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Accelerator Design

JUAS'22 - Lecture and Mini-Workshop - Bastian Haerer (KIT)

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How to build an accelerator?

Idea:

You learned all the basics.

You are experts.

Transverse beam dynamics

Longitudinal beam dynamics

Transverse linear imperfections

Transverse non-linear effects

Synchrotron radiation

Now we take a "piece of paper" and apply it for an actual design!

—> You will split up into teams of 3-4 persons and work on a case study.

Scope: Design a top-factory

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Particle collider for precision measurements of the top quark mass

- Measurements at the $t\bar{t}$ pair production threshold
- Produce at least 100000 $t\bar{t}$ pairs per year for sufficient statistics
- The circumference of the machine must not exceed 100 km
- Synchrotron radiation power is limited to 50 MW per beam

Based on these boundary conditions… **propose a collider design!**

Tutor Team

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- Bastian Haerer (lectures)
- Adrian Oeftiger (workshop showrunner)
- Kévin André, Marc Wenskat, Carsten Mai (tutors)
- Bernhard Holzer (“the free electron”)

Topics I - Basic parameter set and general design aspects (Adrian, Carsten)

- Beam energy, cross section, luminosity
- No. of bunches, particles per bunch, β*, emittance
- General layout, magnet technology, basic cell layout, dipole filling factor
- Synchrotron radiation power, resistive wall impedance induced by power loss

Topic II - Synchrotron radiation emission and RF sections (Kévin, Marc)

- Synchrotron radiation power, critical energy, beam current
- Momentum compaction factor, transition energy, RF voltage, synchronous phase
- Number of RF cavities, length of RF section, synchrotron tune
- Damping times, equilibrium emittance, energy spread, bunch length

Topic III - Lattice design in MAD-X (Bastian, Bernhard)

- Design a basic cell according to beam requirements, implement a MAD-X model of the cell, close the ring
- Calculate synchrotron radiation integrals with MAD-X and equilibrium beam parameters
- Include dispersion suppressors and straight sections
- Include RF cavities and calculate equilibrium beam parameters with MAD-X

Like in real life: Expert-groups should talk to each other!

Timetable - Lectures

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(COURSE 1)

WEEK #3

	24 Jan. Monday	25 Jan. Tuesday	26 Jan. Wednesday	27 Jan. Thursday	28 Jan. Friday
MORNING	**WRITTEN EXAMINATION** **Transverse beam dynamics**	**Transverse linear imperfections** *H. Bartosik*	**Linacs** *D. Alesini*	**Synchrotron Radiation** *R. Ischebeck*	**Synchrotron Radiation** *R. Ischebeck*
		Synchrotron Radiation *R. Ischebeck*	**Synchrotron Radiation** *R. Ischebeck*	**Synchrotron Radiation** **R. Ischebeck**	**Synchrotron Radiation** **R. Ischebeck**
	WRITTEN EXAMINATION **Longitudinal beam dynamics**	**Synchrotron Radiation** *R. Ischebeck*	**Synchrotron Radiation** *R. Ischebeck*	**Synchrotron Radiation** *R. Ischebeck*	**Synchrotron Radiation** **(exam preparation)** *R. Ischebeck*
AFTERNOON	**Dedicated session on COLLIDERS** 0) Intro 1) LHC & future high-energy circular colliders *(O. Bruning)* 2) Electron-positron circular colliders *(F. Zimmermann)* 3) Future high-energy linear colliders *(L. Rinolfi)* 4) The US Electron-Ion Collider *(T. Satogata)* 5) Muon collider *(D. Schulte)*	**Transverse nonlinear effects** *H. Bartosik*	**Transverse nonlinear effects** *H. Bartosik*	**Synchrotron Radiation** *R. Ischebeck*	**Synchrotron Radiation** **(exam preparation)** *R. Ischebeck*
		Transverse nonlinear effects *H. Bartosik*	**Transverse nonlinear effects** *H. Bartosik*	**Accelerator design** *B. Holzer & B. Härer*	**Accelerator design** *B. Holzer & B. Härer*
		Transverse nonlinear effects *H. Bartosik*	**Transverse nonlinear effects** *H. Bartosik*	**Accelerator design** *B. Holzer & B. Härer*	**Accelerator design** *B. Holzer & B. Härer*
			Transverse nonlinear manipulation Seminar *M. Giovannozzi*	**Accelerator design** *B. Holzer & B. Härer*	**Accelerator design** *B. Holzer & B. Härer*

Timetable - Workshop

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(COURSE 1)

WEEK #4

	31 Jan. Monday	1 Feb. Tuesday	2 Feb. Wednesday	3 Feb. Thursday	4 Feb. Friday
MORNING	**Collective effects (mainly space charge & instabilities)** *M. Migliorati*	**Collective effects (mainly space charge & instabilities)** *M. Migliorati*	**Collective effects (mainly space charge & instabilities)** *M. Migliorati*	**Collective effects (mainly space charge & instabilities)** *M. Migliorati*	**Virtual visit ESRF: Intro, Scientific case & Facility** *J-L. Revol*
	Collective effects (mainly space charge & instabilities) *M. Migliorati*	**Collective effects (mainly space charge & instabilities)** *M. Migliorati*	**Collective effects (mainly space charge & instabilities)** *M. Migliorati*	**Collective effects (mainly space charge & instabilities)** *M. Migliorati*	**Virtual visit ESRF: Control room & Beamline** *J-L. Revol*
	Collective effects (mainly space charge & instabilities) *M. Migliorati*	**Collective effects (mainly space charge & instabilities)** *M. Migliorati*	**Collective effects (mainly space charge & instabilities)** *M. Migliorati*	**Collective effects (mainly space charge & instabilities)** *M. Migliorati*	
AFTERNOON	**Accelerator design Workshop** *B. Holzer & B. Härer*	**Accelerator design Workshop** *B. Holzer & B. Härer*	**Accelerator design Workshop** *B. Holzer & B. Härer*	**Accelerator design Workshop** *B. Holzer & B. Härer*	**Accelerator design Workshop** *B. Holzer & B. Härer*
	Accelerator design Workshop *B. Holzer & B. Härer*	**Accelerator design Workshop** *B. Holzer & B. Härer*	**Accelerator design Workshop** *B. Holzer & B. Härer*	**Accelerator design Workshop** *B. Holzer & B. Härer*	**Accelerator design Workshop** *B. Holzer & B. Härer*
	Repurposing the LHC Seminar J. Jowett	**Beam-based impedance measurements** Seminar N. Biancacci	**Novel High Gradient Particle Accelerators** Seminar *R. Assmann*	**CERN LIU Project: Beam dynamics aspects & solutions** Seminar G. Rumolo	**Accelerator design Workshop** *B. Holzer & B. Härer*

Discussion session

Timetable - Examination

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(COURSE 1)

WEEK #5

	7 Feb. Monday	8 Feb. Tuesday	9 Feb. Wednesday	10 Feb. Thursday	11 Feb. Friday
MORNING	**ORAL EXAMINATION Accelerator design**	**Cyclotrons & FFAs** *B. Jacquot*	**Injection / Extraction** *N. Carmignani*	**WRITTEN EXAMINATION** **Synchrotron Radiation**	**WRITTEN EXAMINATION** **Subject 5 (TBA mid week 4)**
	ORAL EXAMINATION Accelerator design	**Cyclotrons & FFAs** *B. Jacquot*	**Injection / Extraction** *N. Carmignani*	**WRITTEN EXAMINATION** **Subject 4 (TBA mid week 4)**	**CLOSING SESSION - Course 1 -**
	ORAL EXAMINATION Accelerator design	**Cyclotrons & FFAs** *B. Jacquot*	**Injection / Extraction** *N. Carmignani*		
AFTERNOON	**ORAL EXAMINATION Accelerator design**	**Cyclotrons & FFAs** *B. Jacquot*			
	Free-Electron Lasers Seminar *E. Prat Costa*	**Cyclotrons & FFAs** *B. Jacquot*			
		Cyclotrons & FFAs *B. Jacquot*			

Boundary conditions for examination

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- Oral group examination in 15 min slots
- 9 min presentation + 2-3 min questions by tutors

- Be present in Zoom at your time slot.
- The rest of the time you are free to study for the exams.
- In the afternoon session the “best team per topic” gets the chance to present again for the whole audience.

