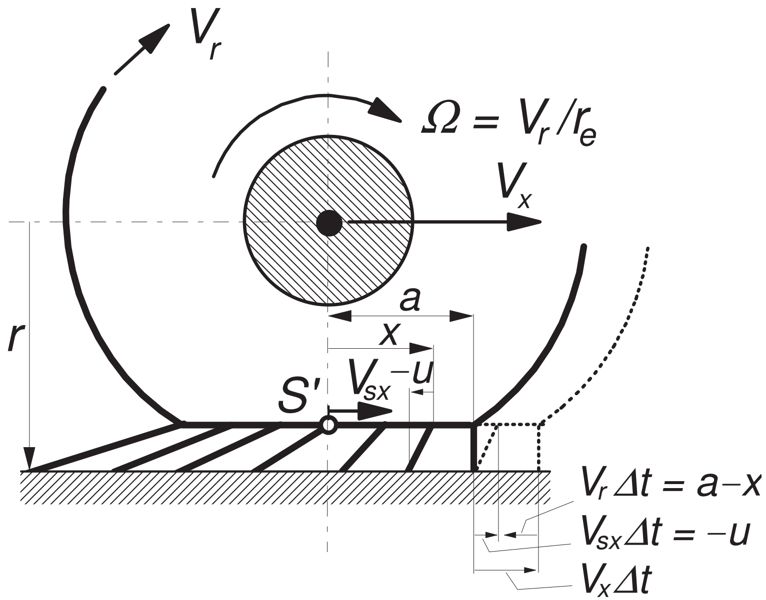


comparison with experiments… (page 98/100)



note: contact length calculation using page 126

Pure longitudinal slip



tread band has a velocity V_r with respect to the wheel centre (the rolling speed).

- by definition: $V_r = \Omega r_e$
- a bristle moves backwards through the contact region with velocity V_r

absolute sliding velocity in contact patch

$V_{sx} = V_x - V_r$

consider a time increment Δt:

$u = -V_{sx}\Delta t$ and $a - x = V_r \Delta t$

after elimination of Δt:

$$u = -(a-x)\frac{V_{sx}}{V_r} = -(a-x)\frac{V_{sx}}{V_x - V_{sx}} = (a-x)\frac{\kappa}{1+\kappa}$$

we may introduce a “theoretical slip”

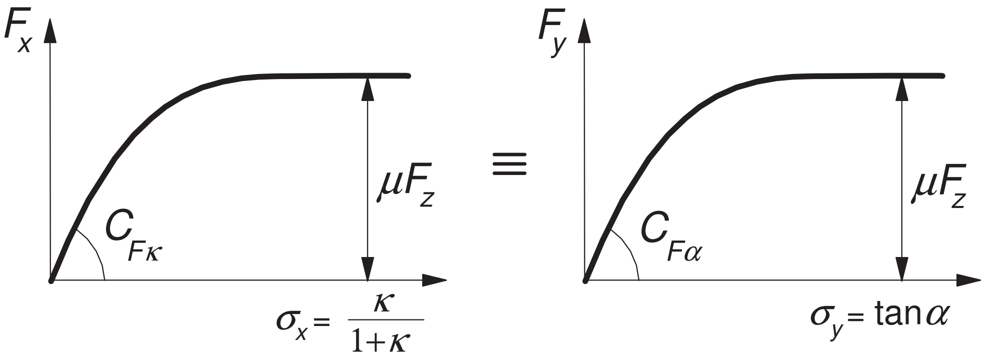
$$\sigma_x = -\frac{V_{sx}}{V_r} = \frac{\kappa}{1+\kappa}$$

then: $u = (a-x)\sigma_x$

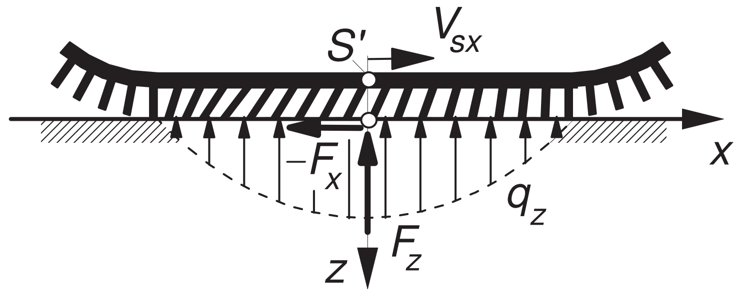
in lateral direction we had:

$v = (a-x)\tan\alpha = (a-x)\sigma_y$

if we assume equal bristle stiffness $\left(c_{py} = c_{px}\right)$ and friction coefficient $\left(\mu_y = \mu_x\right)$ then $F_y = f(\sigma_y)$ will be identical to $F_x = f(\sigma_x)$!



bristle deflection during braking:



using results obtained for the lateral force calculation:

linear characteristics:

$F_x = 2c_{px}a^2\kappa$

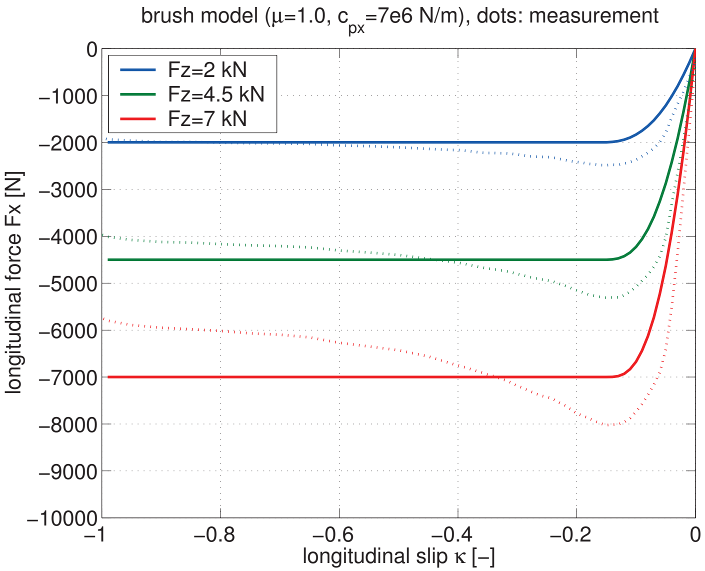
longitudinal slip stiffness:

$$C_{f\kappa} = \left(\frac{\partial F_x}{\partial \kappa}\right)_{\kappa=0} = 2c_{px}a^2$$

full sliding:

$$\kappa_{sl} = \frac{-1}{1 \pm \theta_x} \qquad \text{where: } \theta_x = \frac{2c_{px}a^2}{3\mu F_z}$$

comparison with experiments… (page 108)



note:
- no difference between peak and locked wheel friction coefficient, force remains constant
- longitudinal slip stiffness $C_{f\kappa}$ too low

Combined slip

simplified analysis:
- equal bristle stiffness $\left(c_p = c_{py} = c_{px}\right)$
- equal friction coefficient $\left(\mu = \mu_y = \mu_x\right)$

tip deflections of the bristle:

$$u = -(a-x)\frac{V_{sx}}{V_r} = (a-x)\sigma_x \qquad \text{(longitudinal)}$$

$$v = -(a-x)\frac{V_{sy}}{V_r} = (a-x)\sigma_y \qquad \text{(lateral)}$$

with:

$$\kappa = -\frac{V_{sx}}{V_x}\,, \quad \tan\alpha = -\frac{V_{sy}}{V_x} \quad \text{and } V_x = V_r + V_{sx}$$

we get:

$$\sigma_x = \frac{\kappa}{1+\kappa} \quad \text{and} \quad \sigma_y = \frac{\tan\alpha}{1+\kappa}$$

the point of transition from adhesion to sliding:

$$c_p\sqrt{u^2+v^2}=\mu q_z$$

or:

$$c_p(a-x_t)\sqrt{\sigma_x^2+\sigma_y^2}=\mu\frac{3F_z}{4a}\left\{1-\left(\frac{x_t}{a}\right)^2\right\}$$

introduce $\sigma=\sqrt{\sigma_x^2+\sigma_y^2}$ and find x_t:

$$x_t=\frac{4c_pa^3\sigma}{3\mu F_z}-a$$

or using the adhesion parameter:

$$\lambda=\frac{a-x_t}{2a}=1-\theta\sigma$$

where:

$$\theta=\frac{2c_pa^2}{3\mu F_z}$$

total sliding starts at $\sigma_{sl}=\dfrac{1}{\theta}$

using similarity (again…) with pure lateral force calculation and replacing θ_y by θ and σ_y by σ the magnitude of the force becomes:

when $\sigma<\sigma_{sl}$

$F=\mu F_z(1-\lambda^3)$ or $F=\mu F_z\theta\sigma\{3-3|\theta\sigma|+(\theta\sigma)^2\}$

when $\sigma>\sigma_{sl}$

$F=\mu F_z$

the individual components of the force:

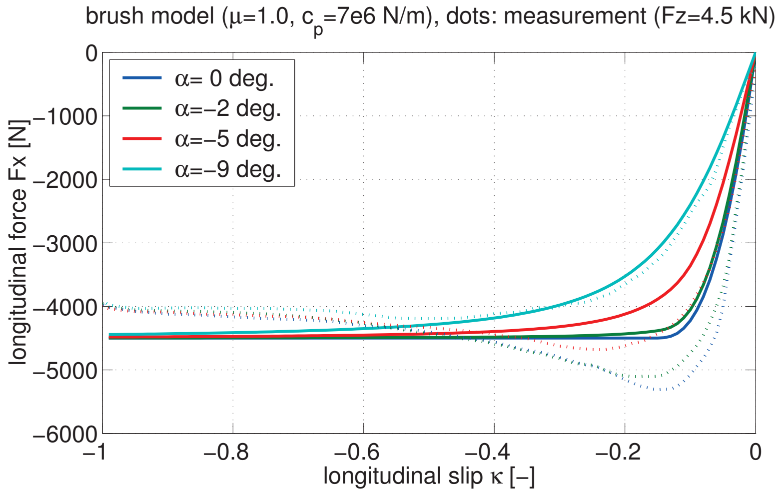
$F_x=F\dfrac{\sigma_x}{\sigma}$ and $F_y=F\dfrac{\sigma_y}{\sigma}$

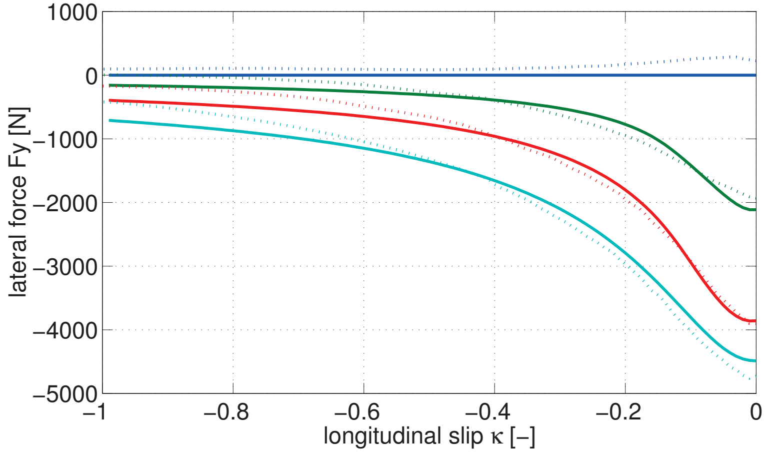
note: this is only valid due to the assumption that the bristle stiffness and friction coefficient are the same in fore/aft and lateral direction: the resulting force will be opposite to the local tread element deflection.

The self-aligning moment can be calculated using the expression for the pneumatic trail and again replacing θ_y by θ and σ_y by σ.

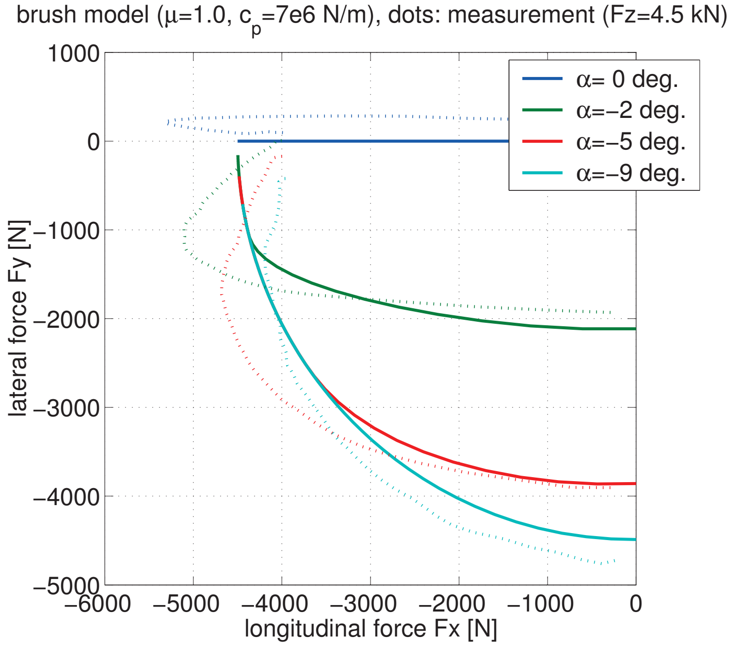
$$M_z=-t(\sigma)\cdot F_y$$

comparison with experiments… (page 111)





comparison with experiments… (page 113)



Limitations/enhancements

some limitations of the brush model:

- equal friction coefficient in longitudinal and lateral direction
- in the longitudinal direction peak and locked wheel friction coefficient identical
- longitudinal slip stiffness too low
- self-aligning moment too small
- dependency of characteristics on vertical load not correct
- effect of inclination angle?
- **...**

“base” brush model can be useful:

- for a qualitative analysis of the tyre behaviour
- when very limited measurement data is available

possible enhancements:

- separate stiffness of tread elements and carcass (introduction of a flexible carcass)
- increase the accuracy of the vertical pressure distribution
- velocity and pressure dependent friction law
- multiple, parallel rows of bristles
- …

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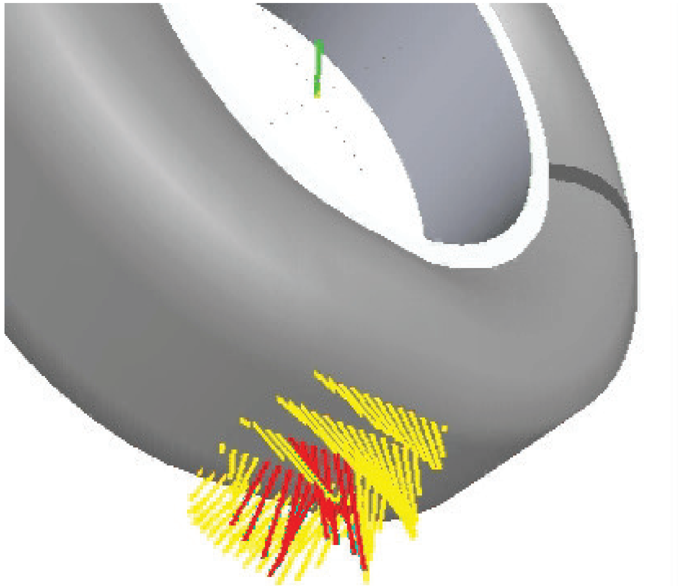
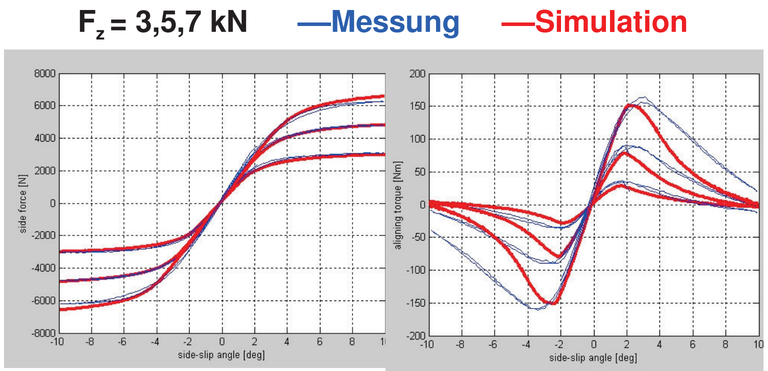
more advanced brush models
example: F-Tire (prof. Gipser, Germany)

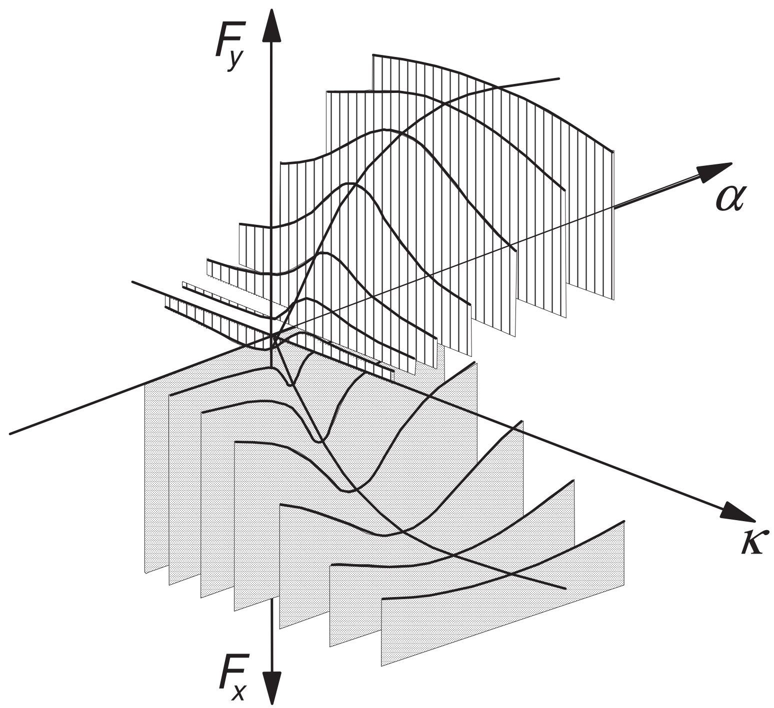




Book Pacejka
- chapter 2.5
- chapter 3.2 up to 3.2.4 (pages 90-112)

Next time…
- the Magic Formula tyre model

6. The Magic Formula
- Magic Formula tyre model
  –pure longitudinal slip
  –pure lateral slip
  –combined slip
- practicalities
- rolling resistance



Tyre models in full vehicle simulations

some notes:

- emphasis on an accurate representation of the forces generated by the tyre; details on the contact patch may be less important (this statement will be true in particular for level or smooth road surfaces)

- tyre model should be fast:
  -tractor semi-trailer, road train: > 10 tyres
  -real-time applications: driving simulator, HIL

- continuously varying inputs $\kappa, \alpha, \gamma, F_z$

- model should be robust for extreme inputs

"inputs:"

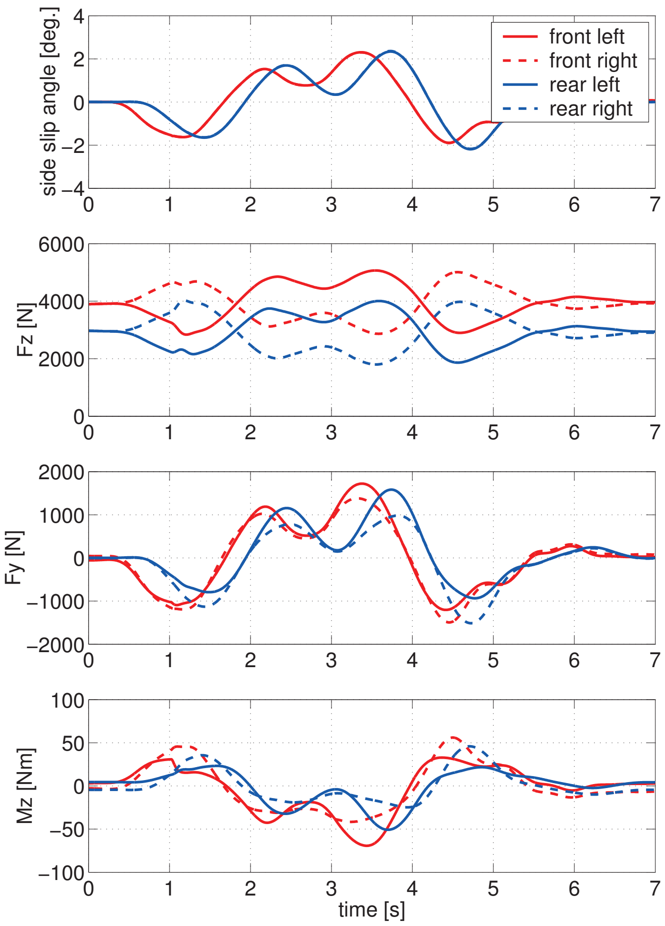
longitudinal slip κ
side slip angle α
inclination angle γ
vertical load F_z

→ Magic Formula →

"outputs:"

F_x longitudinal force
F_y lateral force
M_z self aligning moment

continuously varying conditions…
(lane change simulation)

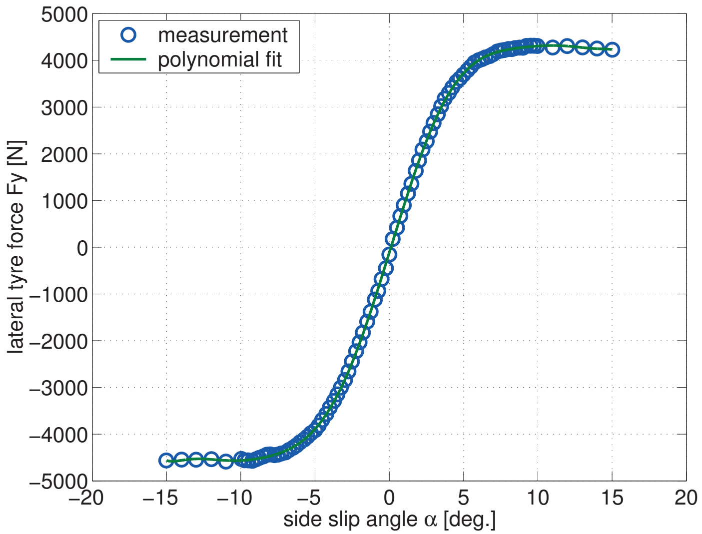


(semi-)empirical tyre models

mathematical model to represent measurements
- smoothing: measurements may be noisy
- data reduction/compression: less storage space required

(bad) example: ”polynomial” tyre model:

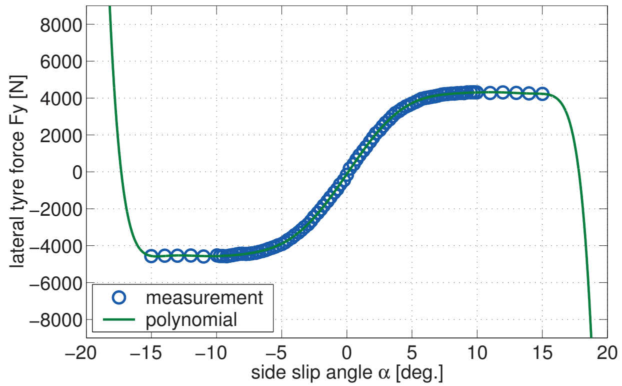
$$F_y = \sum_{i=1}^{n} c_i \alpha^i$$

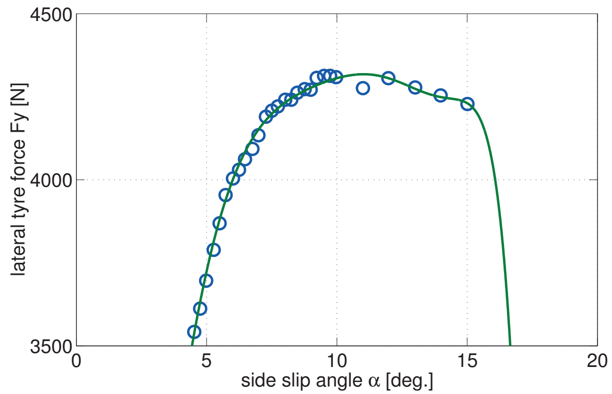


measurement data: 91 data points α, F_y

polynomial: 11 coefficients $(n = 11)$

extrapolation properties… ($|\alpha| > 15$ deg.)





outside measurement range:
opposite signs side slip angle and lateral force: unrealistic, simulation may fail (unstable)!!!

