Vehicle Dynamics 4L150 guided self-study exercise - 2010

The Vehicle Dynamics exam consists of two parts:

1- a written exam, no books or hand-outs are allowed at the exam
2- a written report on the guided self-study exercise, to be finalised 15 January 2011 latest. There is no second chance to redo this report in the academic year 2010/2011!

The final note will be a combination of 1 (2/3) and 2 (1/3).
To pass: 1 has to be ≥ 5 in any case!

This document describes the guided self-study exercise. It consists of three parts:

- half car simulation model using MATLAB/Simulink
- handling dynamics using the linear single track model
- brush tyre model

With respect to the report to be made on this exercise:

- keep it clear, logical and concise
- explain how you got to the results
- for equations: give references (e.g. slide number) or derivations
- number equations and graphs
- make sure that various lines in a graph can be distinguished clearly
- do a final check!!!

The finished report has to be delivered at the DCT secretary (room WH 0.143) or at my desk (room WH 0.130).

Success!

Igo Besselink

1 - Half car simulation with Matlab/Simulink

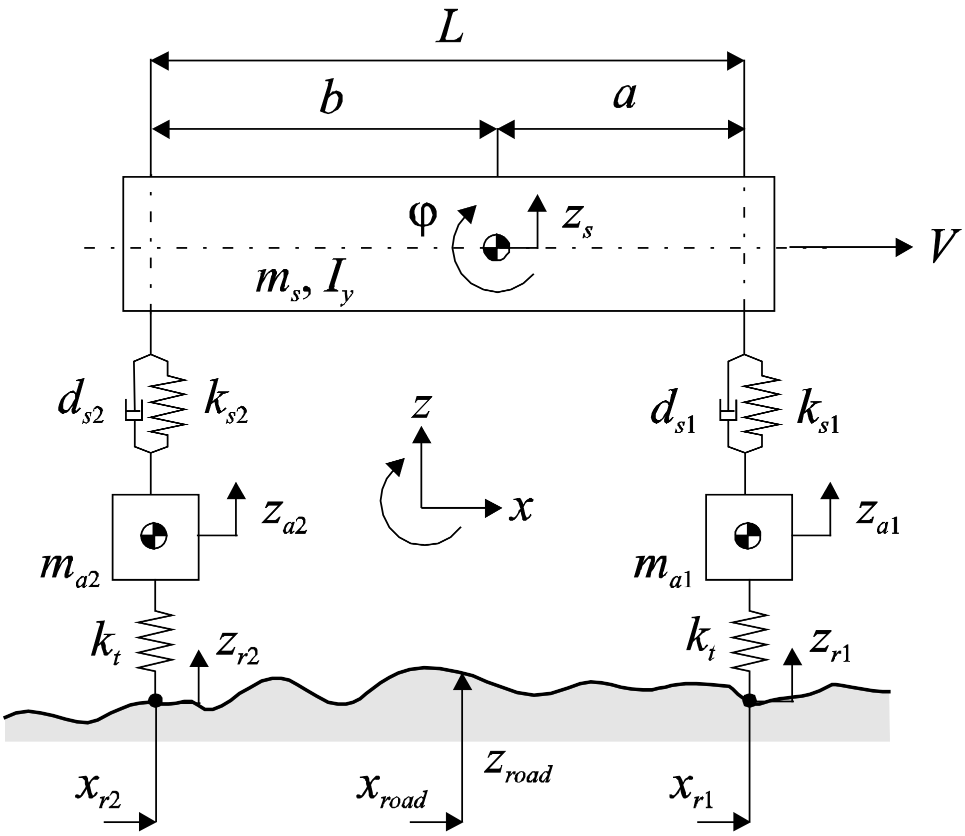

Figure 1: *Half car model.*

Of a vehicle the following parameters are known:

- Wheelbase L = 2.6 m
- Measured mass under the front left tyre: 365 kg
- Measured mass under the rear left tyre: 240 kg
- Moment of inertia vehicle body, I_y = 1000 kg m^2
- Front left unsprung mass, m_{a1} = 45 kg
- Rear left unsprung mass, m_{a2} = 40 kg
- Front left spring stiffness, k_{s1} = 24000 N/m
- Rear left spring stiffness, k_{s2} = 21500 N/m
- Front left suspension damping, d_{s1} = 1800 Ns/m
- Rear left suspension damping, d_{s2} = 1500 Ns/m
- Vertical tyre stiffness, k_t = 200000 N/m
- Gravity g = 9.81 m/s^2

a) Calculate the mass of half of the vehicle body, i.e. the sprung mass m_s of the half vehicle model. Also calculate the horizontal distances from the vehicle body's centre of gravity to the front and rear axle, distances *a* and *b* respectively.

b) Set up/derive the equations of motion for the half car model. Include gravity acting on the various bodies. As a reference you can have a look at page 10/11 and 46/47 of the "Vertical Dynamics 2008" handout.

