Concrete Beam Design

Concrete beam section capacity designed in accordance with ACI 318-14.

Assumptions

[ASSUME] ACI 318-14 controls member design per sections referenced below

[ASSUME] Beam is subject to flexural and/or shear load demands only

[ASSUME] Beam is non-prestressed

[ASSUME] Beam does not qualify as a deep beam or a one-way joist system

[ASSUME] Torsional effects can be neglected according to ACI 318-14 9.5.4.1

[ASSUME] Shear reinforcement is required according to ACI 318-14 9.6.3

[ASSUME] Normal weight concrete is used

Inputs

Specified compressive strength of concrete;	$f_c' = 4000 \text{ psi}$
Yield strength of reinforcement steel;	$f_y = 60 \text{ ksi}$
Modulus of elasticity of reinforcement steel;	$E_s = 29000 \text{ ksi}$
Beam section width;	$b = 12 \text{ in}$
Beam height;	$h = 24 \text{ in}$

Beam Longitudinal Reinforcement Properties

Reinforcement

Bar #	$A_s(\text{in}^2)$	$d_b(\text{in})$	$x(\text{in})$	$y(\text{in})$
Bar #0	0.44	0.75	1.875	22.125
Bar #1	0.44	0.75	6	22.125
Bar #2	0.44	0.75	10.125	22.125

Number of transverse shear reinforcement bars at spacing s;	$num_v = 0$

Spacing of transverse reinforcement; $s = 8 \text{ in}$

Size of transverse reinforcement; $\text{Size}_{\text{v}} = \#3$

Beam Section

1. Flexural Capacity

Crushing strain of concrete:

$\varepsilon_c = 0.003$ [ACI 318-14 22.2.2.1]

Yield strain of reinforcement steel:

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60 \text{ ksi}}{29000 \text{ ksi}}$$

$$\therefore \varepsilon_y = 0.002069$$

Concrete stress assumed to be uniformly distributed in the compression block:

[ACI 318-14 22.2.2.1]

$$f'_{c-comp} = 0.85 \cdot f'_c = 0.85 \ \cdot 4000 \text{ psi}$$

$$\therefore f'_{c-comp} = 3400 \text{ psi}$$

Equivalent rectangular compressive stress block depth ratio:

$$\beta_1 = \max\left(0.65, \min\left(0.85, 0.85 - 0.05 \cdot \left(\frac{f_c^{'} - 4000}{1000} \right) \right) \right)$$

[ACI 318-14 Table 22.2.2.4.3]

$$= \max\left(0.65 \ , \min\left(0.85 \ , 0.85 \ - 0.05 \ \cdot \left(\frac{4000 \text{ psi} - 4000}{1000} \right) \right) \right)$$

$$\therefore \beta_1 = 0.85$$

Neutral axis depth required for section equilibrium:

$c = 2.284 \text{ in}$

1.1. Individual Reinforcement Analysis

With the equilibrium-producing neutral axis depth (c) computed, the stress and strain at capacity can be calculated for each reinforcement bar.

1.1.1. Example Calculation

For example, first bar (Bar #0) has the following properties:

Cross sectional steel area:

$A_{s1} = 0.44 \text{ in}^2$

Depth from the top of the beam:

$d_1 = 22.12 \text{ in}$

Which yield the following results assuming that strain is proportional to the distance from the neutral axis (c):

Average strain in the reinforcing bar:

$$\varepsilon_{s1} = \frac{\varepsilon_c \cdot d_1 - c}{c} = \frac{0.003 \ \cdot 22.12 \text{ in} - 2.284 \text{ in}}{2.284 \text{ in}}$$

$$\therefore \varepsilon_{s1} = 0.02606$$

Average calculated stress in the reinforcing bar assuming a linear stress-strain relationship:

$$f_{s1-calc'd} = E_s \cdot \varepsilon_{s1} = 29000 \text{ ksi} \cdot 0.02606$$

$$\therefore f_{s1-calc'd} = 755.9 \text{ ksi}$$

The design stress in the reinforcing bar limited by yield strength of the steel:

$$
\begin{aligned}
f_{s1} &= \max\left(\left(-f_y\right), \min\left(f_y, f_{s1-calc'd}\right)\right) \\
&= \max\left(\left(-60\ \text{ksi}\right), \min\left(60\ \text{ksi}, 755.9\ \text{ksi}\right)\right) \\
&\therefore f_{s1} = 60\ \text{ksi}
\end{aligned}
$$

1.1.2. Summary of All Reinforcement

The above calculations can be applied to all of the reinforcing bars to produce the following results:

Stress and strain in each of the longitudinal reinforcement bars

Reinforcement Results

Bar #	$A_s(\text{in}^2)$	ε_s	$f_s(\text{ksi})$	$y(\text{in})$
Bar #0	0.44	0.02606	60	22.12
Bar #1	0.44	0.02606	60	22.12
Bar #2	0.44	0.02606	60	22.12

