Exercises for Week 6

The work handed in should be entirely your own. You can consult Dummit and Foote, Artin and/or the class notes but nothing else. To receive full credit, justify your answer in a clear and logical way. Due Oct. 16.

Reading. With Dummit and Foote, please read Sections 3.1, 3.2. Alternatively, read Artin Section 2.12.

1. Consider $S_{n-1} \subset S_n$ as permutations of the first $n-1$ letters. Show that S_{n-1} is not a normal subgroup.

2. If H, K are two groups, show that $H \times \{1_K\}, \{1_H\} \times K$ are normal subgroups of $H \times K$. Identify the corresponding quotient groups.

3. In the general linear group $GL_3(\mathbb{R})$, consider the subgroups

$$H := \left\{ \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} \middle| a, b, c \in \mathbb{R} \right\}, \qquad K := \left\{ \begin{pmatrix} 1 & 0 & a \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \middle| a \in \mathbb{R} \right\},$$

   Show that K is a normal subgroup of H, and identify the quotient group with a more elementary one.

4. Let H and K be subgroups of a group G. Show that the intersection $xH \cap yK$ of two cosets of H and K is either empty or else a coset for the subgroup $H \cap K$.