

Math H1b Quiz 5

Don't forget to write down clearly your **Name**:

and **ID number**:

1. True or False (5 points). Mark the box in front of a correct answer.

- ☐ If both $y = f(x)$ and $y = xf(x)$ are solutions of a second order differential equation, then they are linearly independent.
- ☐ The differential equation $ay'' + by' + cy = 0$, when $b^2 - 4ac = 0$, has only one solution.
- ☐ The differential equation $5y'' + xy' = y$ is non-homogeneous.
- ☐ $y = \sin x$ is a particular solution of $y'' + y' + y = \cos x$.
- ☐ If $y = f(x)$ and $y = g(x)$ are both solutions of $y'' + y = x$, then $y = f(x) + g(x)$ is also a solution of the same equation.

2. Initial Value Problem (10 points). Consider the following second order differential equation

$$y'' - 6y' + 8y = 0,$$

(1) Use $y = e^{rx}$ as a text solution, determine the conditions on r for y to be a solution. Write down the form of a general solution of the differential equation.

(2) If we are given the initial conditions $y(0) = 2$, $y'(0) = 2$, find the solution of the initial value problem.

3. Series solution (5 points). Consider the differential equation

$$y'' - 2xy' + y = 0,$$

and let $y = \sum_{n=0}^{\infty} c_n x^n$ be a series solution. Write down the recursive relation on the coefficients c_n ($n = 0, 1, 2, \dots$) determined by the differential equation.