Maximimum tensile strain in reinforcement steel:
$\varepsilon_{t,max} = 0.02606$

Estimated distance from extreme compression fiber to centroid of longitudinal reinforcement. Taken as the maximum y position.:
$d_{est} = 22.12\ \text{in}$

The total area of tensile longitudinal steel counting towards minimum steel area. This is conservatively calculated as the sum of all steel area which is in the lower half of the beam section AND in tension at the section capacity strain profile::
$A_{s-tension} = 1.32\ \text{in}^2$

1.2. Minimum Longitudinal Steel

[ACI 318-14 9.6.1.2(a)]

$$A_{s-min1} = \frac{3 \cdot \sqrt{f_c'}}{f_y \cdot 1000 \text{ lbs/kip}} \cdot b \cdot d_{est}$$

$$= \frac{3 \cdot \sqrt{4000 \text{ psi}}}{60 \text{ ksi} \cdot 1000 \text{ lbs/kip}} \cdot 12 \text{ in} \cdot 22.12 \text{ in}$$

$$\therefore A_{s-min1} = 0.8396 \text{ in}^2$$

[ACI 318-14 9.6.1.2(b)]

$$A_{s-min2} = \frac{200}{f_y \cdot 1000 \text{ lbs/kip}} \cdot b \cdot d_{est}$$

$$= \frac{200}{60 \text{ ksi} \cdot 1000 \text{ lbs/kip}} \cdot 12 \text{ in} \cdot 22.12 \text{ in}$$

$$\therefore A_{s-min2} = 0.885 \text{ in}^2$$

Minimum flexural reinforcement:

$$A_{s-min} = \max\left(A_{s-min1}, A_{s-min2}\right) = \max\left(0.8396 \text{ in}^2, 0.885 \text{ in}^2\right)$$

$$\therefore A_{s-min} = 0.885 \text{ in}^2$$

$$Check\ A_{s-tension} > A_{s-min}$$

$$1.32 \text{ in}^2 > 0.885 \text{ in}^2$$

$$\therefore OK$$

1.3. Section Flexural Capacity

Net moment contribution from reinforcement steel:
$\Sigma A_{si} f_{si} y_{si} = 1752 \text{ k} - \text{in}$

Net moment contribution from concrete compression:

$$M_{nc} = \frac{0.85 \cdot \frac{f_c'}{1000 \text{ lbs/kip}} \cdot b \cdot \beta_1 \cdot c \cdot \beta_1 \cdot c}{2}$$

$$= \frac{0.85 \cdot \frac{4000 \text{ psi}}{1000 \text{ lbs/kip}} \cdot 12 \text{ in} \cdot 0.85 \cdot 2.284 \text{ in} \cdot 0.85 \cdot 2.284 \text{ in}}{2}$$

$$\therefore M_{nc} = 76.87 \text{ k} - \text{in}$$

Nominal moment capacity of beam section:

$$M_n = \Sigma A_{si} f_{si} y_{si} - M_{nc} = 1752\ \mathrm{k-in} - 76.87\ \mathrm{k-in}$$

$$\therefore M_n = 1675\ \mathrm{k-in}$$

Strength reduction factor for moment capacity:

$$\phi_m = \max(\ 0.65, \min(\ 0.9, 0.65 + 0.25 \cdot (\ \frac{\varepsilon_{t,max} - \varepsilon_y}{0.005 - \varepsilon_y})\ )\ )$$ [ACI 318-14 Table 21.2.2]

$$= \max(\ 0.65\ , \min(\ 0.9\ , 0.65\ + 0.25\ \cdot (\ \frac{0.02606\ - 0.002069}{0.005\ - 0.002069})\ )\ )$$

$$\therefore \phi_m = 0.9$$

Design moment capacity of the beam section:

$$\phi M_n = \frac{\phi_m \cdot M_n}{12\ \mathrm{in/ft}} = \frac{0.9\ \cdot 1675\ \mathrm{k-in}}{12\ \mathrm{in/ft}}$$

$$\therefore \phi M_n = 125.6\ \mathrm{k-ft}$$

2. Shear Capacity

Effective area of each transverse reinforcing leg:

$A_{vi} = 0.11\ \mathrm{in}^2$

Effective area of all transverse reinforcing bars within spacing s:

$$A_v = num_v \cdot A_{vi} = 0\ \cdot 0.11\ \mathrm{in}^2$$

$$\therefore A_v = 0\ \mathrm{in}^2$$

2.1. Section Shear Capacity

Maximum allowed concrete shear strength:

$f'_{cv-max} = 100\ \mathrm{psi}$ [ACI 318-14 22.5.3.2]

