Math 120a — Exam 1

1. Find an equation of the plane passing through $(1, 1, 1)$, $(0, 2, 3)$, and $(1, 2, 0)$.

2. Compute the directional derivative of $g(x, y) = 5x^3y - 2y^2 + xy$ at $(2, -1)$ in the direction from $(2, -1)$ to $(3, 0)$.

3. Find the equation for the tangent line to the curve $\mathbf{r}(t) = \langle t^2, t^3, \ln(t^2 + 1)\rangle$ at the point $\mathbf{r}(1)$.

4. Find the equation of the tangent plane to $f(x, y) = x\sin(x^2y)$ at the point $(2, \pi/8, 2)$.

5. Find where the following functions are continuous; justify each answer.

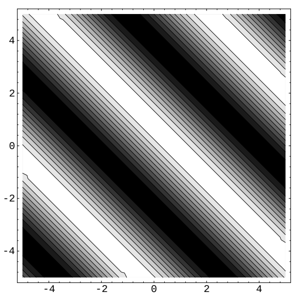
(a) $f(x, y) = \begin{cases} \dfrac{xy}{x^2 + y^2} & \text{for } (x, y) \neq (0, 0) \\ 0 & \text{for } (x, y) = (0, 0) \end{cases}$

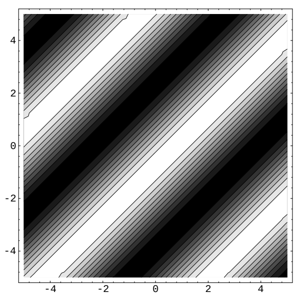
(b) $g(x, y) = \begin{cases} \dfrac{x^2y^2}{x^2 + y^2} & \text{for } (x, y) \neq (0, 0) \\ 0 & \text{for } (x, y) = (0, 0) \end{cases}$

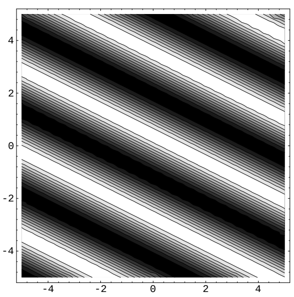
(c) $h(x, y) = \begin{cases} \sin(y + e^{xy}) & \text{for } (x, y) \neq (0, 0) \\ 0 & \text{for } (x, y) = (0, 0) \end{cases}$

6. Find every value of t for which the planes $x - y + tz = 1$ and $2x + ty - 4z = 7$ are parallel.

7. One of these is the contour plot of $f(x, y) = \sin(x + y) + \cos(x + y)$. Say which and give a reason for your choice.

(a) 

(b) 

(c) 

8. Suppose $z(x, y) = f(x + y)g(x - y)$ for differentiable functions f and g. Show

$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 2g(x - y)f'(x + y)$$