Vehicle Dynamics 4L150 guided self-study - exercise 1 (2008)

Half vehicle model simulation with Matlab

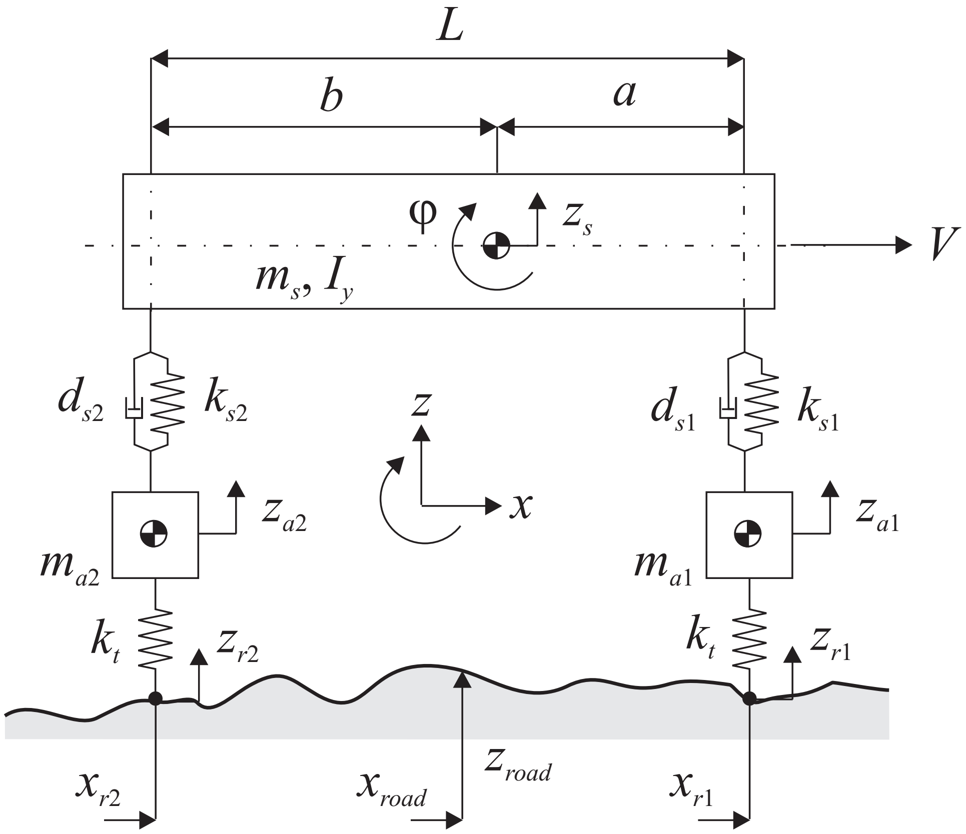

Figure 1: *Half vehicle model.*

Of a vehicle the following parameters are known:

- Wheelbase L = 2.6 m
- Measured mass under the front left tyre: 365 kg
- Measured mass under the rear left tyre: 240 kg
- Moment of inertia vehicle body, I_y = 1000 kg m^2
- Front left unsprung mass, m_{a1} = 45 kg
- Rear left unsprung mass, m_{a2} = 40 kg
- Front left spring stiffness, k_{s1} = 24000 N/m
- Rear left spring stiffness, k_{s2} = 21500 N/m
- Front left suspension damping, d_{s1} = 1800 Ns/m
- Rear left suspension damping, d_{s2} = 1500 Ns/m
- Vertical tyre stiffness, k_t = 200000 N/m

1. Calculate the mass of half of the vehicle body, i.e. the sprung mass m_s of the half vehicle model. Also calculate the horizontal distances from the vehicle body's centre of gravity to the front and rear axle, distances *a* and *b* respectively.

2. Build a half car model (Figure 1) of this vehicle. Proceed according to the following steps:
   a. Derive the equations of motion.
   b. The equations of motion can be written in the following form:
      $\mathbf{M}\ddot{\mathbf{z}} + \mathbf{D}\dot{\mathbf{z}} + \mathbf{K}\mathbf{z} = \mathbf{F}\mathbf{u}$
      Write down the mass matrix **M**, the damping matrix **D**, the stiffness matrix **K** and the excitation matrix **F**.
   c. Write down the equations of motion in state-space form:
      $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$
      $\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)$
      The output column **y** should at least contain the accelerations of the vehicle body and unsprung masses, the suspension travel of the front and rear axles and the dynamic tyre loads front and rear.
   d. Create an m-file containing the numerical values of the vehicle model parameters and the state-space matrices determined under c.
   e. Calculate the (damped) natural frequencies and damping ratios of the system. If your model is implemented correctly you should get the damped natural frequencies that are given in figure 2. In figure 2 the mode shapes are also shown. Give the correct names to these mode shapes. Do we have 'pure' bounce and pitch modes or does any coupling occur? Where do the poles of the motion lie for these two modes?

