Math 120, Practice exam 2

The best way to use this practice exam is to take it as if it were the real test. Set aside 90 minutes to work on it without interruption. Don't use the book or notes or calculators, and don't peak at the solutions until you're done.

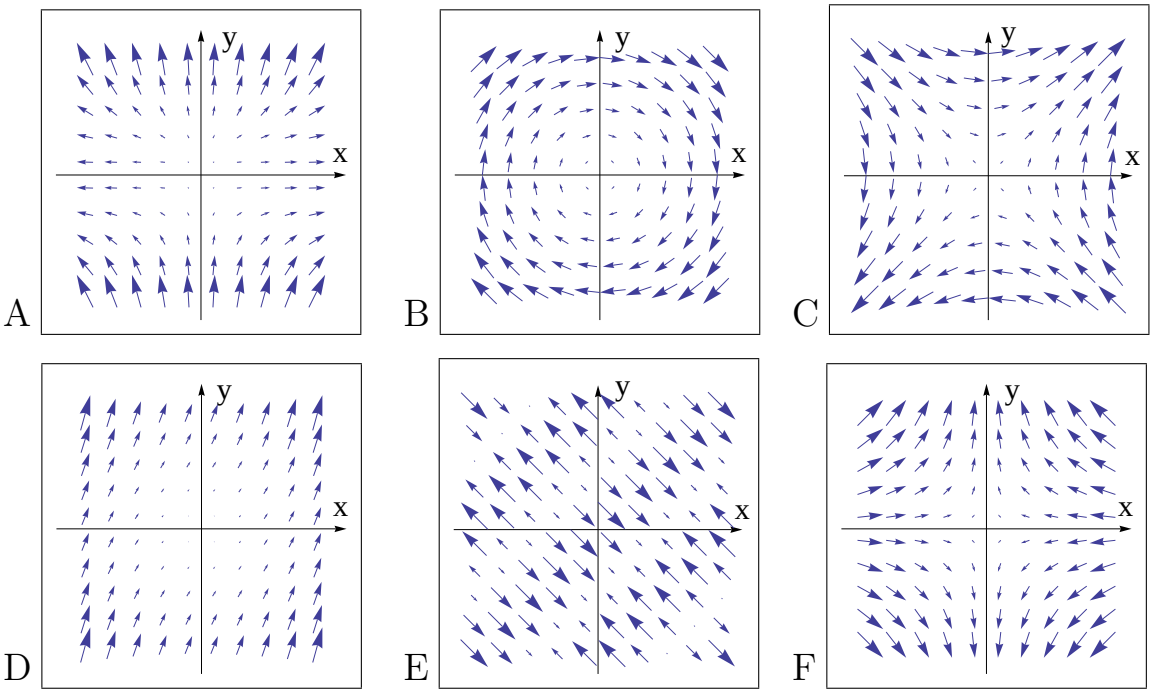
1. [25 pts] Find the absolute maximum and minimum values of the function $f(x, y) = x^3 + xy + 3y$ on the region between $y = x^2$ and $y = 1$.

2. [25 pts] Let D be the region inside the circle $x^2 + y^2 = 2$ and above the line $y = 1$. Evaluate

$$\iint_D \frac{1}{(x^2 + y^2)^{3/2}}\, dA$$

3. [20 pts] Each vector field 1. – 4. is represented by one of the pictures A – F. Assign to each vector field its corresponding graph. You do not need to justify your choices.

   1. $\mathbf{F}_1 = \nabla f$, where $f(x, y) = xy$ Graph _____
   2. $\mathbf{F}_2 = \nabla f$, where $f(x, y) = \sin(x - y)$ Graph _____
   3. $\mathbf{F}_3 = \langle\, x\,,\, y^2 \,\rangle$ Graph _____
   4. $\mathbf{F}_4 = \langle\, \ln(x^2 + y^2 + 1)\,,\, x^2 + y^2 \,\rangle$ Graph _____



4. Let $\mathbf{F}(x, y) = (4xy - 3x^2 \cos y)\,\mathbf{i} + x^3 \sin y\,\mathbf{j}$.

   (a) [5 pts] Is the vector field $\mathbf{F}$ conservative? Give reasons to support your answer.

   (b) [25 pts] Evaluate $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the triangular path from $(0, 0)$ to $(1, 2)$ to $(0, 1)$ to $(0, 0)$.

5. [20 pts] Evaluate the integral $\displaystyle\int_C x^2 ds$, where C is the circle $x^2 + y^2 = 4$, oriented counterclockwise.

6. [30 pts] Evaluate $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the curve given by $\mathbf{r}(t) = \langle\, t\,,\, \sin(\pi t) \,\rangle$ for $-1 \le t \le 1$, and

$$\mathbf{F} = \left\langle\, y\cos(x^2y^2) + x^2,\ x\cos(x^2y^2) + y^2 \,\right\rangle.$$