Math 113 Quiz 3

Don't forget to write down clearly your **Name**: and **ID number:**

1. True or False (10 points) Check the box in front of a correct statement.

- ☐ The trivial subgroup $\{1_G\}$ inside any group G is always normal.
- ☐ Any subgroup of a cyclic group is normal.
- ☐ The left coset space S_3/S_2 has a group structure.
- ☐ The left coset space $GL(2,\mathbb{R})/SL(2,\mathbb{R})$ has a group structure.
- ☐ If $\phi : G \to H$ is a surjective group homomorphism, then $\ker(\phi) = \{1_G\}$.
- ☐ The inclusion map $\mathbb{R}^* \subset \mathbb{R}$ is an injective group homomorphism.
- ☐ The exponential map $\mathbb{R} \mapsto U(1), a \mapsto e^{2\pi i a}$ is a surjective group homomorphism. Here $U(1)$ stands for the unit norm complex numbers under the usual complex multiplication.
- ☐ S_3 can be generated by one element in it.
- ☐ The free group on two letters is an abelian group.
- ☐ The map $\mathbb{Z}/(12) \mapsto \mathbb{Z}/(3), \overline{a} \mapsto \overline{a}$ is a group homomorphism.

2. The first isomorphism theorem (10 points). Answer the following question and justify your answer.

Consider the Heisenberg group H and its subgroup K,

$$H := \left\{ \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} \middle| a, b, c \in \mathbb{R} \right\} \qquad K := \left\{ \begin{pmatrix} 1 & 0 & b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \middle| b \in \mathbb{R} \right\}$$

under the usual matrix multiplication.

(a). Determine if H and K are abelian groups or not. If not, give examples of elements in them that do not commute under multiplication.

(b). Show that the map

$$\phi : H \to \mathbb{R}^2, \qquad \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} \mapsto (a, c)$$

is a group homomorphism, and determine the kernel of ϕ.

(c) Use the first isomorphism to identify H/K with a more familiar group.