Monday 7 February

9:00 - 9:15	group 1
9:15 - 9:30	group 3
9:30 - 9:45	group 5
9:45 - 10:00	group 7
10:00 - 10:15	group 9
— 15 min buffer —	
10:30 - 10:45	group 2
10:45 - 11:00	group 4
11:00 - 11:15	group 6
11:15 - 11:30	group 8
11:30 - 11:45	group 10

Content overview

- We will review key aspects of previous lectures.
- We will discuss aspects of electron and hadron storage rings.
- Different lattice types and applications.

Context of the workshop: electron-positron collider for $t\bar{t}$ production

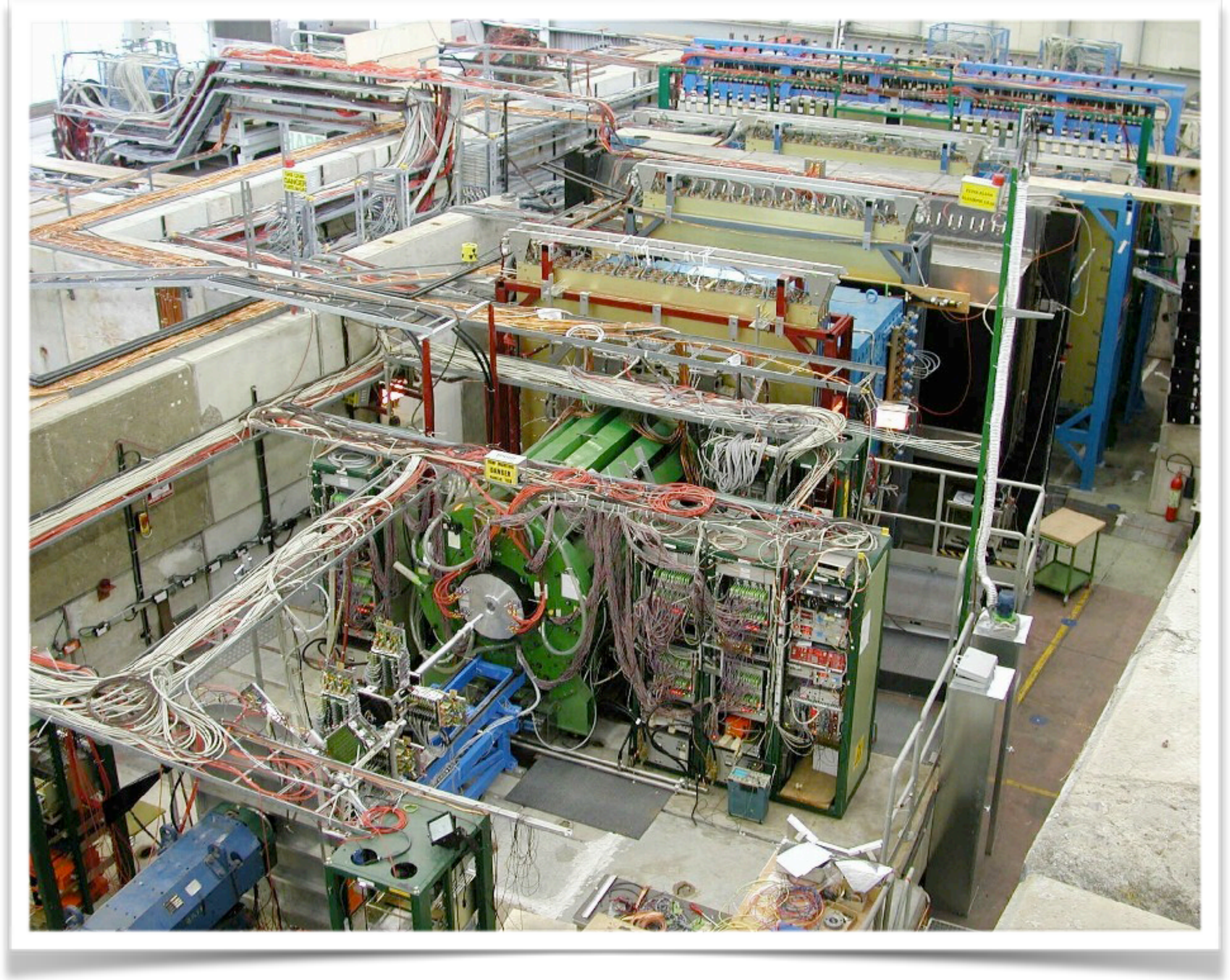
—> Design of a high-energy storage ring as preparation for the workshop.