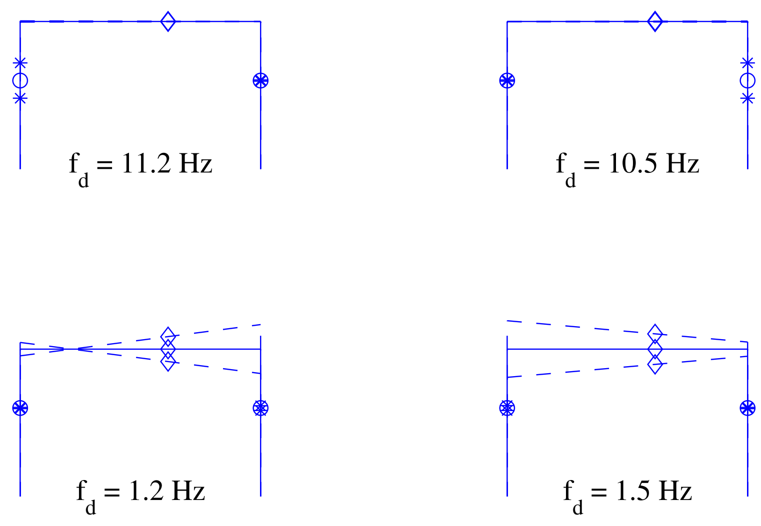

Figure 2: *Mode shapes and damped natural frequencies of the half vehicle model (note: front axle is on the right of the figures).*

3. We want to analyse the vehicle response while driving over a 10 mm step obstacle with 80 km/h. The step is positioned 4 m ahead of the rear tyres of the vehicle. The simulation is done for 3 seconds with a time step of 1 ms.
    a. Determine expressions for the x-position of the front and rear tyre as a function of time (x_{r1} and x_{r2} in Figure 1). The step road surface may be defined using a lookup table and interpolation function. Use the following function syntax: `zr1 = interp1(x_road,z_road,xr1)`; where `x_road` is a vector containing the horizontal coordinates and `z_road` the corresponding road height. Make a plot of the road inputs z_{r1} and z_{r2} as a function of time. Check that the rear tyre encounters the step at a later time than the front tyre.
    b. Perform a simulation to determine the dynamic response of the vehicle. Use the MATLAB command `[Y] = lsim(sys,U,t)`. The time vector is specified by `t`, in this case: `t=[0:0.001:3]`. The input matrix `U` contains the road heights z_{r1} and z_{r2} for each time step. The state-space object `sys` can be created using the following MATLAB command: `sys=ss(A,B,C,D)`. Note that all initial conditions are zero. Plot the responses of the following signals versus time:
        - vertical acceleration of the sprung mass
        - body pitch acceleration
        - vertical acceleration of the unsprung masses
        - front and rear spring travel
        - dynamic wheel loads front and rear
    c. Explain the results obtained under b.

4. Now the half car model will be used to drive over a measured road surface with 80 km/h for 500 meters. The file `mroad.mat` contains the measured heights of the road profile per 0.05 m of travelled distance. The first column of this file contains the x-coordinates, the second column the road heights of the left track and the third the heights of the right track.
    a. Determine the required simulation time and use again a time step of 1 ms. Replace the step road surface from the previous exercise with the measured road profile. Calculate the road inputs z_{r1} and z_{r2} as a function of time and make a plot of them.
    b. Perform a simulation to determine the dynamic response of the vehicle. Calculate the RMS (root mean square value) and minimum and maximum values of the following output signals:
        - vertical acceleration of the sprung mass
        - pitch acceleration of the sprung mass
        - vertical acceleration of the unsprung masses
        - front and rear spring travel
        - dynamic wheel loads front and rear

You can use the MATLAB functions min, max and std for this purpose. Can you confirm that the maximum/minimum values are about 3 to 4 times the standard deviation?

c. Analyse the results in the frequency domain. Plot the power spectral density (PSD) of the following two signals:
   - vertical acceleration of the sprung mass
   - pitch acceleration of the sprung mass

   Use the MATLAB function [Pxx,F]=pwelch(X,[],[],[],Fs) for obtaining the power spectral density of a signal X, where Fs is the sample frequency. Explain the existence of null points in the power spectral densities.

If there is some time left you can perform some additional experiments: e.g. what happens if we modify the dampers, springs, forward velocity, (un)sprung mass, etc.