Math H1b Quiz 3

Don't forget to write down clearly your Name:

and **ID number**:

1. True or False (4 points). Mark the box in front of a correct answer.

- \square The logistic population model $\frac{dP(t)}{dt} = kP(1-M/P)$ is a first order differential equation.
- \square The differential equation y'(x) = y + x is separable.
- \square The direction field of y' = y + x at the point (1, -1) has slope 0.
- \square If $\{a_n\}$ is a sequence with $\lim_{n\to\infty} a_n = 0$, then the series $\sum_{n=1}^{\infty} a_n$ converges.

2. Multiple choices (6 points). Mark the box in front of the correct answer.

- (1) Which of the following differential equation describes the population growth model in an environment with unlimited resources?
- (2) What is an integration factor I(x) for the differential equation $y' = 2xy + x^2$? $\Box \quad I(x) = e^{2x} \quad \Box \quad I(x) = e^{-2x} \quad \Box \quad I(x) = e^{x^2} \quad \Box \quad I(x) = e^{-x^2}$
- (3) Which of the following sequences $\{a_n\}$ do **not** have a limit?
 - $\square \quad a_n = \frac{(-1)^n}{n} \qquad \square \quad a_n = \frac{1}{n} \qquad \square \quad a_n = (-1)^n \qquad \square \quad a_n = 1$

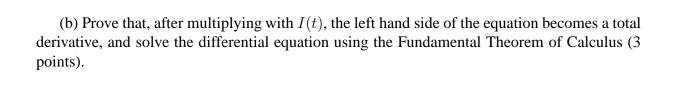
3. Differential equation (5 points). We will consider the effect of integration factor for first order linear differential equations.

Let P(t) be the performance level of someone learning a skill as a function of the training time t. A reasonable model for learning is given by

$$\frac{dP}{dt} + kP(t) = kM,$$

where k is a positive constant.

(a) Find the integration factor I(t) for this differential equation (2 points).



4. Sequences and series. (5 points) Use your favorite way to rewrite the decimal number $x=2.\overline{45}=2.454545\cdots$ in the form $x=\frac{M}{N}$, where $M,N\in\mathbb{Z}$ are integers.