Math 113 Quiz 4

Don't forget to write down clearly your **Name**: and **ID number:**

1. True or False (10 points) Check the box in front of a correct statement.

- ☐ If G is a group of order 5, then the coset space G/G has 5 elements.
- ☐ An isometry of a Euclidean vector space that fixes zero must be linear.
- ☐ The group $O(2,\mathbb{R})$ is abelian.
- ☐ A composition of isometries is also an isometry.
- ☐ The cyclic subgroup C_n in D_n is normal.
- ☐ The dihedral group D_6 is cyclic.
- ☐ The symmetric group on n-letters S_n acts transitively on that set of n-letters.
- ☐ The additive group $\mathbb{R}$ has a subgroup of order 10.
- ☐ The stabilizer subgroup in D_6 of a vertex in a regular hexagon has order 2.
- ☐ The left-multiplication action $GL(n,\mathbb{R})$ on $\mathbb{R}^n$ is transitive.

2. Group action (10 points). Answer the following questions and justify your answer.
Consider a map of sets "·"

$$G \times S \to S, \quad (g,s) \mapsto g \cdot s.$$

(a) Write down the conditions for this map to be a group action. (2 points)

(b) Let H be a group. Consider $G = H \times H$ and $S = H$ as in (a). Prove that

$$(H \times H) \times H \to H, \quad ((h_1, h_2), h) \mapsto (h_1, h_2) \cdot h := h_1 h h_2^{-1}$$

is a group action. (3 points)

(c) Prove that, if $\phi : G' \to G$ is a group homomorphism and G acts on S as above, then the map

$$G' \times S \to S, \quad (g', s) \mapsto \phi(g') \cdot s$$

defines a group action of G' on S. (3 points)

(d) Consider the diagonal group homomorphism $\Delta : H \to H \times H$ and the action we considered in (b). What is this group action of H on itself you obtain if you apply the construction in (c) for this homomorphism? (No proof needed for this part. 2 points.)