# **Exam Vehicle Dynamics (4L150)** 22-11-2005, 9:00-12:00 am

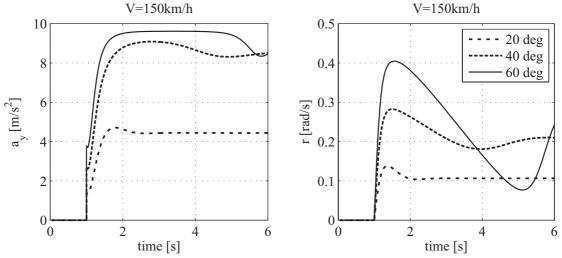
# **General remarks**

- Answers may also be given in Dutch
- Calculator is allowed
- No laptop, (lecture) notes, books, etc. are allowed

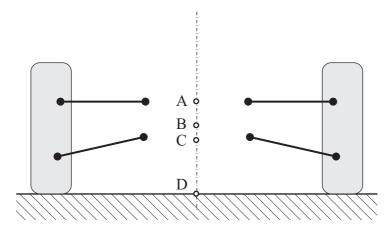
## A. Multiple-choice questions

- 1) Up to what level of lateral acceleration is the linear single track vehicle model valid?
  - a)  $0.4 \text{ m/s}^2$
  - b)  $4 \text{ m/s}^2$
  - c)  $40 \text{ m/s}^2$
  - d)  $400 \text{ m/s}^2$
- 2) A car drives 100 km/h. What happens if the rear wheels of this car block during straight-line braking?
  - a) vehicle continues to drive in a straight line
  - b) vehicle does not respond anymore to steering wheel input
  - c) rear of the vehicle brakes away
  - d) front wheels block as well due to extra load transfer
- 3) In the BBC TV program TopGear a professional driver, called "the Stig", tests cars on a circuit. For a certain passenger car he has trained on a dry circuit, but due to heavy rainfall the friction coefficient between tyre and road is suddenly halved. TopGear's Stig, who always drives through the corners of the circuit at the limit of grip, knows exactly what the maximum speeds for the corners are for the dry circuit, but not for the wet circuit. Therefore he asks his engineer what the speed in a specific corner should be that he normally (dry circuit) drives at 100 km/h. Consider you are his engineer. What do you answer?
  - a) about 20 km/h
  - b) about 50 km/h
  - c) about 70 km/h
  - d) about 90 km/h
- 4) Below several options that influence the under- and oversteer behaviour of a vehicle are listed. Which combination of options reduces the understeer tendency of an understeered vehicle most?
  - a) decrease front steering compliance, decrease roll stiffness front axle
  - b) increase front steering compliance, increase roll stiffness front axle
  - c) increase front steering compliance, decrease roll stiffness front axle
  - d) decrease front steering compliance, increase roll stiffness front axle
- 5) Which of the following statements is not true:
  - a) anti-dive helps to keep the vehicle level during braking
  - b) the anti-dive percentage is dependent on the fore/aft brake force distribution
  - c) too much anti-dive leads to an uncomfortable vehicle
  - d) anti-dive results in a much shorter stopping distance
- 6) Statement: an ESP system will correct excessive understeer by applying a brake force at the inner rear wheel of the car. This is:
  - a) true, it will result in an increase of the cornering radius
  - b) true, but perhaps not so effective due to the low vertical load on this tyre
  - c) not true, it will only slow the vehicle down
  - d) not true, the outer front tyre should be braked instead

In the figure below the lateral acceleration and yaw velocity response are shown for step steering wheel inputs of 20, 40 and 60 degrees at a vehicle forward velocity of 150 km/h.

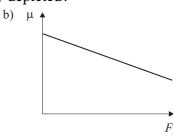


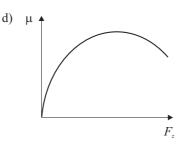
- 7) The cause of the oscillations at large steering angles is:
  - a) the system becomes unstable
  - b) the damping of the system decreases
  - c) the relaxation length decreases
  - d) the driver cannot maintain the steering wheel angle anymore
- 8) Consider again the figure above 7. What is the reason that the lateral acceleration does not exceed about  $9.5 \text{ m/s}^2$ 
  - a) forward velocity is not high enough
  - b) tyre peak friction limit is reached
  - c) steering wheel angle is not large enough
  - d) nonlinear increase of tyre force
- 9) Construct the roll centre of the double wishbone suspension shown below. Which point (A, B, C or D) corresponds to the roll centre?



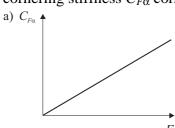
10) In which figure is the relation between measured vertical tyre load  $F_z$  and peak friction coefficient  $\mu$  correctly depicted?

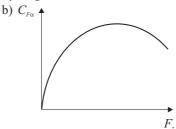
a) μ • F,

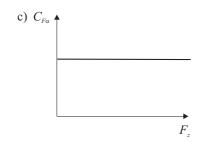


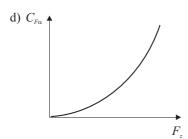


11) In which figure is the relation between measured vertical tyre load  $F_z$  and tyre cornering stiffness  $C_{F\alpha}$  correctly depicted?









