Advanced Linear Algebra Quiz 1

Don't forget to write down clearly your **Name**: and **Net Id:**

1. True or False (5 points) Mark "T" (True) or "F" (False) in front of each statement.

____ The set of integers $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$ is a field.

____ Every vector space contains a unique zero vector.

____ The usual Euclidean spaces $\mathbb{R}^3$ has infinitely many subspaces.

____ If v is a nonzero vector, the set $\{v\}$ is linearly dependent.

____ A finite-dimensional vector space can have an infinite spanning set.

2. Lagrangian Interpolation (5 points) Consider the points $a_0 = -1, a_1 = 1, a_2 = 3$. Find polynomials $e_0(x),\ e_1(x),\ e_2(x)$ in $P_2(\mathbb{R})$ satisfying, for any $0 \leq i, j \leq 3$,

$$e_i(a_j) = \begin{cases} 1 & i = j, \\ 0 & i \neq j. \end{cases}$$

Write the function $f(x) = x + 2$ as a linear combination of $e_0(x),\ e_1(x),\ e_2(x)$.