c) Translate the equations of motion into a block diagram and build a Simulink model of the half car model. For the report the Simulink block diagram is sufficient, as long as it is clearly structured and well documented (e.g. minimal number of crossing signal lines, follows the physical lay-out of the system and using clear names). For some inspiration you can have a look at page 38 of the handout, which shows a quarter car block diagram. This model ('quarter car.mdl') is also provided in a slightly modified form with this exercise.

d) Test your model on a flat road surface. Make plots of the vertical acceleration of the centre of gravity and sprung masses, suspension/tyre deflection and vertical tyre force. What is happening? Check the values by means of a calculation and show that you can explain the steady-state response. Adapt the model so that all dynamics disappear at the start of the simulation by introducing a preload force in the springs. Calculate the required preload force and provide the numbers in the report.

e) Test your model using a 0.1 m step in road height after 20 m of forward driving with a velocity of 10 m/s. Check that the front tyre encounters the obstacle first. Make plots of the vertical tyre force, suspension deflection and centre of gravity position and pitch angle.

Now the half car model will be used to drive over a measured road surface with 80 km/h for 500 meters. The file mroad.mat contains the measured heights of the road profile per 0.05 m of travelled distance. The first column of this file contains the x-coordinates, the second column the road heights of the left track and the third the heights of the right track.

f) Plot the power spectra density of the road profile. As an example see page 22, figure 2.11 and page 24. Can you observe the -2 slope using log-log axis? What statement can you make with respect to the road class?

g) Perform a simulation with the half car model with 80 km/h and 500 meters of track. In all “To Workspace” blocks use a fixed sample time of 1 ms. Record the:

- accelerations of the vehicle body (vertical and pitch) and the unsprung masses.
- suspension deflections front and rear
- vertical tyre force

Determine the peak and RMS values of the requested signals. What is the ratio between the peak value and the RMS value?

h) Make power spectral density plots of:

- the vertical acceleration of the centre of gravity of the sprung mass
- the pitch acceleration of the sprung mass
- the vertical acceleration of the unsprung mass front and rear

Explain the results (hint: have a look at page 50/51 of the handout).

2 - Handling dynamics using the linear single track model

A vehicle has undergone a similar test programme as shown in lecture 5, for both an unloaded condition (driver only) and a fully loaded condition (driver + passengers and cargo). The data is stored in a number of *.mat files and graphs are available when running the MATLAB-file *plot_measurements_unloaded.m* and *plot_measurements_loaded.m* respectively. In the *.mat files SI units are used: N, kg, m, rad, s. So angles are in radians and the velocity is expressed in m/s! When creating the plots the conversion to km/h and degrees is made. Note that the steering wheel angle δ_s and not the steer angle δ of the front wheel is measured, these angles are related by the steering ratio, $\delta_s = i_s \cdot \delta$ and $i_s = 17$. The wheel base of the vehicle equals $l = 2.51$ m.

When the handling tests were executed the vehicle was put on scales, with the following results:

unloaded vehicle	front axle: 784.0 kg	rear axle: 522.5 kg
loaded vehicle	front axle: 809.5 kg	rear axle: 840.0 kg

a) Based on loads on the axles determine the location of the centre of gravity. Calculate the distances a and b for both the unloaded and loaded condition.

Steady state cornering

b) Provided with this exercise are two measurement files *steady_state_circular_unloaded.mat* and *steady_state_circular_loaded.mat.* Make a plot of the corner radius for the different tests and determine an average corner radius R (various possibilities exist to check this, see VD sheets page 112).

c) For the steady state circular test we have information on the steering angle δ_s and vehicle side slip angle β as a function of the lateral acceleration a_y. Determine the cornering stiffness C_1 and C_2 so that the single track vehicle model agrees with the measurement results in the linear range (0 to 4 m/s^2). Create plots in which the model is compared with the measurements. Hint: modify both *plot_measurements* files and add the results of the linear bicycle model to the existing graphs. Do this for both the loaded and unloaded vehicle, the cornering stiffness may vary between the unloaded and loaded situation.

d) Calculate the understeer gradient. Is the vehicle understeered, oversteered or neutral steer? What about the differences between the unloaded and loaded vehicle?

e) Make a plot of the yaw velocity gain for the velocity range 0 to 100 km/h, both experiment and model in one plot. An example is shown on the VD sheets page 114. Again do this for the unloaded and loaded vehicle and add the graphs to both *plot_measurements* files (in the section dealing with steady-state cornering).

Random steering

f) The file *plot_measurements* also displays the transfer functions of the vehicle. The numerical data can be found in the files *pseudo_random_unloaded.mat* and *pseudo_random_loaded.mat.* The transfer functions of the vehicle were measured at an average forward velocity of 100 km/h. Create a state-space model of the linear bicycle model (see also VD sheets page 99). Calculate the transfer functions between the steering angle δ_s and lateral acceleration a_y and yaw velocity *r*. Use the values of the cornering stiffness C_1 and C_2 obtained under c) and estimate the yaw moment of inertia I_{zz} using the rules of thumb given on VD sheets page 70. Again plot both the measured transfer function and calculated transfer function in a single plot (magnitude and phase). You may tune the value of yaw moment of inertia I_{zz} to get the best match. Hint: again the easiest way to create this plot is to modify both *plot_measurements* files and add the transfer function of the model. Create the plots for both the unloaded and loaded case.

