Quiz 1

Don't forget to write down clearly your **Name**:

1. Multiple choices (10 points). Mark the box in front of the correct answer.

(1) Which of the following equals the integral $\int_0^2 \sqrt{4-x^2}dx$?
☐ π ☐ 2π ☐ 3π ☐ 4π

(2) Which of the following equals the integral $\int_{-\pi}^{\pi} \cos 2x dx$?
☐ 1 ☐ π ☐ 0 ☐ -1

(3) Integration by parts is a consequence of the Fundamental Theorem of Calculus together and which of the following differentiation rule that is the most relevant?

☐ $(f(g(x)))' = f'(g(x))g'(x)$ ☐ $(f(x)+g(x))' = f'(x)+g'(x)$
☐ $(f(x)g(x))' = f'(x)g(x)+f(x)g'(x)$ ☐ $(af(x))' = af'(x)$ if $a \in \mathbb{R}$.

(4) When integrating $\int \sqrt{a^2-x^2}dx$ and we use the substitution rule $x = a\cos\theta$, which of the following could the range of θ be?

☐ $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ ☐ $0 \leq x \leq \pi$
☐ $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$ ☐ $0 \leq x \leq 2\pi$

(5) Perform long division for the rational expression $\frac{x^2-4x+4}{x-2}$. What should you get?
☐ $x+2$ ☐ $x+4$ ☐ $x-4$ ☐ $x-2$

2. Use your favorite way to prove the following integration formula (5 points).

$$\int e^x \sin x dx = \frac{1}{2}e^x(\sin x - \cos x) + C$$

3. Prove the following statement (5 points). Let $f(x)$ be a continuous function that only takes positive values. Assume that $\int_{-\infty}^{\infty} f(x)dx$ converges. Use the meaning of (improper) integration to show that, for any two real numbers $a, b \in \mathbb{R}$ and $a \neq b$, we have

$$\int_{-\infty}^{a} f(x)dx + \int_{a}^{\infty} f(x)dx = \int_{-\infty}^{b} f(x)dx + \int_{b}^{\infty} f(x)dx.$$