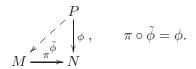
Homework 1

January 11, 2019

(i) In ring theory, a module P over a ring A is called *projective* for any given surjective A-module map $\pi: M \twoheadrightarrow N$ and any map $\phi: P \longrightarrow N$, there is a map $\tilde{\phi}: P \longrightarrow M$ making the diagram commute



Reprove Lemma 3.1 of the textbook with the notion of "free resolutions" replaced by "projective resolutions" (an exact sequence each of whose term is a projective module). As you can see, since free modules are always projective, this generalized exercise contains Lemma 3.1 as a specical case.

- (ii) Prove that, for a family of A-modules M_i , $i \in I$ and N, we have $\operatorname{Ext}_A^{\bullet}(\oplus_{i \in I} M_i, N) \cong \prod_{i \in I} \operatorname{Ext}_A^{\bullet}(M_i, N)$.
- (iii) Prove that $\operatorname{Tor}_A^i(M,N)\cong\operatorname{Tor}_A^i(N,M)$ for any modules over a commutative ring A. You may find it useful to show first that $\operatorname{Tor}_A^i(M,N)=0$, i>0, if either M or N is a free A-module.
- (ii) Exercises 3, 6, 8, 11 of Hatcher, Section 3.1.