Magic Formula basics

a different approach…

<u>notion:</u>
the base tyre characteristics $F_x = f(\kappa)$, $F_y = f(\alpha)$ and $M_z = f(\alpha)$ have a sinusoidal shape, with a "stretched" horizontal axis for large values of slip

this consideration is the basis for a tyre model known as the "Magic Formula"

some notes:

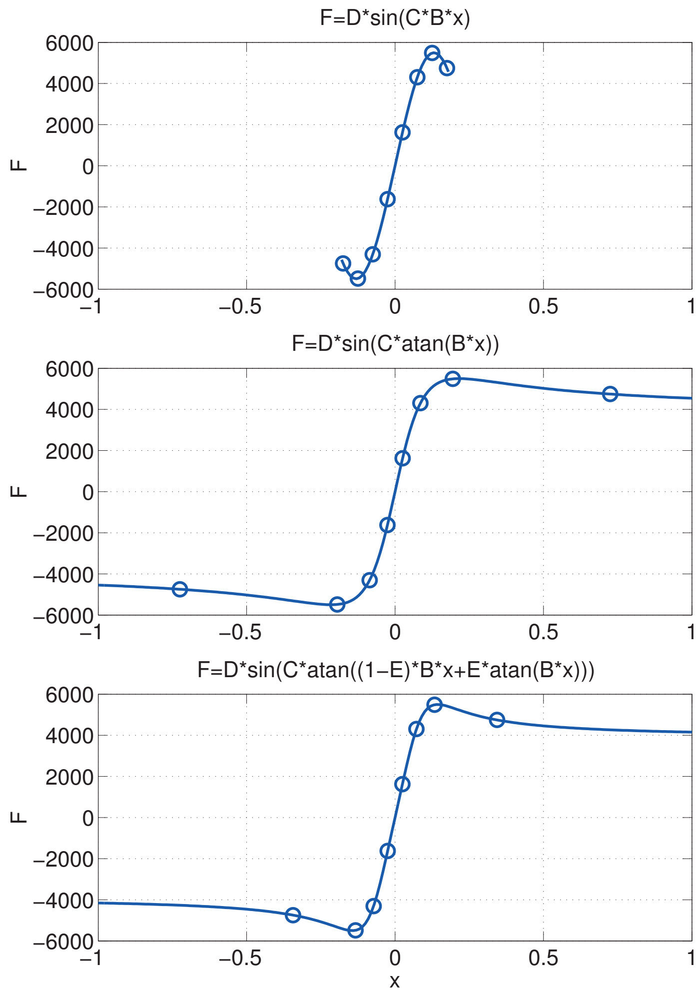
- first versions developed by Egbert Bakker (PD&E, Helmond) and professor Pacejka (TU Delft)

- probably the most popular tyre model for vehicle handling simulations (worldwide!)

base Magic Formula:

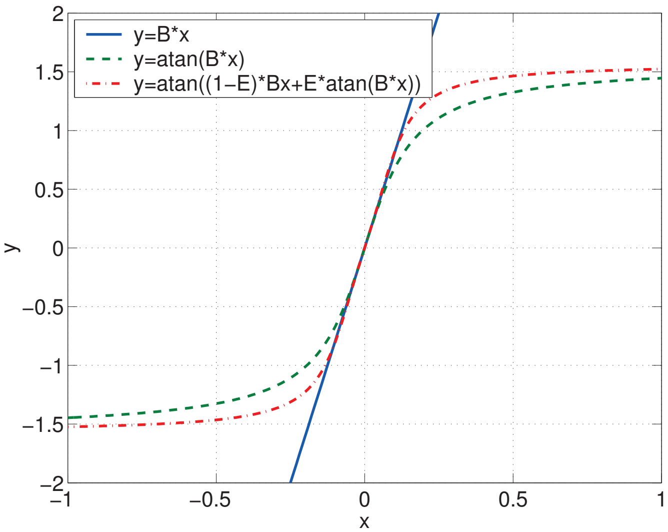
$$F = D\sin(C\arctan((1-E)Bx + E\arctan(Bx)))$$

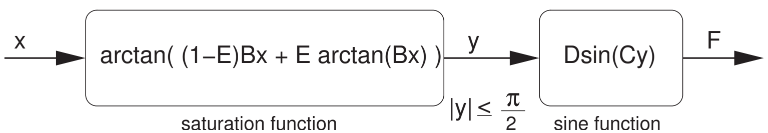
stretching the sine…



parameters in this example:
$B = 8, C = 1.5, D = 5500, E = -2$

arctan-function results in saturation of the input to the sine function





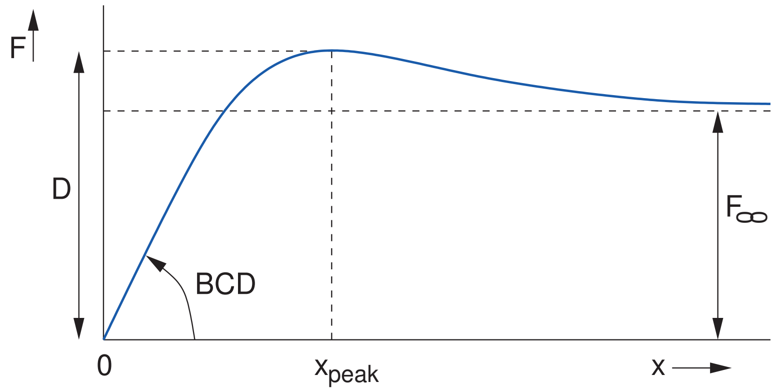
Magic Formula coefficients:

- D determines the peak value
- C determines the limit value when $x \to \infty$

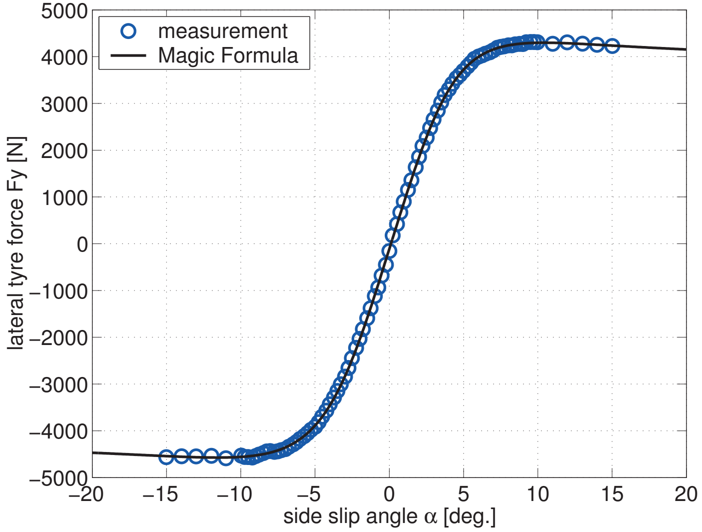
$$C = 2 - \frac{2}{\pi}\arcsin\left(\frac{F_\infty}{D}\right) \qquad \text{note: } C \geq 1$$

- BCD determines the slope near the origin
- B, E & C determine the location of the peak

$$E = \frac{Bx_{peak} - \tan(\pi/2C)}{Bx_{peak} - \arctan(Bx_{peak})} \qquad \text{note: } E \leq 1$$



curve fitting using non-linear, constrained optimisation techniques (iterative process)



to account for offsets and asymmetry:

- introduce $\Delta\alpha$, ΔF and different E for positive and negative side slip angles

$$F_y = f_{MF}(\alpha + \Delta\alpha) + \Delta F$$

fit: $B = 10.1068$ rad^{-1}
$C = 1.3056$
$D = 4434.9$ N
$E_{\alpha,pos} = -0.9947 \quad E_{\alpha,neg} = -0.6086$
$\Delta\alpha = 0.004$ rad (=0.2317 deg)
$\Delta F = -137.01$ N

longitudinal characteristics (straight line braking)

note: curves are dependent on vertical load F_z!

introduce dimensionless load increment df_z

$$df_z = \frac{F_z - F_{zn}}{F_{zn}}$$

where F_{zn} is the nominal (rated) load of the tyre

$$F_{x0} = D_x \sin(C_x \arctan((1-E_x)B_x\kappa_x + E_x \arctan(B_x\kappa_x)))$$

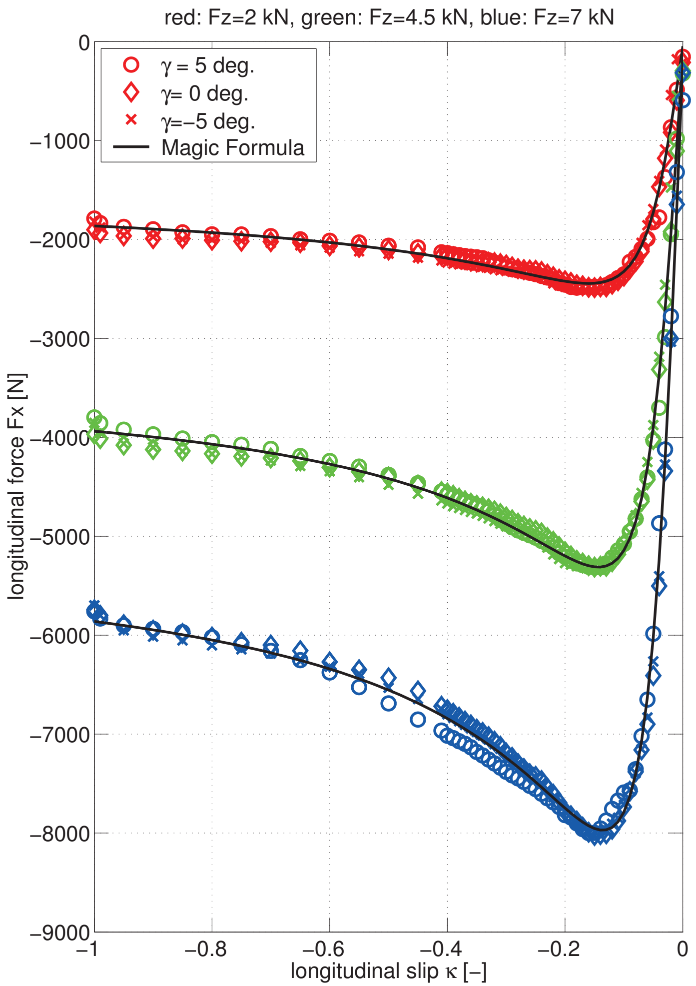
where:

$$\kappa_x = \kappa + S_{Hx} = \kappa + p_{Hx1} + p_{Hx2}df_z$$
$$D_x = F_z\mu_x\lambda_{\mu x} = F_z(p_{Dx1} + p_{Dx2}df_z)\lambda_{\mu x}$$
$$C_x = p_{Cx1}$$
$$E_x = p_{Ex1} + p_{Ex2}df_z + p_{Ex3}df_z^2$$
$$K_x = B_xC_xD_x = F_z(p_{Kx1} + p_{Kx2}df_z)\lambda_{Kx}$$

note:

- $K_x = C_{f\kappa}$: longitudinal slip stiffness
- B_x is calculated from C_x, D_x and K_x
- equations somewhat simplified w.r.t. book Pacejka for educational reasons…

result after fitting coefficients:



data reduction:

495 measurement points ⇒ 11coefficients

numerical values:

$F_{zn} = 4000$ N, $p_{Cx1} = 1.5676$

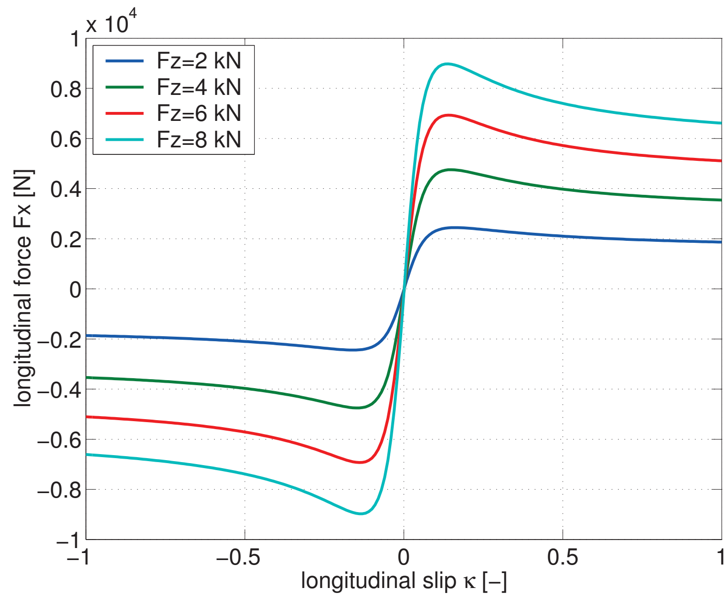
$p_{Dx1} = 1.1887$, $p_{Dx2} = -0.0665$

$p_{Ex1} = 0.1620$, $p_{Ex2} = -0.1578$, $p_{Ex1} = 0.1532$

$p_{Hx1} = -0.0153$, $p_{Hx2} = -0.0024$

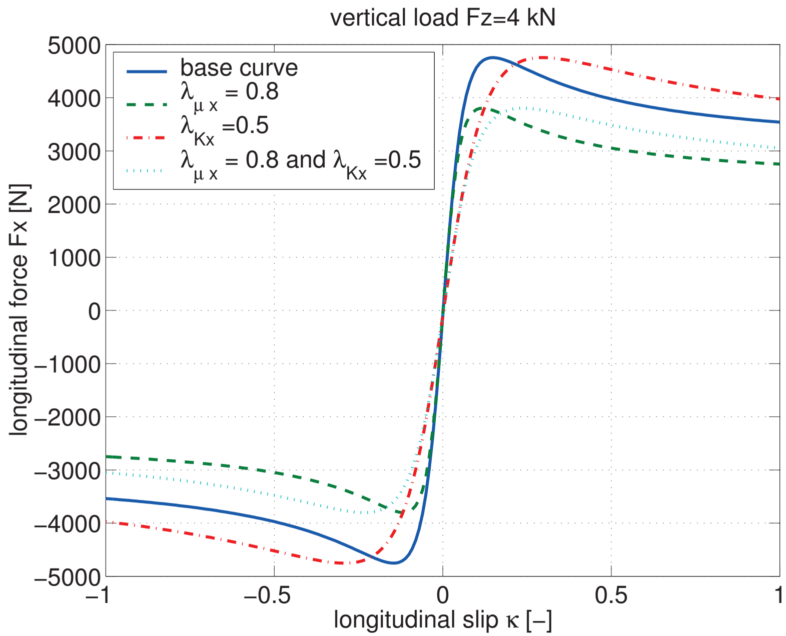
$p_{Kx1} = 21.13$, $p_{Kx2} = 0.3673$

interpolation/extrapolation (load, braking/driving)



scaling coefficients $\lambda_{Kx}, \lambda_{\mu x}$:

- value equals 1 during fitting process
- may be used to adjust tyre characteristics (e.g. tuning of a full simulation model, correction for different road types, etc.)



lateral characteristics (pure cornering)

$$F_{y0} = f_{MF}(\alpha, \gamma, F_z)$$

again the Magic Formula can be used:

$$F_{y0} = D_y \sin(C_y \arctan((1-E_y)B_y\alpha_y + E_y \arctan(B_y\alpha_y))) + S_{Vy}$$

where:

$$\alpha_y = \alpha + S_{Hy}$$

$$C_y = p_{cy1}$$

$$D_y = F_z\left(p_{Dy1} + p_{Dy2}df_z\right)\left(1 - p_{Dy3}\gamma^2\right)\lambda_{\mu y}$$

$$E_y = \left(p_{Ey1} + p_{Ey2}df_z\right)\left\{1 - \left(p_{Ey3} + p_{Ey4}\gamma\right)sign(\alpha_y)\right\}\lambda_{Ey}$$

$$K_y = p_{Ky1}F_{zn}\sin\left(2\arctan\left(\frac{Fz}{p_{Ky2}F_{zn}}\right)\right)\left(1 - p_{Ky3}\gamma^2\right)$$

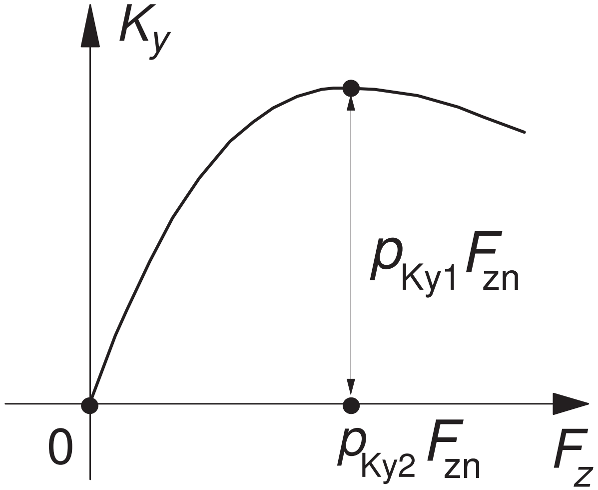
$$B_y = \frac{K_y}{C_y D_y}$$

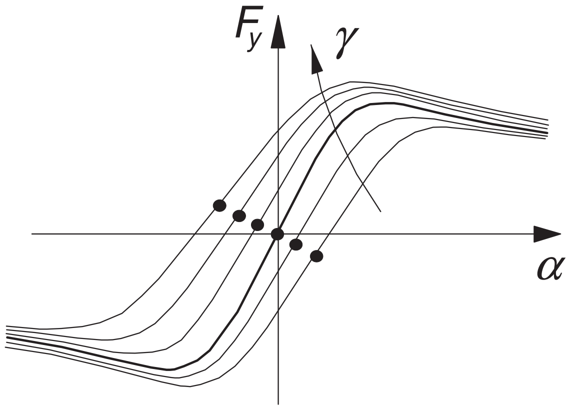
$$S_{Hy} = \left(p_{Hy1} + p_{Hy2}df_z\right)\lambda_{Hy} + p_{Hy3}\gamma\lambda_{K y\gamma}$$

$$S_{Vy} = F_z\left(p_{Vy1} + p_{Vy2}df_z\right)\lambda_{Vy} + \left(p_{Vy3} + p_{Vy4}df_z\right)\gamma\lambda_{K y\gamma}$$

note: $K_y = C_{f\alpha}$: cornering stiffness formula for the cornering stiffness:

$$K_y = p_{Ky1} F_{zn} \sin\left(2\arctan\left(\frac{Fz}{p_{Ky2} F_{zn}}\right)\right)\left(1 - p_{Ky3}\gamma^2\right)$$



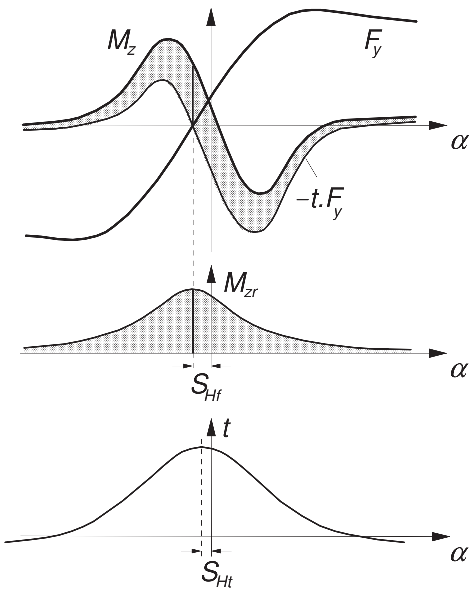


<u>self-aligning moment</u> M_z

in early versions of the Magic Formula the self-aligning moment was fitted using a similar approach as the F_y characteristic.

more recently this was changed in the concept of using a pneumatic trail and residual moment (easier to handle combined slip)

$$M_z = -F_{y0} \cdot t_0 + M_{zr0}$$

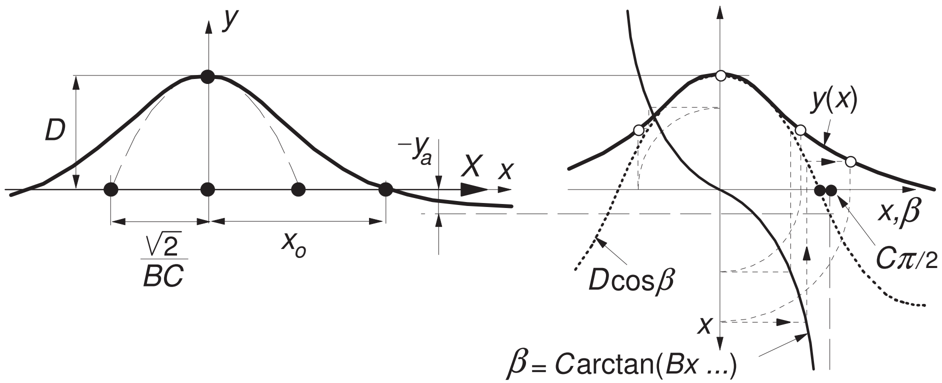


for the description of the pneumatic trail and residual moment a <u>cosine</u> type of Magic Formula is used

$$t_0 = D_t \cos(C_t \arctan((1-E_t)B_t\alpha_t + E_t \arctan(B_t\alpha_t)))$$

and

$$M_{zr0} = D_r \cos(C_r \arctan(B_r\alpha_r))$$



<u>combined slip</u>

the reduction of longitudinal forces and lateral forces for combined slip conditions is taken into account by applying a weighting function to the "pure" characteristics

in addition a "braking induced plysteer force" $S_{Vy\kappa}$ is taken into account for the lateral force

$$F_x = G_{x\alpha} \cdot F_{x0}$$
$$F_y = G_{y\kappa} \cdot F_{y0} + S_{Vy\kappa}$$

weighting functions $G_{x\alpha}, G_{y\kappa}$ may reach values between 0 and slightly over 1

this method was developed by Bayle (Michelin)

load dependency on $G_{x\alpha}, G_{y\kappa}$ was introduced by Van Oosten (TNO) weighting function for F_x:

$$G_{x\alpha} = \frac{\cos(C_{x\alpha}\arctan((1-E_{x\alpha})B_{x\alpha}\alpha_s + E_{x\alpha}\arctan(B_{x\alpha}\alpha_s)))}{\cos(C_{x\alpha}\arctan((1-E_{x\alpha})B_{x\alpha}S_{Hx\alpha} + E_{x\alpha}\arctan(B_{x\alpha}S_{Hx\alpha})))}$$

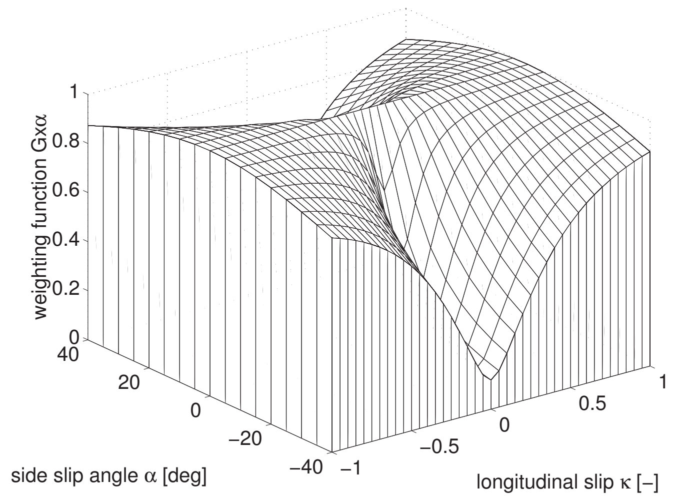
were:

