Exercise 2

longitudinal

$$ma_x = F_x - F_{\text{drag, aero}}$$

driving: $F_x = \dfrac{P}{V} - F_{\text{rolling resistance}}$

adhesion limit: $|F_x| \le \mu_x F_z$

$$F_z = mg + c_l \cdot V^2$$

$$F_{\text{drag,aero}} = c_d \cdot V^2$$

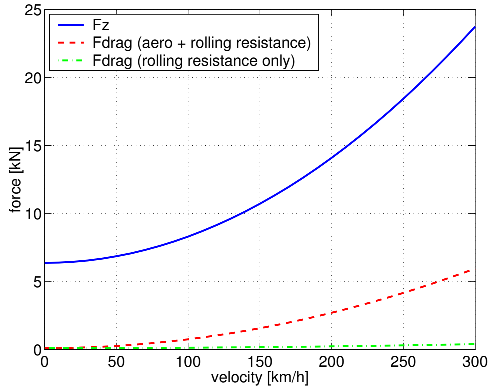
$$F_{\text{rolling resistance}} = 400 \cdot \frac{mg + c_l \cdot V^2}{mg + c_l \cdot (300/3.6)^2}$$

lateral

$$m|a_y| \le \mu_y F_z$$

parameters:
m=650 kg, $c_l = 2.5$, $c_d = 0.8$, $g = 9.81$

question 1
plot F_z and $F_{\text{drag, aero}} + F_{\text{rolling resistance}}$ as a function of forward velocity V



F_z standing still: 6.38 kN
F_z at 300 km/h: 23.74 kN

so increase by factor 3.7 *(see VD sheets page 72, a factor 4 is mentioned)*

rolling resistance 400 N at 300 km/h

so rolling resistance coef.: 400/23738=0.0169 (approx. 1.7 % which is o.k)

release of the throttle at 300 km/h:

$P = 0$ (engine power/torque zero)

$$ma_x = -F_{\text{drag, aero}} - F_{\text{rolling resistance}}$$

$F_{\text{drag, aero}} + F_{\text{rolling resistance}}$ = 5956 N

so a_x = -9.16 m/s^2

this is comparable to emergency braking on a normal road car…

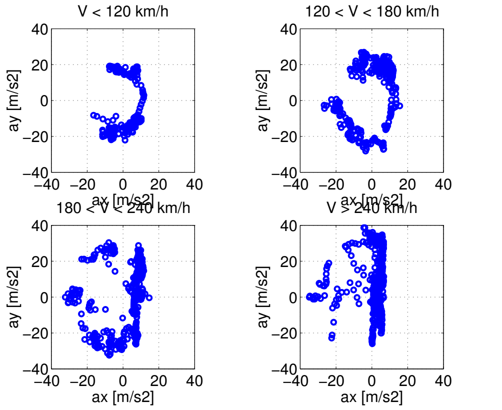
question 2
acceleration boundaries

longitudinal acceleration:
< 180 km/h: tyre grip limits acceleration
> 180 km/h: engine power limits acceleration

lateral acceleration:
- tyre friction: $m|a_y| \le \mu_y F_z$ and F_z increases with forward velocity

longitudinal deceleration:
- aerodynamic drag increases with forward velocity: $F_{\text{drag,aero}} = c_d \cdot V^2$
- braking: tyre friction $|F_x| \le \mu_x F_z$ and F_z increases with forward velocity

explain this figure:

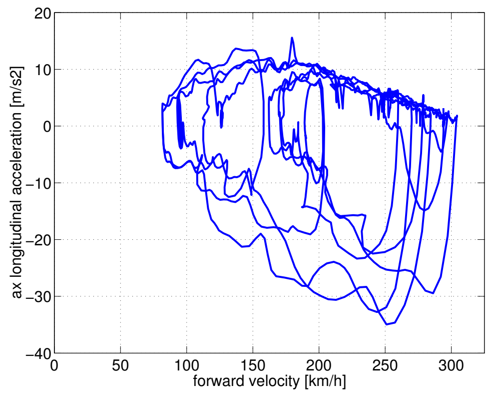


- friction circle limits the accelerations: circular shape (not only the tyre but also applies to the full vehicle)
- higher forward velocity: increasing down force bigger accelerations possible in lateral direction a_y and a_x (braking)
- on driving side $a_x > 0$, limitation at higher forward velocities due to engine power limitation
- parts of the friction circle may be missing due to the lay-out of the circuit

<u>question 3</u>
engine power

crude approximation: assume a top speed of 300 km/h.

problem: vehicle still has the capability to accelerate at 300 km/h, it has to slow down because it reaches a corner.



other approach:

$$m a_x = \frac{P}{V} - F_{\text{rolling resistance}} - F_{\text{drag, aero}}$$

for a given speed we determine a_x from the graph and calculate $F_{\text{drag, aero}} + F_{\text{rolling resistance}}$

200 km/h, a_x = 11 m/s^2 ⇒ P=546 kW
250 km/h, a_x = 6 m/s^2 ⇒ P=547 kW
300 km/h, a_x = 1 m/s^2 ⇒ P=560 kW
310 km/h, a_x = 0 m/s^2 ⇒ P=550 kW

so $P \approx$550 kW (= 747 hp)

engine torque:

$$M = \frac{P}{\omega} \quad \text{and} \quad \omega = 2\pi \cdot \frac{17000}{60}$$