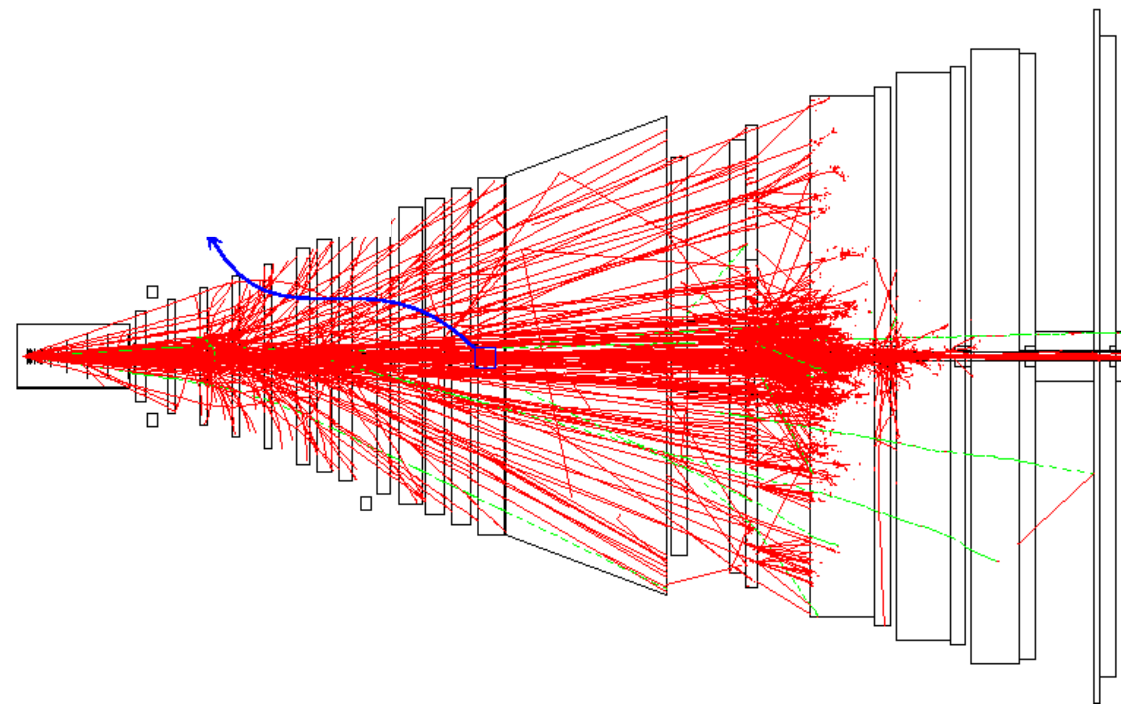
Fixed target vs. beam-beam collisions

Fixed target experiments

- high event rate
- limited energy reach

$$E_{lab} \propto \sqrt{E_{beam}}$$



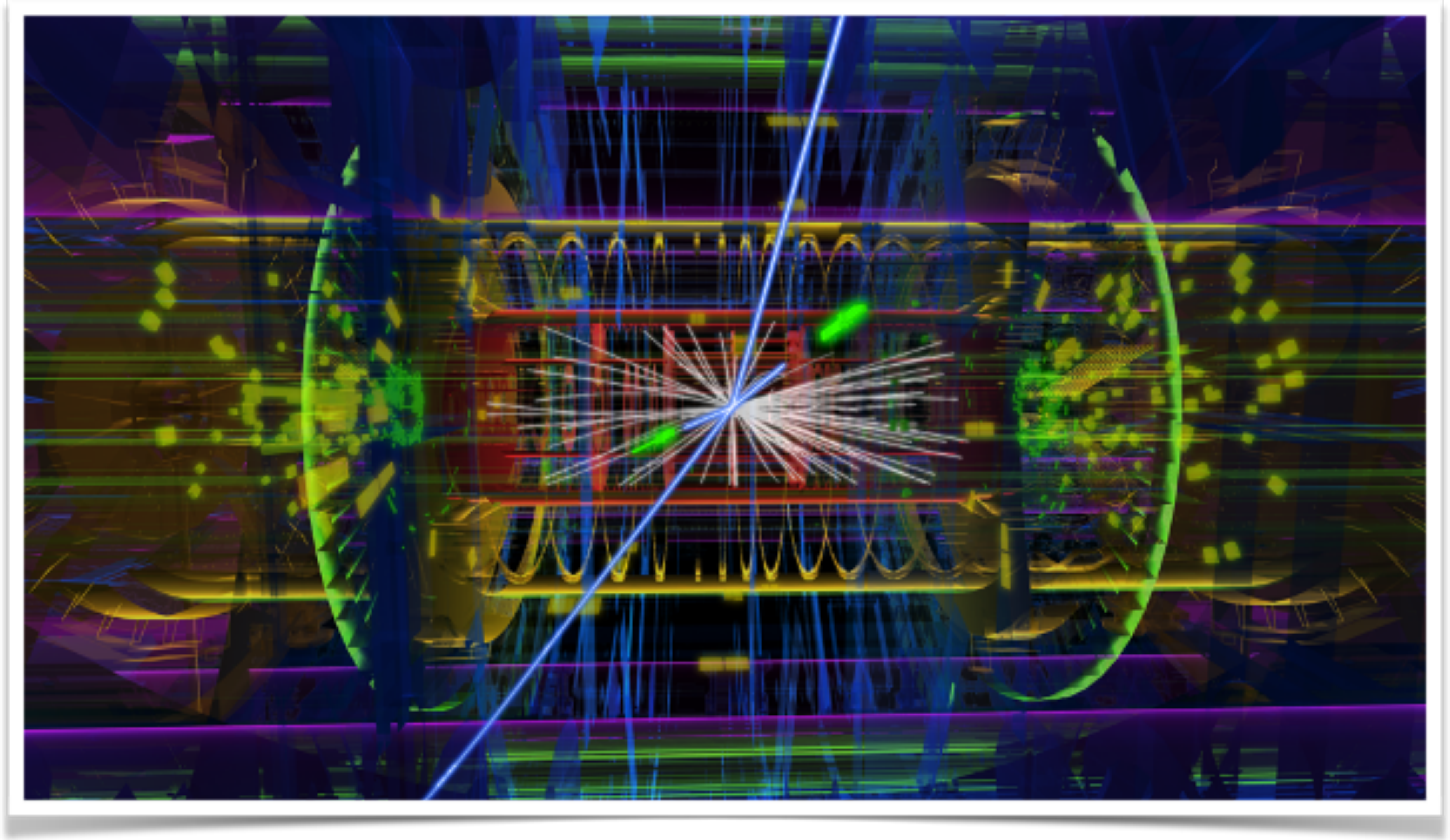


fixed target event p + W → xxxxx

Beam-beam collisions

- low event rate (luminosity)
- high energy reach

$$E_{lab} = E_{beam\ 1} + E_{beam\ 2}$$



ATLAS event display:
H → $e^+ + e^- + \mu^+ + \mu^-$

Choice of particle species

Hadrons

- Heavier, easier to reach high energies

  -> discovery machines

- Don't radiate (much)

m_p = 938 MeV/c² $\quad E = 10\,\mathrm{GeV} \quad \rightarrow \gamma_p = 11$

m_e = 0.511 MeV/c² $\quad \rightarrow \gamma_e = 19570$

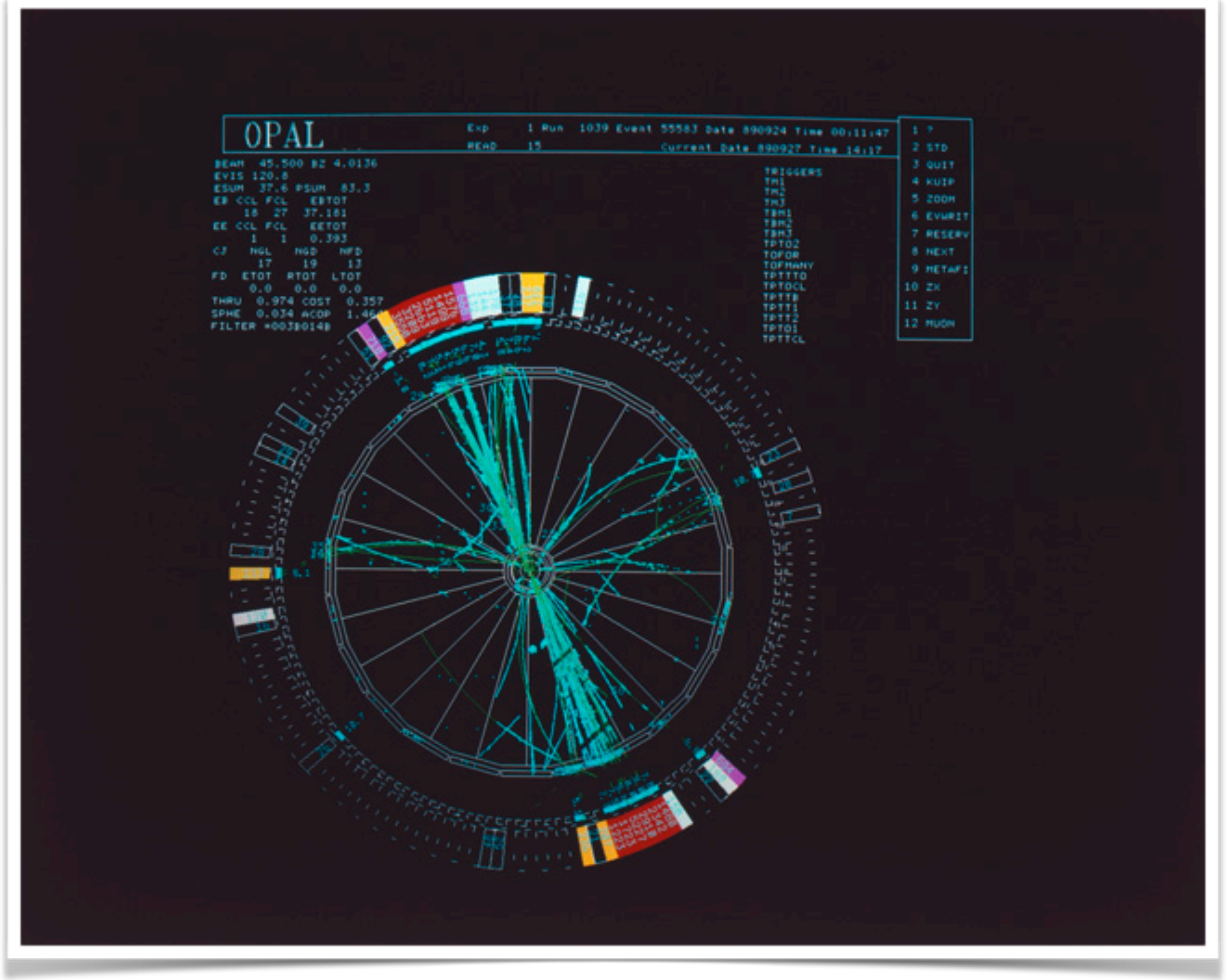
$$P_\gamma = \propto \frac{\gamma^4}{\rho^2}$$