g) As was shown in lecture 4 adding the relaxation length of the tyre improves the accuracy of the model, see VD sheets page 120-123. Extend the state space description as given on page 99 so that it includes the tyre relaxation behaviour. Give the A, B, C and D matrix.

h) Similar to f) compare the measured transfer functions with the model results, now including the tyre relaxation length σ_1, σ_2 . The relaxation length is dependent on the vertical load on the tyres, but the lateral stiffness of the tyres is (almost) constant (stiffness *k* in VD sheets page 120). Calculate the relaxation for the front and rear tyre by dividing the cornering stiffness through the tyre lateral stiffness, so:

$$\sigma_1 = \frac{C_1}{k} \quad \text{and} \quad \sigma_2 = \frac{C_2}{k}$$

Tune the magnitude of the tyre lateral stiffness k (and possibly the vehicle yaw moment of inertia I_{zz}) to get the best match with the measurements for both the loaded and unloaded case. Provide the plots of the transfer functions, has the model become more accurate?

Eigenfrequency analysis

i) By analysing the eigenvalues of the matrix A, we can determine the frequency and damping ratio of the system (see VD sheets page 101). Since the system dynamics of the vehicle are velocity dependent, this analysis has to be performed at different forward velocities. Using the results from f) and h) calculate the frequencies and damping ratios at 50, 100 and 150 km/h for the following four conditions:

- unloaded, no relaxation length
- loaded, no relaxation length
- unloaded, including relaxation length
- loaded, including relaxation length

Discuss the results.

J-turn

j) With the same vehicle a J-turn manoeuvre has been performed, both for the loaded and unloaded condition, the measurement data is given in the files Jturn_unloaded and Jturn_loaded respectively. Use the measured steering input as input for the simulation model and calculate the lateral acceleration and yaw velocity. The forward velocity may assumed to be constant and equals 100 km/h. Plot the simulation model results on top of the measurement results (see also VD sheets page 133). Compare the model with and without relaxation length, do you see an improvement? Make separate plots for the unloaded and loaded case.

Double lane change

k) Do the same as j) but now for the double lane change manoeuvre (files lane_change_unloaded.mat and lane_change_loaded.mat). Note that during the experiment the forward velocity is not constant and does not equal 100 km/h, calculate the average value. Compare the accuracy of the simulation model for part j) and k). What is the reason for the difference in accuracy?

3 - *The brush tyre model*

In the lectures on tyre modelling (5, 6 and 7) it is explained that the tyre can be seen as a function block with multiple inputs and outputs, see for example the figure below.

Title:
tyre_magic_formula.eps
Creator:
fig2dev Version 3.2 Patchlevel 3d
Preview:
This EPS picture was not saved
with a preview included in it.
Comment:
This EPS picture will print to a
PostScript printer, but not to
other types of printers.

In lecture 6 the brush tyre model has been discussed and in this case effect of the inclination angle γ is neglected. In this exercise you are asked to program the brush model yourself and to make plots of the resulting tyre characteristics. The parameters are given in the table below.

parameter	description	value
r_f	free tyre radius	0.3 m
c_z	tyre vertical stiffness	250000 N/m
c_p	tread element stiffness	$9{\cdot}10^6$ N/m^2
μ	friction coefficient	1.2

Tyre model parameters

a) Make a plot of half of the contact length a as a function of the vertical force F_z using the empirical equation given on VD sheets page 181 (suggested range: 0 to 10 kN).

The next thing to be done is to program the brush model as a MATLAB function, typically the calling syntax would be:

```
[Fx,Fy,Mz] = brush(kappa,alpha,Fz)
```

So you create a file “brush.m”, which will be similar to the following lines:

```
function [Fx,Fy,Mz] = brush(kappa,alpha,Fz)
... (your algorithm)
Fx = ... (provide the right equations here)
Fy = ... (provide the right equations here)
Mz = ... (provide the right equations here)
return
```

This function can then be called from the MATLAB command line. The function should be able to handle combined slip conditions.

b) Program the function and include the listing of `brush.m` in the report. How do you handle the case of complete wheel lock? (κ exactly equal to -1)

c) Make plots of the pure slip characteristics at a vertical load of 4000 N:

- Longitudinal force F_x versus pure longitudinal slip κ
- Lateral force F_y versus pure sideslip angle α
- Aligning moment M_z versus pure sideslip angle α

You can use the graphs on VD sheets page 190 and 194 as a reference.

d) Make plots of the combined slip characteristics at a vertical load of 4000 N. For the side slip angle α use a value of 0, 5, 20 and 90 degrees, vary the longitudinal slip κ in the range from -1 to 1.

- Longitudinal force F_x versus longitudinal slip κ
- Lateral force F_y versus longitudinal slip κ
- Aligning moment M_z versus longitudinal slip κ

You can use the graphs on VD sheets page 198 as a reference. Do you think the tyre model results at a side slip angle of 90 degrees are realistic? Yes or no, and why…