## Math H1b Quiz 3

Don't forget to write down clearly your **Name**: and **ID number:**

**1. True or False (4 points).** Mark the box in front of a correct answer.

- ☐ The logistic population model $\frac{dP(t)}{dt} = kP(1-M/P)$ is a first order differential equation.
- ☐ The differential equation $y'(x) = y + x$ is separable.
- ☐ The direction field of $y' = y + x$ at the point $(1, -1)$ has slope $0$.
- ☐ If $\{a_n\}$ is a sequence with $\lim_{n\to\infty} a_n = 0$, then the series $\sum_{n=1}^{\infty} a_n$ converges.

**2. Multiple choices (6 points).** Mark the box in front of the correct answer.

(1) Which of the following differential equation describes the population growth model in an environment with unlimited resources?

☐ $P'(t) = kP$ ☐ $P'(t) = kP(1-M/P)$
☐ $P'(t) = kP(1-M/P) - c$ ☐ $P'(t) = k\ln(P/M)P$

(2) What is an integration factor $I(x)$ for the differential equation $y' = 2xy + x^2$?
☐ $I(x) = e^{2x}$ ☐ $I(x) = e^{-2x}$ ☐ $I(x) = e^{x^2}$ ☐ $I(x) = e^{-x^2}$

(3) Which of the following sequences $\{a_n\}$ do **not** have a limit?
☐ $a_n = \frac{(-1)^n}{n}$ ☐ $a_n = \frac{1}{n}$ ☐ $a_n = (-1)^n$ ☐ $a_n = 1$

**3. Differential equation (5 points).** We will consider the effect of integration factor for first order linear differential equations.

Let $P(t)$ be the performance level of someone learning a skill as a function of the training time $t$. A reasonable model for learning is given by

$$\frac{dP}{dt} + kP(t) = kM,$$

where $k$ is a positive constant.

(a) Find the integration factor $I(t)$ for this differential equation (2 points).

(b) Prove that, after multiplying with $I(t)$, the left hand side of the equation becomes a total derivative, and solve the differential equation using the Fundamental Theorem of Calculus (3 points).

**4. Sequences and series. (5 points)** Use your favorite way to rewrite the decimal number $x = 2.\overline{45} = 2.454545\cdots$ in the form $x = \frac{M}{N}$, where $M, N \in \mathbb{Z}$ are integers.