Electrons & positrons

- Beam dynamics driven by emission of synchrotron radiation

- Elementary particles -> precision measurements

Event display of OPAL at LEP



https://cds.cern.ch/record/2554156

Linear vs. circular collider

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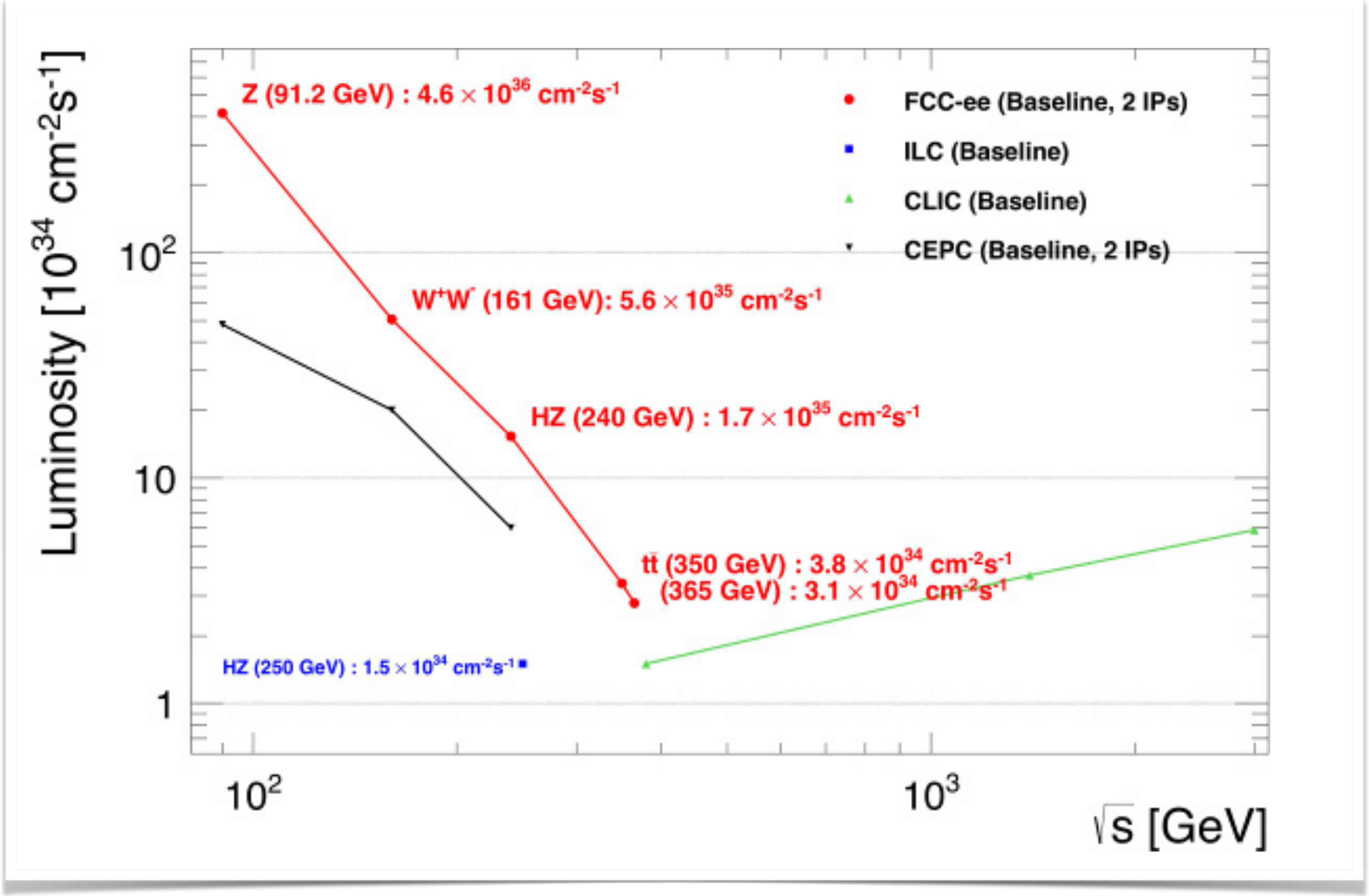
Linear collider

- no synchrotron radiation
- only one experiment at a time
- single use of particle bunches

Circular colliders

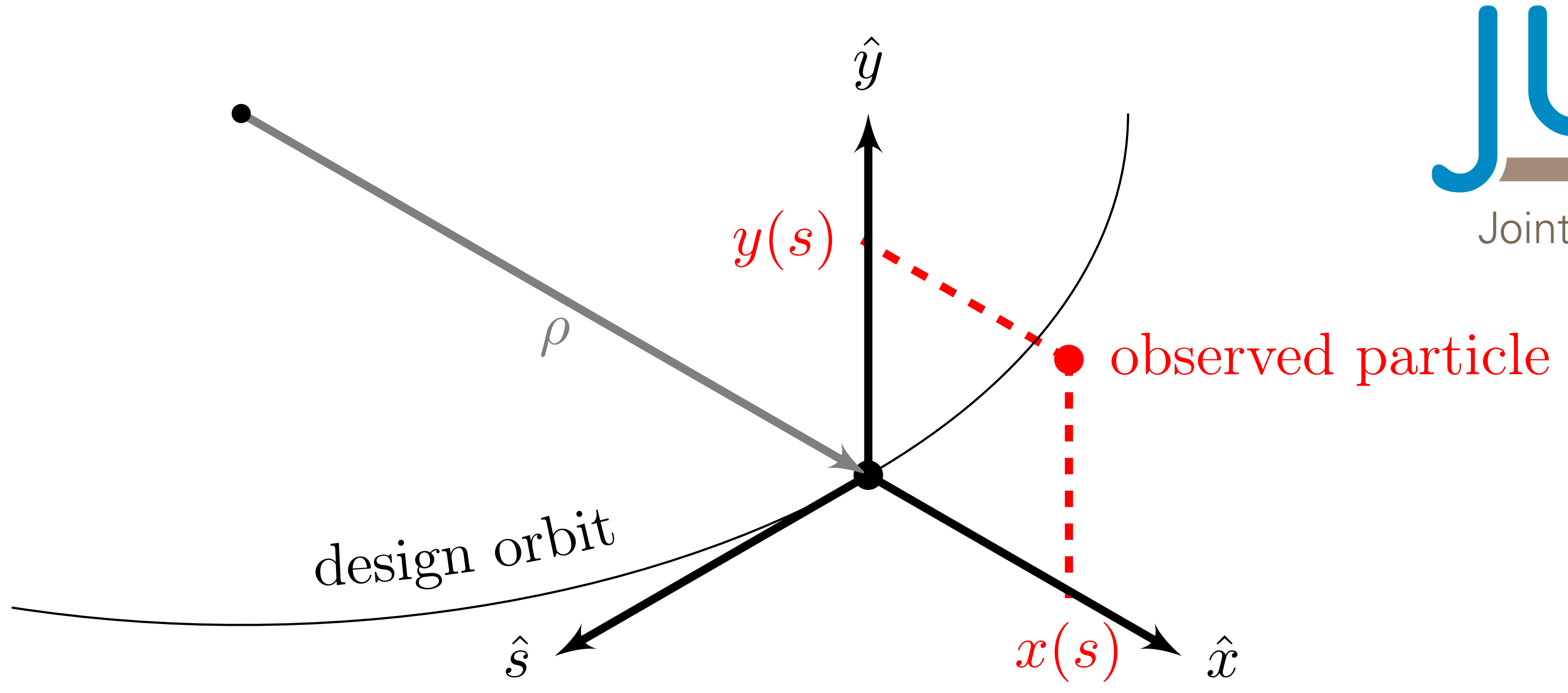
- multiple experiments
- bunches can be collided multiple times
- SR radiation power increases $P \propto \gamma^4$

Trade-off between SR power and luminosity



FCC-ee Design Report: Baseline luminosities expected to be delivered for different e^+e^- collider projects

Dipole fields define geometry



Condition for circular orbit

- Lorentz force $F_{\mathrm{L}} = evB$
- Centripetal force $F_{\mathrm{centr}} = \dfrac{\gamma m_0 v^2}{\rho}$

$$\boxed{\frac{p}{e} = B\,\rho}$$

"Beam rigidity"

The strength of the dipole magnets and the size of the machine define the maximum momentum (or energy) of the particles that can be carried in the machine.

