Math 62CM: Syllabus

Introduction

This is an advanced class which requires a significant amount of work. It should not be difficult if you attend lectures, read lecture notes and make serious efforts to do all homeworks. In general in Mathematics there are no difficult topics, only topics which are not properly explained or structured. Sometimes, looking at the problem from the right angle makes the subject much more understandable. As it is stated in the course description on *Stanford Course Explore*, this is a proof based class. This is slightly misleading. Of course, it is proof based because in Mathematics everything should be proof based, or otherwise this is not Mathematics but something else. But the main feature of this class is that it introduces many new concepts, which are important in Mathematics and its applications. When a new concept is introduced, it is not sufficient to be able to repeat the definition, as to properly understand it one needs to see this the notion in action, i.e. seeing many examples and doing several exercises. That's the main goal of homeworks. Besides homeworks, the lecture notes have also several exercises, some of which easy, some more difficult. If you have any questions please come to discuss with the instructor or the TA during office hours. If you cannot come during regular office hours, please contact us to make special arrangements. From time to time there will be posted additional non-mandatory homeworks. We highly recommend you to attempt doing these additional problems and then discuss with us at office hours.

We will be using the Campuswire platform for communications. Please accept the invitation which will be sent to you. In particular, you will receive before each lecture the list of sections in the Lecture notes which need to be reviewed before the lecture.

The 63CM class which will be taught in the Spring quarter will be a natural continuation of 62CM, and in particular it will use some of the machinery developed in 62CM.

The list of topics and approximate schedule

1. Week 1. Jan. 6-10

   - Dual vector space.
   - Elements of topology in a vector space. Continuous maps and functions.
   - Differential and gradient of a smooth function.
   - Gradient vector field.
   - Fields of various objects: vector fields, differential 1-forms, plane fields, etc.
   - Vector fields as first order partial differential operators.

2. Week 2: Jan.13-20

   - Exact and closed 1-forms.
   - Smooth maps and their differentials. Pull-back operator.
   - Hyperplane fields and Pfaffian equations and the problem of integrability.
   - Diffeomorphisms and curvilinear coordinates.

3. Week 3: Jan. 21-24

   - Inverse and Implicit Function Theorems.
   - Riemann integral for functions and differential 1-forms.
   - Integration of 1-forms along curves.
   - Integral of closed and exact forms.
   - Homotopic paths.

4. Week 4: Jan. 27-31

    - Linear and multilinear functions.
    - Tensor product.
    - Symmetric and skew-symmetric tensors.
    - Exterior product and its properties.
    - Orientation, Determinants and Volume.
    - Dualities.

5. Week 5: Feb. 3-7

    - Differential k-forms.
    - Pull-back, exterior product and other operations on differential forms.
    - Exterior differential, its properties and computation. Closed and exact forms.
    - Vector analysis operations: $grad, curl$ and div.
    - Lie derivative and Cartan's formula.

6. Week 6: Feb. 10-14

    - Integration of functions over domains in $\mathbb{R}^n$ and Fubini's theorem.
    - Integration of n-forms over bounded domains in $\mathbb{R}^n$.
    - Change of variables formula.
    - Computation and applications of multiple integrals.

7. Week 7: Feb. 18-21

    - Manifolds: definitions, constructions and examples. Manifolds with boundary.
    - -Tangent spaces and tangent bundles.
    - Vector bundles and their homomorphisms.

- Orientation and co-orientation.
- Integration of k-forms over k-dimensional submanifolds.

8. Week 8: Feb. 24-28

   - Technical tools: partition of unity and smoothing.
   - Stokes's formula and its special cases. Integration of closed and exact forms.
   - Proof of Stokes's theorem.
   - Vector analysis. Computations in curvilinear coordinates.
   - Integral formulas of vector analysis. Work and Flux.

9. Week 9: March 3-7

   - Homotopy. Simply connected and k-connected domains.
   - Complex-valued 1-forms, holomorphic functions and Cauchy integral formula.
   - Winding and linking numbers.

10. Week 10: March 10-14

    - Degree of a map
    - De Rham cohomology and applications.
    - Applications to problems in Physics.

Important dates

January 20 (Mon) Martin Luther King, Jr., Day (holiday, no classes)

January 24 (Fri, 5 p.m.) Final day to drop the class (I hope nobody does)

February 12 (Wed) Take-home Midterm is posted. It is due Tuesday, February 18

February 17 (Mon) Presidents' Day (holiday, no classes)

February 28 (Fri, 5 p.m.) Course withdrawal deadline

March 20 (Th, 8:30am) Final Exam

Access and Accommodations

Stanford is committed to providing equal educational opportunities for disabled students. Disabled students are a valued and essential part of the Stanford community. We welcome you to our class. If you experience disability, please register with the Office of Accessible Education (OAE). Professional staff will evaluate your needs, support appropriate and reasonable accommodations, and prepare an Academic Accommodation Letter for faculty. To get started, or to re-initiate services, please visit http://oae.stanford.edu/. If you already have an Academic Accommodation Letter, please use the vanity URL https://goto.stanford.edu/math62cmoae to upload it and detail the specific accommodations you will need in this course. Letters are preferred by the end of week 2, and at least two weeks in advance of any exam, so we may partner with you and OAE to identify any barriers to access and inclusion that might be encountered in your experience of this course. New accommodation letters, or revised letters, are welcome throughout the quarter; please note that there may be constraints in fulfilling last-minute requests.