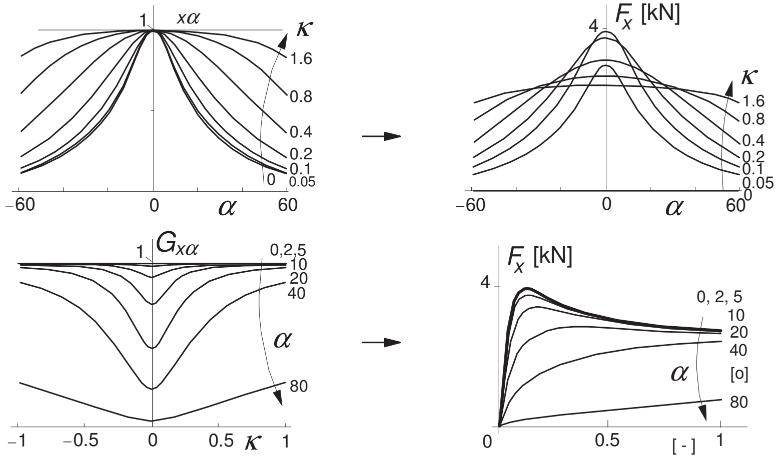
$$\alpha_s = \alpha + S_{Hx\alpha}$$
$$B_{x\alpha} = r_{Bx1}\cos(\arctan(r_{Bx2}\kappa))$$
$$C_{x\alpha} = r_{Cx1}$$
$$E_{x\alpha} = r_{Ex1} + r_{Ex2}df_z$$
$$S_{Hx\alpha} = r_{Hx1}$$

notes:

- $\alpha = 0 \quad \Rightarrow \quad G_{x\alpha} = 1$
- $G_{x\alpha}$ as a function of α has the cosine-Magic Formula shape
- $B_{x\alpha}$ is a function of κ and alters the base cosine-Magic Formula shape
  $\kappa \to \infty \quad \Rightarrow \quad B_{x\alpha} = 0 \quad \Rightarrow \quad G_{x\alpha} = 1$
- $E_{x\alpha}$ is sometimes ignored or may be considered almost constant (dependency on F_z only for special tyres)

$$\underbrace{F_x(\kappa,\alpha,F_z)}_{combined} = \underbrace{G_{x\alpha}(\kappa,\alpha)}_{weighting} \cdot \underbrace{F_{x0}(\kappa,F_z)}_{pure}$$





weighting function for F_y:

$$G_{y\kappa} = \frac{\cos\left(C_{y\kappa}\arctan\left(\left(1-E_{y\kappa}\right)B_{y\kappa}\kappa_s + E_{y\kappa}\arctan\left(B_{y\kappa}\kappa_s\right)\right)\right)}{\cos\left(C_{y\kappa}\arctan\left(\left(1-E_{y\kappa}\right)B_{y\kappa}S_{Hy\kappa} + E_{y\kappa}\arctan\left(B_{y\kappa}S_{Hy\kappa}\right)\right)\right)}$$

were:

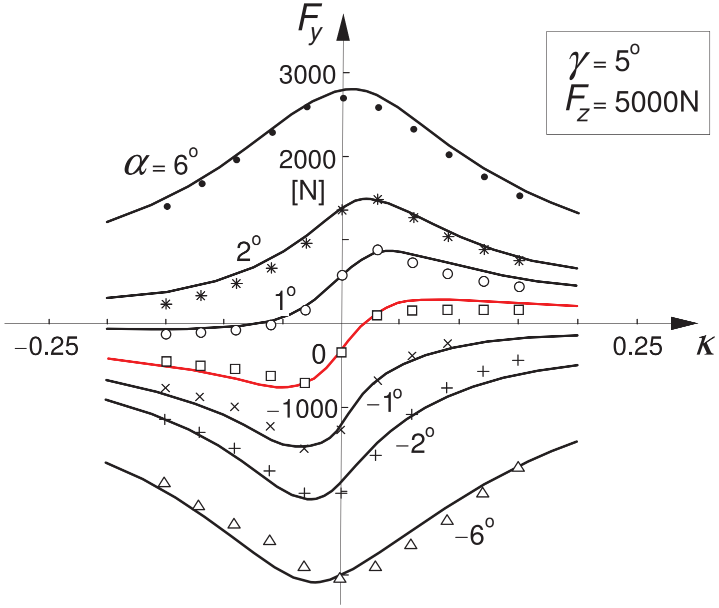
$$\kappa_s = \kappa + S_{Hy\kappa}$$
$$B_{y\kappa} = r_{By1}\cos\left(\arctan\left(r_{By2}\left(\alpha - r_{By3}\right)\right)\right)$$
$$C_{y\kappa} = r_{Cy1}$$
$$E_{y\kappa} = r_{Ey1} + r_{Ey2}df_z$$
$$S_{Hy\kappa} = r_{Hy1} + r_{Hy2}df_z$$

notes:

- structure $G_{y\kappa}$ is very similar to $G_{x\alpha}$ when replacing $\alpha \to \kappa$ and $\kappa \to \alpha$
- weighting function independent of γ
- again F_z dependency may be disregarded

braking induced plysteer

- clearly visible for small side slip angles
- cannot be handled by weighting function



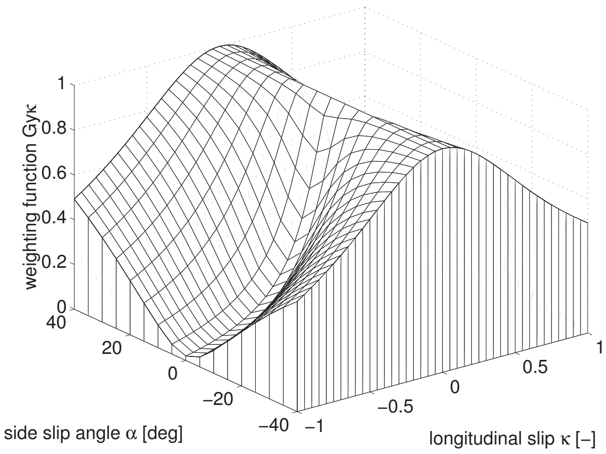
sine Magic Formula…

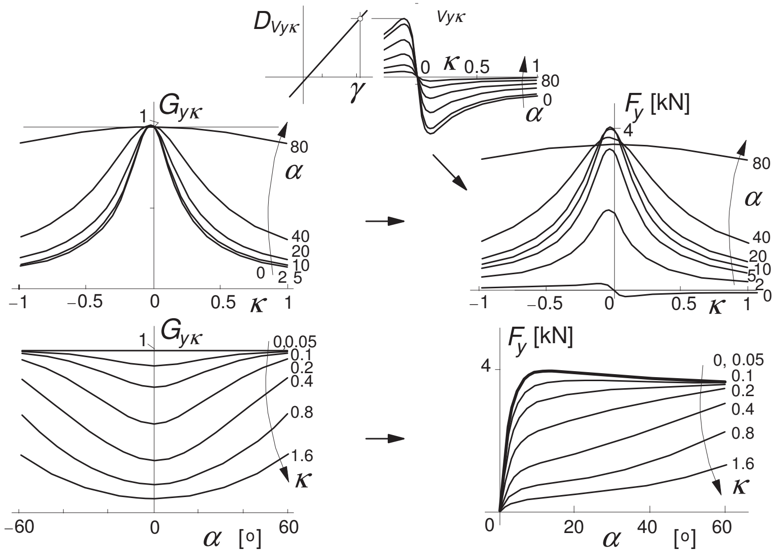
$$S_{Vy\kappa} = D_{Vy\kappa}\sin\left(r_{Vy5}\arctan\left(r_{Vy6}\kappa\right)\right)$$

where

$$D_{Vy\kappa} = f(\gamma, F_z)\cdot\cos\left(\arctan\left(r_{Vy4}\alpha\right)\right)$$

$$\underbrace{F_y(\kappa,\alpha,\gamma,F_z)}_{combined} = \underbrace{G_{y\kappa}(\kappa,\alpha)}_{weighting} \cdot \underbrace{F_{y0}(\alpha,\gamma,F_z)}_{pure} + \underbrace{S_{vy\kappa}(\kappa,\alpha,\gamma,F_z)}_{braking\_induced\_plysteer}$$





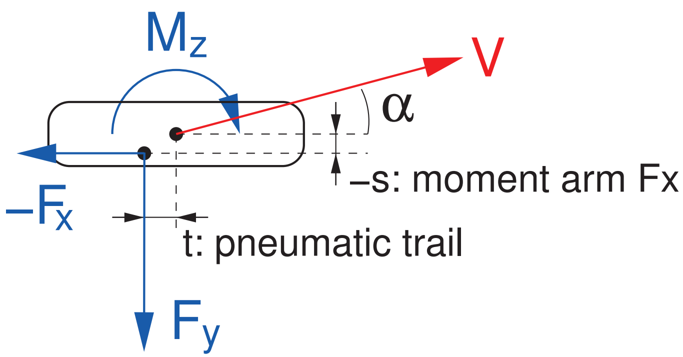
self-aligning moment under braking:

$$M_z = -t(\alpha_{t,eq}) \cdot F_y + M_{zr}(\alpha_{r,eq}) + s(F_y,\gamma) \cdot F_x$$

pneumatic trail t and residual moment M_{zr} are calculated using equivalent side slip angles, incorporating the effect of longitudinal slip

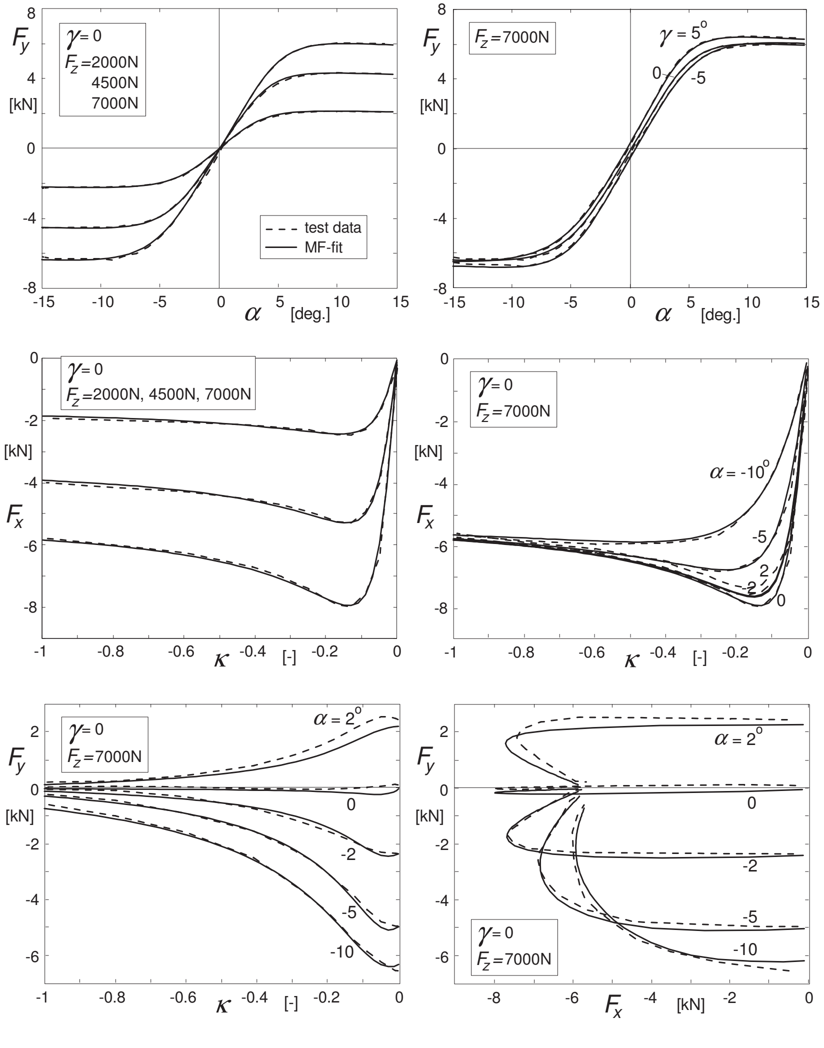
$$\alpha_{t,eq} = \sqrt{\alpha_t^2 + \left(\frac{K_x}{K_y}\right)^2 \kappa^2} \cdot \mathrm{sgn}(\alpha_t)$$

in addition we have a moment arm s for the longitudinal force to account of a lateral offset between the longitudinal force and contact center

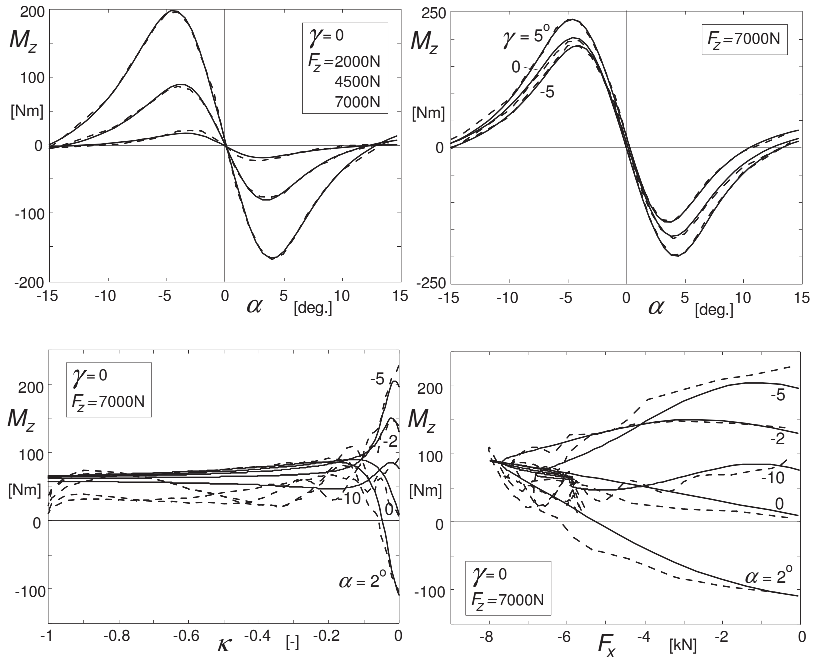


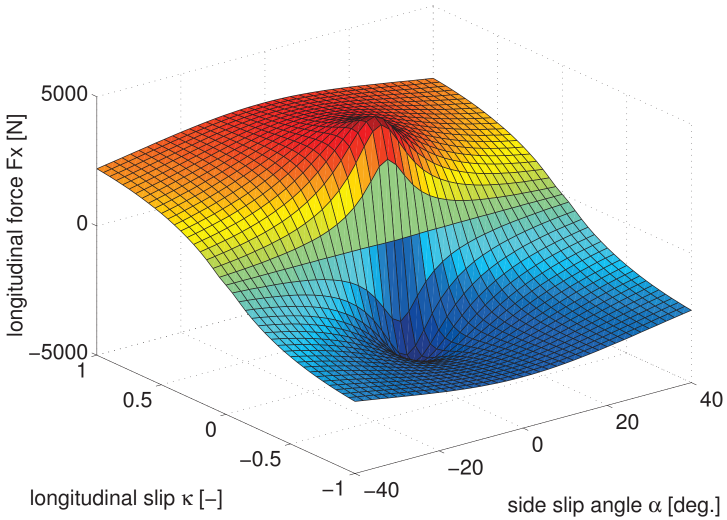
a lot of equations… but does it work?

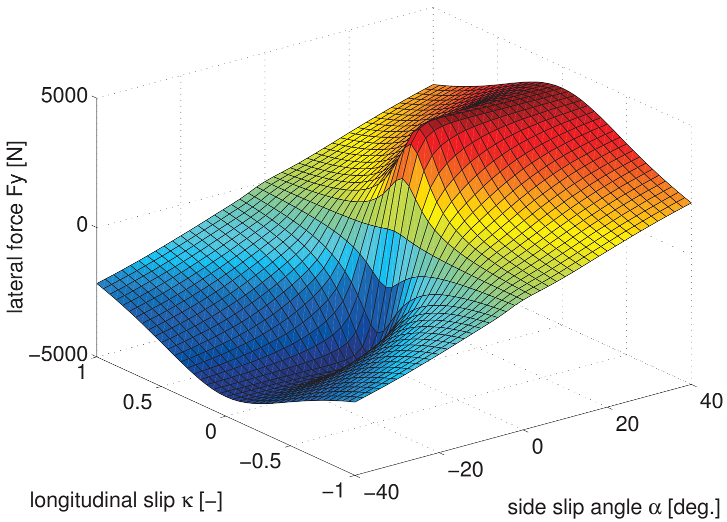
- longitudinal and lateral force



- self-aligning moment




5000
0
−5000
longitudinal force Fx [N]
1
0.5
0
−0.5
−1
longitudinal slip κ [–]
−40
−20
0
20
40
side slip angle α [deg.]


5000
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lateral force Fy [N]
1
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longitudinal slip κ [–]
−40
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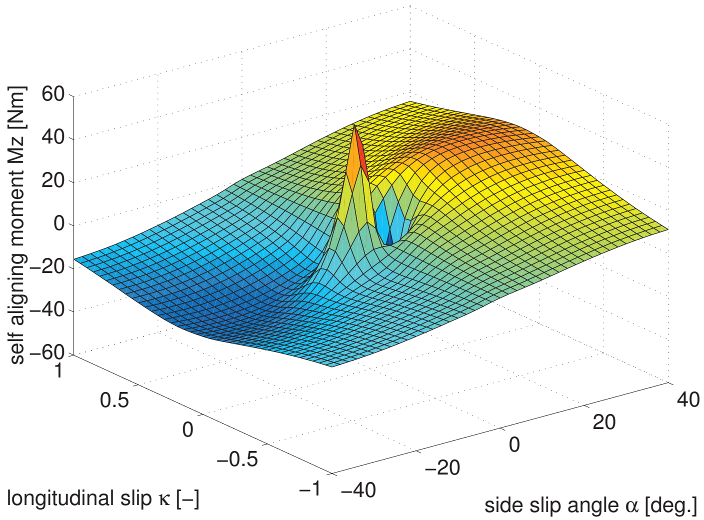
0
20
40
side slip angle α [deg.]


60
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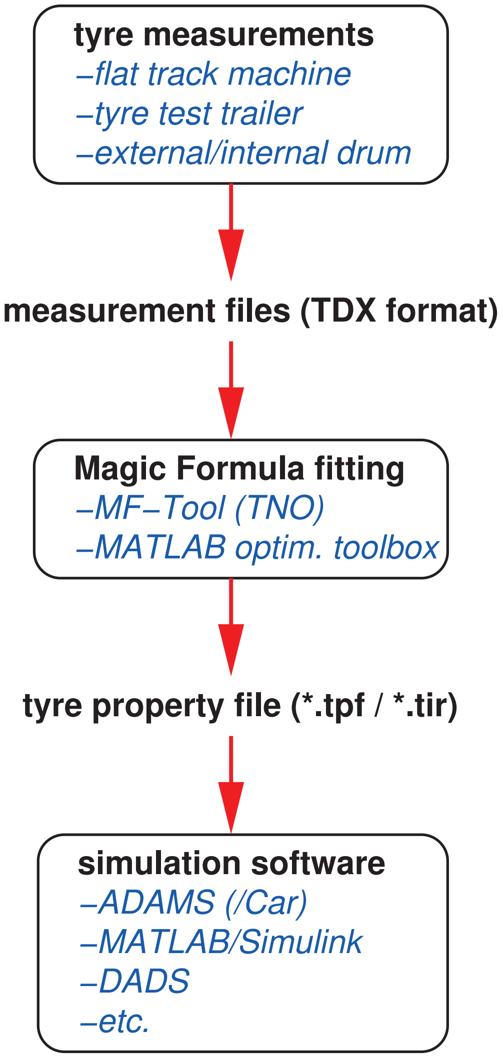
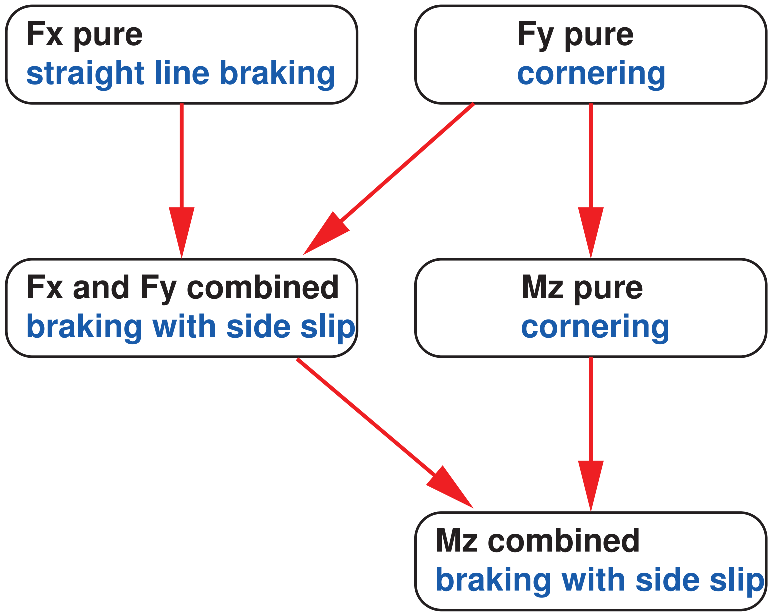
−20
−40
−60
self aligning moment Mz [Nm]
1
0.5
0
−0.5
−1
longitudinal slip κ [–]
−40
−20
0
20
40
side slip angle α [deg.]

Practical aspects
data flow



practical aspects: fitting

Magic Formula fitting
stepwise approach



additional notes:

- Pacejka has extended the Magic Formula to include turn slip and parking behaviour, see chapter 4.3.3

- Magic Formula describes steady-state tyre characteristics (valid up to 0.5 - 1 Hz). extensions are possible to include relaxation behaviour and/or dynamics up to 60 Hz. (will be discussed in Advanced Vehicle Dynamics)

Rolling resistance

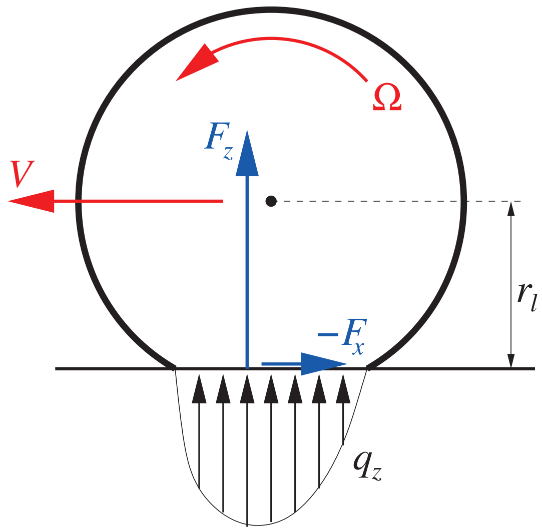
a tyre will deform in the contact zone…

in case of a rotating tyre:

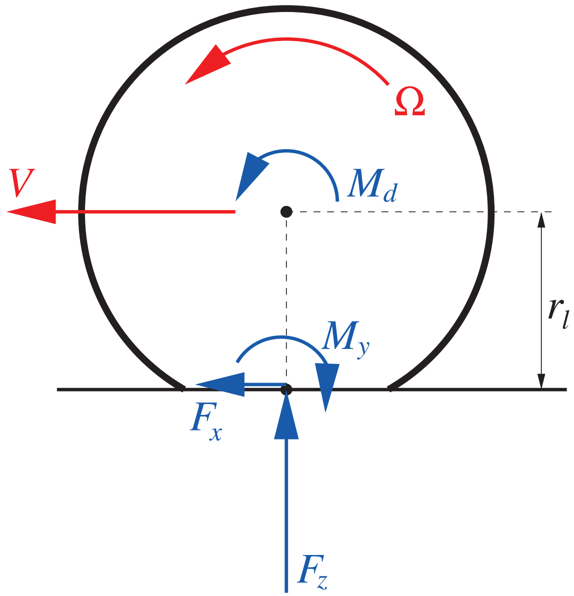
- material continuously moves through the contact zone and internal damping and hysteresis will result in energy dissipation

energy is required to maintain a constant forward velocity $\Rightarrow$ rolling resistance

for a rolling tyre the contact pressure distribution will be (slightly) asymmetric: the resulting vertical tyre force F_z will be ahead of the wheel centre



when translating the vertical force to the tyre contact centre a moment will arise: the rolling resistance moment M_y



dynamics of the wheel:

$$I_p\dot{\Omega} = -F_x r_l - M_y + M_d$$

where:

F_x longitudinal force

M_d drive moment

(e.g. engine, negative in case of braking)

I_p wheel + tyre polar moment of inertia

r_l loaded radius

for a free rolling tyre ($M_d = 0$) at constant velocity ($\dot{\Omega} = 0$) we get:

$$F_x = -\frac{M_y}{r_l}$$

the longitudinal force F_x due to rolling resistance is usually expressed as a fraction of the vertical force F_z

$$F_{x,rr} = -f_r F_z$$

f_r is the <u>rolling resistance coefficient</u>

examples

- asphalt: f_r= 0.01-0.02
- grass: f_r= 0.05
- soft soil: f_r up to 0.5

note:

The correct way to introduce rolling resistance in a simulation model which includes rotating wheels is by means of a rolling resistance moment M_y (e.g. $M_y = f_r r_l F_z$) which is opposite to the angular velocity of the wheel!

