

### Math 185 Quiz 3

Don't forget to write down clearly your **Name**:

and **ID number**:

**1. True or False (10 points).** Mark “T” (True) in front of a correct statement and “F” (False) in front of a wrong one.

\_\_\_ The series  $\sum_{k=0}^{\infty} \left(\frac{1}{2} + \frac{i}{2}\right)^k$  converges.

\_\_\_ Let  $\{f_n(z)\}$  be a sequence of analytic functions on a domain that converges uniformly to  $f(z)$ , then  $f(z)$  is analytic.

\_\_\_ Any Taylor series  $\sum_{k=0}^{\infty} a_n z^n$  has a positive radius  $R > 0$  of convergence.

\_\_\_ The Taylor series for the function  $\frac{1}{1-z}$  at the point  $z = -1$  has radius of convergence 2.

\_\_\_ The function  $f(z) = z^2$  is analytic at infinity.

**2. Taylor series (5 points).** Find the Taylor series expansion for the analytic function

$$f(z) = \sin z$$

at  $z = -\pi$ , and find the radius of convergence.

**3. Uniqueness principle (5 points).** Prove that the identity

$$\sin\left(z + \frac{\pi}{3}\right) = \frac{1}{2} \sin z + \frac{\sqrt{3}}{2} \cos z$$

holds for any complex numbers  $z \in \mathbb{C}$ .