Math 120, Practice exam 1

The best way to use this practice exam is to take it as if it were the real test. Set aside 90 minutes to work on it without interruption. Don't use the book or notes or calculators, and don't peak at the solutions until you're done.

1. Let $\mathbf{v} = \langle 1, 2, 1\rangle$ and $\mathbf{w} = \langle -2, -1, 1\rangle$.

   (a) [10 pts] Find the angle between $\mathbf{v}$ and $\mathbf{w}$.

   (b) [10 pts] Find the vector projection $\mathrm{proj}_{\mathbf{v}}\mathbf{w}$ of $\mathbf{w}$ onto $\mathbf{v}$.

2. [30 pts] Find an equation of the plane that contains both the point $(1, -2, 5)$ and the line given by

$$\frac{x-2}{2} = \frac{y+5}{-5} = \frac{z-4}{-1}.$$

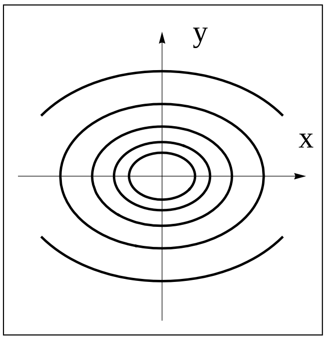
3. The position of particle A is given by the vector function $\mathbf{r}_A(t) = \langle e^t, e^{-t}, \sqrt{2}\, t\rangle$.

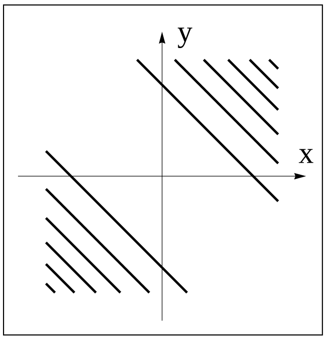
   (a) [10 pts] Does the space curve given by $\mathbf{r}_A(t)$ intersect the curve given by $\mathbf{r}_B(s) = \langle s, s, e^s\rangle$? Give reasons for your answer.

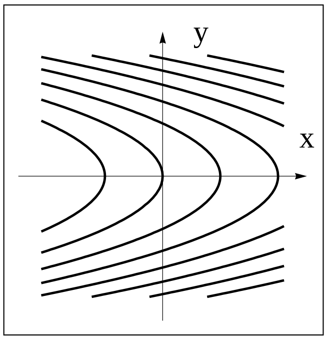
   (b) [10 pts] Find parametric equations of the tangent line to the space curve given by $\mathbf{r}_A(t)$ at $t = 2$.

   (c) [10 pts] Find the total distance traveled by particle A from the point $(1, 1, 0)$ to the point $(e, \frac{1}{e}, \sqrt{2})$.

4. [15 pts] Each level curve diagram I - III corresponds to one of the surfaces in (a) - (e). Assign to each diagram its corresponding surface. You do not need to justify your choices.







I: Surface ______ II: Surface ______ III: Surface ______

(a) $z = (x + y)^2$

(b) $z = \cos(x - y^2)$

(c) $z = x - y$

(d) $z = x + y^2$

(e) $z = \ln(x^2 + 2y^2)$

5. For each function, find the limit as $(x, y) \to (0, 0)$, or show that it does not exist.

(a) [10 pts] $f(x, y) = \dfrac{y^3 \sin(x)}{2x^2 + y^2}$

(b) [10 pts] $g(x, y) = \dfrac{1 - e^{x^2+y^2}}{x^2 + y^2}$

(c) [10 pts] $h(x, y) = \dfrac{x^2 + y^2}{x}$

6. [25 pts] Find an equation for the tangent plane to the surface

$$z = \ln\left(y \cos(x)\right) - e^{3x}$$

at the point $(x, y, z) = (0, 1, -1)$.