Math 185 Quiz 1

Don't forget to write down clearly your **Name**: and **ID number:**

1. True or False (10 points). Mark "T" (True) in front of a correct statement and "F" (False) in front of a wrong one.

____ The equation $x^2 + 1 = 0$ has one solution over $\mathbb{C}$.

____ $5i > 3i$.

____ $|5i| > |3i|$.

____ $\overline{e^{\frac{i\pi}{2}}} = e^{\frac{3i\pi}{2}}$

____ Every non-zero complex number is invertible under multiplication.

2. Find out the real and imaginary parts (5 points). Identify the map $f(z) = \frac{1}{z} : \mathbb{C}^* \to \mathbb{C}^*$ ($\mathbb{C}^* := \mathbb{C}\backslash\{0\}$) as a map between $\mathbb{R}^2\backslash\{(0,0)\}$, so that $f(x+iy) = u(x,y) + iv(x,y)$, where $u(x,y)$ and $v(x,y)$ are real-valued. Find the functions $u(x,y)$ and $v(x,y)$.

3. Prove the following trigonometric formula (5 points). Please use the formula

$$e^{i\theta}e^{i\phi} = e^{i(\theta+\phi)},$$

to rederive the following double-angle relations,

$$\sin(2\theta) = 2\sin\theta\cos\theta, \qquad \cos(2\theta) = \cos^2\theta - \sin^2\theta.$$