Math 225 Quiz 2

Don't forget to write down clearly your **Name**: and **ID number:**

1. True or False (5 points) Mark "T" (True) or "F" (False) in front of each statement.

____ The number of vectors in any two finite generating sets for a finite-dimensional vector space are the same.

____ The dimension of $M_{m\times n}(\mathbb{F})$ is mn.

____ A linear map $T : V \longrightarrow W$ must satisfy $T(0_V) = 0_W$.

____ If $S, T : V \longrightarrow W$ are two linear maps, and $a, b \in \mathbb{F}$ are scalars, then $aT + bS$ is also linear.

____ If $T : V \longrightarrow W$ is an onto linear map, then $\mathrm{Ker}(T) = \{0_V\}$.

2. Find a basis (5 points). Consider the collection of all traceless 2×2 matrices over a field $\mathbb{F}$:

$$\left\{ A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_2(\mathbb{F}) \;\middle|\; \mathrm{Tr}(A) = a + d = 0 \right\}.$$

Please give a basis of this vector space, and find the dimension of the space.