- 12) Which of the following statements about the effective rolling radius is <u>not</u> correct?
  - a) depends on the vertical load
  - b) is always larger than the loaded radius
  - c) relates forward velocity and angular velocity for a free rolling tyre
  - d) determines the location of the slip point
- 13) In vehicle models the tyre relaxation length is included to get a better correspondence with measurement results. This is because:
  - a) the driver cannot steer infinitely fast
  - b) it takes some time to develop a side force
  - c) the tyre needs to roll a certain distance to develop a side force
  - d) a mathematical trick is needed to get the phase shift right

14) The base Magic Formula can be written as:

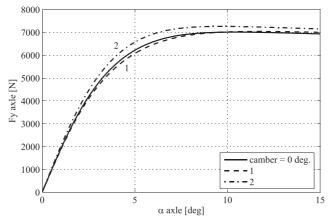
$$F_{y} = D\sin(C\arctan((1-E)B\alpha + E\arctan(B\alpha)))$$

For the design of the Formula Student vehicle some Magic Formula coefficients were provided by the tyre manufacturer. At the nominal load  $F_{z0}$  (5797.935 N) we have

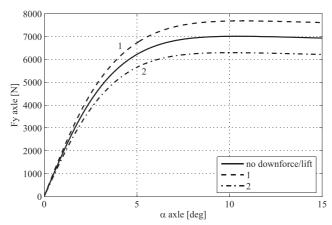
$$C = 0.199182$$
,  $D = 52.7007 \cdot F_{z_0}$  and  $BCD = 182.130974 \cdot F_{z_0}$ ,  $\alpha$  in radians

This shows that:

- a) the friction coefficient of the tyre is larger than 50, which is fine
- b) the cornering stiffness is very low
- c) the measurements have been executed with great accuracy
- d) it is garbage, these coefficients don't really make sense!
- 15) The figure below shows the effective axle characteristic of a vehicle. If or how does this characteristic change if the axle has negative camber?

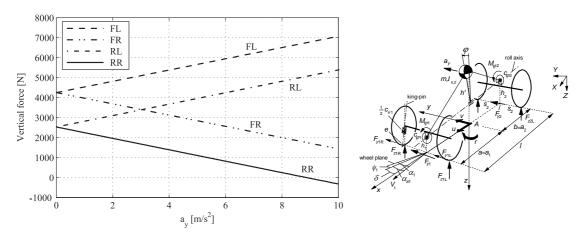


- a) line does not change
- b) line 1 is correct
- c) line 2 is correct
- d) the correct answer is not shown
- 16) The figure below shows the effective axle characteristic of a vehicle. If or how does this characteristic change in case of downforce?

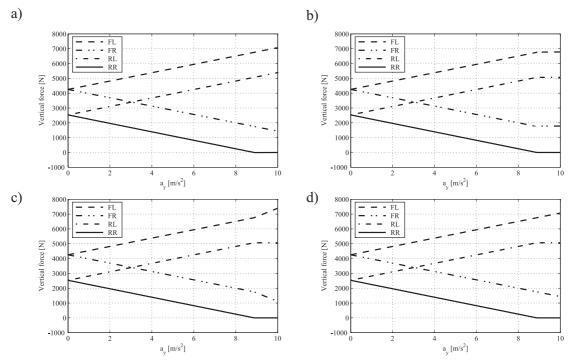


- a) line does not change
- b) line 1 is correct
- c) line 2 is correct
- d) the correct answer is not shown

17) A student has performed a steady-state cornering analysis with the two track vehicle model. He shows the following plot of vertical forces versus lateral acceleration to his professor (FL = front left tyre, FR = front right tyre, RL = rear left tyre, RR = rear right tyre).



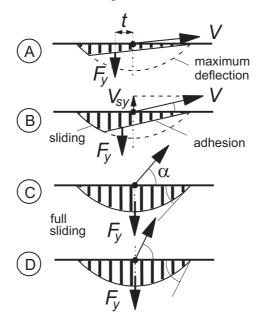
The professor comments however that this plot cannot be correct because the vertical load on the rear right tyre (RR) cannot become negative, because this would mean that the road pulls on the tyre. What is the correct plot?



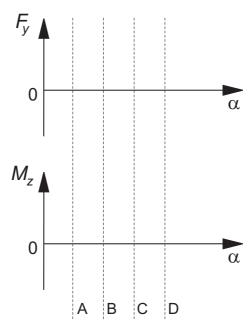
- 18) An understeered vehicle is driving through a highway corner (steady-state conditions). Suddenly the driver starts to brake hard without locking the wheels while keeping the steering wheel in the same position. What happens?
  - a) corner radius will become smaller
  - b) corner radius will stay the same
  - c) corner radius will become larger
  - d) vehicle will continue to drive in a straight line (cornering radius infinitely large)

#### **B.** Brush model

With the Brush model it is possible to qualitatively describe the pure sideslip characteristics of a tyre. The deformation pattern of the Brush model for different sideslip angles is shown below (A through D).

