Math H1b Quiz 4

Don't forget to write down clearly your **Name**: and **ID number:**

1. True or False (4 points). Mark the box in front of a correct answer.

- ☐ If $\{b_n\}$ is a positive sequence, then the series $\sum_{n=1}^{\infty}(-1)^n b_n$ converges.
- ☐ If $\{a_n\}$ is a sequence whose limit is zero, then the series $\sum_{n=1}^{\infty} a_n$ converges.
- ☐ If $f(x)$ is a positive function with $\int_0^{\infty} f(x)dx < \infty$, then the series $\sum_{n=1}^{\infty} f(n)$ converges.
- ☐ The series $\sum_{n=1}^{\infty} a_n$ converges if and only if $\sum_{n=10^{100}}^{\infty} a_n$ converges.

2. Multiple choices (6 points). Mark the box in front of the correct answer.

(1) Which of the following series is absolutely convergent?
☐ $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ ☐ $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3}$ ☐ $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ ☐ $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n}$

(2) On which of the following sequences is the Root Test inconclusive?
☐ $a_n = (1+\frac{1}{n})^n$ ☐ $a_n = \frac{(-1)^{n-1}}{n^n}$ ☐ $a_n = \frac{1}{3^n}$ ☐ $a_n = \left(\frac{3n+3}{2n+5}\right)^n$

(3) Which of the following power series has its radius of convergence equal to $\mathbb{R}$?
☐ $\sum_{n=0}^{\infty} x^n$ ☐ $\sum_{n=0}^{\infty} nx^{n-1}$ ☐ $\sum_{n=0}^{\infty}(-1)^{n+1}\frac{x^{n+1}}{n+1}$ ☐ $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

3. Taylor Series (4 points). Find the Taylor series expansion for the function $f(x) = x^3$ near $x = 1$.

4. Maclaurin Series (6 points). Consider the Maclaurin series for $f(x) = \sin x$ and $g(x) = \cos x$.

(a). Write down the best degree-five polynomial approximations for the functions $f(x)$ and $g(x)$.

(b). Use part (a) to find the first three terms of the Maclaurin expansion for $h(x) = \sin(2x)$. Compare it directly with the result you get by considering $f(2x)$ to show that these expressions agree.