Answers Exam Vehicle Dynamics (4L150)

1. Multiple-choice questions

1)	A
2)	C
3)	D
4)	B
5)	C
6)	B
7)	B
8)	C
9)	C
10)	A

2. Brush model

a) As shown in the given figure, the deformation of the bristles is linear. So we write:
$v = Ax + B$
At $x = 0$, $v = a\tan\alpha$, so: $B = a\tan\alpha$.
At $x = a$, $v = 0$, so: $Aa + a\tan\alpha = 0 \quad \Rightarrow \quad A = -\tan\alpha$
Thus: $v = -x\tan\alpha + a\tan\alpha = \tan\alpha(a - x)$
Check: $v(a) = 0$ and $v(-a) = 2a\tan\alpha$

b) The equation for the lateral force can be derived as follows:

$$F_y = \int_{-a}^{a} q_y(x)dx = \int_{-a}^{a} c_{py} v(x)\,dx = c_{py}\tan\alpha \int_{-a}^{a} (a - x)\,dx$$

$$= c_{py}\tan\alpha \left[\left(ax - \frac{1}{2}x^2\right)\right]_{-a}^{a} = c_{py}\tan\alpha\left(\left(a^2 - \frac{1}{2}a^2\right) - \left(-a^2 - \frac{1}{2}a^2\right)\right) = 2c_{py}a^2\tan\alpha$$

c) The equation for the self-aligning moment can be derived as follows:

$$M_z = \int_{-a}^{a} q_y(x)x\,dx = \int_{-a}^{a} c_{py} v(x)\,x\,dx = c_{py}\tan\alpha \int_{-a}^{a} (a - x)\,x\,dx = c_{py}\tan\alpha \int_{-a}^{a} \left(ax - x^2\right)dx$$

$$= c_{py}\tan\alpha \left[\left(a\frac{1}{2}x^2 - \frac{1}{3}x^3\right)\right]_{-a}^{a} = c_{py}\tan\alpha\left(\left(\frac{1}{2}a^3 - \frac{1}{3}a^3\right) - \left(\frac{1}{2}a^3 + \frac{1}{3}a^3\right)\right)$$

$$= -\frac{2}{3}c_{py}a^3\tan\alpha$$

d) Cornering stiffness: $$C_{f\alpha} = \left.\frac{\mathrm{d}F_y}{\mathrm{d}\alpha}\right|_{\alpha=0} = \left.2c_{py}a^2\left(1 + \tan^2\alpha\right)\right|_{\alpha=0} = 2c_{py}a^2$$

Self-aligning stiffness: $$C_{m\alpha} = -\left.\frac{\mathrm{d}M_z}{\mathrm{d}\alpha}\right|_{\alpha=0} = \left.\frac{2}{3}c_{py}a^3\left(1 + \tan^2\alpha\right)\right|_{\alpha=0} = \frac{2}{3}c_{py}a^3$$

e) Magnitude of the pneumatic trail:

$$t = -\frac{M_z}{F_y} = -\frac{-\frac{2}{3}c_{py}a^3\tan\alpha}{2c_{py}a^2\tan\alpha} = \frac{a}{3}$$

Remark: Answers for which $\tan\alpha$ is linearised are also correct.

3. Rear wheel steering

a) $m\left(\dot{v}+ur\right)=F_{y1}+F_{y2}$; $I\dot{r}=aF_{y1}-bF_{y2}$

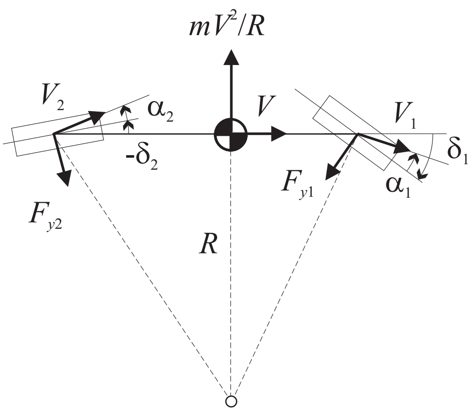
b) $\alpha_1=\delta_1-\dfrac{1}{u}\left(v+ar\right)$; $\alpha_2=\delta_2-\dfrac{1}{u}\left(v-br\right)$; $\beta=-\dfrac{v}{u}$

