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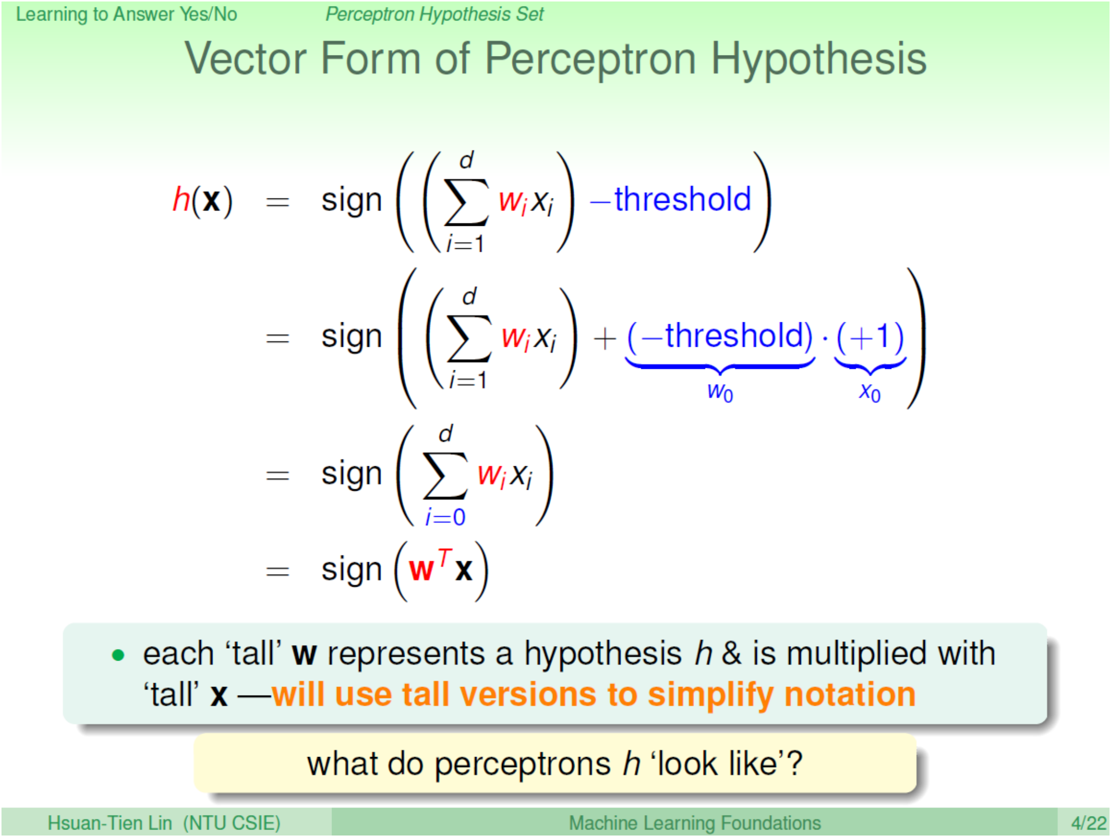
Lecture 2: Learning to Answer Yes/No

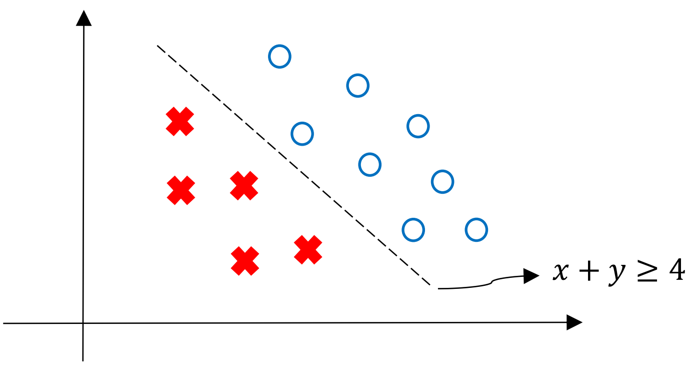
About threshold in PLA

1. Why use threshold?

The following is page 4 of 02_handout of Machine Learning Foundations.



PLA find a hyperplane to separate data with different values(-1 and 1), but the hyperplane which separate data usually doesn't contain origin.



We need to add one dimension to the data. If data is of dimension two, we want to find a good $h(x) = \operatorname{sign}(w_0 + w_1x_1 + w_2x_2)$. In the example above, $x + y \geq 4$ will separate the data perfectly.

$$x + y \geq 4 \iff x + y - 4 \geq 0 \leftrightarrow \mathrm{h(x)} = \mathrm{sign(-4 + x + y)} \qquad (a)$$

Now $x + y \geq 4$ corresponds to $(-4, 1,1)$ a 3-dimensional vector. And $(-4, 1,1)$ also correspond to a plane $-4\mathrm{p} + \mathrm{q} + \mathrm{r} = 0$ in 3-dimensional space which contains the origin.

$$\mathrm{h(x)} = \mathrm{sign(-4 + x + y)} \leftrightarrow (-4,1,1) \leftrightarrow -4\mathrm{p} + \mathrm{q} + \mathrm{r} = 0 \qquad \text{(b)}$$

Combine (a) and (b), we have:

$$x + y \geq 4 \leftrightarrow -4\mathrm{p} + \mathrm{q} + \mathrm{r} = 0$$

That is, a line in 2-dimensional space (not necessary contains the origin) corresponds to a plane (contains the origin) in 3-dimensional space.

Note that $-4\mathrm{p} + \mathrm{q} + \mathrm{r} = 0 \leftrightarrow x + y \geq 4$, while $4\mathrm{p} - \mathrm{q} - \mathrm{r} = 0 \leftrightarrow x + y \leq 4$ in this correspondence.

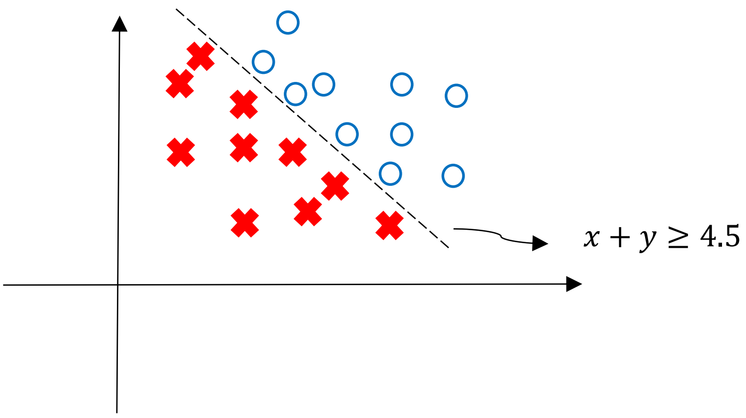
Therefore, to find a good $\mathrm{h(x)}$ is equal to find a good hyperplane.

2. Integral coefficient $w_{0,0}$?

In PLA, we have:

$$W_{t+1} \leftarrow W_t + y_{n(t)} X_{n(t)}$$

We usually let $W_0 = \left(w_{0,0}, w_{0,1}, \dots, w_{0,d}\right) = (0,0, \dots ,0)$, and we have $x_{n(t),0} = 1$ for all data. Thus we have $w_{t,0} \epsilon \mathbb{Z}$ for all t. What if we have data like this:



What $w_{t,0}$ will we get when PLA stop?

In fact, we will get some W_t close to $(-9,2,2)$ since

$$x + y \geq 4.5 \iff 2x + 2y \geq 9$$

And by properties of real numbers, we can find a good W_t even if all $w_{t,0} \epsilon \mathbb{Z}$.