Math 113 Quiz 5

Don't forget to write down clearly your **Name**: and **ID number:**

1. True or False (10 points) Check the box in front of a correct statement.

- ☐ If G is a group and H its subgroup, then the left translation of G on G/H is transitive.
- ☐ The group $SO(3, \mathbb{R})$ is a normal subgroup of $O(3, \mathbb{R})$.
- ☐ The group $SO(3, \mathbb{R})$ is abelian.
- ☐ Any finite subgroup of $SO(3, \mathbb{R})$ is abelian.
- ☐ The rotational symmetry groups for the cube and octahedron are isomorphic.
- ☐ Any ring is commutative.
- ☐ The set $\mathbb{N}$ under the usual $+$ and $\times$ forms a ring.
- ☐ The group ring FG is commutative if the group G is abelian.
- ☐ The cyclic group $\mathbb{Z}/(8)$ is a module over the ring $\mathbb{Z}$.
- ☐ The polynomial ring $F[x]$ is commutative.

2. Symmetry group (5 points). Answer the following questions and justify your answer.
Show that the rotational symmetry group of the regular cube inside $\mathbb{R}^3$ has 24 elements.

3. Isomorphic rings (5 points). Answer the following questions and justify your answer.

In class we have defined the path ring $R := F(1 \longrightarrow 2)$ as the F-vector space spanned by

$$R \cong F(1) \oplus F(2) \oplus F(1|2),$$

where (1), (2) are the lazy paths stationed at the vertices, while $(1|2)$ is the path from vertex 1 to 2. Prove that this ring is isomorphic to the 2×2 upper triangular matrix ring by explicitly defining an isomorphism

$$\phi : R \to U := \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \middle| a, b, c \in F \right\}.$$