Field strength defined by coil current, gap height $\Big\}$ $B = \dfrac{\mu_0\, n\, I}{h}$

—> *keep the beam dimensions small !!!*

Bending angle and particle momentum

- The integrated dipole strength (along "s") defines the momentum of the particle beam.

$$d\theta = \frac{ds}{\rho} \approx \frac{dl}{\rho} = \frac{B\,dl}{B\rho} = \frac{e}{p_0} B\,dl \qquad \Rightarrow \int B\,dl = 2\pi \frac{p_0}{e}$$

Example: LHC 7 TeV proton storage ring

- B = 8.3 T
- N = 1232
- l = 14.3 m

$$\int B\,dl \approx N\,l\,B = 2\pi \frac{p_0}{e}$$

$$p_0 = \frac{N\,l\,B\,e}{2\pi} = 7\,\frac{\text{TeV}}{c}$$

Quadrupole magnets for focusing

Equation of motion: $\boldsymbol{x'' + K\,x = 0}$

Define in hor. plane: $K = 1/\rho^2 - k$

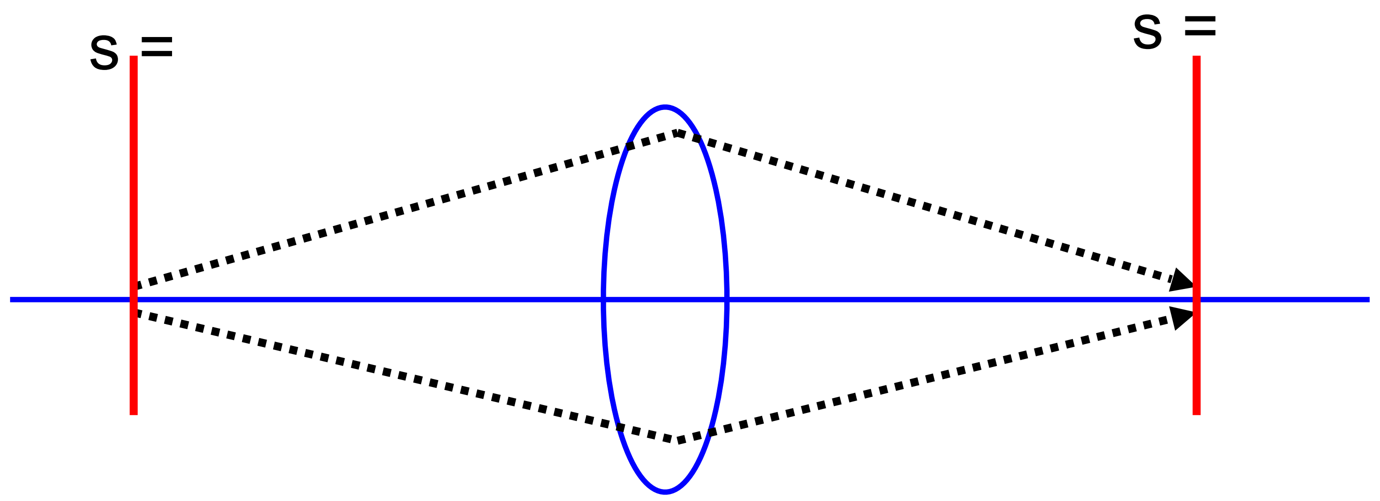
… in vert. plane: $K = k$

Differential equation of harmonic oscillator … with spring constant K

Solution: Hor. Focusing Quadrupole K > 0:

$$x(s) = x_0 \cdot \cos(\sqrt{|K|}s) + x_0' \cdot \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}s)$$

$$x'(s) = -x_0 \cdot \sqrt{|K|} \cdot \sin(\sqrt{|K|}s) + x_0' \cdot \cos(\sqrt{|K|}s)$$



For convenience we like to express that in matrix form:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{foc} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$

Transfer matrices

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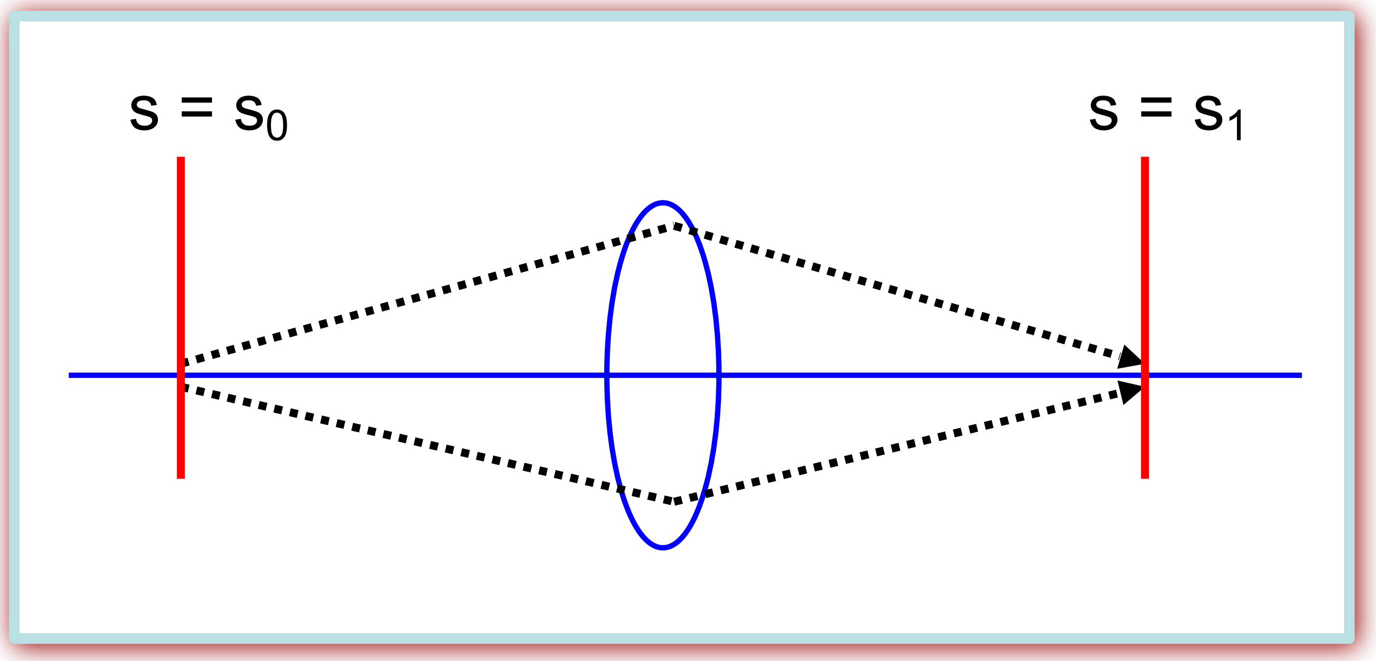
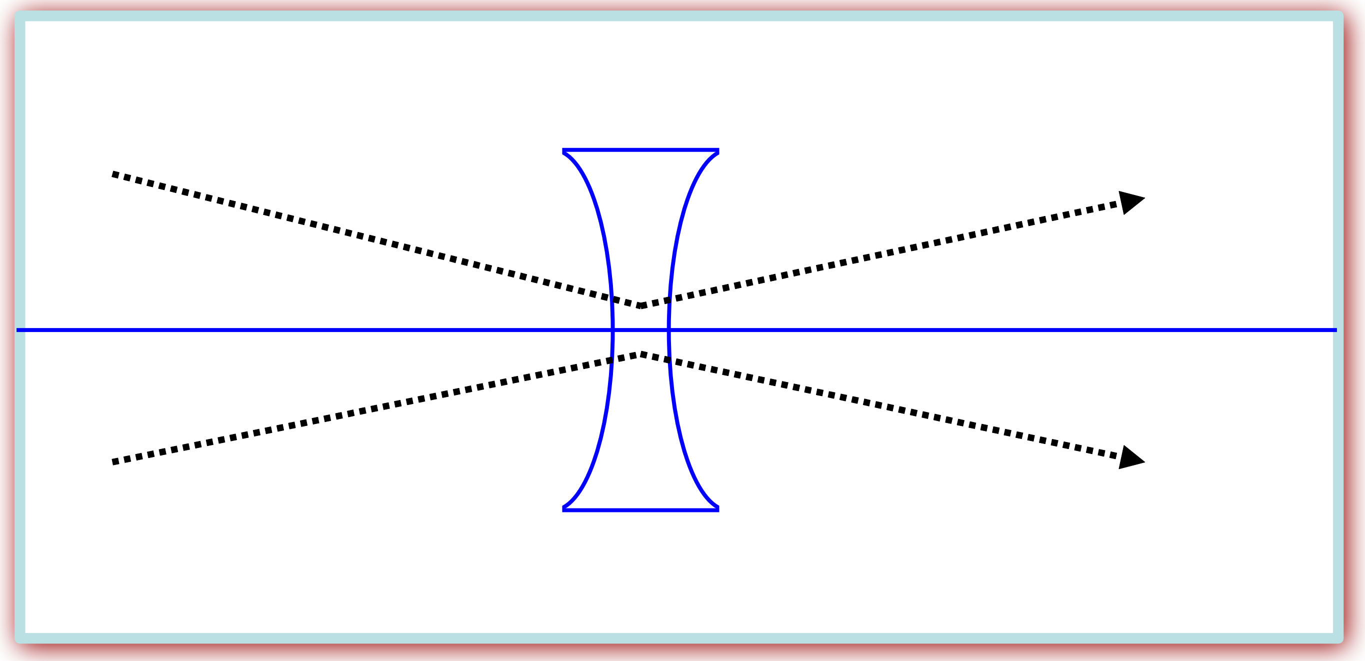
hor. focusing quadrupole: ***K < 0***