Book Pacejka

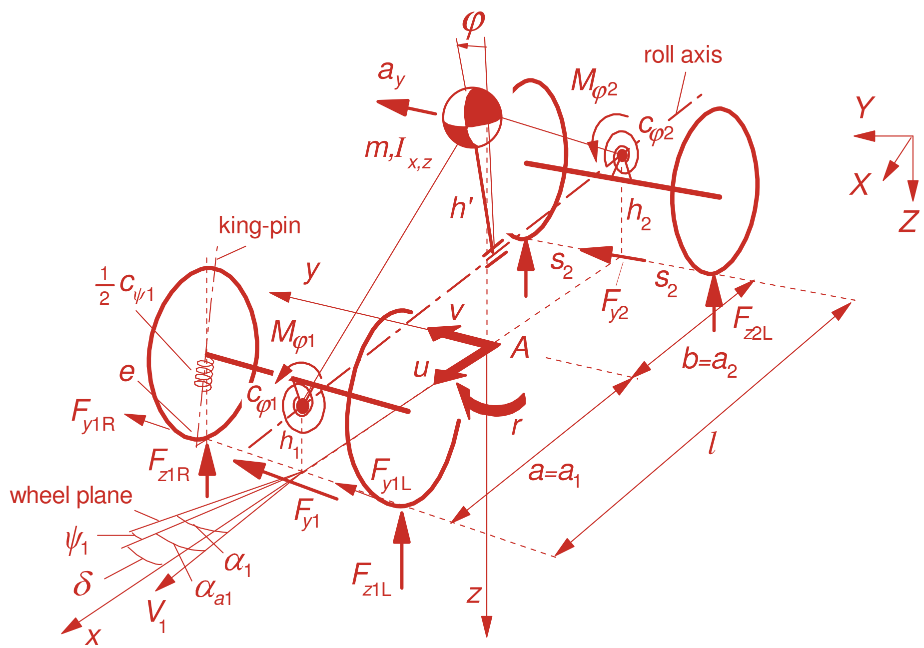
- chapter 4.3 (page 165-209) excluding 4.3.3 and 4.3.5

Next time…

- two track vehicle model

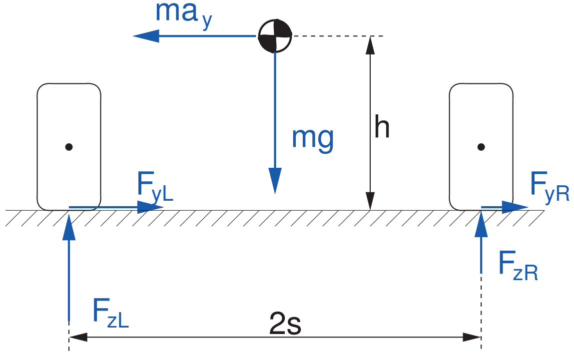
7. Two track vehicle model including body roll

- effective axle characteristics
- suspension, roll centre - roll stabiliser
- vehicle model with four tyres and roll degree of freedom, equations of motion

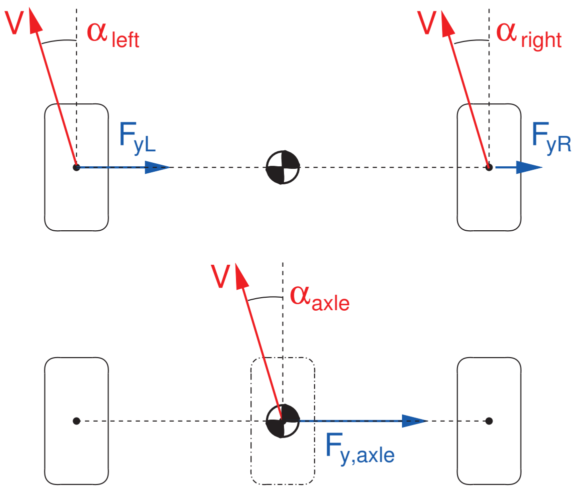


Effective axle characteristics

rear view



top view



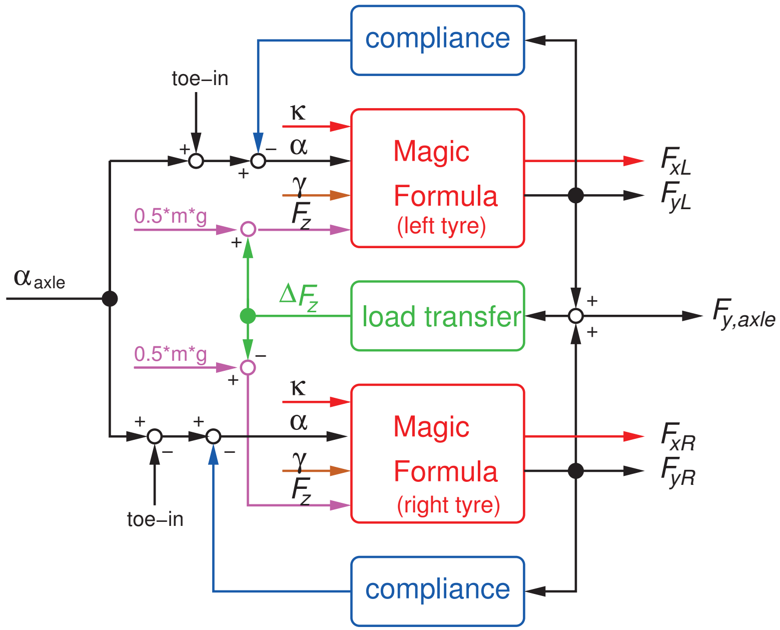
$F_{y,axle} = F_{yL} + F_{yR}$

characteristics:

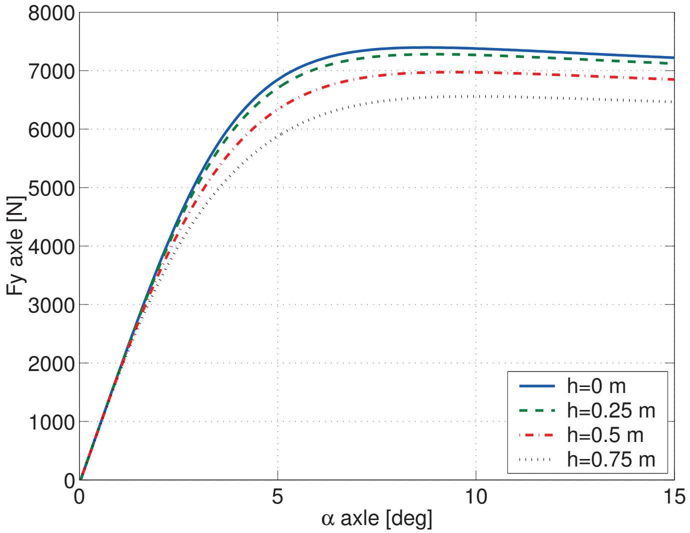
- single axle
- centre of gravity height taken into account
- two identical tyres
- steady-state analysis

default parameters:

- m=800 kg
- $2s$=1.5 m
- h=0.6 m
- g=9.81 m/s^2
- 195/65 R15 passenger car tyre



effect of C.G. height… (load transfer)



due to the C.G. height the vertical load on the left and right tyre is not identical anymore:

$$\Delta F = \frac{h}{2s} m a_y = \frac{h}{2s} F_{y,axle}$$

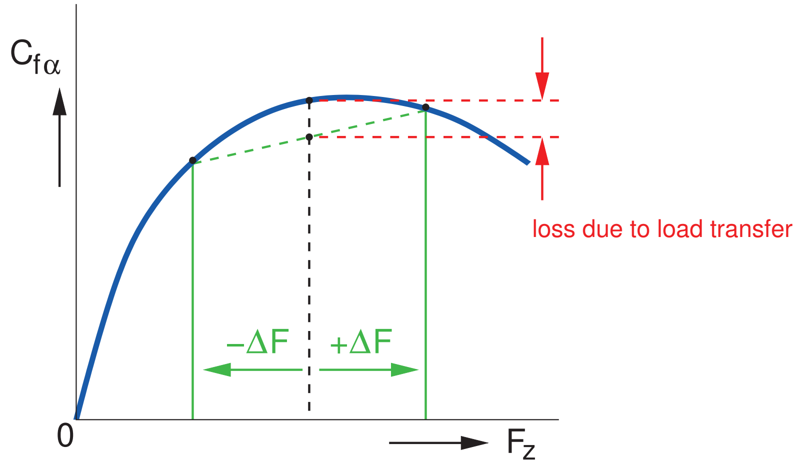
$$F_{zL} = 0.5mg + \Delta F \qquad F_{zR} = 0.5mg - \Delta F$$

tyre characteristics are dependent on the vertical load:

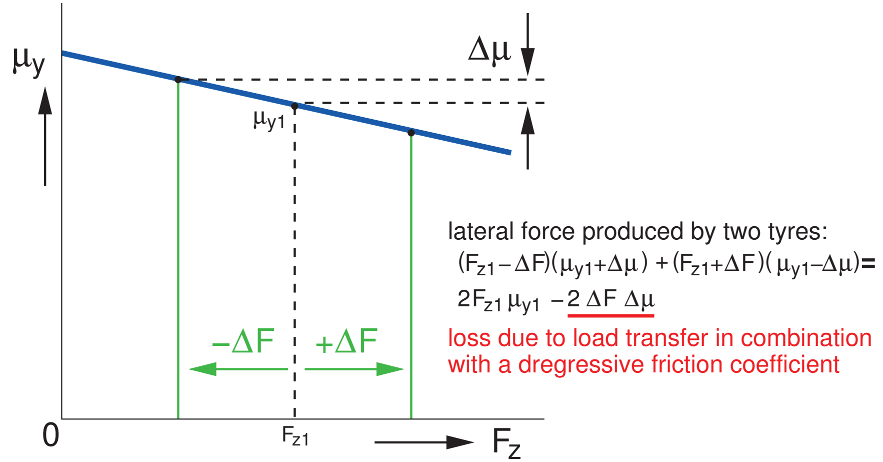
- degressive friction coefficient with vertical load
- saturation of the cornering stiffness

explanation: (see also page 99)

cornering stiffness



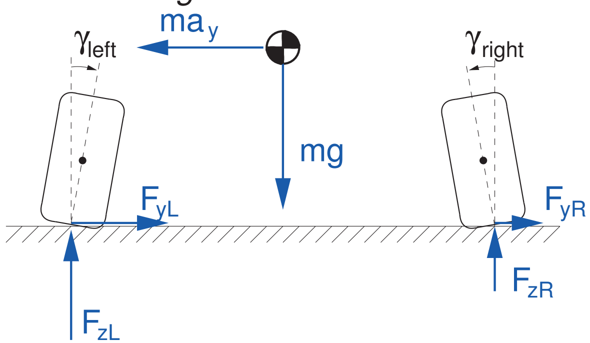
peak lateral friction coefficient

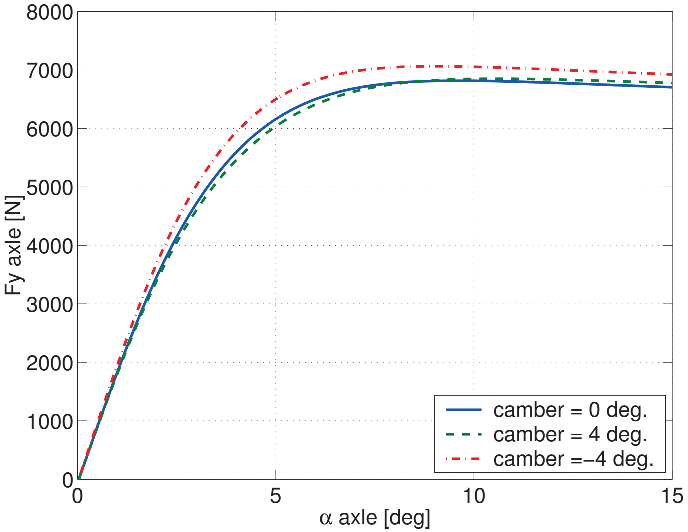


note: it is very obvious that a low C.G. is beneficial to maximise the lateral acceleration!

effect of camber…

rear view: negative camber

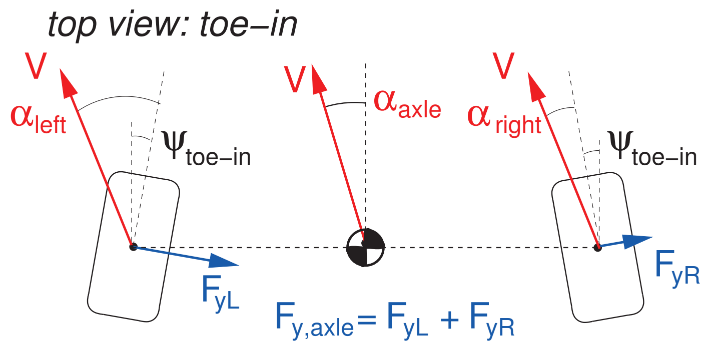


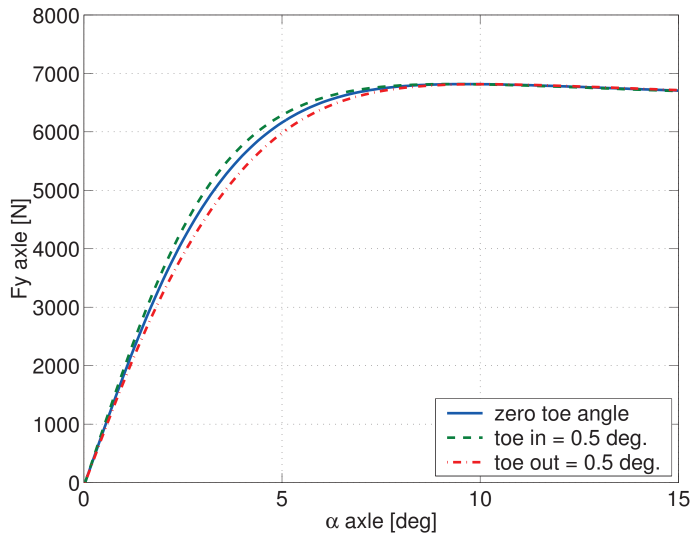


negative camber on the highly loaded wheel results in an upward shift of the F_y vs. α curve (see page 102 and 105)

- uneven tyre wear may pose a limit

effect of a toe-in angle…





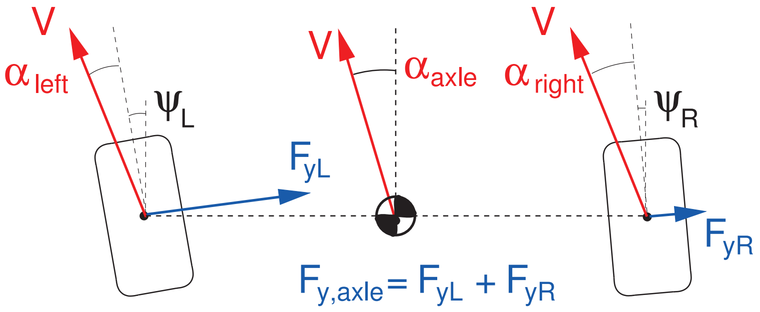
large toe angles:

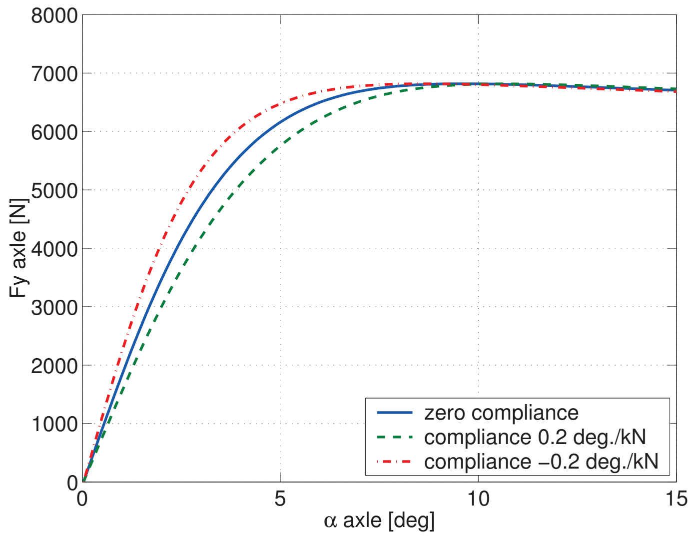
- cause additional drag
- tyre wear may impose a limit

effect of compliance steer…

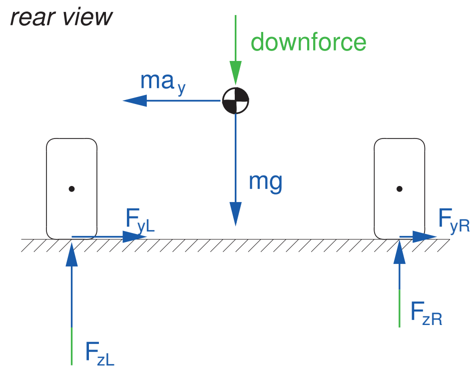
- suspension may deflect under the influence of lateral force (and self-aligning moment and/or longitudinal force…)

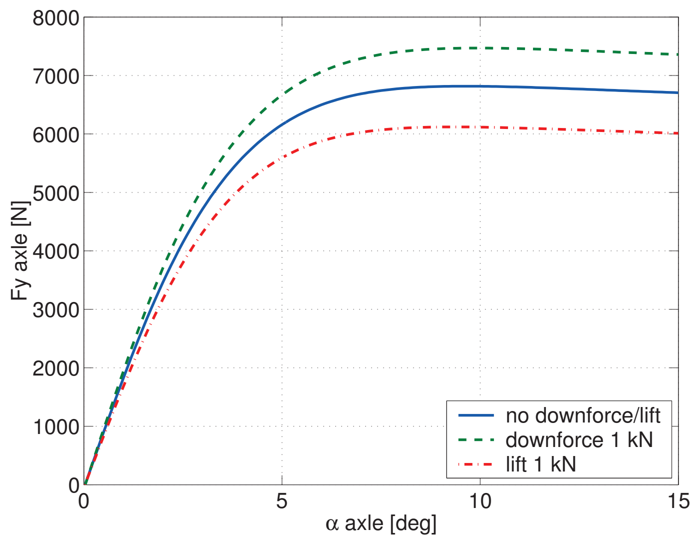
top view: (positive) steering compliance





aerodynamic down force/lift…





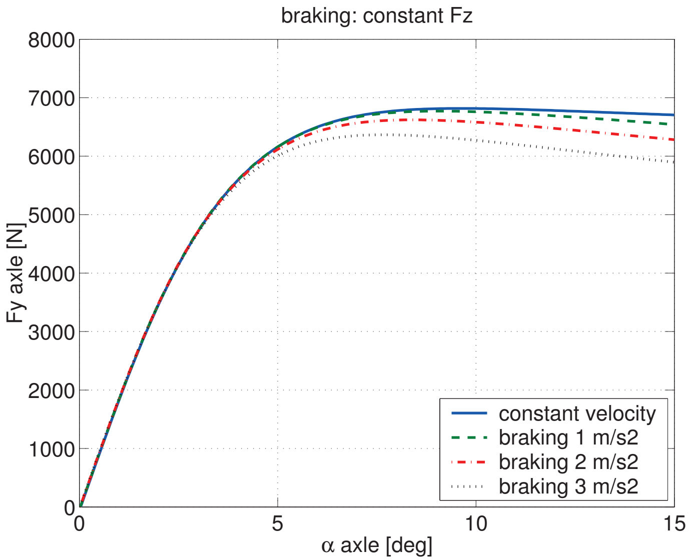
F-1 racing car: down force distribution over the front and rear axle will completely determine the under/oversteer behaviour at high speeds
(note: velocity dependent handling diagram!)

braking…
assumption: identical longitudinal force left/right

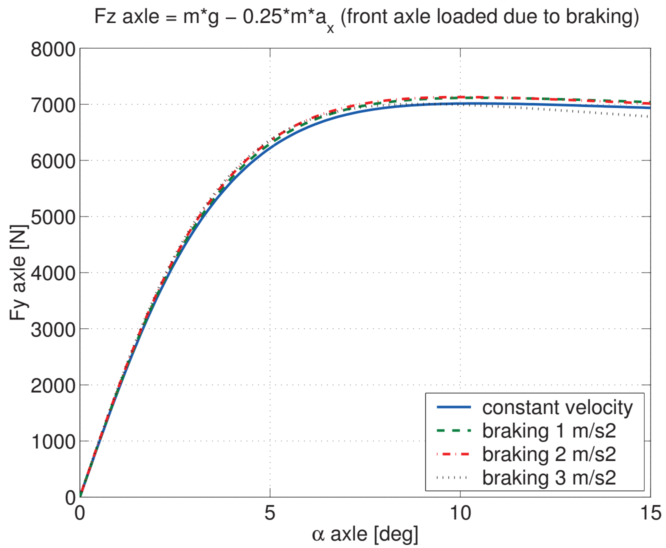
effects:
- combined slip
- on a vehicle: load transfer front/rear axle

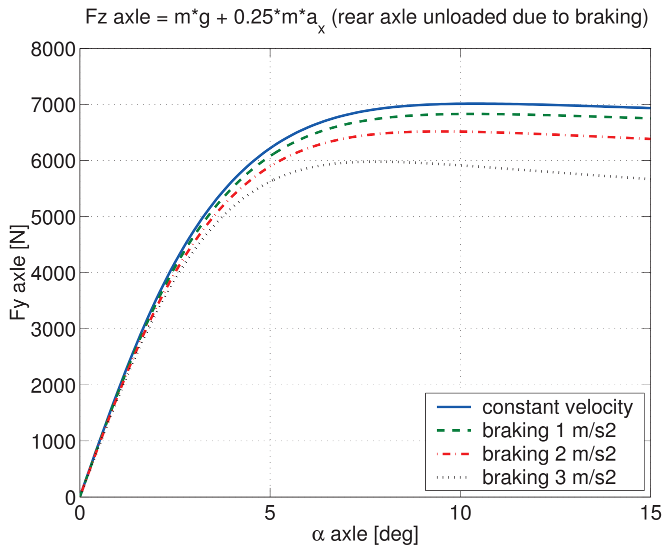
note:
- only moderate braking considered
  (no locking of unloaded wheel)
- power on/off has same effects

without load transfer front/rear:



including load transfer front/rear:





The roll centre

a *four* wheeled vehicle:

- for the calculation of the vertical tyre forces we have four unknowns and three equations

statically undetermined construction:

