

Homework 2

January 23, 2019

- (i) Compute the cohomology ring of $\mathbb{R}P^2$ and the Klein bottle K with \mathbb{Z}_2 coefficients as follows.
 - (a) Construct $\mathbb{R}P^2$ and K by identifying various edges of the standard square.
 - (b) Find a suitable triangulation of the square, and use the definition of simplicial (Δ) cohomology to compute the ring structure.
- (ii) Find suitable triangulations of the closed unit disk $D^2 = \{z \in \mathbb{C} \mid |z| \leq 1\}$ with 2 distinct open small balls removed. Use the triangulation to compute its cohomology ring structure.
- (iii) Show that the Euler number is multiplicative: $\chi(X \times Y) = \chi(X)\chi(Y)$, if X and Y are finite cell-complexes.
- (iv) Exercises 2, 3, 7, 11 of Hatcher, Section 3.2.