$$M_{foc} = \begin{pmatrix} \cos(\sqrt{|K|}s) & \frac{1}{\sqrt{|K|}}\sin(\sqrt{|K|}s \\ -\sqrt{|K|}\sin(\sqrt{|K|}s) & \cos(\sqrt{|K|}s) \end{pmatrix}_0$$



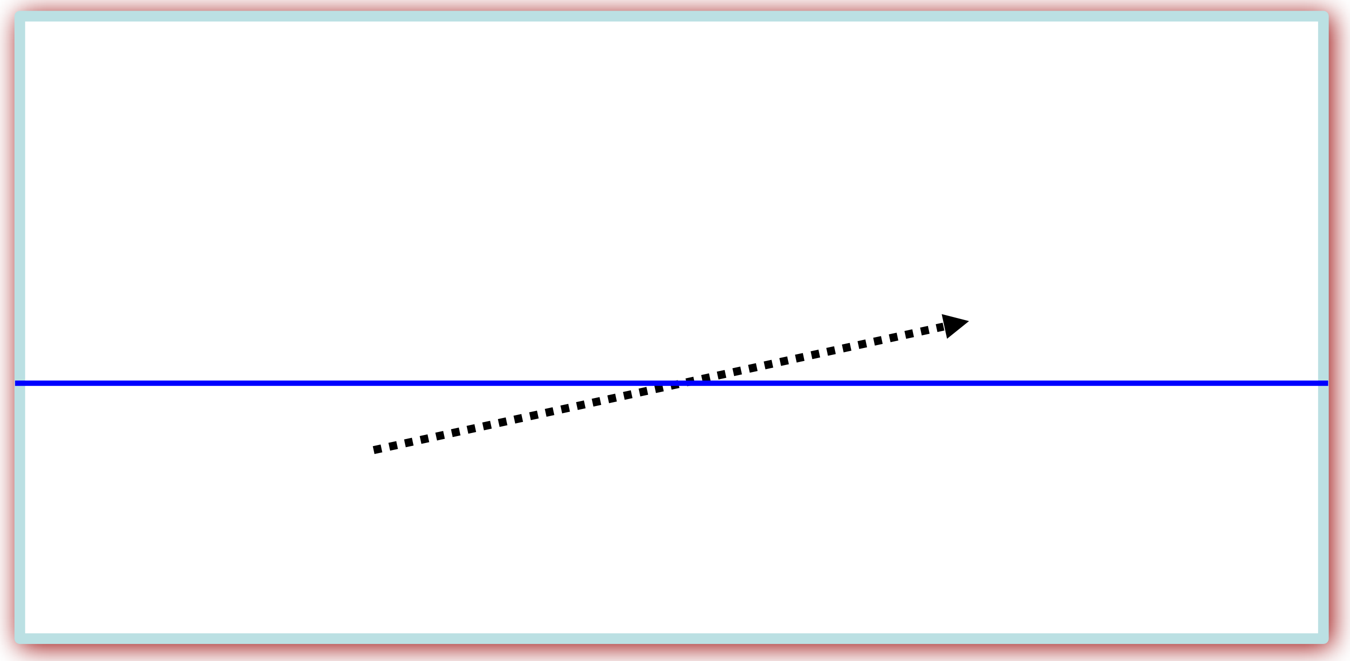
hor. defocusing quadrupole: ***K < 0***

$$M_{defoc} = \begin{pmatrix} \cosh\sqrt{|K|}l & \frac{1}{\sqrt{|K|}}\sinh\sqrt{|K|}l \\ \sqrt{|K|}\sinh\sqrt{|K|}l & \cosh\sqrt{|K|}l \end{pmatrix}$$



drift space: ***K = 0***

$$M_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$



Thin lens approximation

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matrix of a quadrupole lens

$$M = \begin{pmatrix} \cos\sqrt{|k|}l & \frac{1}{\sqrt{|k|}}\sin\sqrt{|k|}l \\ -\sqrt{|k|}\sin\sqrt{|k|}l & \cos\sqrt{|k|}l \end{pmatrix}$$

in many practical cases we have the situation:

$$f = \frac{1}{kl_q} >> l_q$$

... focal length of the lens is much bigger than the length of the magnet

limes: $l_q \rightarrow 0$ ***while keeping*** $k\, l_q = const$

$$M_x = \begin{pmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{pmatrix} \qquad M_z = \begin{pmatrix} 1 & 0 \\ \frac{-1}{f} & 1 \end{pmatrix} \qquad f = \frac{1}{kl_q}$$

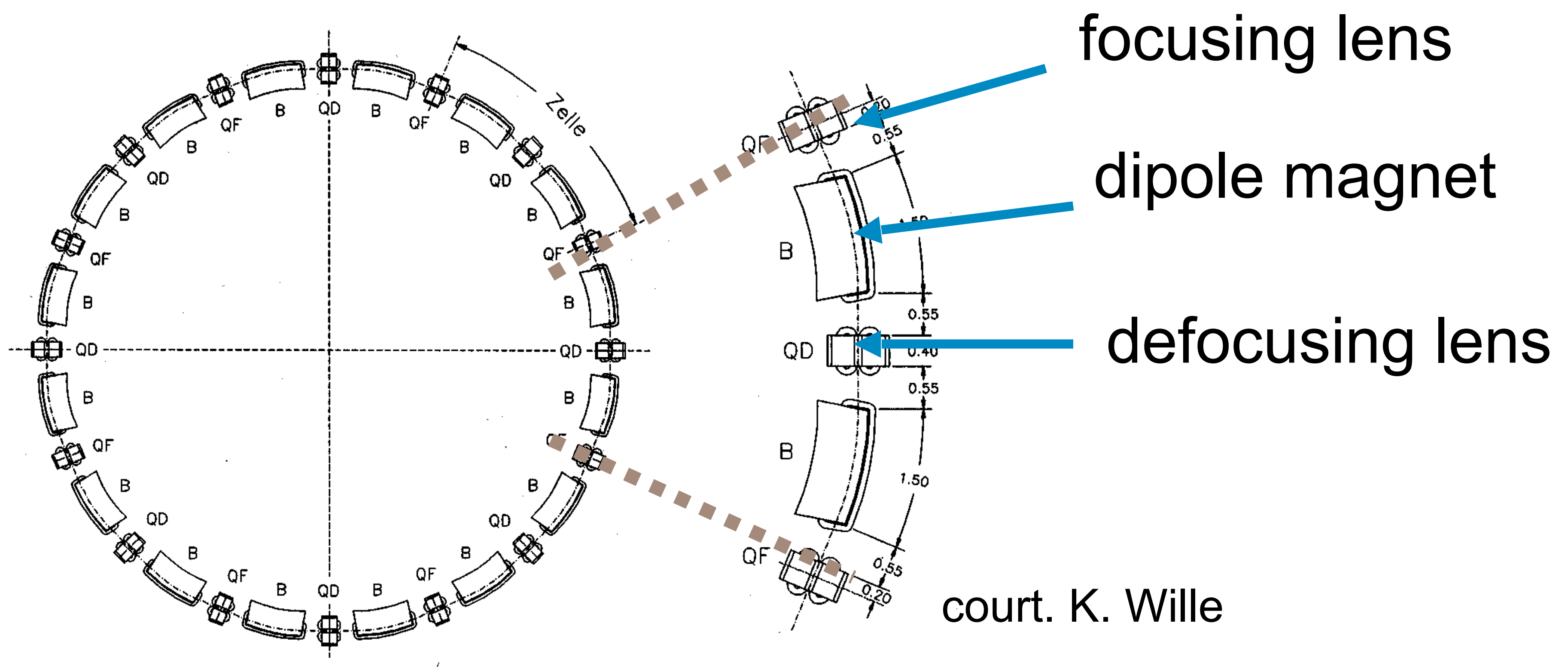
... useful for fast (and in large machines still quite accurate) „back of the envelope calculations“ ... and for the mini - workshop !

