Math 113 Quiz 2

Don't forget to write down clearly your **Name**: and **ID number:**

Note: $\{\pm 1, \pm i, \pm j, \pm k\}$ stands for the *quaternionic group*, with the relations $i^2 = j^2 = k^2 = -1$, $ij = k$, $jk = i$ and $ki = j$.

1. Multiple choices (10 points). Mark the box in front of the correct answer.

(1) Which of the following groups is abelian?
☐ S_3 ☐ $GL(2,\mathbb{R})$ ☐ $\mathbb{Z}/(12)$ ☐ $\{\pm 1, \pm i, \pm j, \pm k\}$

(2) Which of the following groups is **not** cyclic ?
☐ $\mathbb{Z}$ ☐ $\mathbb{Z}/(3)$ ☐ $\{\pm 1, \times\}$ ☐ $\left\{\begin{pmatrix} \pm 1 & 0 \\ 0 & \pm 1 \end{pmatrix}\right\}$

(3) Which of the following groups is **not** a normal subgroup of the ambient group?

☐ $S_2 \subset S_3$ ☐ $SL(2,\mathbb{R}) \subset GL(2,\mathbb{R})$
☐ $5\mathbb{Z} \subset \mathbb{Z}$ ☐ $\{\pm 1, \pm i\} \subset \{\pm 1, \pm i, \pm j, \pm k\}$

(4) Which of the following map is a group homomorphism?

☐ $\exp : (\mathbb{R}, +) \longrightarrow (\mathbb{R}, \times)$ ☐ $\det : M(2,\mathbb{R}) \longrightarrow (\mathbb{R}, +)$
☐ $\ln : (\mathbb{R}^{>0}, \times) \longrightarrow (\mathbb{R}, +)$ ☐ $\phi : (\mathbb{Z}, +) \longrightarrow (\mathbb{Z}, +),\ \phi(a) = a + 2$

(5) Which of the following groups is **not** isomorphic to $\mathbb{Z}/(6)$?
☐ The subgroup $\langle \overline{2} \rangle$ of $\mathbb{Z}/(12)$ ☐ $\{e^{\frac{2\pi i k}{6}} | k = 0, 1, \ldots, 5\}$
☐ $\left\{\begin{pmatrix} \cos(\frac{2k\pi}{6}) & -\sin(\frac{2k\pi}{6}) \\ \sin(\frac{2k\pi}{6}) & \cos(\frac{2k\pi}{6}) \end{pmatrix} \Big| k = 0, 1, \ldots, 5\right\}$ ☐ S_3

2. Answer the following question and justify your answer (6 points). If G is a group of order 6, is it possible for it to have an element of order (a) 5? (b) 3? (c) 2?

If it's not, give the reason. If yes, please give an example of such a group (groups) and an element (elements).

3. Prove the following statement (4 points). Let G be a group and g_0 be a fixed element of G. Define the conjugation map

$$C_{g_0} : G \to G, C_{g_0}(g) := g_0 g g_0^{-1}.$$

Show that it is an isomorphism of G onto itself.