Selected Practice Midterm Problems

February 27, 2020

Problem 1. Show that the planes $x + 2y + 3z = 1$ and $4x + y - 2z = 1$ are orthogonal to each other.

Problem 2 Find the length of the helix $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}$ as a function, starting at the point $(1, 0, 0)$.

Problem 3 (a) Find the directional derivative of $f(x, y, z) = xy^2 + x^2z + yz^3$ at the point $(-1, 0, 1)$, in the direction given by the vector $\langle 1, 2, -2\rangle$.

(b) Find the tangent plane to the surface $xy^2 + x^2z + yz^3 = 1$ at the point $(-1, 0, 1)$.

Problem 4 If $\mathbf{r}(t)$ is the trajectory equation of an object moving on the sphere $x^2 + y^2 + z^2 = 1$, show that $\mathbf{r}'(t) \cdot \mathbf{r}(t) \equiv 0$.

Problem 5. Let D be the region between the circles $(x - 1)^2 + y^2 = 1$ and $(x - 2)^2 + y^2 = 4$, and above the x-axis (where $y \geq 0$). Evaluate the integral

$$\iint_D y dA.$$

Problem 6 Find the volume of the solid bounded by the surfaces $z = x^2 + y^2$ and $z = 4 - x^2 - y^2$.

Problem 7 Find the derivative $\frac{dz}{dt}$ where

$$z = \ln(x^2 + y^2), \quad x = 2\sin t, \quad y = 2\cos t.$$

Problem 8 If $f(x, y) = xy + 3$, find the gradient vector ∇f at $(1, 2)$, and find the tangent line equation of the level curve $f(x, y) = 5$.

Problem 9 Compute the double integral $\iint_D x dA$ where D is the region bounded by $x = y - 2$ and $y = x^2$.

Problem 10. Let $f(x, y) = 3x^2 + y^2 + 6xy + 8y$. Find and classify the critical points of f(x, y).