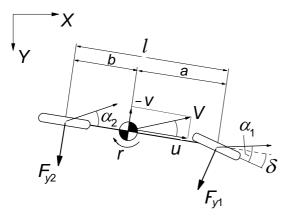


Copy the figure below and draw the lateral force and aligning moment characteristics of the brush model for pure sideslip. Indicate in these drawings where the above mentioned deformation patterns A through D occur. Use the Pacejka axis system, i.e. the default axis system in the lecture notes.



# C. Steady-state cornering

To analyse the steady-state cornering behaviour of a vehicle (mass m, moment of inertia I), the single track vehicle model is used, see figure below.



From the vehicle under consideration the following data is known:

- wheelbase: L = 2.5 m
- vertical force at the front axle:  $F_{z1} = 8000 \text{ N}$
- vertical force at the rear axle:  $F_{z2} = 6000 \text{ N}$
- front and rear axle cornering stiffnesses:  $C_1 = C_2 = 100000 \text{ N/rad}$
- steering ratio:  $i_s = \delta_s/\delta = 10$

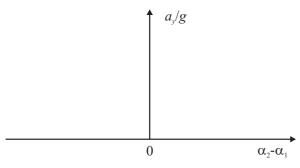
You may assume that the gravitational acceleration (g) equals about  $10 \text{ m/s}^2$ 

- a) Derive/give the equations of motion of the single track vehicle model.
- b) The understeer coefficient  $\eta$  is defined as:

$$\eta = \frac{F_{z1,static}}{C_1} - \frac{F_{z2,static}}{C_2} .$$

Calculate  $\eta$  and comment if the vehicle is understeered, neural steered or oversteered?

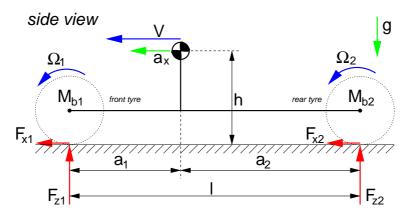
c) Copy the figure given below and sketch herein the handling diagram of the vehicle model. How is the understeer coefficient related to the handling diagram?



From now on we consider the vehicle driving a radius *R* of 100 m with a constant forward velocity of 72 km/h

- d) Calculate the required steering wheel angle ( $\delta_s$ ).
- e) Calculate the tyre side forces  $F_{v1}$  and  $F_{v2}$
- f) What are in this case the tyre sideslip angles ( $\alpha_1$  and  $\alpha_2$ )?
- g) Calculate the yaw velocity and the steady-state yaw velocity gain.

## D. Straight line braking



A 2-dimensional model as shown in the picture will be used to analyse the problem. Shown are the longitudinal acceleration  $a_x$  and forces acting on the tyres ( $F_x$  and  $F_z$ ) according to the ISO sign conventions. Obviously during braking the longitudinal acceleration  $a_x$  and longitudinal tyre force  $F_x$  will be negative. In the revolute joint between the wheel and chassis a braking moment  $M_b$  is applied ( $M_{b1}$  and  $M_{b2}$  for the front and rear tyre respectively). The vehicle mass equals m, the moment of inertia of the wheels may be ignored and the radius R of the front and rear tyre is the same.

a) Derive analytical expressions for the vertical force on the front and rear tyre ( $F_{z1}$  and  $F_{z2}$ ) as a function of the acceleration  $a_x$ . Check the result: during braking  $a_x$  will be negative and the vertical force on the front tyre will increase and the vertical force on the rear tyre will decrease with respect to the static situation.

b) The longitudinal force  $F_x$  of the tyre is a nonlinear function of the slip ratio  $\kappa$  and vertical load  $F_z$ . Make a sketch of the longitudinal force characteristic for three different vertical forces and a slip ratio ranging from wheel lock ( $\kappa = -1$ ) to free rolling ( $\kappa = 0$ ). Mark the different curves with: "low  $F_z$ ", "medium  $F_z$ " and "high  $F_z$ ".

c) Ideally both the front and rear tyres operate close to the peak in the longitudinal slip characteristic. Assume that the peak friction coefficient between tyre and road equals  $\mu_{x,peak}$  and may be considered constant. What is the relation between  $\mu_{x,peak}$  and acceleration  $a_x$  of the vehicle during braking?

d) Using the peak friction coefficient  $\mu_{x,peak}$  for both front and rear tyres, derive an analytical expression for the optimal brake moment distribution p.

$$p = \frac{M_{b1}}{M_{b1} + M_{b2}}$$

(for example p = 0.7 would indicate that 70 % of the total brake moment is applied to the front brakes and 30 % to the rear brakes) Hint: use the results of a) and c).

e) Suppose that the brake moment distribution p has been determined on a dry road surface and stays fixed, what happens on a slippery road surface (low  $\mu$ )? What about the brake moment to be applied? Will the front or rear wheels lock-up first? Is it possible to obtain the maximum deceleration as derived under c)? Give a short explanation.