Math 120 Practice Final 2

1. Evaluate these limits

(a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}$

(b) $\displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{x^2+y^2-z^2}{\sqrt{x^4+y^4+z^4}}$

(c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y\sin(x^2-y^3)}{x^2+y^2}$

2. Find the directional derivative of $f(x,y,z) = xy + z^3$ at $(3,-2,-1)$ in the direction pointing toward the origin.

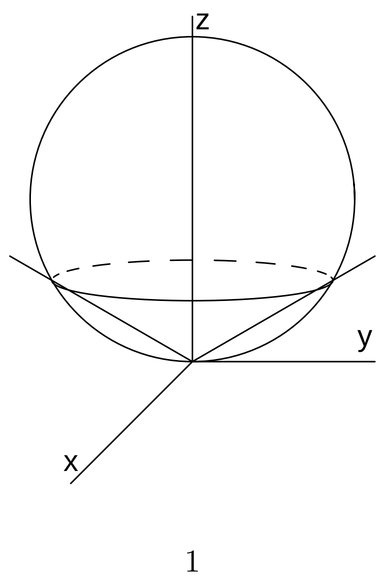
3. Find the absolute max and absolute min of $f(x,y) = xy - x^2y$ on the rectangle $0 \le x \le 2, 0 \le y \le 1$.

4. Evaluate the integral $\displaystyle\int_0^1\int_0^1\int_{x^2}^1 \frac{xye^z}{z}dzdxdy$.

5. Find $\dfrac{df}{dt}$ at $t=0$ if $f(x,y) = 2x^2y+1, x = r(t)+2$, and $y = s(t)^2$. Use these values of $r(t), r'(t), s(t)$, and $s'(t)$,

t	-3	-2	-1	0	1	2	3
$r(t)$	4	1	-2	-1	3	3	2
$r'(t)$	-2	-3	-1	5	1	-1	-2
$s(t)$	-4	-2	0	-2	1	4	9
$s'(t)$	2	1	-1	1	2	5	9

6. Find the volume of the solid that lies above $2z = \sqrt{x^2+y^2+z^2}$ and inside $x^2+y^2+(z-8)^2 = 8^2$.



7. Suppose C is the curve composed of the line segments from $(1,0,0)$ to $(1,3,0)$ to $(-1,0,3)$ to $(1,0,0)$, oriented counterclockwise. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x,y,z) = \langle \sin(\sin(x)), 3x+6z, x^2 e^z \rangle$.

8. Evaluate $\int_C \frac{x}{x^2+y^2}dx + \frac{y}{x^2+y^2}dy$ where C is the curve parameterized by $\mathbf{r}(t) = \cos(t)\mathbf{i} + 3\sin(t)\mathbf{j}$, for $0 \leq t \leq \pi/2$.

9. Let $\mathbf{F}(x,y) = \frac{-y\mathbf{i} + (x-2)\mathbf{j}}{x^2-4x+4+y^2}$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the circle with center $(2,0)$ and radius 1, oriented counterclockwse.

10. Suppose S is the surface $z = x^2+y^2$, $0 \leq z \leq 4$ with downward pointing normal vectors, and $\mathbf{F}(x,y,z) = \langle 3x + (z-4)\arctan(y^3-y^2), yz + (z-4)\arctan(x^3-x^2), z \rangle$.

(a) Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

10 (b) Recall S is the surface $z = x^2+y^2$, $0 \leq z \leq 4$ with downward pointing normal vectors, and $\mathbf{F}(x,y,z) = \langle 3x + (z-4)\arctan(y^3-y^2), yz + (z-4)\arctan(x^3-x^2), z \rangle$. Evaluate $\iint_S \text{curl}(\mathbf{F}) \cdot d\mathbf{S}$.

11. Suppose $f(x,y)$ is a real-valued function with continuous second partial derivatives and $f(x,y) < 0$ for all (x,y) in the plane. Suppose C is any simple closed curve in the plane, oriented counterclockwise. Evaluate $\int_C \frac{1}{f^2}\nabla f \cdot d\mathbf{r}$.