Transformation through a system of lattice elements

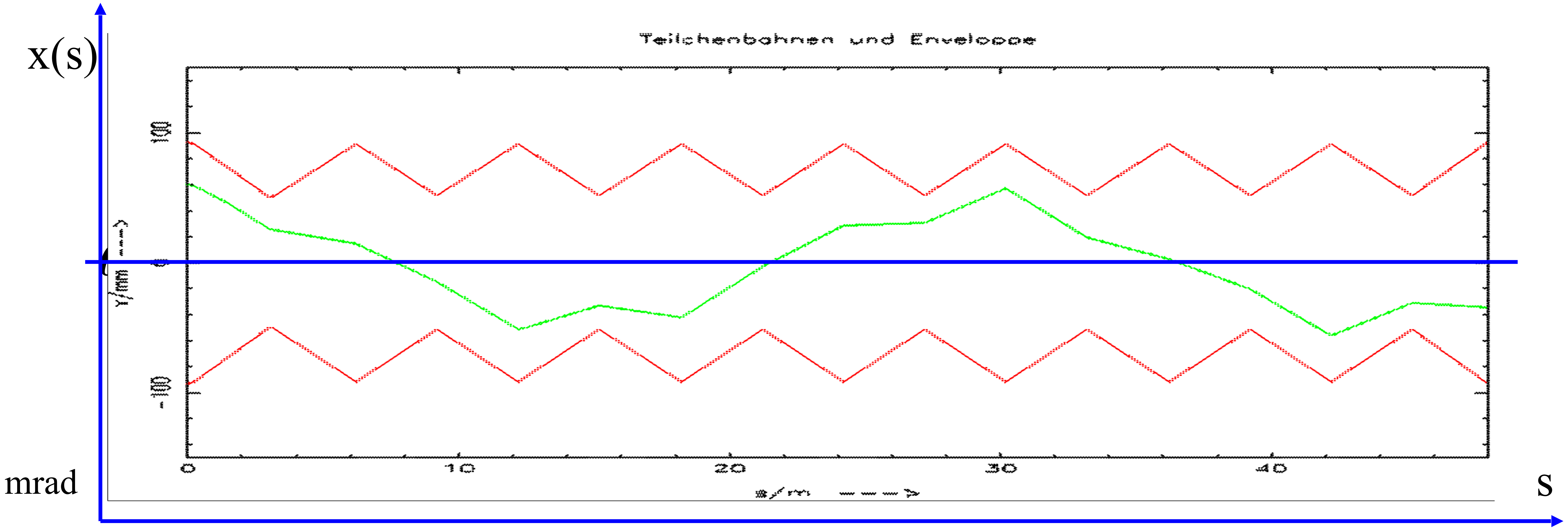
$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s2} = M_{1->2} \begin{pmatrix} x \\ x' \end{pmatrix}_{s2}$$

combine the single element solutions by multiplication of the matrices …

$$M_{total} = \ldots * M_{QD} * M_D * M_{Bend} * M_D * M_{QF}$$



… which is especially useful for period sub-structures, e.g a basic lattice cell.



typical values in a strong foc. machine:
$x \approx$ mm, $x' \leq$ mrad

Transfer matrix as function of optics functions

general solution of Hill´s equation

$$\begin{cases} x(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}\cos\{\psi(s)+\phi\} \\ x'(s) = \dfrac{-\sqrt{\varepsilon}}{\sqrt{\beta(s)}}\left[\alpha(s)\cos\{\psi(s)+\phi\} + \sin\{\psi(s)+\phi\}\right] \end{cases}$$

remember the trigonometrical gymnastics: sin(a + b) = … etc

$$x(s) = \sqrt{\varepsilon}\sqrt{\beta_s}\left(\cos\psi_s\cos\phi - \sin\psi_s\sin\phi\right)$$

$$x'(s) = \frac{-\sqrt{\varepsilon}}{\sqrt{\beta_s}}\left[\alpha_s\cos\psi_s\cos\phi - \alpha_s\sin\psi_s\sin\phi + \sin\psi_s\cos\phi + \cos\psi_s\sin\phi\right]$$

starting at point s(0) = s_0, where we put $\Psi(0) = 0$

$$\left.\begin{aligned} \cos\phi &= \frac{x_0}{\sqrt{\varepsilon\beta_0}} \quad , \\ \sin\phi &= -\frac{1}{\sqrt{\varepsilon}}\left(x_0'\sqrt{\beta_0} + \frac{\alpha_0 x_0}{\sqrt{\beta_0}}\right) \end{aligned}\right\}$$

inserting above …

$$x(s) = \sqrt{\frac{\beta_s}{\beta_0}} \left\{\cos\psi_s + \alpha_0 \sin\psi_s\right\} x_0 + \left\{\sqrt{\beta_s\beta_0} \sin\psi_s\right\} x_0'$$

$$x'(s) = \frac{1}{\sqrt{\beta_s\beta_0}} \left\{(\alpha_0 - \alpha_s)\cos\psi_s - (1+\alpha_0\alpha_s)\sin\psi_s\right\} x_0 + \sqrt{\frac{\beta_0}{\beta_s}} \left\{\cos\psi_s - \alpha_s \sin\psi_s\right\} x_0'$$

which can be expressed ... for convenience ... in matrix form

$$\begin{pmatrix} x \\ x' \end{pmatrix}_s = M \begin{pmatrix} x \\ x' \end{pmatrix}_0$$

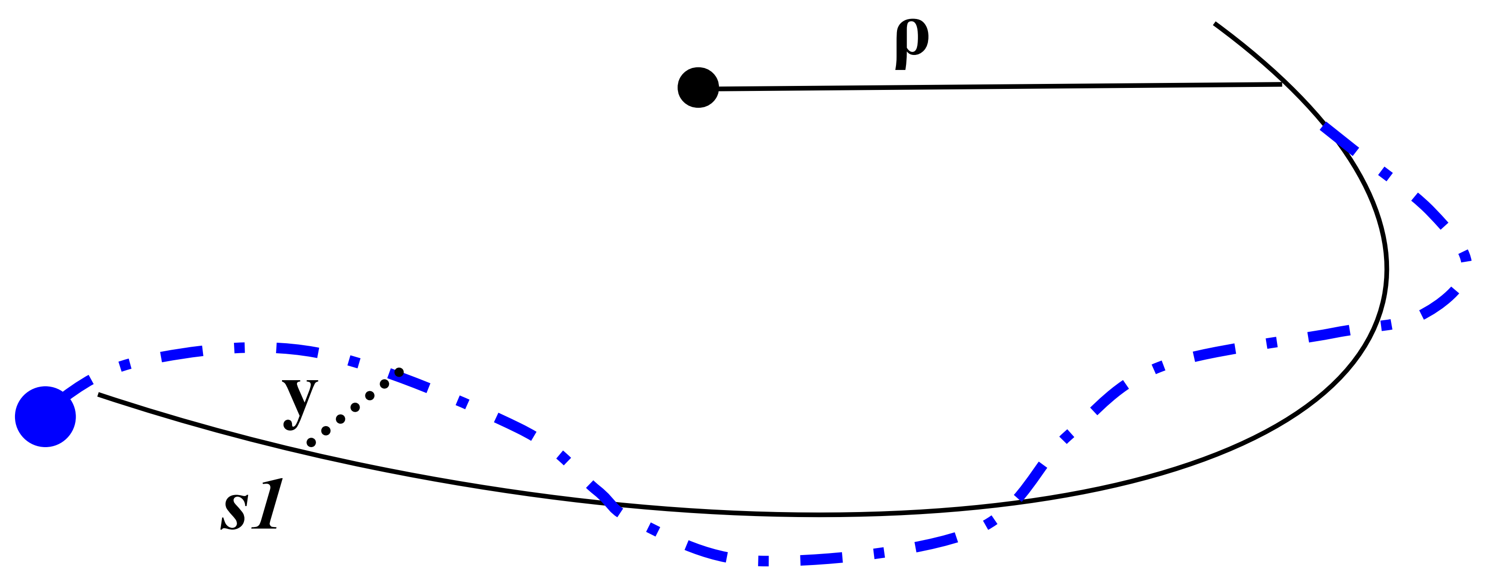
$$M = \begin{pmatrix} \sqrt{\frac{\beta_s}{\beta_0}} \left(\cos\psi_s + \alpha_0 \sin\psi_s\right) & \sqrt{\beta_s\beta_0} \sin\psi_s \\ \frac{(\alpha_0 - \alpha_s)\cos\psi_s - (1+\alpha_0\alpha_s)\sin\psi_s}{\sqrt{\beta_s\beta_0}} & \sqrt{\frac{\beta_0}{\beta s}} \left(\cos\psi_s - \alpha_s \sin\psi_s\right) \end{pmatrix}$$

