Math H1b Quiz 2

Don't forget to write down clearly your **Name**: and **ID number:**

1. Multiple choices (10 points). Mark the box in front of the correct answer.

(1) What is the arclength integral for $f(x) = x$ from $x = 0$ to $x = 1$?
☐ 1 ☐ $\sqrt{2}$ ☐ π ☐ 2

(2) If we rotate the curve $y = \sqrt{9 - x^2}$, $-3 \leq x \leq 3$ about the x-axis, what's the surface area obtained?
☐ π ☐ 4π ☐ 9π ☐ 36π

(3) If four particles of equal mass are placed on the points $(1, 0)$, $(-1, 0)$, $(0, 1)$ and $(0, -1)$ what's the coordinates of the center of mass?
☐ $(0, 0)$ ☐ $(1, 1)$ ☐ $(0, -1)$ ☐ $(1, 0)$

(4) Which of the following is **not** a probability density function?

☐ $f(x) = \begin{cases} 0.1 & x \in [0, 10] \\ 0 & \text{otherwise} \end{cases}$ ☐ $f(x) = \begin{cases} 0.2e^{-0.2x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$

☐ $f(x) = \begin{cases} xe^{-x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$ ☐ $f(x) = \begin{cases} \cos x & x \in [0, \frac{5}{2}\pi] \\ 0 & \text{otherwise} \end{cases}$

(5) If a random variable has probability density function $f(x), x \in \mathbb{R}$, what's the mean μ of it?
☐ $\mu = \int_{-\infty}^{\infty} f(x)$ ☐ $\mu = \int_{-\infty}^{\infty} xf(x)$
☐ $\mu = \int_{-\infty}^{\infty} x^2 f(x)$ ☐ $\mu = \int_{-\infty}^{\infty} f^2(x)$

2. We will use the Brazilian bug model to rederive the arclength formula (10 points). Suppose a bug is crawling along the x-axis in unit speed ($1m/s$), and a uniform light is placed below it. There is a screen in the plane along the wire $y = f(x),\ (f(x) > 0)$ to catch its shadow.

(a). When the bug is at the point $x = a$, what's the direction of the velocity of the bug shadow on the wire-screen? You can express the direction in terms of the tangent of the angle it makes with the x-axis.

(b). What's the shadow's instantaneous speed, i.e., its magnitude of velocity, when the bug is at $x = a$? (Hint: Draw a triangle expressing the relation between the bug's speed and the shadow's speed.)

(c). Use the speed-distance equation from physics $d = \int_0^a |v(t)|dt$ to find the formula for the arclength of the curve $y = f(x)$ when x lies in $[0, a]$.