c) $\dfrac{mV^2}{R}=C_1\alpha_1+C_2\alpha_2$; $0=aC_1\alpha_1-bC_2\alpha_2$,

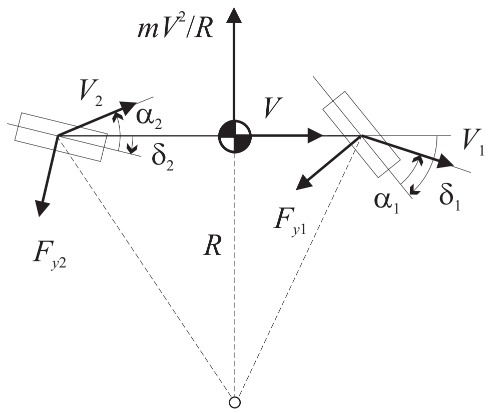
$\alpha_1=\delta_1+\beta-\dfrac{a}{R}$ $\alpha_2=\delta_2+\beta+\dfrac{b}{R}$

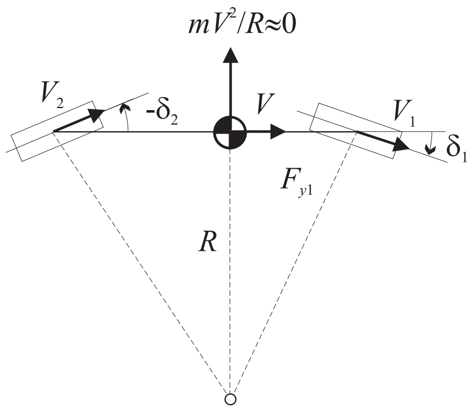
d)

low velocity



high velocity





e) $\dfrac{mV^2}{CR}=\alpha_1+\alpha_2=\delta_1+K\delta_1+2\beta$; $0=\alpha_1-\alpha_2=\delta_1-K\delta_1-\dfrac{2a}{R}$

$\beta=0$

$$\frac{mV^2}{CR}=(1+K)\delta_1=(1+K)\frac{2a}{R(1-K)} \;\rightarrow\; \frac{mV^2}{2aC}=\frac{(1+K)}{(1-K)} \;\rightarrow\; K=\frac{\dfrac{mV^2}{2aC}-1}{\dfrac{mV^2}{2aC}+1}$$

4. Straight line braking

a) $\Sigma F_x = ma_x \Leftrightarrow ma_x = F_{x1} + F_{x2}$

$\Sigma F_z = 0 \Leftrightarrow F_{z1} + F_{z2} - mg = 0 \quad \Rightarrow \quad F_{z2} = mg - F_{z1}$

$\Sigma M = 0 \Leftrightarrow a_1 F_{z1} - a_2 F_{z2} + ma_x h = 0$

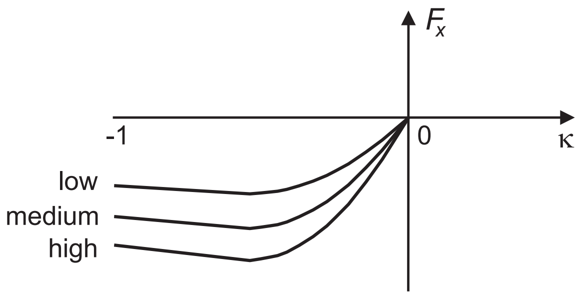
$$a_1 F_{z1} - a_2 \left(mg - F_{z1} \right) + ma_x h = 0$$