Concrete shear strength:

$$f'_{cv} = \min\left(\sqrt{f'_c}, f'_{cv-max}\right) = \min\left(\sqrt{4000\ \text{psi}}, 100\ \text{psi}\right)$$

$$\therefore f'_{cv} = 63.25\ \text{psi}$$

Shear capacity contribution of concrete:

$$V_c = \frac{2 \cdot f'_{cv} \cdot b \cdot d_{est}}{1000\ \text{lbs/kip}}$$

$$= \frac{2 \cdot 63.25\ \text{psi} \cdot 12\ \text{in} \cdot 22.12\ \text{in}}{1000\ \text{lbs/kip}}$$

$$\therefore V_c = 33.58\ \text{kips}$$

Calculated shear capacity contribution of reinforcement steel:

[ACI 318-14 22.5.10.5.3]

$$V_{s-calc} = \frac{A_v \cdot f_y \cdot d_{est}}{s} = \frac{0\ \text{in}^2 \cdot 60\ \text{ksi} \cdot 22.12\ \text{in}}{8\ \text{in}}$$

$$\therefore V_{s-calc} = 0\ \text{kips}$$

Maximum allowed shear capacity contribution of reinforcement steel:

[ACI 318-14 22.5.1.2]

$$V_{s-max} = \frac{8 \cdot f'_{cv} \cdot b \cdot d_{est}}{1000\ \text{lbs/kip}}$$

$$= \frac{8 \cdot 63.25\ \text{psi} \cdot 12\ \text{in} \cdot 22.12\ \text{in}}{1000\ \text{lbs/kip}}$$

$$\therefore V_{s-max} = 134.3\ \text{kips}$$

Shear capacity contribution of reinforcement steel:

$$V_s = \min\left(V_{s-calc}, V_{s-max}\right) = \min\left(0\ \text{kips}, 134.3\ \text{kips}\right)$$

$$\therefore V_s = 0\ \text{kips}$$

Nominal shear capacity of the beam section:

$$V_n = V_c + V_s = 33.58\ \text{kips} + 0\ \text{kips}$$

$$\therefore V_n = 33.58\ \text{kips}$$

Strength reduction factor for shear capacity:

$\phi_v = 0.75$ [ACI 318-14 Table 21.2.1]

Design shear capacity of the beam section:

$$\phi V_n = \phi_v \cdot V_n = 0.75 \;\cdot 33.58 \text{ kips}$$
$$\therefore \phi V_n = 25.19 \text{ kips}$$

2.2. Shear Reinforcement Limits

2.2.1. Minimum Shear Reinforcement Area

[ACI 318-14 9.6.3.3(a)]

$$A_{v-min1} = \frac{0.75 \cdot \sqrt{f'_c}}{f_y \cdot 1000 \text{ lbs/kip}} \cdot b \cdot s$$
$$= \frac{0.75 \;\cdot \sqrt{4000 \text{ psi}}}{60 \text{ ksi} \cdot 1000 \text{ lbs/kip}} \cdot 12 \text{ in} \cdot 8 \text{ in}$$
$$\therefore A_{v-min1} = 0.07589 \text{ in}^2$$

[ACI 318-14 9.6.3.3(b)]

$$A_{v-min2} = \frac{50 \cdot b \cdot s}{f_y \cdot 1000 \text{ lbs/kip}} = \frac{50 \;\cdot 12 \text{ in} \cdot 8 \text{ in}}{60 \text{ ksi} \cdot 1000 \text{ lbs/kip}}$$
$$\therefore A_{v-min2} = 0.08 \text{ in}^2$$

Minimum shear reinforcing at spacing s:

$$A_{v-min} = \max\left(A_{v-min1}, A_{v-min2}\right) = \max\left(0.07589 \text{ in}^2, 0.08 \text{ in}^2\right)$$
$$\therefore A_{v-min} = 0.08 \text{ in}^2$$

$$Check\ A_v \;>\; A_{v-min}$$
$$0 \text{ in}^2 \;>\; 0.08 \text{ in}^2$$
$$\therefore NOT\ PASSING$$

2.2.2. Maximum Spacing of Shear Reinforcement

Limiting shear reinforcement steel capacity:

[ACI 318-14 Table 9.7.6.2.2]

$$V_{s-lim} = \frac{4 \cdot \sqrt{f_c'} \cdot b \cdot d_{est}}{1000 \text{ lbs/kip}}$$

$$= \frac{4 \cdot \sqrt{4000 \text{ psi}} \cdot 12 \text{ in} \cdot 22.12 \text{ in}}{1000 \text{ lbs/kip}}$$

$$\therefore V_{s-lim} = 67.17 \text{ kips}$$

$$\rightarrow V_s \le V_{s-lim}$$

Maximum allowed spacing of shear reinforcement:

[ACI 318-14 9.7.6.2.2]

$$s_{max} = \min\left(\frac{d_{est}}{2}, 24\right) = \min\left(\frac{22.12 \text{ in}}{2}, 24\right)$$

$$\therefore s_{max} = 11.06 \text{ in}$$

$$Check\ s < s_{max}$$

$$8 \text{ in} < 11.06 \text{ in}$$

$$\therefore OK$$