- tyre and suspension deflections need to be considered to calculate the vertical force distribution

to analyse the cornering behaviour of a vehicle on a flat road, the concept of a “roll centre” is introduced:

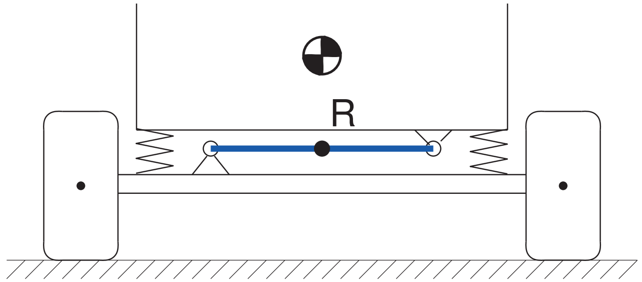
- simplified analysis: may not be valid for high lateral acceleration levels (validity will also depend on the suspension type)
- is important for a basic understanding of the effects of vehicle roll
- vertical tyre deflection neglected (rigid tyres)
- may be the source of some debate…

roll centre definition [SAE]:

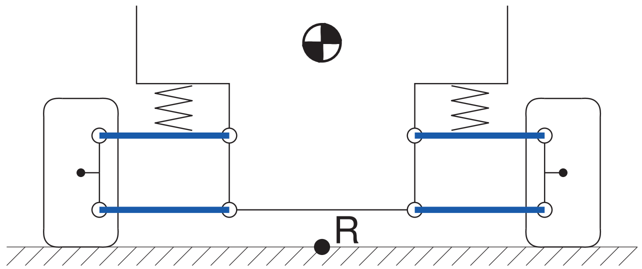
The roll centre is the point in the traverse vertical plane through any pair of wheel centres at which the lateral force may be applied to the sprung mass (=vehicle body) without producing suspension roll.

examples:

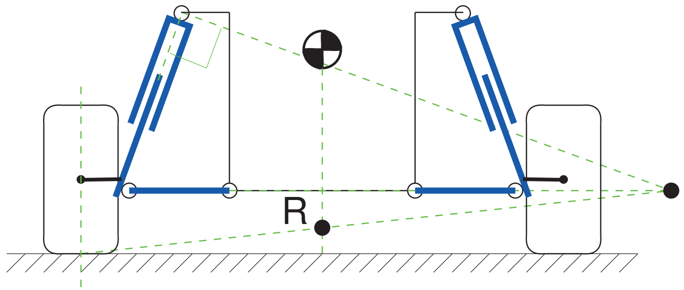
suspension with Panhard rod



double wishbone suspension



McPherson suspension

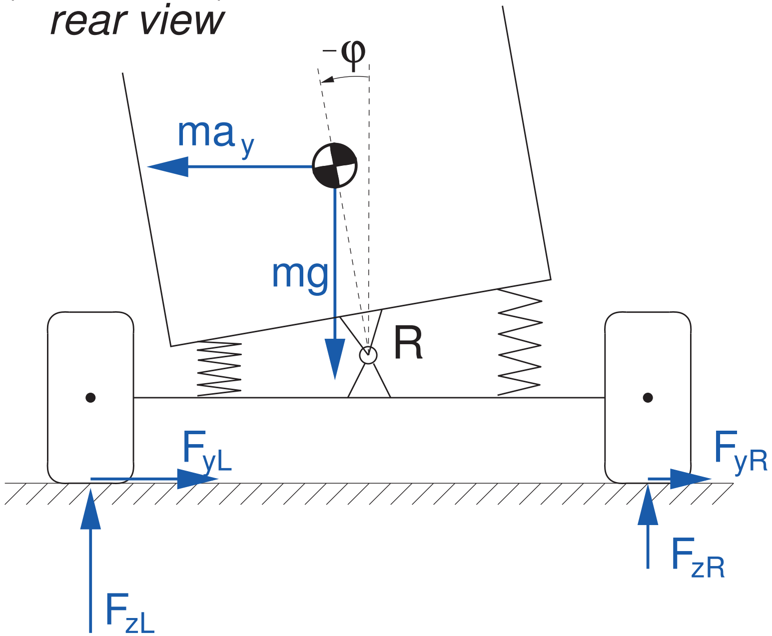


roll centre height:
- generally between road and wheel centre
- possibly below road level
- front usually lower than rear

detailed analysis: roll centre can move e.g. vertically a function of suspension deflection, and laterally for high lateral acceleration

axle + body under lateral acceleration
(R: roll centre)

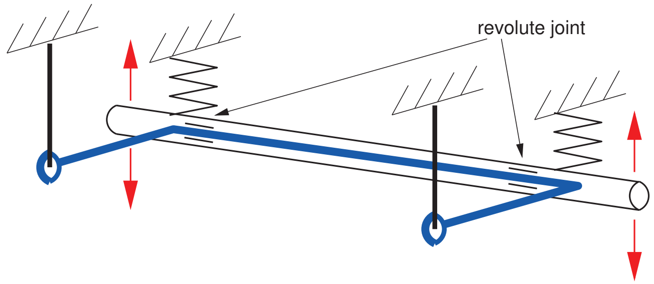
rear view

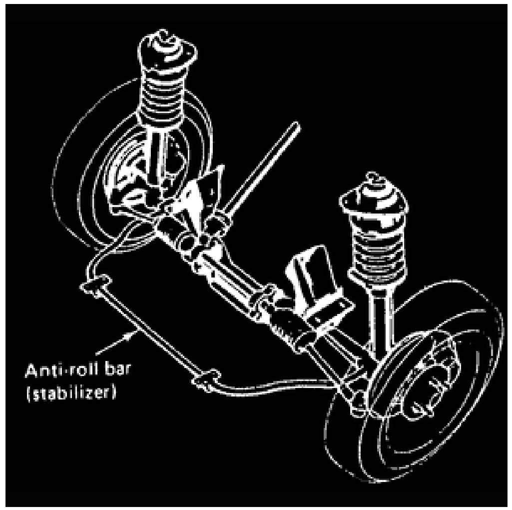


roll stiffness may be increased by introducing a roll stabiliser (also called: anti-roll bar)

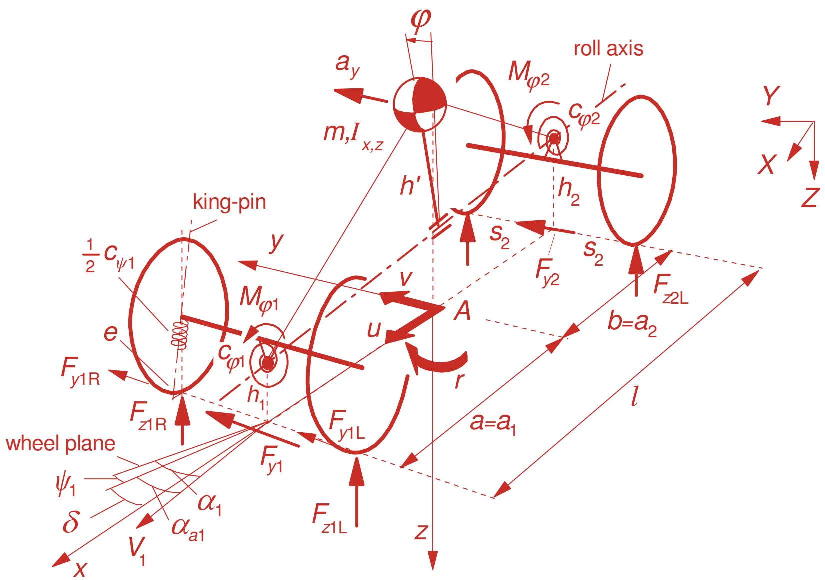
roll stabiliser:
- increases roll stiffness, reduction of body roll
- affects vehicle over/understeer behaviour (will be shown…)

roll stabiliser

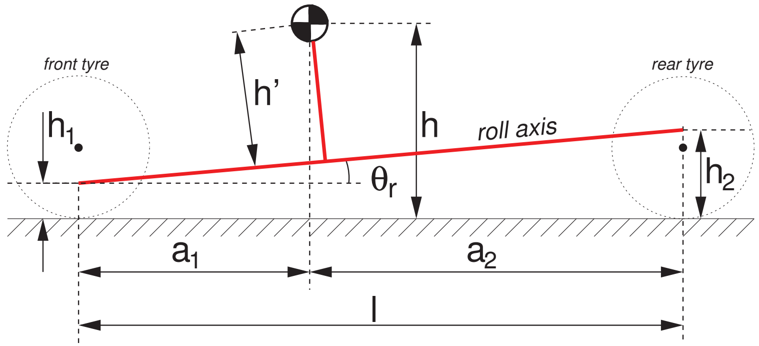




Roll-axis vehicle model



side view



steady state cornering analysis

roll angle:

$$\varphi = \frac{-ma_y h'}{c_{\varphi 1} + c_{\varphi 2} - mgh'}$$

load transfer: $(i = 1,2)$

$$\Delta F_{zi} = \sigma_i m a_y$$

$$\sigma_i = \frac{1}{2s_i}\left(\frac{c_{\varphi i} h'}{c_{\varphi 1} + c_{\varphi 2} - mgh'} + \frac{l - a_i}{l} h_i\right)$$

$$F_{ziL} = \left(\frac{l-a_i}{2l}\right) mg + \Delta F_{zi} \,,\; F_{ziR} = \left(\frac{l-a_i}{2l}\right) mg - \Delta F_{zi}$$

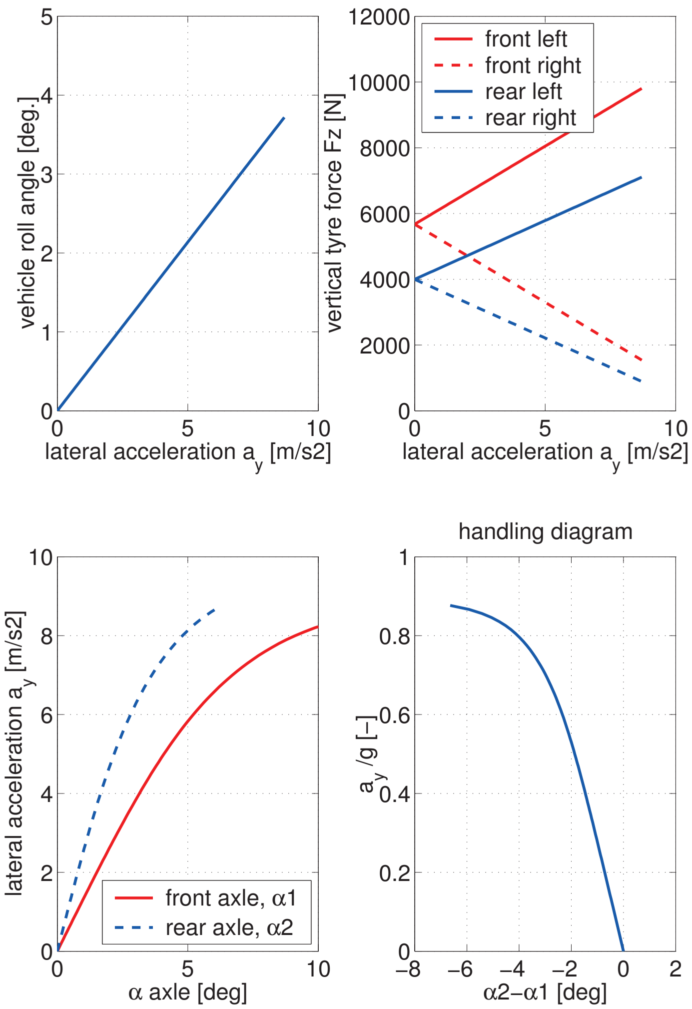
lateral tyre forces: $(i = 1,2)$

$$F_{yiL} + F_{yiR} = F_{yi,axle} = \left(\frac{l-a_i}{l}\right) m a_y$$

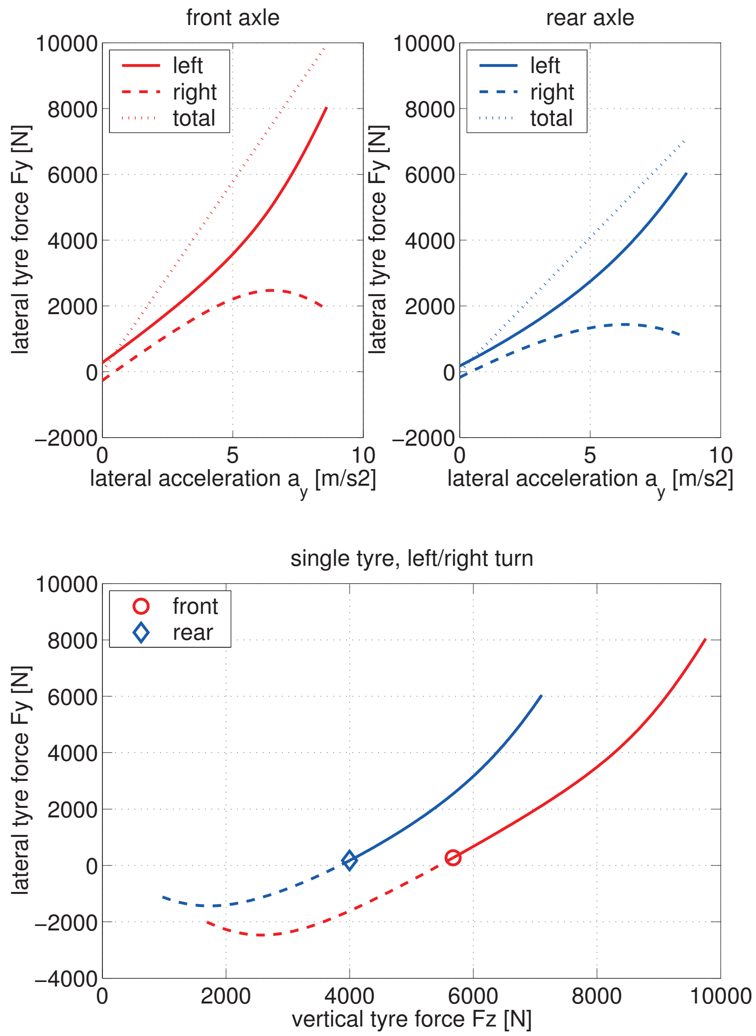
calculation sequence:

- select lateral acceleration level a_y
- calculate the vertical tyre forces and required lateral tyre force for each axle $F_{yi,axle}$
- (iteratively) determine α_{axle} so that the required $F_{yi,axle}$ is obtained using the Magic Formula; possibly including corrections for toe-in and compliance

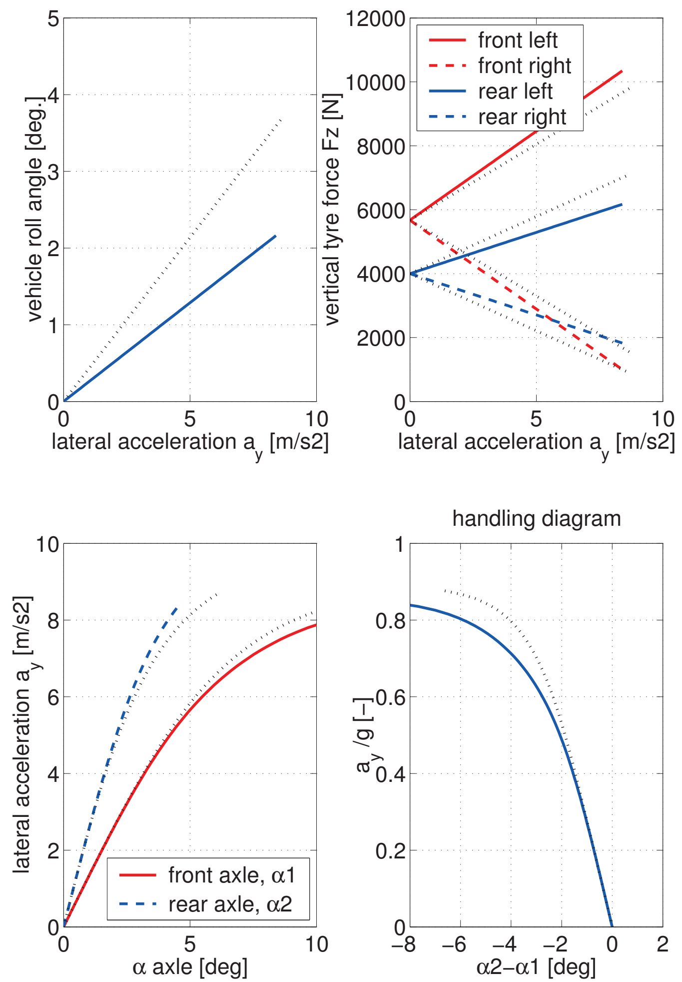
example (baseline)



distribution of the lateral tyre forces:

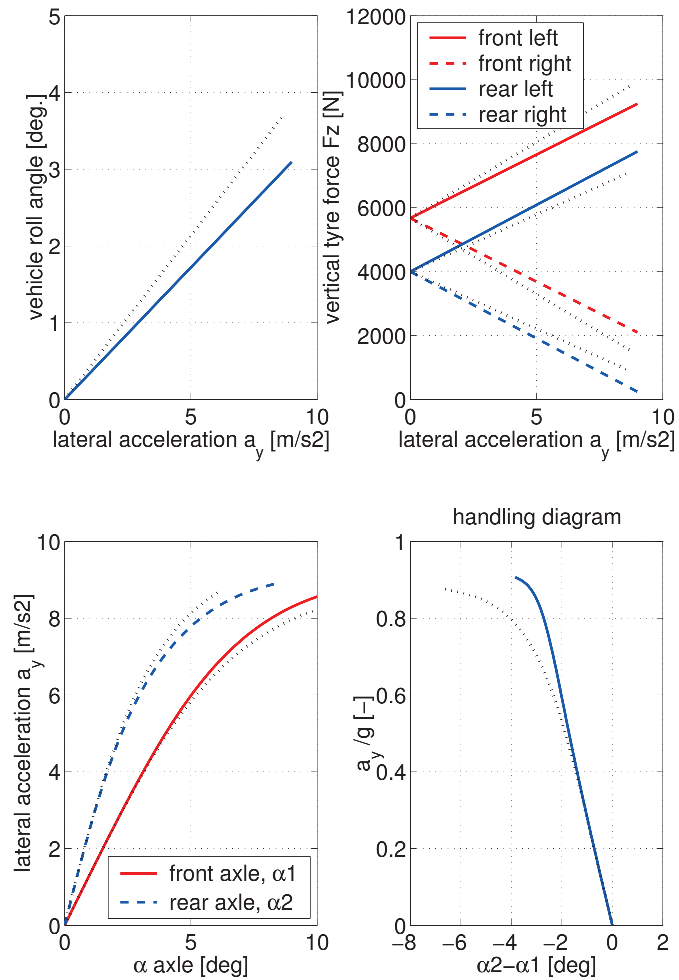


increase front axle roll stiffness $c_{\varphi 1}$…

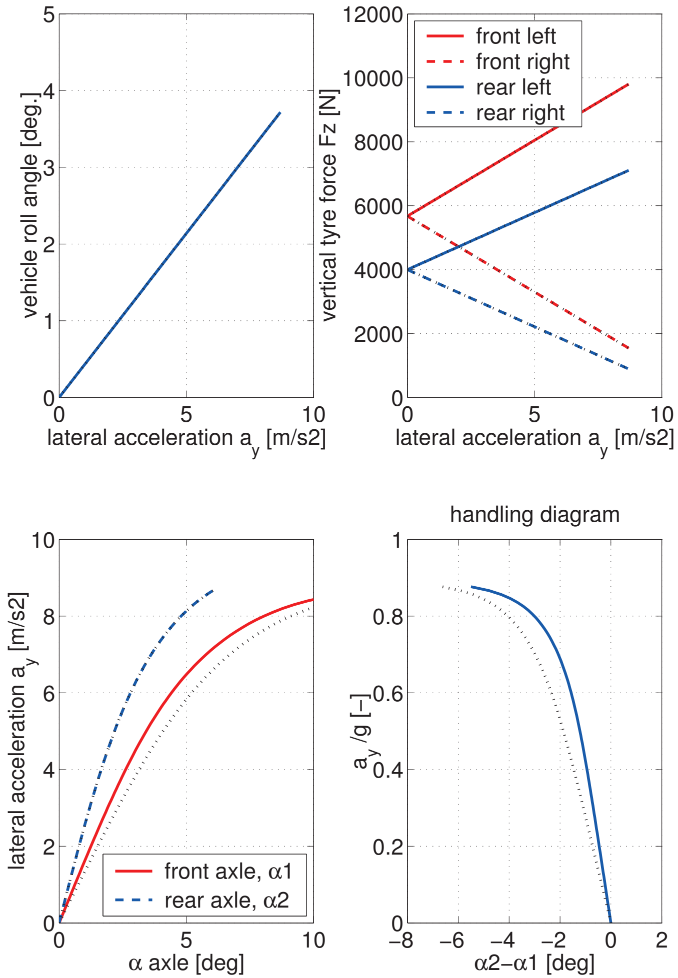


more understeer (dotted line: baseline vehicle)

increase height roll centre rear axle h_2…



less understeer (dotted line: baseline vehicle) reduced steering compliance on front axle…



less understeer (dotted line: baseline vehicle)

equations of motion

- small roll angle φ
- small roll axis inclination θ_r

kinetic energy:

$$T = \frac{1}{2} m \left\{ (u - h'\varphi r)^2 + (v + h'\dot{\varphi})^2 \right\} + \frac{1}{2} I_x \dot{\varphi}^2 + \frac{1}{2} I_y (\varphi r)^2 + \frac{1}{2} I_z \left(1 - \varphi^2\right) r^2 + \left(I_z \theta_r - I_{xz}\right) r \dot{\varphi}$$

potential energy:

$$U = \frac{1}{2}\left(c_{\varphi 1} + c_{\varphi 2}\right)\varphi^2 - \frac{1}{2} m g h' \varphi^2$$

modified Lagrange equations:

$$\frac{d}{dt}\frac{\partial T}{\partial u} - r\frac{\partial T}{\partial v} = \sum F_x$$

$$\frac{d}{dt}\frac{\partial T}{\partial v} + r\frac{\partial T}{\partial u} = \sum F_y$$

$$\frac{d}{dt}\frac{\partial T}{\partial r} - v\frac{\partial T}{\partial u} + u\frac{\partial T}{\partial v} = \sum M_z$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\varphi}} - \frac{\partial T}{\partial \varphi} + \frac{\partial U}{\partial \varphi} = Q_\varphi \quad \left(= -\left(k_{\varphi 1} + k_{\varphi 2}\right)\dot{\varphi}\right)$$

resulting equations of motion:

$$m\dot{u} - mrv - mh'\varphi\dot{r} - 2mh' r\dot{\varphi} = \sum F_x$$

$$m\dot{v} + mru + mh'\ddot{\varphi} - mh' r^2\varphi = \sum F_y$$

$$I_z\dot{r} + (I_z\theta_r - I_{xz})\ddot{\varphi} - mh'(\dot{u} - rv)\varphi = \sum M_z$$

$$(I_x + mh'^2)\ddot{\varphi} + mh'(\dot{v} + ru) + (I_z\theta_r - I_{xz})\dot{r}$$
$$-(mh'^2 + I_y - I_z)r^2\varphi + (k_{\varphi 1} + k_{\varphi 2})\dot{\varphi} + (c_{\varphi 1} + c_{\varphi 2} - mgh')\varphi = 0$$

