Math 151b Midterm Exam

February 4, 2018

Notes. No discussions allowed for the exam. You have one day to finish the exam (Thu 5:00pm-Friday 5:00pm). No late exams shall receive a grade.

You may use any result from Hatcher unless instructed to repeat the proof.

(i) Prove that, if $\{h^*\}$ is a reduced cohomology theory on CW complexes and $\delta : h^k \longrightarrow h^{k+1}$ is the coboundary map, then there is an isomorphism

$$h^*(X) \longrightarrow h^{*+k}(\Sigma^k X)$$

that is natural with respect to any continuous map of CW complexes.

(ii) Let $\mathbb{k}$ be a field and (C_*, ∂) be a chain complex of $\mathbb{k}$-vector spaces. Let (C^*, δ) be the dual complex such that $C^k := \mathrm{Hom}(C_k, \mathbb{k})$ and $\delta := \partial^*$.

(a) Prove that $\mathrm{H}_n(C_*, \partial) \cong \mathrm{Hom}(\mathrm{H}^n(C^*, \delta))$. (Point out all the differences with the proof of the Universal Coefficient Theorem we did in class).

(b) Prove that if X is a finite CW complex, and $\mathbb{k}$ is a field, then

$$\sum_m (-1)^m \dim_{\mathbb{k}} \mathrm{H}^*(X; \mathbb{k})$$

does not depend on the choice of the field $\mathbb{k}$, and is equal to the Euler number $\chi(X)$ of X (the alternating sums of cell-numbers according to their dimensions).

(iii) Prove that, if $n > m$, then any continuous map $f : \mathbb{R}P^n \longrightarrow \mathbb{R}P^m$ induces the zero map on cohomology in each positive degree.

(iv) Hatcher, Exercise 6 and 26 of Hatcher, Chapter III.3.