Exercises for Week 1

The work handed in should be entirely your own. You can consult Dummit and Foote, Artin and/or the class notes but nothing else. To receive full credit, justify your answer in a clear and logical way. Due Sept. 11.

Reading. If you have Dummit and Foote, read Sections 1.1 and 2.1. With Artin, read Sections 2.1, 2.2.

1. Determine if the matrix multiplication structure on $\mathrm{M}(n, \mathbb{R})$ is associative/commutative. Is it a group under multiplication, why? (Caution: what's the difference when $n = 1$ and $n \neq 1$?)

2. Show that given any set S and a fixed element $x_0 \in S$, the map $(x, y) \mapsto x_0$, $\forall x, y \in S$ defines a commutative, associative law of composition.

3. Let $G := \{a + b\sqrt{2} | (a, b) \in \mathbb{Q}^2 \backslash (0, 0)\}$. Show that G is a group under multiplication.

4. Prove that if all elements of a group G satisfy $x^2 = 1$, then G is abelian.

5. Show by definition that $SL(2, \mathbb{R})$ is a subgroup of $GL(2, \mathbb{R})$.

6. List all subgroups of S_3.

7. Let G be a group and g_0 be a fixed element of G. Show that the set $Z_G(g_0) := \{g \in G | g g_0 g^{-1} = g_0\}$ is a subgroup of G. This is called the *centralizer* of g_0 in G.