## A Summary of Commonly Used Math Notations

- $\in$ : This indicates an element belonging to a set. Example "5 is a natural number" can be written as " $5 \in \mathbb{N}$ ". Compare with the subset notation below.
- $\subset$ : This indicates the inclusion relation between sets. For instance, " $\{1,2\}$  is a subset of  $\{1,2,3,4\}$ " can be abbreviated as  $\{1,2\} \subset \{1,2,3,4\}$ . There are some variations of this notation, like  $\supset$  (containment),  $\subseteq$  (subset, may be equal),  $\supseteq$  (containment, may be equal),  $\subseteq$  (subset and not equal)  $\supseteq$ . Notice that this is a relationship between sets, while the notation  $\in$  is between an element and a set. For instance,

$$\{5\} \subset \{1, 2, 3, 4, 5\}, \qquad 5 \in \{1, 2, 3, 4, 5\}$$

both tell you that the element 5 is in the set  $\{1, 2, 3, 4, 5\}$ . But  $\{5\} \in \{1, 2, 3, 4, 5\}$  is NOT mathematically correct.

- $\forall$ : This means "for any" or "for all." Example: "For any vector v in a vector space V, a scalar multiple of it is still in the vector space" can be written as " $\forall v \in V$ , and  $\forall c \in \mathbb{F}$ ,  $cv \in \mathbb{F}$ ."
- $\exists$ : This means "there exists." For example: "For any  $\epsilon > 0$ , there exists a  $\delta > 0$  such that..." can be written as " $\forall \epsilon > 0$ ,  $\exists \delta > 0$  s.t. ..." A negation of this symbol is  $\nexists$ , meaning "there does not exist."
- ⇒ means the statement before the arrow implies the statement after the arrow. ⇔ indicates the equivalence of statements.
- $\mathbb{N}$ : the set of natural numbers  $\mathbb{N} = \{0, 1, 2, 3, 4, \dots\}$ .  $\mathbb{Z}$ : the set of integers.  $\mathbb{Q}$ : the set of rational numbers (this is the first example of a field).  $\mathbb{R}$ : the set of real numbers.  $\mathbb{C}$ : the set of complex numbers.
- $\sum$  and  $\prod$ : meaning taking sum/product of all terms behind the symbol satisfying some conditions. For instance, summing over all natural numbers from 0 to 100 can be written as

$$0 + 1 + \dots + 100 = \sum_{k=0}^{100} k.$$

• Greek letters :  $\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta, \theta, \kappa, \lambda$  etc. Used as alternatives for English letters. In math different alphabets are usually used to represent concepts of different nature.

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- $\cup$ : union of sets  $A \cup B = \{x | x \in A \text{ or } x \in B\}.$
- $\bullet \ \cap : \text{intersection of sets } A \cap B = \{x | x \in A \text{ and } x \in B\}.$