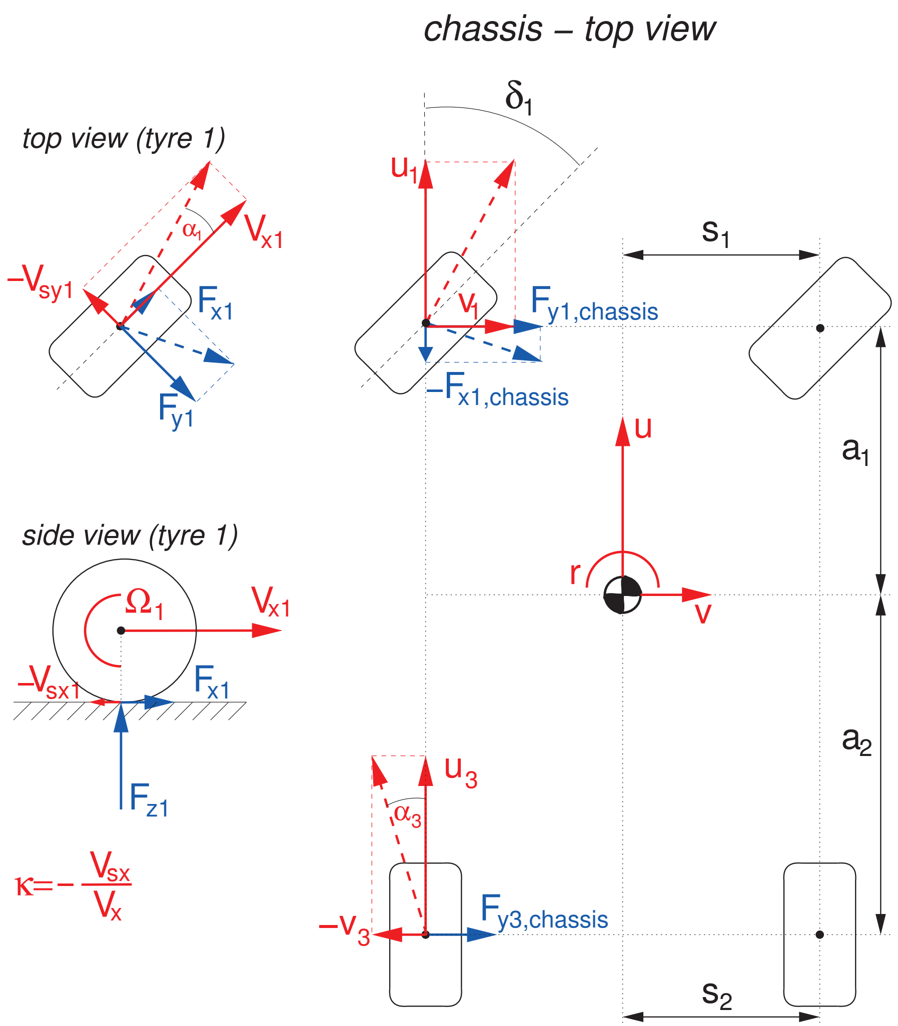
or:

$$\begin{bmatrix} m & 0 & -mh'\varphi & 0 \\ 0 & m & 0 & mh' \\ -mh'\varphi & 0 & I_z & I_z\theta_r - I_{xz} \\ 0 & mh' & I_z\theta_r - I_{xz} & I_x + mh'^2 \end{bmatrix} \begin{matrix} \dot{u} \\ \dot{v} \\ \dot{r} \\ \ddot{\varphi} \end{matrix} = \mathbf{f}$$

$$\mathbf{f} = \begin{pmatrix} \sum F_x + mrv + 2mh' r\dot{\varphi} \\ \sum F_y - mru + mh' r^2\varphi \\ \sum M_z - mh' rv\varphi \\ -k\dot{\varphi} - (c - mgh')\varphi - mh' ru + (mh'^2 + I_y - I_z)r^2\varphi \end{pmatrix}$$

with: $c = c_{\varphi 1} + c_{\varphi 2}$ and $k = k_{\varphi 1} + k_{\varphi 2}$

tyre slip forces



<u>calculation sequence</u>

initial conditions: $u, v, r, \varphi, \dot{\varphi}, \Omega_1, \Omega_2, \Omega_3, \Omega_4$

left front wheel slip forces:

$u_1 = u + rs_1$ and $v_1 = v + ra_1$

$$V_{x1} = u_1 \cos\delta_1 + v_1 \sin\delta_1$$
$$V_{sy1} = -u_1 \sin\delta_1 + v_1 \cos\delta_1$$

$$\kappa_1 = -\frac{V_{sx1}}{V_{x1}} = -\frac{V_{x1} - \Omega_1 r_e}{V_{x1}}$$
$$\alpha_1 = -\arctan\left(\frac{V_{sy1}}{V_{x1}}\right)$$

F_{x1}, F_{y1}, M_{z1}=MagicFormula($\kappa_1, \alpha_1, \gamma_1, F_{z1}$)

$$F_{x1,chassis} = F_{x1}\cos\delta_1 - F_{y1}\sin\delta_1$$
$$F_{y1,chassis} = F_{x1}\sin\delta_1 + F_{y1}\cos\delta_1$$

wheel angular velocity:

$$I_p\dot{\Omega}_1 = -F_{x1}r + M_{1,engine} - M_{1,brake}$$

for convenience: $r = r_e = 0.3$ m (rigid wheel/tyre)

sum chassis forces, moments:

$$\sum F_x = F_{x1,chassis} + F_{x2,chassis} + F_{x3,chassis} + F_{x4,chassis}$$
$$\sum F_y = F_{y1,chassis} + F_{y2,chassis} + F_{y3,chassis} + F_{y4,chassis}$$
$$\begin{aligned}\sum M_z = \; & F_{x1,chassis}s_1 + F_{y1,chassis}a_1 + M_{z1} \\ & - F_{x2,chassis}s_1 + F_{y2,chassis}a_1 + M_{z2} \\ & + F_{x3,chassis}s_2 - F_{y3,chassis}a_2 + M_{z3} \\ & - F_{x4,chassis}s_2 - F_{y4,chassis}a_2 + M_{z4}\end{aligned}$$

vertical equilibrium

- load transfer due to roll:

$$\Delta F_{z1,roll} = \frac{(F_{y1,chassis} + F_{y2,chassis})h_1 - c_{\varphi1}\varphi - k_{\varphi1}\dot{\varphi}}{2s_1}$$
$$\Delta F_{z2,roll} = \frac{(F_{y3,chassis} + F_{y4,chassis})h_2 - c_{\varphi2}\varphi - k_{\varphi2}\dot{\varphi}}{2s_2}$$

- load transfer due to braking:

$$\Delta F_{z,brake} = \frac{h}{2l}\Sigma F_x = \frac{h}{2l} m a_x$$

vertical tyre force:

$$F_{z1} = \frac{a_2}{2l} mg + \Delta F_{z1,roll} - \Delta F_{z,brake}$$

$$F_{z2} = \frac{a_2}{2l} mg - \Delta F_{z1,roll} - \Delta F_{z,brake}$$

$$F_{z3} = \frac{a_1}{2l} mg + \Delta F_{z2,roll} + \Delta F_{z,brake}$$

$$F_{z4} = \frac{a_1}{2l} mg - \Delta F_{z2,roll} + \Delta F_{z,brake}$$

note that we have an algebraic loop!

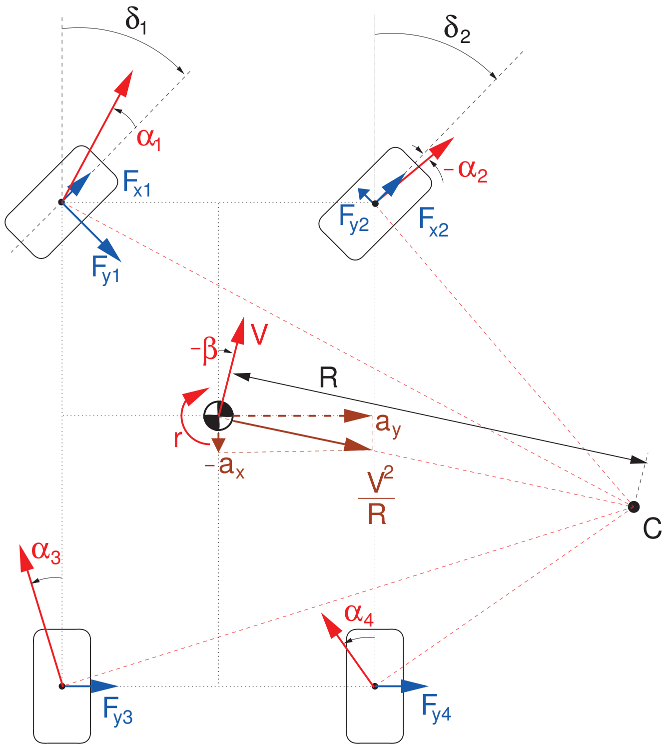
F_z is required to calculate F_x and F_y
but to calculate F_z we need F_x and F_y

possible solutions:

- let SIMULINK iteratively solve the equations (slow…)
- use F_z of the previous integration time step (fast, but results may depend on time step)
- include pitch dynamics of the vehicle
- use a filter to approximate the pitch dynamics
- …

steady state cornering, fixed radius R

chassis – top view



note:

- in order to maintain a constant velocity V we need to drive the (front) wheels…
- centripetal acceleration can have a component in the longitudinal direction (a_x): will result in load transfer between front and rear axle…

Book Pacejka

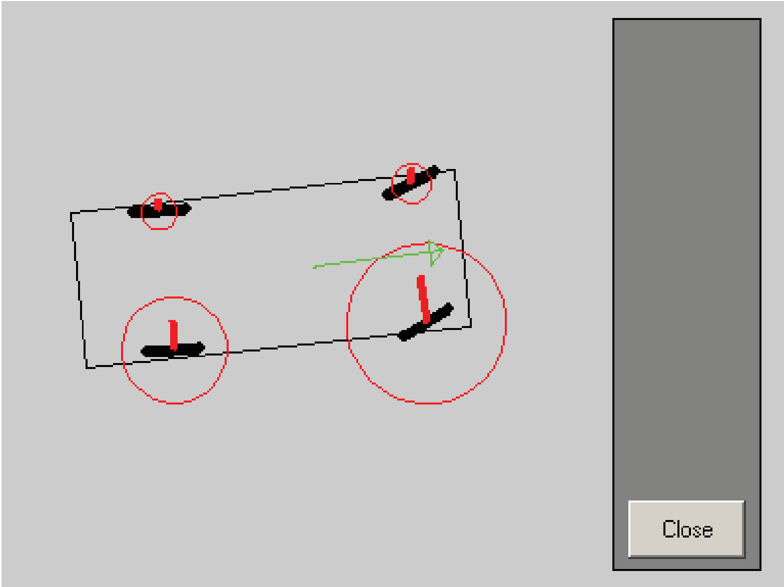
- chapter 1 until 1.3.2 (page 2-22)

Next time…

- development of a MATLAB/Simulink vehicle model
- validation of the two track vehicle model
- steering geometry
- braking in a turn

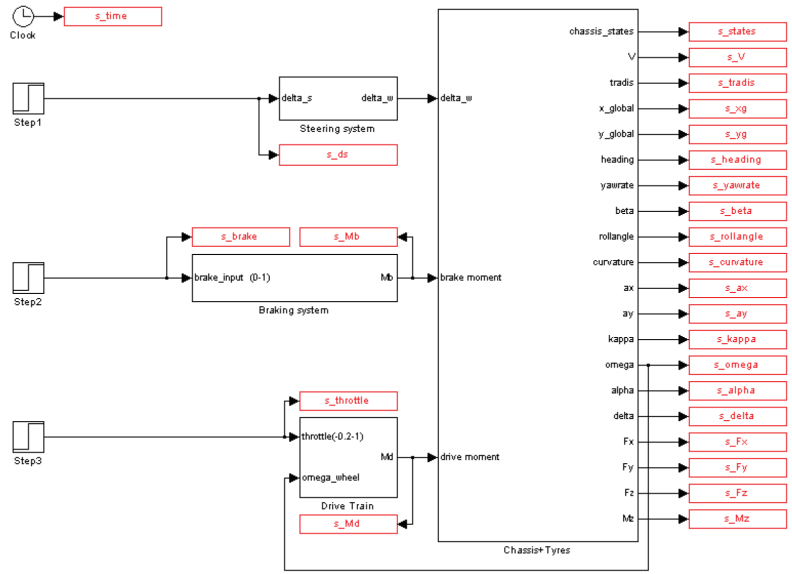
8. Two track vehicle model validation

- MATLAB/Simulink model development
- model validation using driving tests
- braking: straight line, in a turn
- suspension/steering geometry

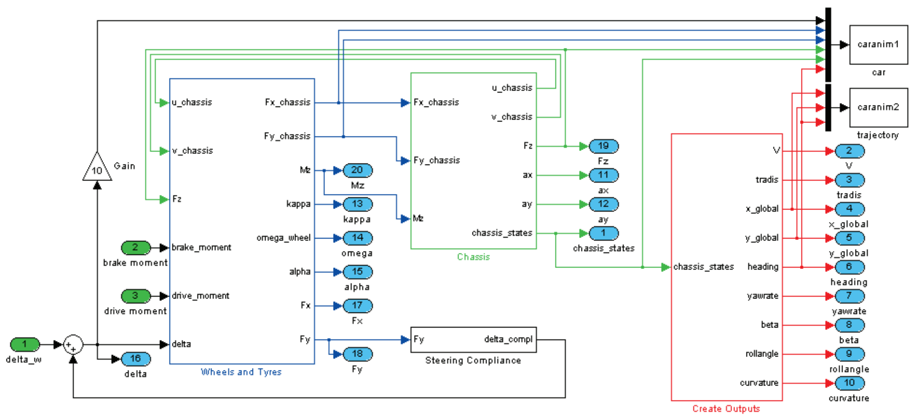


MATLAB/Simulink model development

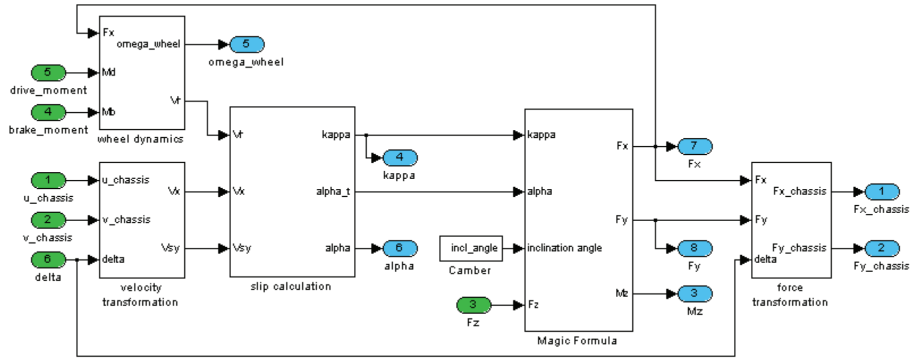
top level:



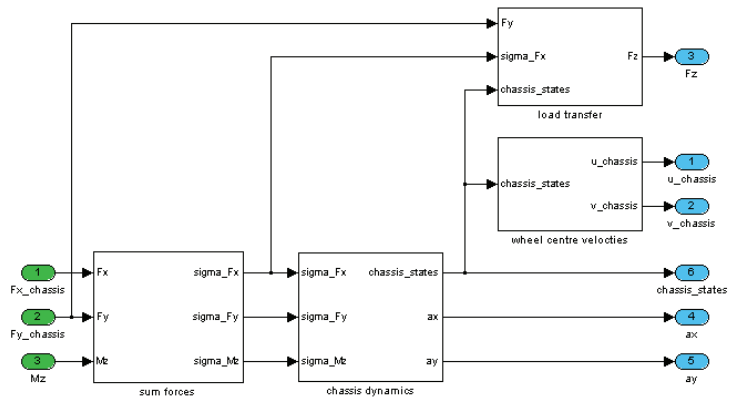
Chassis+Tyres:



Wheels and Tyres:



Chassis:



features:

- 4 vehicle body degrees of freedom: longitudinal, lateral, yaw and roll (u, v, r, φ)
- 4 wheels/tyres all having an independent angular velocity (Ω), brake (M_{brake}) and driving moment (M_{engine})
- full Magic Formula tyre model including combined slip
- includes steering compliance on the front axle (≈steering system flexibility) and rear axle
- constant camber and toe-in settings on front and rear axle
- algebraic loops solved by memory blocks
- no pitch dynamics
- simple braking system model: constant brake moment distribution front-rear axle
- very simple drive train model
- no aerodynamics

Validation

experimental data same presented in lecture 3

<u>model parameters</u>

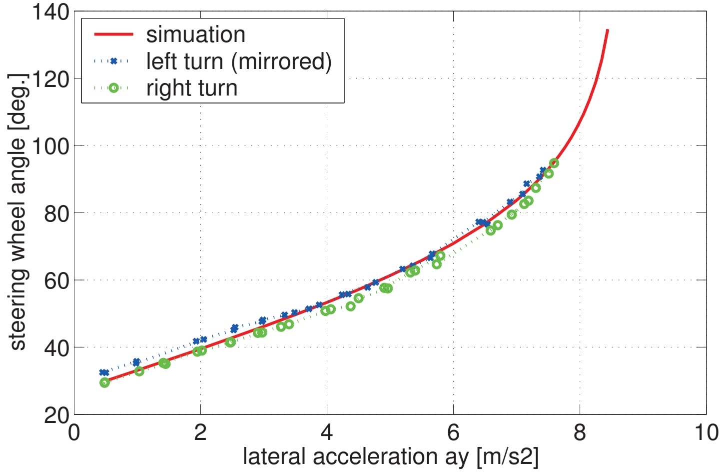
mass: m =1971.8 kg
x-position CG: a_1= 1.1907 m
z-position CG: h= 0.6 m
inertia: I_x= 900 kgm^2, I_y= 3200 kgm^2,
I_z= 3600 kgm^2, I_{xz}= 0 kgm^2

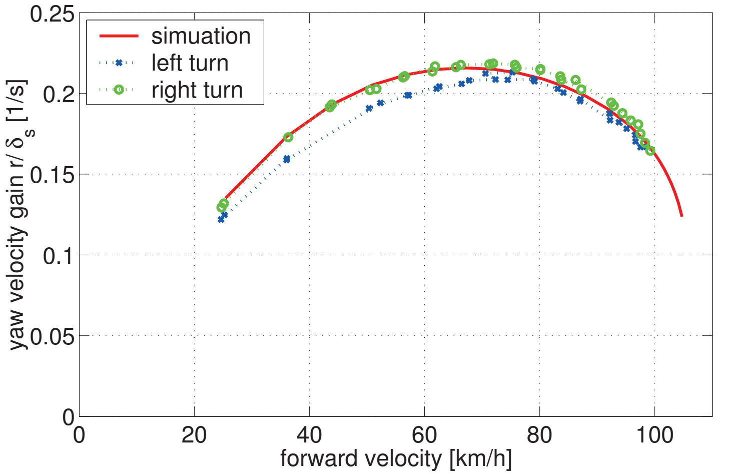
wheelbase: $l(= a_1 + a_2)$ = 2.88 m
front track width: $2s_1$=1.591 m
rear track width: $2s_2$=1.580 m
steer ratio: i_s= 16.19

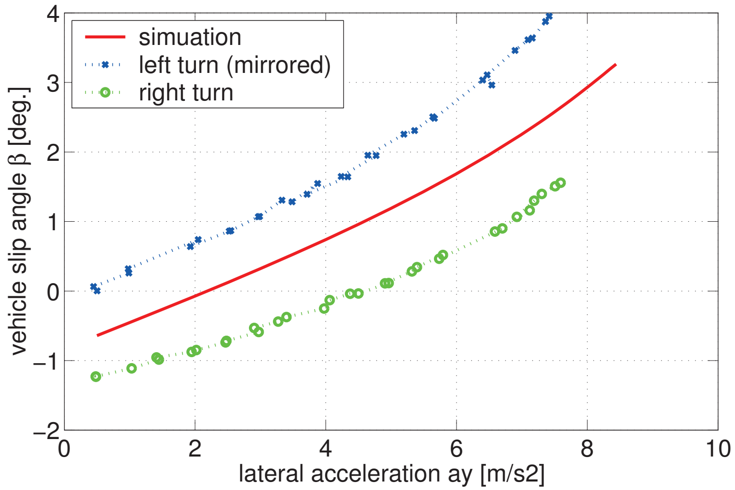
	front	rear
toe-in:	0.2 deg.	0.15 deg
camber:	-0.6 deg.	-0.7 deg.
roll centre z-pos.:	0 m	0.05 m
roll stiffness:	105 kNm/rad	55 kNm/rad
roll damping:	2 kNms/rad	1.5 kNms/rad
steer compliance:	0.29 deg./kN	0.04 deg./kN

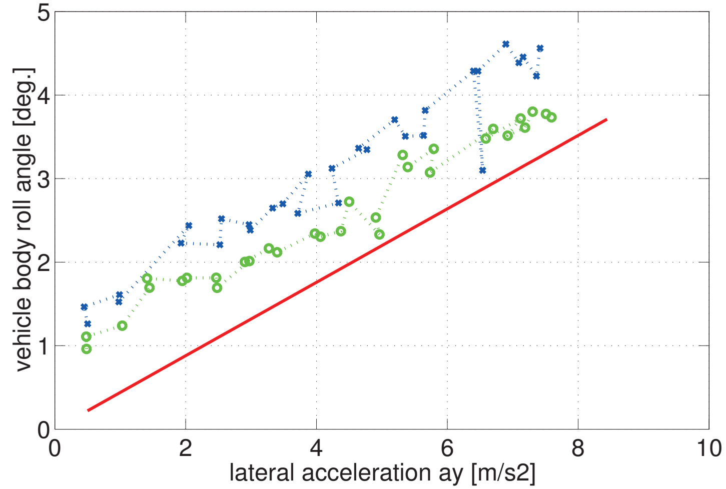
tyres (front and rear): 225/50R16 @ 2.3 bar
F&M: Magic Formula (±160 coefficients…)
relaxation length: 0.5 m

