Homework 6

The work handed in should be entirely your own. You can consult any abstract algebra textbook (e.g. Dummit and Foote, Artin), the course textbook and/or the class notes but nothing else. To receive full credit, justify your answer in a clear and logical way. Due April. 25.

1. Read the textbook Chapter 5 on classification of Coxeter groups.

2. Show that the reflection about a root $\mathbf{r}$

$$S_{\mathbf{r}}(\mathbf{v}) := \mathbf{v} - 2\frac{(\mathbf{v}, \mathbf{r})}{||\mathbf{r}||}\mathbf{r}$$

   is an orthogonal transformation.

3. Suppose Π is a base for Δ.

   (a) If $\mathbf{r} \in \Delta^+$, show that $\mathbf{r} \in \Pi$ if and only if $\mathbf{r}$ is not a strictly positive linear combination of two or more positive roots.

   (b) Use part (a) to give an alternative proof of the uniqueness of Π.

4. Show that the vector ρ we defined in class

$$\rho := \frac{1}{2} \sum_{\mathbf{r} \in \Delta^+} \mathbf{r}$$

   satisfies $(\rho, \mathbf{r}) > 0$ for all $\mathbf{r} \in \Delta^+$.

5. Let H_2^m be the dihedral reflection group on $\mathbb{R}^2$. Find a set of simple roots for H_2^m and compute its Coxeter graph. Compute the determinant of the Coxeter matrix.

6. This is a continuation of the exercise from last week about S_4. Use what you have proven there to compute the Coxeter graph for S_4. Compute the determinant of the Coxeter matrix.

7. Verify the determinant formula for I_4.