M = 309 Nm, which is not particularly high
(Audi A8 V8 diesel: 650 Nm…)

remove wings: $c_l = 0$, $c_d = 0.2$
top speed: $a_x = 0$

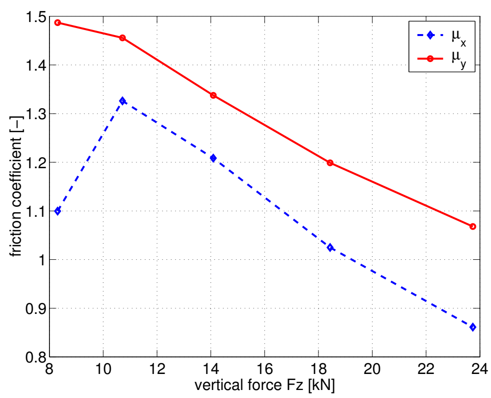
$$\frac{P}{V_{top}} - 0.0169 \cdot mg - c_d V_{top}^2 = 0$$

$P = 550$ kW ⇒ $V_{top} \approx 500$ km/h

<u>question 4</u>
plot the tyre friction coefficient as a function of vertical force F_z

velocity	ax (braking)	ay
100 km/h	-15 m/s^2	19 m/s^2
150 km/h	-24 m/s^2	24 m/s^2
200 km/h	-30 m/s^2	29 m/s^2
250 km/h	-35 m/s^2	34 m/s^2
300 km/h	-40 m/s^2	39 m/s^2

$$\mu_x = -\frac{m a_x + F_{\text{drag, aero}}}{F_z}, \quad \mu_y = \frac{m|a_y|}{F_z}$$

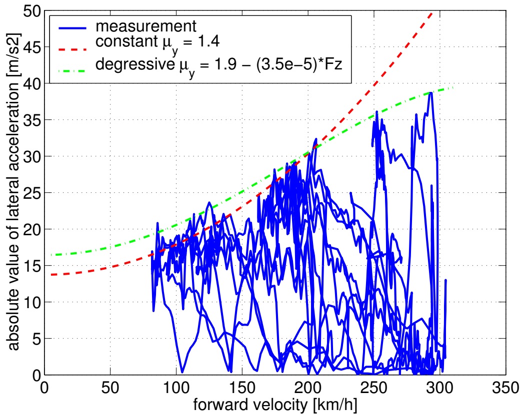


friction coefficient drops a function of the vertical force F_z

explain the linear increase of lateral acceleration with forward velocity:

- down force increases quadratically with forward velocity
- friction coefficient decreases linearly with the vertical load on the tyres

the resulting curve appears to be fairly linear in the forward velocity range of 75 to 300 km/h.



Note:

- the tyre friction coefficient is degressive with vertical load
- friction coefficients not identical in longitudinal and lateral direction (so we have a friction ellipse instead of a friction circle…)

Both findings are confirmed by tests on tyres. This is shown lecture 5, see VD sheets page 154 and 159.

Exercise 1 bonus question

Assume steady-state cornering at a fixed radius R. According to slide 112 the radius equals:

$$R = \frac{V}{r} = \frac{u}{r}$$

Alternatively, the radius can be obtained from the lateral acceleration (again slide 112):

$$R = \frac{V^2}{a_y}$$

A time increment dt later the travelled distance ds equals $Rd\psi$. Thus:

$$d\psi = \frac{ds}{R}$$

The total yaw angle ψ_i of the vehicle at a certain position i can therefore be obtained as follows:

$$\psi_i = \psi_{i-1} + d\psi = \psi_{i-1} + \frac{ds}{R}$$

For the circuit data we assume that the racing driver drives trough corners in a steady-state way, because this is the fastest way to drive around the track.

In addition we assume that distance travelled between two data samples is much smaller than the radius of curvature of the followed path. Then it can be assumed that the radius of curvature does not change much between two data samples.

We can now approximate the radius of curvature of the path between two points by either of the two radii R_i or R_{i-1}.

Alternatively we can use an average radius $\hat{R}$:

$$\hat{R} = \frac{R_i + R_{i-1}}{2} \quad \text{or} \quad \hat{R} = \frac{R_{i+1} + R_{i-1}}{2}$$

The total yaw angle ψ_i of the vehicle at position i can then be obtained as follows:

$$\psi_i = \psi_{i-1} + d\psi = \psi_{i-1} + \frac{s_i - s_{i-1}}{\hat{R}}$$

If the yaw angle of the vehicle is known, the global position (x_i, y_i) of the vehicle can be obtained as follows:

$$x_i = x_{i-1} + (s_i - s_{i-1})\cos\psi_i$$
$$y_i = y_{i-1} + (s_i - s_{i-1})\sin\psi_i$$

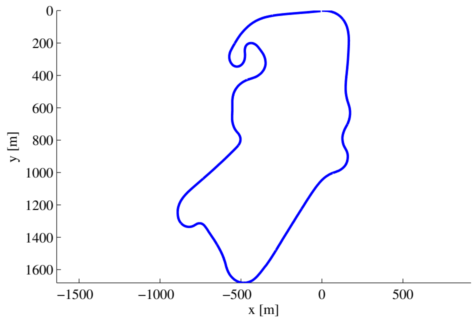
Alternatively ψ can be obtained by integration of the vehicle yaw velocity r:

$$\psi = \int_0^t r\,dt$$

A way to do this in MATLAB is to define a system with transfer function $\mathrm{H}(s) = \frac{1}{s}$ and use the command lsim to do the time simulation.

Example script:
```
H=tf(1,[1 0]);
psi=lsim(H,r,time);
```


If your calculation is correct you should obtain a figure like the one shown below:



Comparison with the different formula 1 circuits shows that the circuit is Silverstone, UK (circuit M).