- steady state circular test

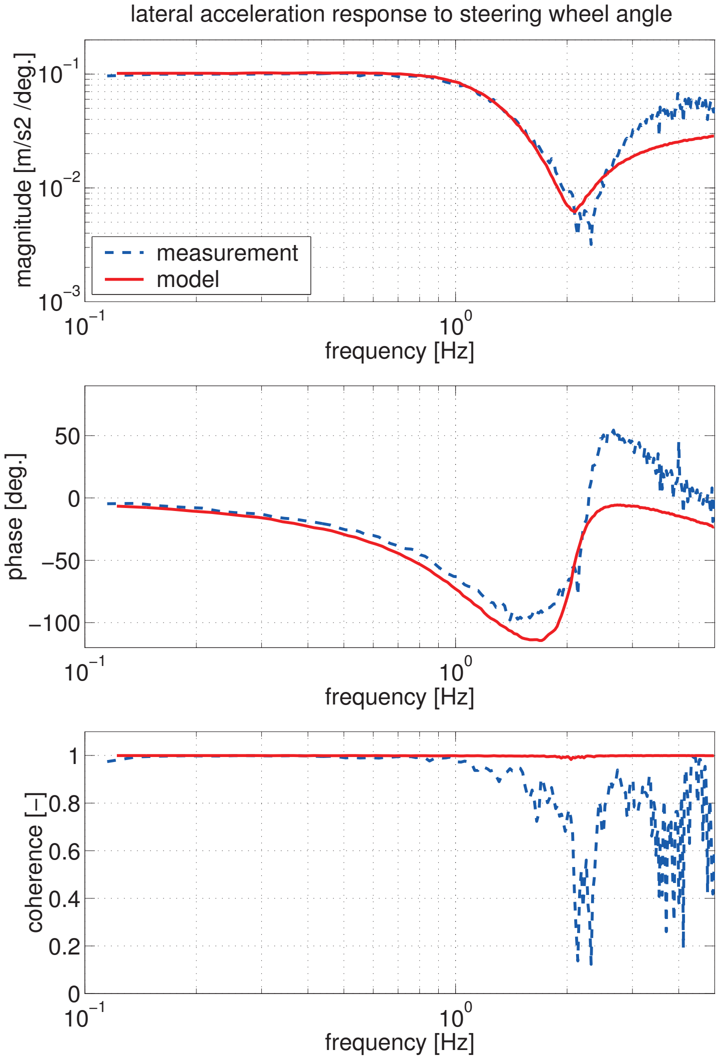


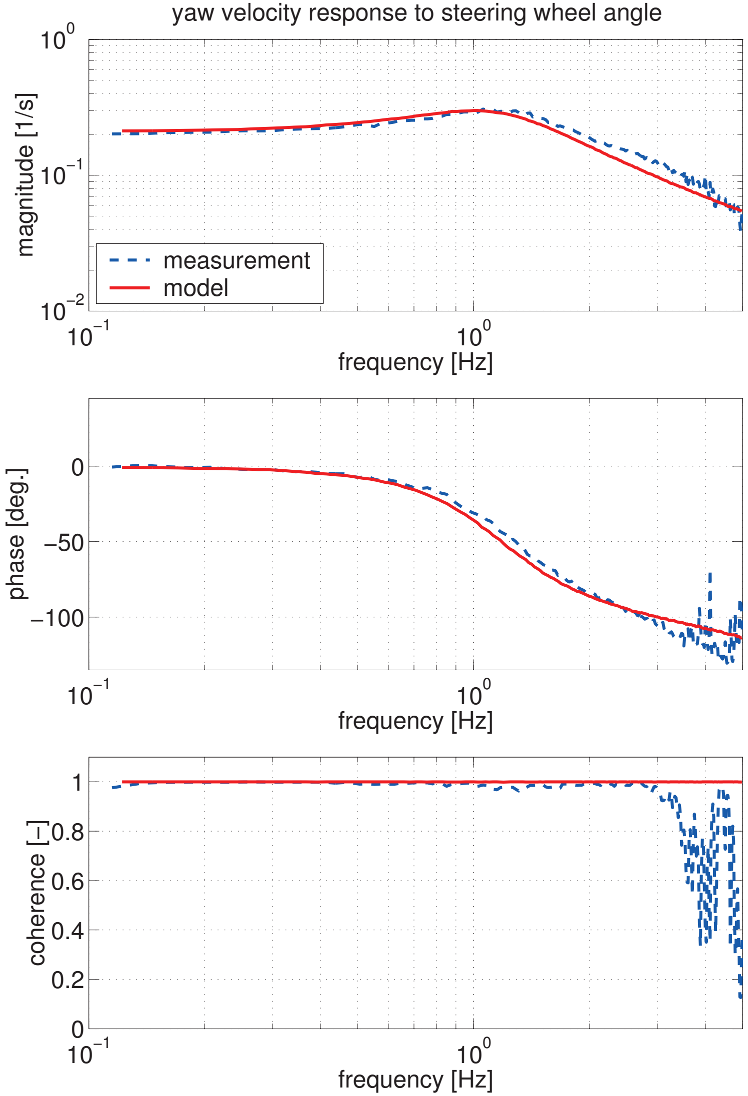


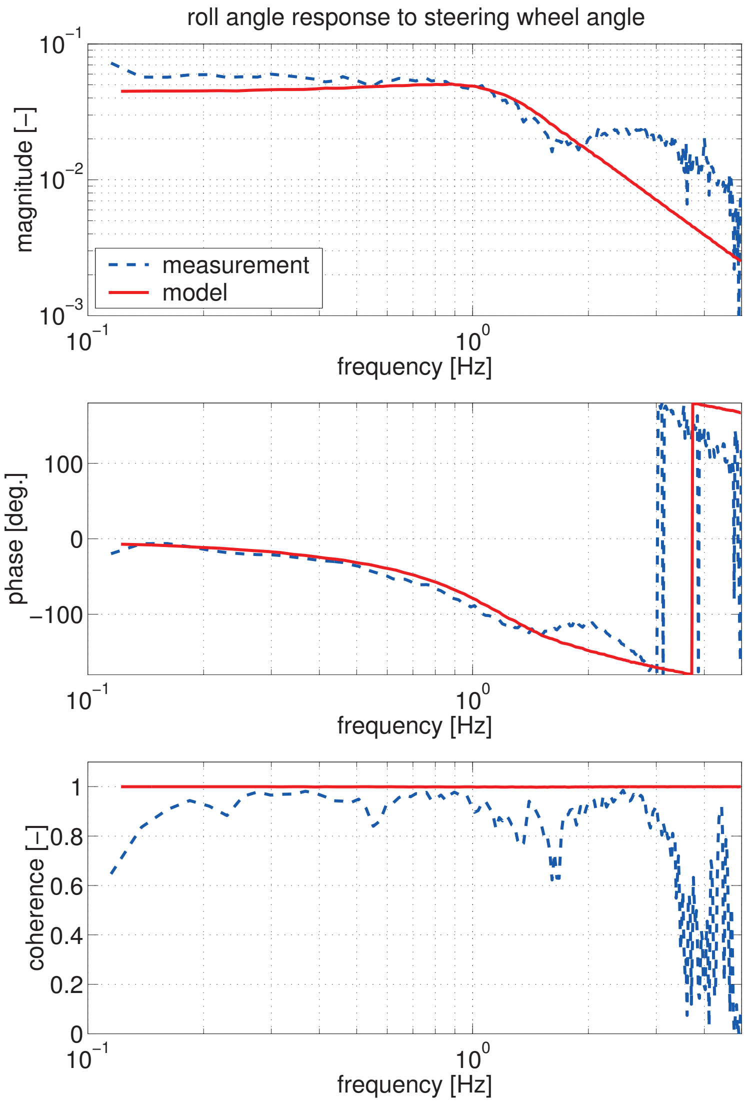


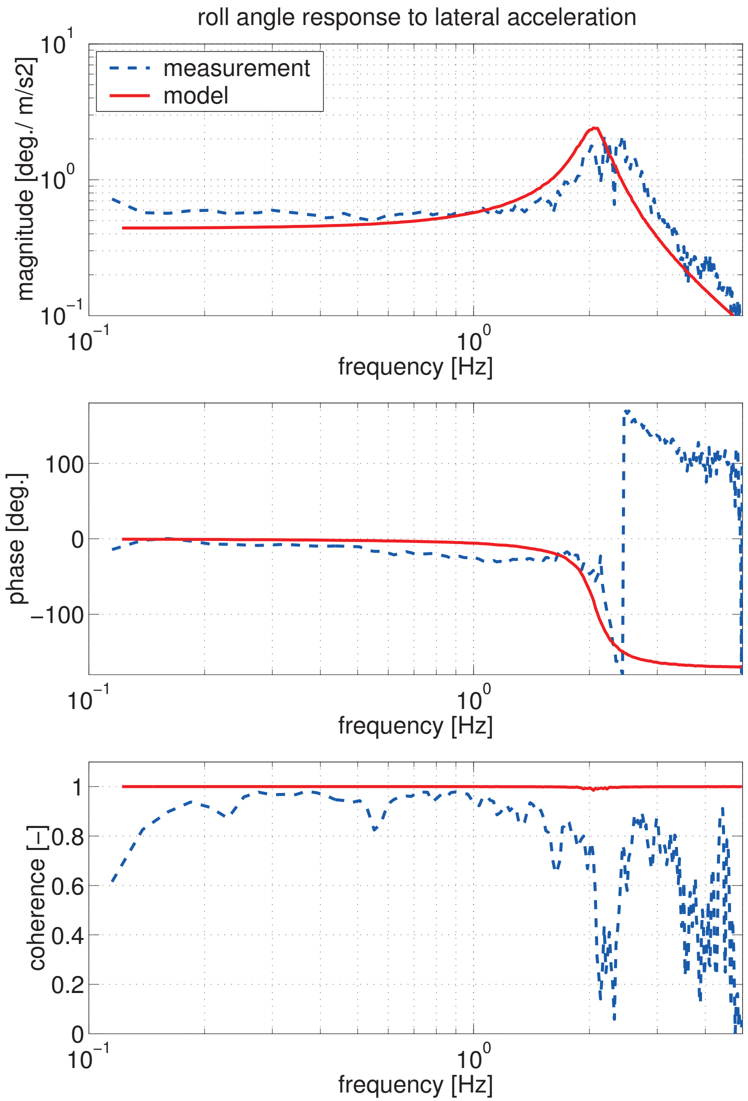


- pseudo random steer (100 km/h)

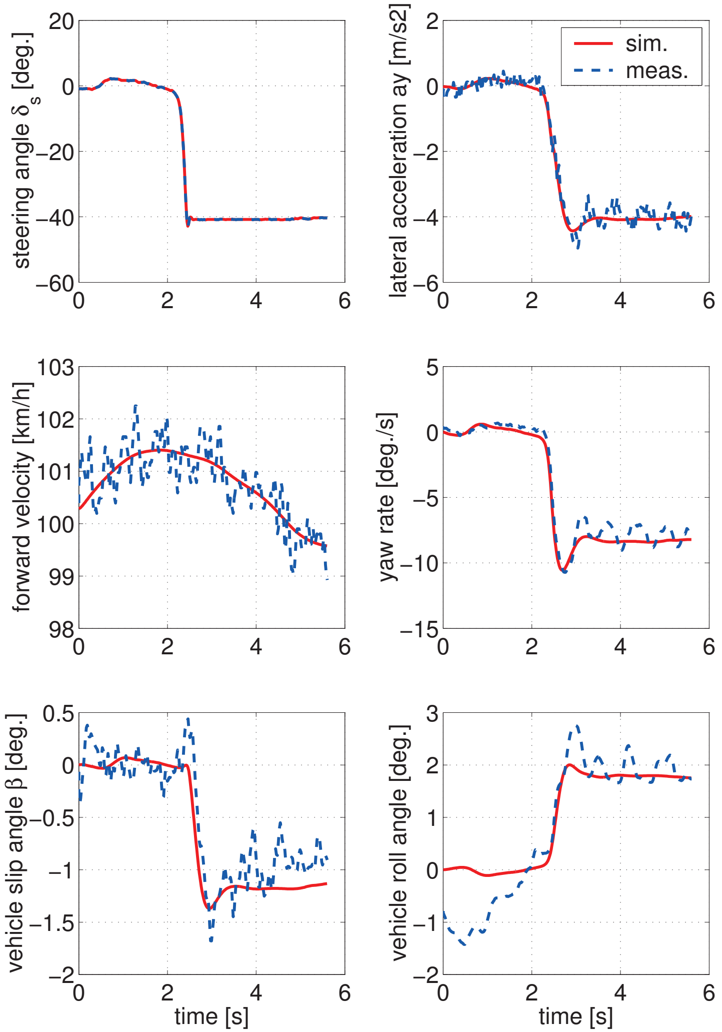




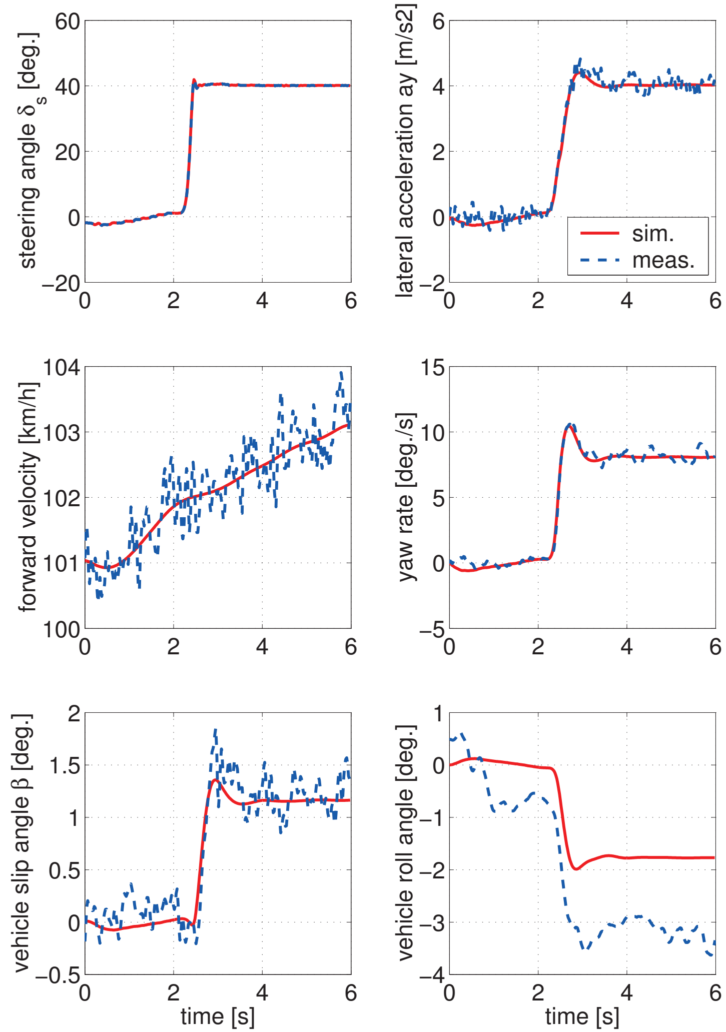

roll angle response to steering wheel angle
magnitude [–]
10^{-1}
10^{-2}
10^{-3}
10^{-1}
10^{0}
frequency [Hz]
measurement
model
phase [deg.]
100
0
−100
10^{-1}
10^{0}
frequency [Hz]
coherence [–]
1
0.8
0.6
0.4
0.2
0
10^{-1}
10^{0}
frequency [Hz]


roll angle response to lateral acceleration
magnitude [deg./ m/s2]
10^{1}
10^{0}
10^{-1}
10^{-1}
10^{0}
frequency [Hz]
measurement
model
phase [deg.]
100
0
−100
10^{-1}
10^{0}
frequency [Hz]
coherence [–]
1
0.8
0.6
0.4
0.2
0
10^{-1}
10^{0}
frequency [Hz]

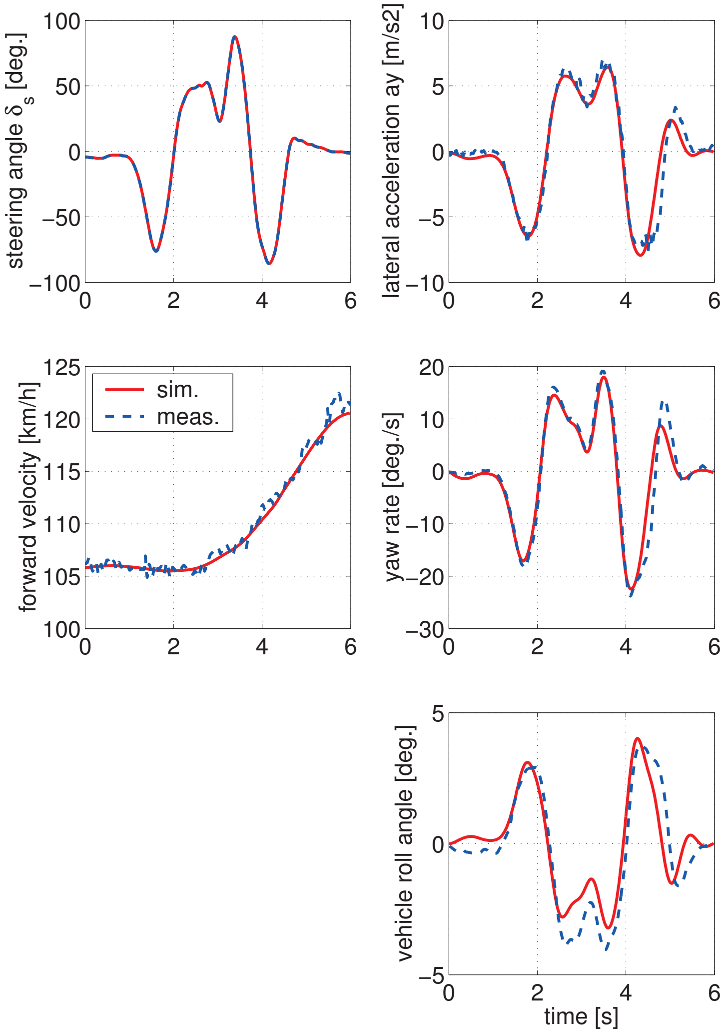
- J-turn (left)



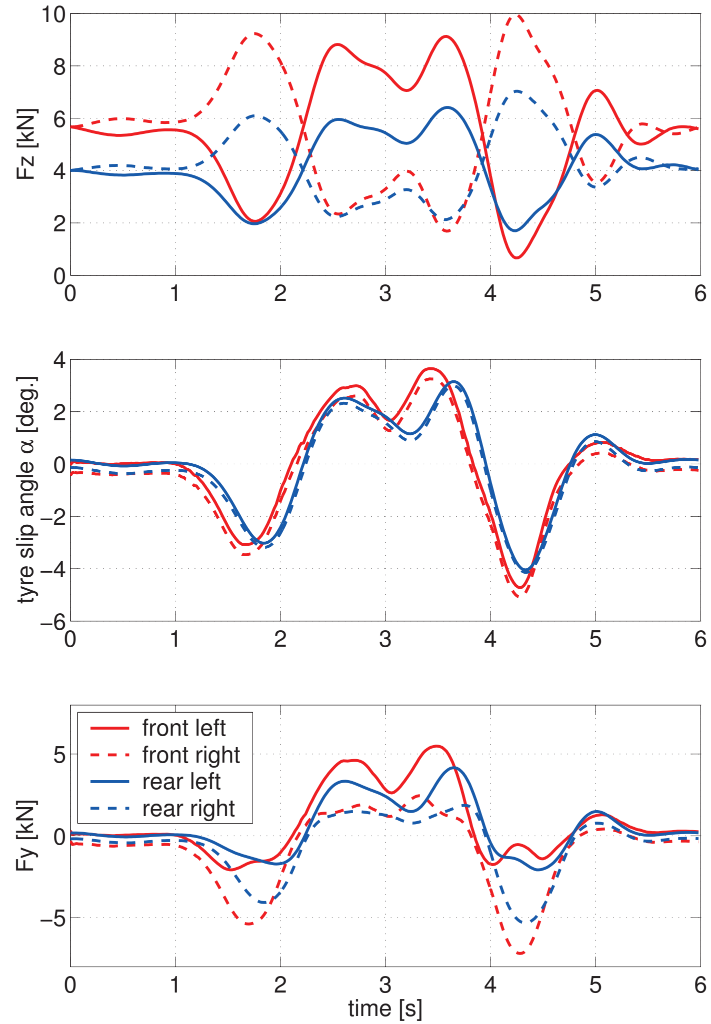
- J-turn (right)



- severe lane change



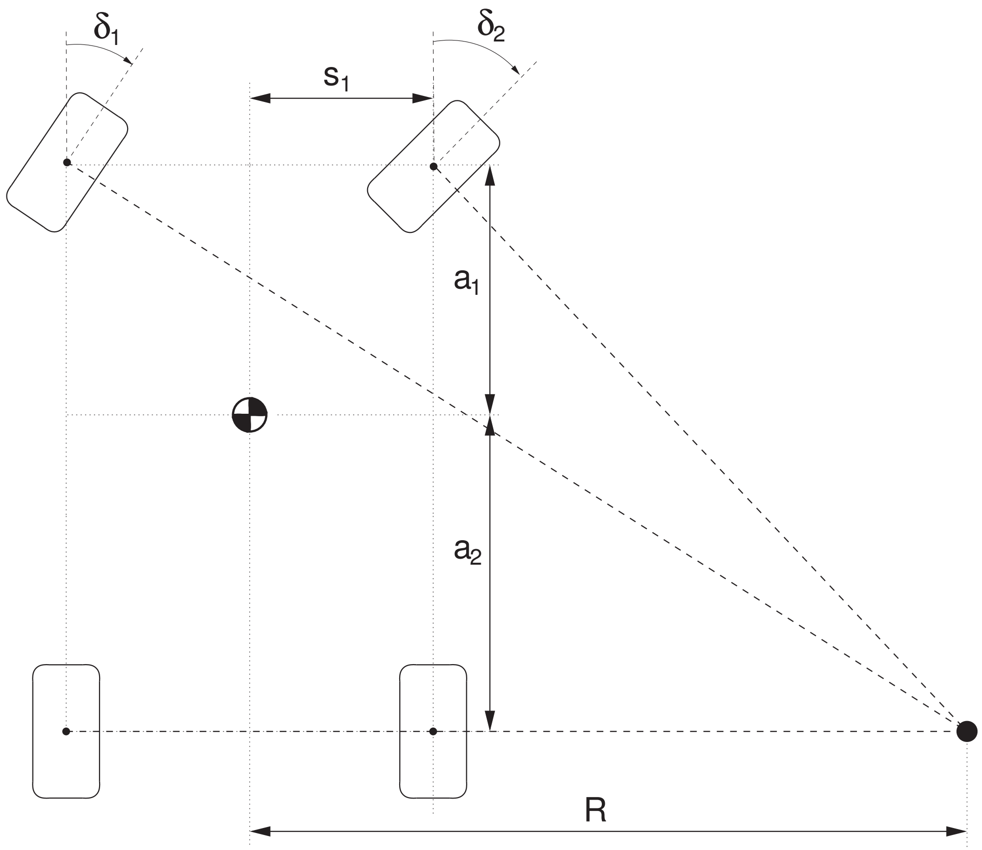
simulation results load transfer…



steering geometry

so far we assumed: $\delta_1 = \delta_2 = \delta$

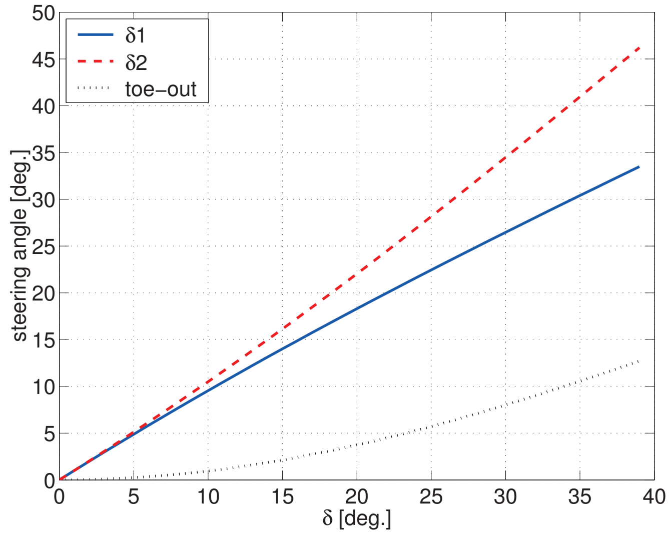
chassis – top view



Ackerman steering:

$$\tan\delta = \frac{l}{R},\ \tan\delta_1 = \frac{l}{R+s_1},\ \tan\delta_2 = \frac{l}{R-s_1}$$

Ackerman steering in case of a right turn:



some notes:

- Ackerman steering can be realised by design of steering linkage
- most cars have less toe-out for large steering angles (e.g. packaging: space in wheel bays)

on race-cars sometimes “parallel steer” or even “reverse Ackerman” steer is employed to maximise the tyre forces