$$F_{z1} = \frac{a_2}{a_1 + a_2} mg - \frac{ma_x h}{a_1 + a_2} = \frac{a_2 mg - ma_x h}{l}$$

$$a_1 \left(mg - F_{z2} \right) - a_2 F_{z2} + ma_x h = 0$$

$$F_{z2} = \frac{a_1}{a_1 + a_2} mg + \frac{ma_x h}{a_1 + a_2} = \frac{a_1 mg + ma_x h}{l}$$

b)



c) $ma_x = F_{x1} + F_{x2} = -\mu_{x,peak} \left(F_{z1} + F_{z2} \right)$

$ma_x = -\mu_{x,peak} mg \quad \Rightarrow \quad a_x = -\mu_{x,peak} g$

d) $$p = \frac{M_{b1}}{M_{b1} + M_{b2}} = \frac{F_{x1} R}{RF_{x1} + RF_{x2}} = \frac{-\mu_{x,peak} F_{z1}}{-\mu_{x,peak} \left(F_{z1} + F_{z2} \right)} = \frac{a_2 mg - ma_x h}{lmg}$$

$$p = \frac{a_2 g - a_x h}{lg} = \frac{a_2 g + \mu_{x,peak} gh}{lg} = \frac{a_2 + \mu_{x,peak} h}{l}$$

e) On low $\mu_{x,peak}$, p to high => too much brake moment on the front axle => front wheels will lock up first => not possible to obtain maximum deceleration. $\mu_x < \mu_{x,peak}$ (over the peak); rear wheels have too little brake torque $\mu_x < \mu_{x,peak}$ (below the peak). Also: before front wheel lock ($\mu_x < \mu_{x,peak}$), rear wheels have too little brake torque.

5. Quarter car

a) The equations of motion

$$m_s\ddot{z}_s = -k_s\left(z_s - z_a\right) - d_s\left(\dot{z}_s - \dot{z}_a\right)$$

$$m_a\ddot{z}_a = k_s\left(z_s - z_a\right) + d_s\left(\dot{z}_s - \dot{z}_a\right) - k_t\left(z_a - z_r\right)$$

b) State space model:

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$$

$$\begin{pmatrix} \ddot{z}_s \\ \ddot{z}_a \\ \dot{z}_s \\ \dot{z}_a \end{pmatrix} = \begin{pmatrix} -\frac{d_s}{m_s} & \frac{d_s}{m_s} & -\frac{k_s}{m_s} & \frac{k_s}{m_s} \\ \frac{d_s}{m_a} & -\frac{d_s}{m_a} & \frac{k_s}{m_a} & -\frac{k_s+k_t}{m_a} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} \dot{z}_s \\ \dot{z}_a \\ z_s \\ z_a \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{k_t}{m_a} \\ 0 \\ 0 \end{pmatrix} z_r$$

$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du}$$

$$\begin{pmatrix} \Delta z \\ \ddot{z}_s \\ \Delta F_z \end{pmatrix} = \begin{pmatrix} 0 & 0 & -1 & 1 \\ -\frac{d_s}{m_s} & \frac{d_s}{m_s} & -\frac{k_s}{m_s} & \frac{k_s}{m_s} \\ 0 & 0 & 0 & -k_t \end{pmatrix} \cdot \begin{pmatrix} \dot{z}_s \\ \dot{z}_a \\ z_s \\ z_a \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ k_t \end{pmatrix} z_r$$

c) When design a suspension three design criteria have to be considered:

- spring travel: required space
- accelerations: they are a measure for the (dis-) comfort
- dynamic vertical tyre force: is a measure for the road holding

d) Spring:

- bump and rebound stops at the end of the stroke, stiffness will increase.
- non-linear spring characteristics: stiffness may increase as a function of deflection.

shock absorber:

- asymmetric behaviour: a low damping constant on compression and a high value on extension.
- friction due to normal forces on the bearings

tyre:

- the tyre may lose contact with the road, vertical force cannot become negative.
- tyre enveloping behaviour on short road obstacles