機器學習基石(林軒田老師)筆記

Lecture 6: Theory of Generalization

Proof $B(N,k) \geq \mathrm{B}(\mathrm{N}-1,\mathrm{k})+\mathrm{B}(\mathrm{N}-1,\mathrm{k}-1)$ ----------- (*)

First, we prove $B(4,3) \geq \mathrm{B}(3,3)+\mathrm{B}(3,2)$.

Here is an example of $\mathrm{B}(3,3)$:

X1	X2	X3
○	○	○
×	○	○
○	×	○
○	○	×
○	×	×
×	○	×
×	×	○

Now, we add a new point X4, with value ○ in all dichotomies.

X1	X2	X3	X4
○	○	○	○
×	○	○	○
○	×	○	○
○	○	×	○
○	×	×	○
×	○	×	○
×	×	○	○

Then note that data in the orange box is an example of $\mathrm{B}(3,2)$. We copy these data, and add × as their value on X4. Then we add these rows.

X1	X2	X3	X4
○	○	○	○
×	○	○	○
○	×	○	○
○	○	×	○
○	×	×	○
×	○	×	○
×	×	○	○
○	○	○	×
×	○	○	×
○	×	○	×
○	○	×	×

To show the set is an example of $B(4,3)$, we need to prove “for any 3 points, they are not shattered”.

Consider {X1, X2, X3}, since every row come from $\mathrm{B}(3,3)$, they are not shattered.

For any 3 points contain X4, for example {X2,X3,X4}, we suppose they are shattered. Then we have every possible values on them. Thus if we fix X4 with value ×
, we still have every possible values on X2 and X3.

X1	X2	X3	X4
○	○	○	○
×	○	○	○
○	×	○	○
○	○	×	○
○	×	×	○
×	○	×	○
×	×	○	○
○	○	○	×
×	○	○	×
○	×	○	×
○	○	×	×

But the values in green box is copied from above with the property $\mathrm{B}(3,2)$. Those values can not shatter X2 and X3, which leads to a contradiction. Thus {X2,X3,X4} is not shattered. Similarly, any other 3 points can be shown that they are not shattered.

We complete the prove of $B(4,3) \geq \mathrm{B}(3,3) + \mathrm{B}(3,2)$.

The argument above also proves $B(N,k) \geq \mathrm{B}(\mathrm{N}-1,\mathrm{k}) + \mathrm{B}(\mathrm{N}-1,\mathrm{k}-1)$, since we didn’t use any particular property of $B(4,3), \mathrm{B}(3,3)$ or $\mathrm{B}(3,2)$.