Braking

a little experiment…

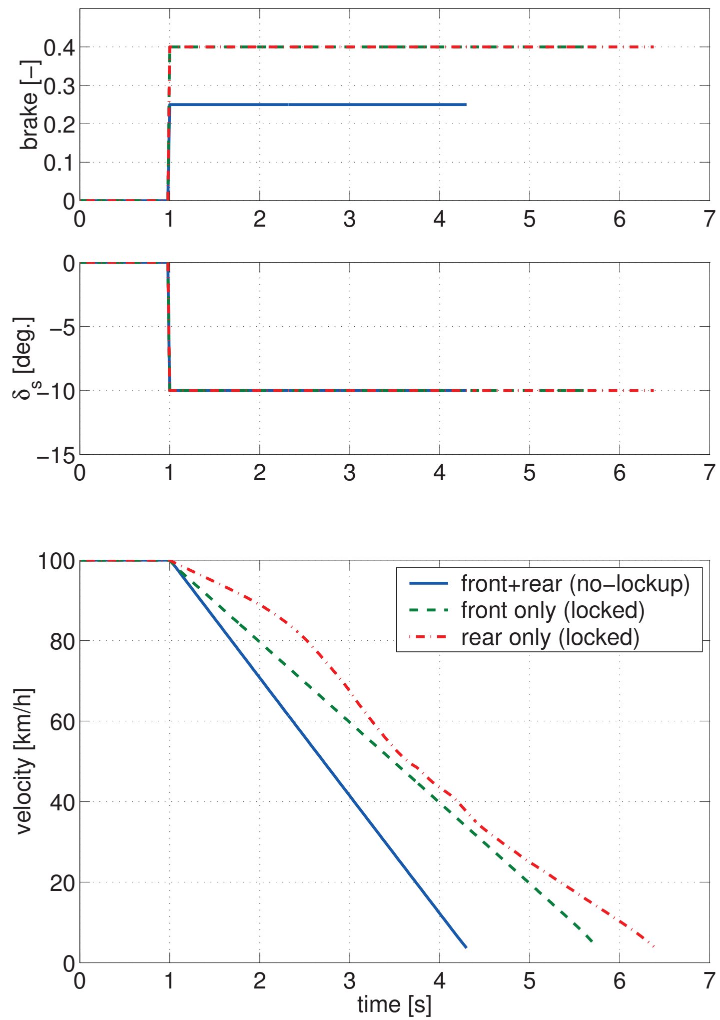
emergency braking manoeuvre + obstacle avoidance:
-very hard braking
-minor steering action (10 deg. steering wheel)

three configurations:

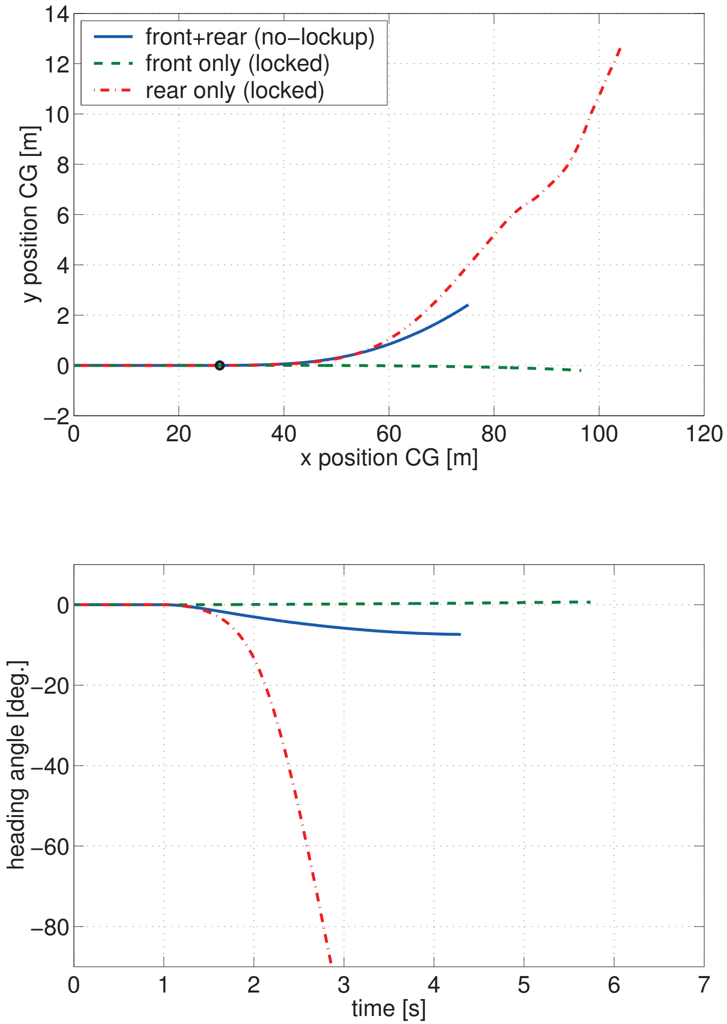
- front and rear brakes
  with a “proper” brake force distribution between front and rear axle, no wheel lock
- front brakes only, lock up of front wheels
- rear brakes only, lock up of rear wheels

(and no engine braking, clutch disengaged)

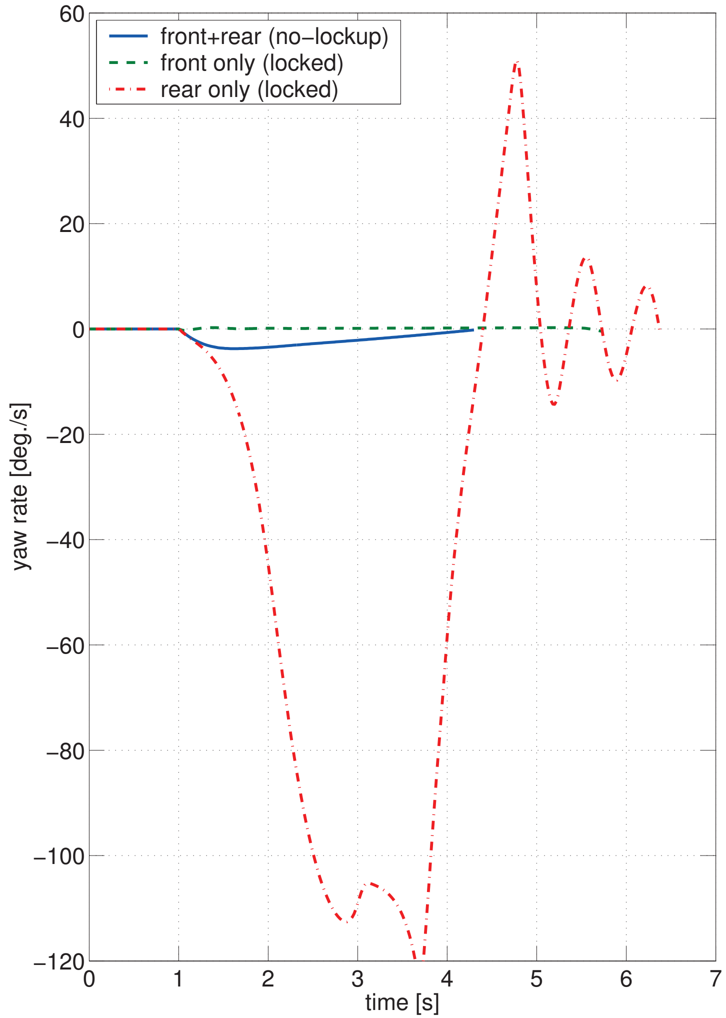
simulation results (1)



simulation results (2)



simulation results (3)



conclusion

- locked front wheels:
  vehicle continues in straight line, yaw velocity remains zero, steering has no effect

- locked rear wheels:
  vehicle out of control, very high yaw velocity, unstable situation

(simplified) explanation:
when a wheel locks $(\kappa = -1)$ the tyre cornering stiffness drops to (almost) zero.

- front wheel lock: excessive understeer
- rear wheel lock: excessive oversteer
  (□ unstable)

“older” cars may have a mechanical device to limit the brake pressure applied to the rear brakes

“newer” cars will have electronic support systems:

- ABS: anti-lock braking system,
- EBD: electronic brake force distribution

racing application…

- rally drivers may use the handbrake (locking the rear wheels) to quickly turn the car in the right direction, e.g. in the case of hairpins

braking in a turn

- initially the vehicle drives on a fixed radius R (in example shown: 100 m)
- constant forward velocity V to achieve a lateral acceleration of 4 m/s^2 (so 72 km/h in this example)
- open loop test: fixed steering wheel angle
- “step” input on brakes
- repeated tests with increased deceleration
- no wheel lock should occur during the test
- standardised in ISO 7975

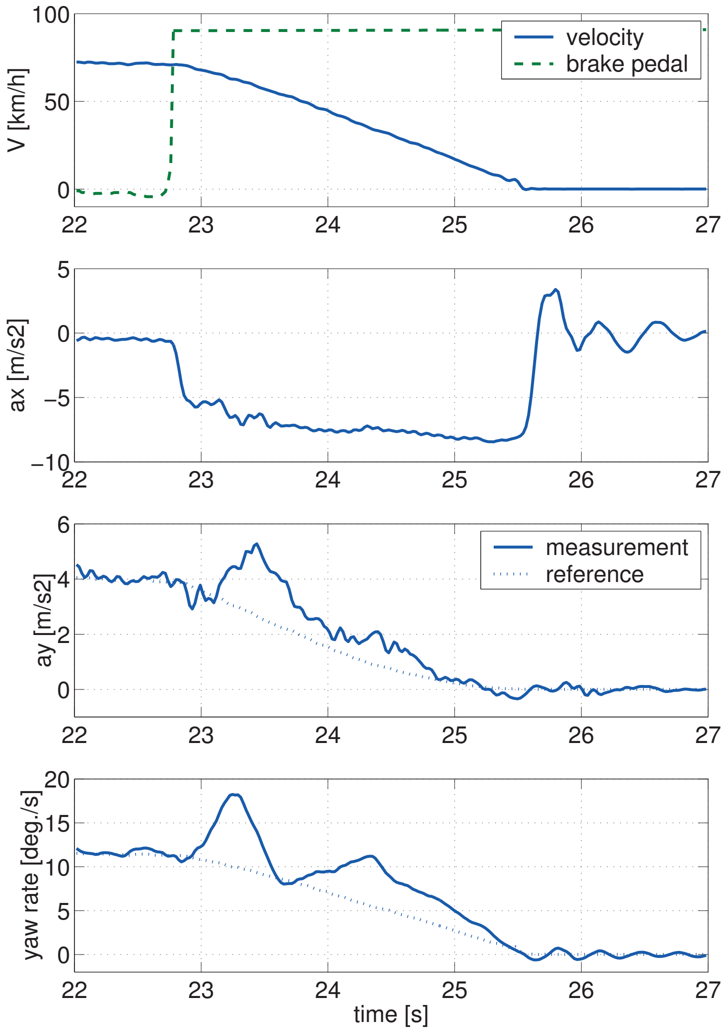
assuming that the vehicle remains on the radius it is possible to calculate reference curves for the lateral acceleration and yaw rate

lateral acceleration: $a_{y,ref}(t) = \dfrac{V(t)^2}{R}$

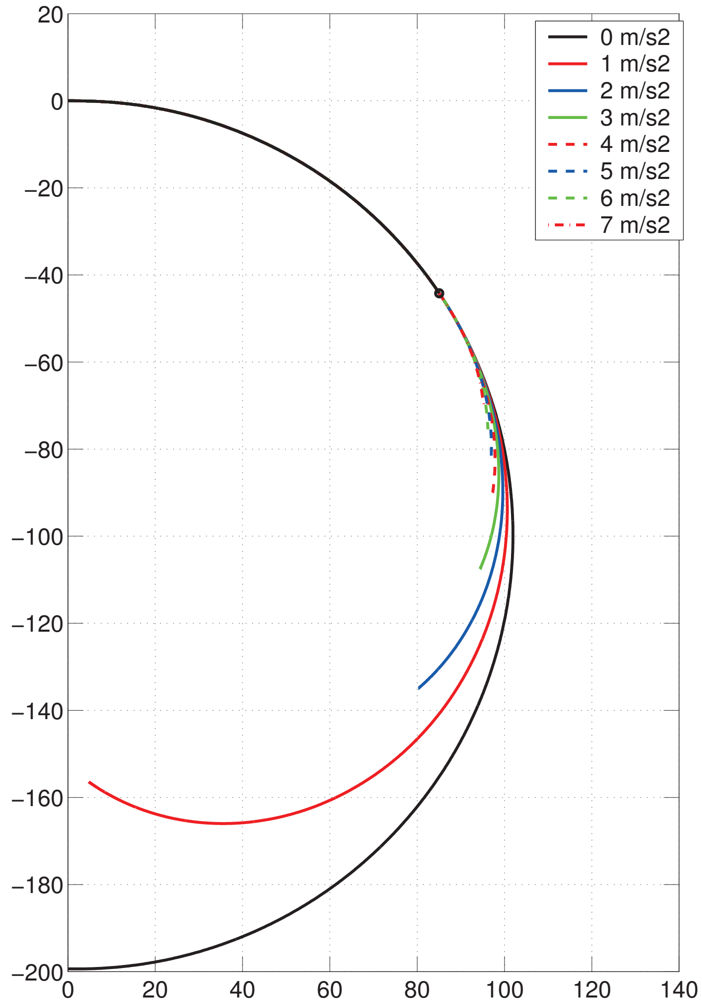
yaw rate: $r_{ref}(t) = \dfrac{V(t)}{R}$

deviations after 1 sec. are considered a good measure for the vehicle performance

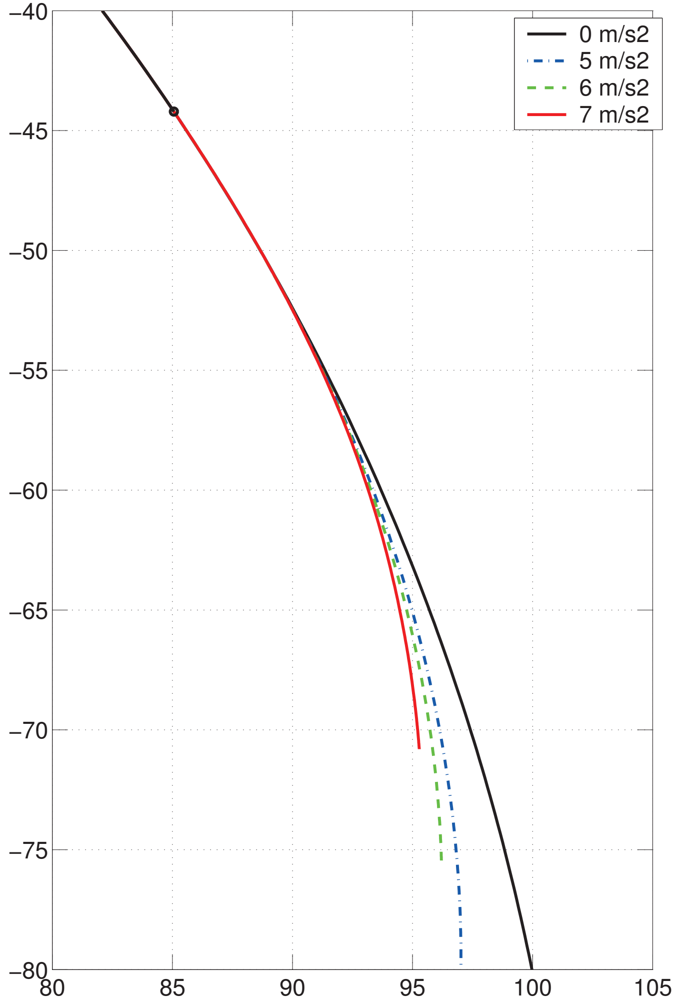
measurements…



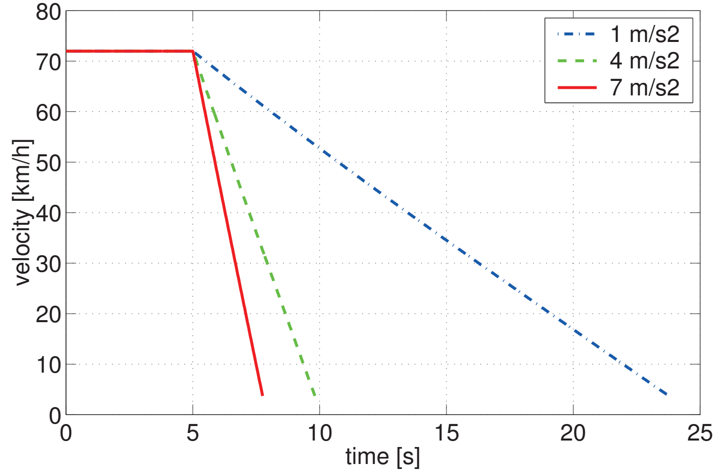
simulations(1), vehicle position [m]

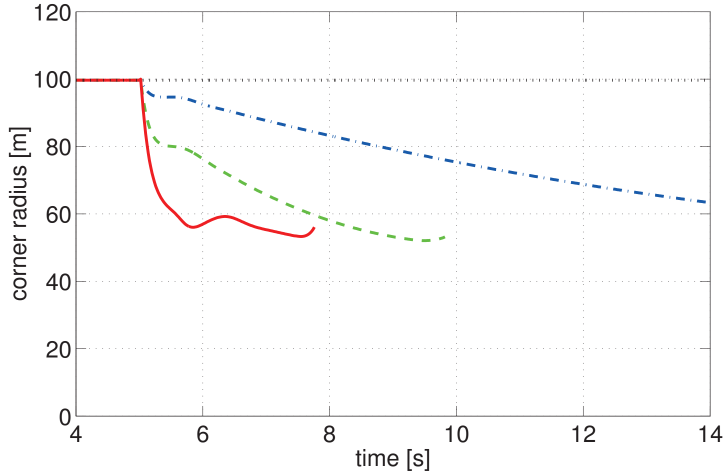


simulations(2), vehicle position [m]

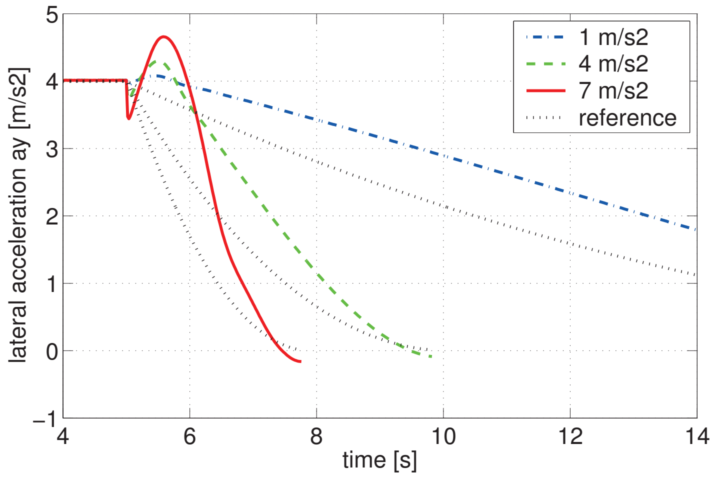


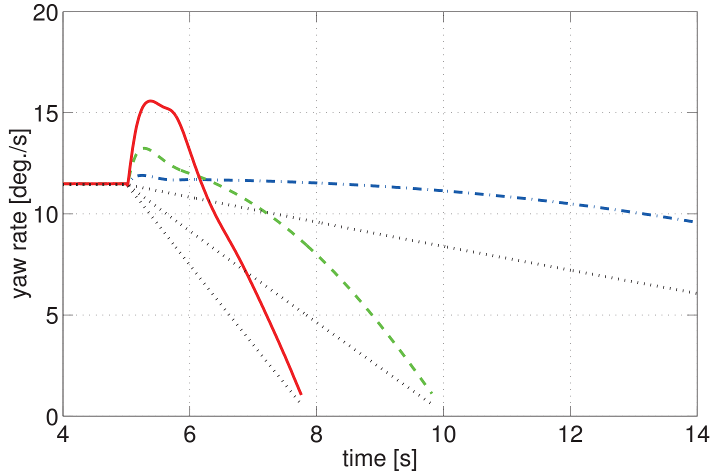
simulations(3)





simulations(4)





explanation…

tyre behaviour obviously!!!

steer angle, corner radius and tyre side slip angles are interrelated:

$$\frac{l}{R} = \delta - \alpha_1 + \alpha_2$$

(page 33, bicycle model)

due to brake application:

- additional load on the front tyres, reduced load on the rear tyres
- tyres have to develop brake forces □ drop in cornering stiffness due to longitudinal slip

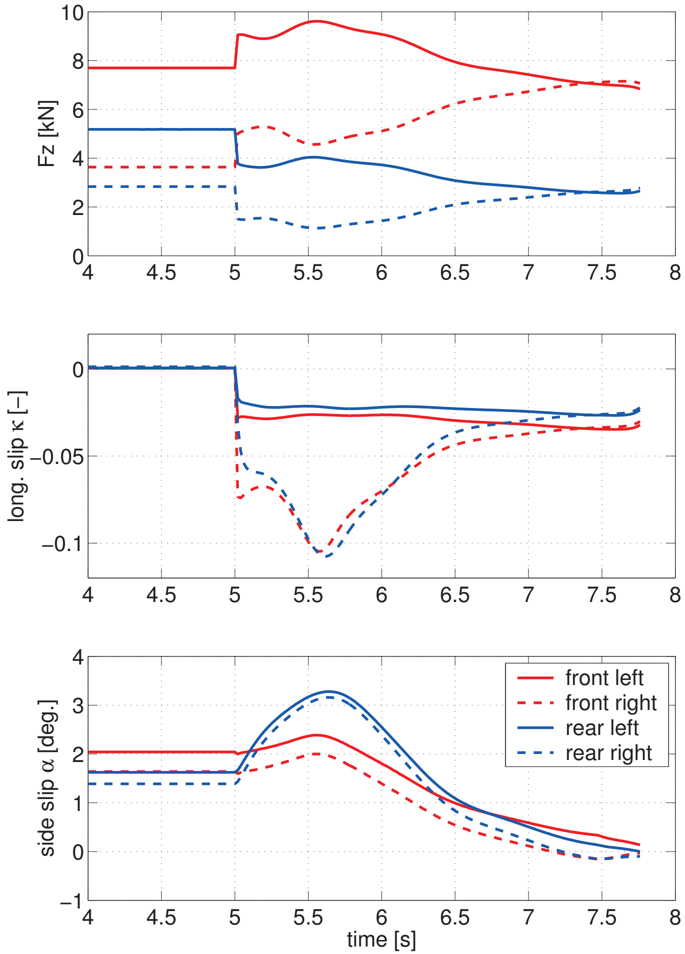
front tyres: reduction in cornering stiffness is compensated by higher vertical load

rear tyres: reduction in cornering stiffness and lower vertical load add up: □ increased tyre side slip angles on the rear axle

consequently the corner radius will become smaller. Due to the tightening of the corner we will get additional load transfer over the axles, which can make things worse!

also: if the vehicle is understeered, reducing the forward velocity will make the vehicle run on a smaller radius

simulation result (7 m/s^2 braking)



<u>"power-off"</u>
deceleration of a vehicle can also be achieved by releasing the throttle (without applying the brakes)

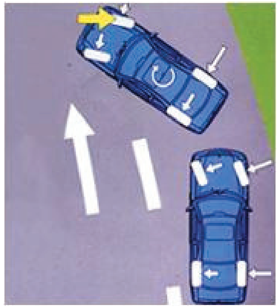
- will give rise to similar vehicle reactions as braking in a turn
- differences will exist between front and rear wheel drive vehicles

also a standardised test: ISO 9816

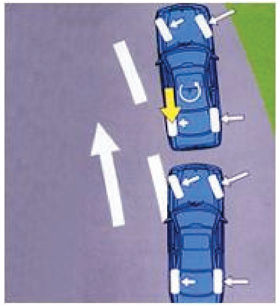
<u>"power-on"</u>
on a rear wheel drive vehicle the application of tractive forces may result in "power-oversteer"...

ESC: electronic stability control

brake wheels individually to keep vehicle on the desired path



correction for oversteer:
brake outer front wheel



correction for understeer:
brake inner rear wheel

steering wheel angle determines a set-point for yaw velocity rate and vehicle side slip angle.

differences with the actual vehicle motion are corrected by individual brake applications

additional sensors: lateral acceleration and yaw rate

Book Pacejka

- chapter 3.4 (page 140-147)

follow-up course:

Advanced Vehicle Dynamics 4J570

- vertical dynamics
- tyre dynamics
- shimmy vibrations
- suspension kinematics and steering system
- commercial vehicle design requirements
- truck ride comfort, vibrations, fatigue, loads, testing (components/full scale)

3 lecturers from DAF Trucks