* ***we can calculate the single particle trajectories between two locations in the ring, if we know the α, β, γ at these positions.***
* ***and nothing but the α β γ at these positions.***
* ***... !***

To make it VERY clear:

going from a point s_1 —> s_2 in an lattice we can either

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s2} = M_{1->2} \begin{pmatrix} x \\ x' \end{pmatrix}_{s2}$$



… calculate the product matrix of the single lattice elements

$$M_{1->2} = M_{QF} \cdot M_d \cdot M_B \cdot M_d \cdot M_{QD} \ldots$$

… or use the matrix expressed as function of the optics parameters at position s_1 and s_2

$$M = \begin{pmatrix} \sqrt{\frac{\beta_s}{\beta_0}} \left(\cos\psi_s + \alpha_0 \sin\psi_s\right) & \sqrt{\beta_s \beta_0} \sin\psi_s \\ \frac{(\alpha_0 - \alpha_s)\cos\psi_s - (1+\alpha_0\alpha_s)\sin\psi_s}{\sqrt{\beta_s \beta_0}} & \sqrt{\frac{\beta_0}{\beta s}} \left(\cos\psi_s - \alpha_s \sin\psi_s\right) \end{pmatrix}$$

The numerical values of the matrix elements are exactly the same.

The optics functions α(s), β(s), γ(s)

— sometimes also called "Twiss" functions —

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(Phil Bryant)

There are two ways of looking at the optics functions:

The first is to regard them as a **parametric way of** expressing the equation of motion and its solution. This interpretation makes the bridge from tracking single ions to the wider view of calculating beam envelopes.

The second is to regard them as **purely geometric parameters** for defining ellipses and hence beam envelopes. Dropping the strict correspondence to individual particles can lead to some interesting extensions such as the inclusion of scattering.

Phase space ellipse

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general solution of Hill equation

$$\begin{cases} (1) \quad x(s) = \sqrt{\varepsilon}\,\sqrt{\beta(s)}\,\cos(\psi(s)+\phi) \\ (2) \quad x'(s) = -\dfrac{\sqrt{\varepsilon}}{\sqrt{\beta(s)}}\left\{\alpha(s)\cos(\psi(s)+\phi)+\sin(\psi(s)+\phi)\right\} \end{cases}$$

from (1) we get

$$\cos(\psi(s)+\phi) = \frac{x(s)}{\sqrt{\varepsilon}\,\sqrt{\beta(s)}}$$

and we know

$$\alpha(s) = \frac{-1}{2}\beta'(s)$$

$$\gamma(s) = \frac{1+\alpha(s)^2}{\beta(s)}$$

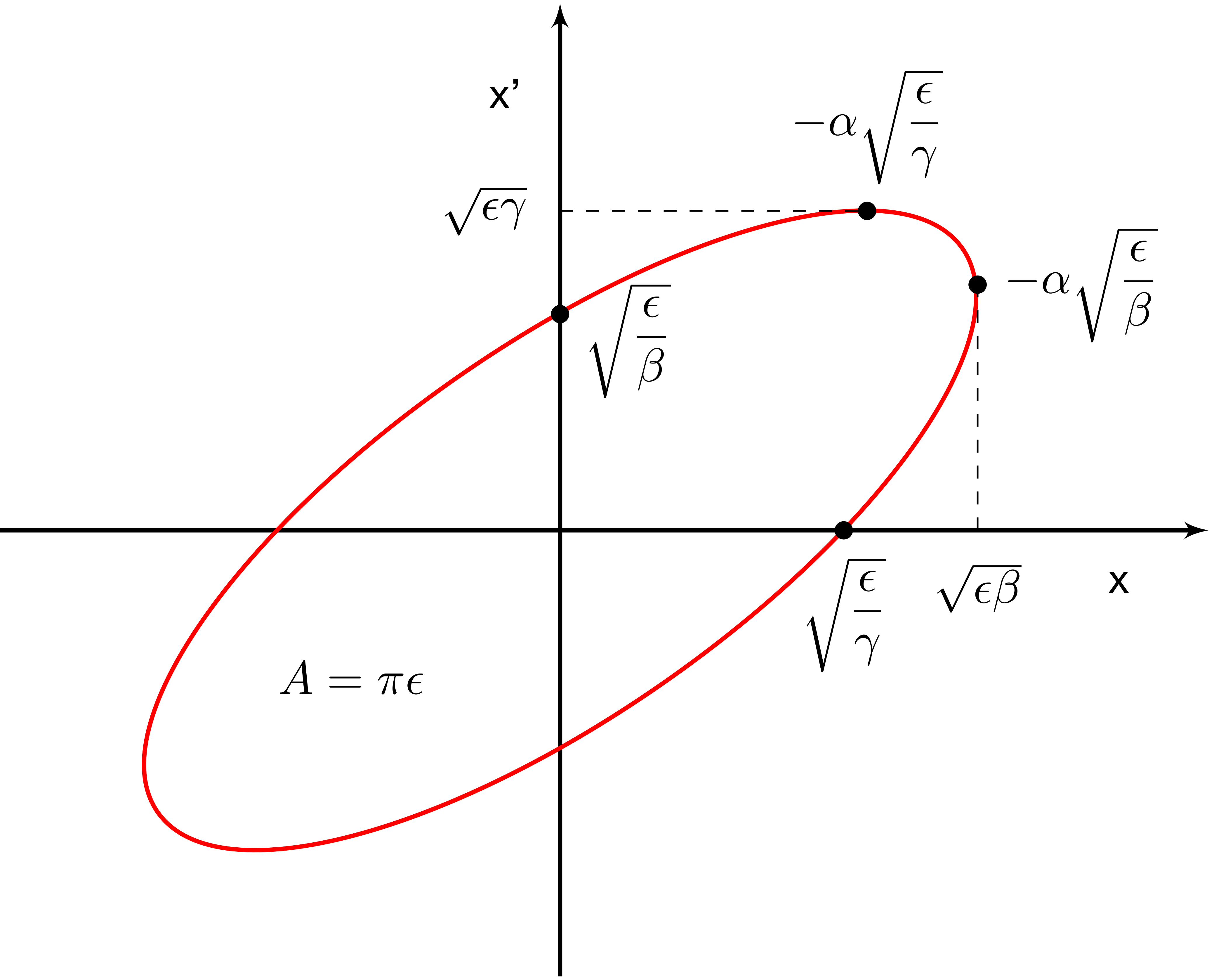
Insert into (2) and solve for ε

$$\varepsilon = \gamma(s)\,x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)\,x'^2(s)$$

* *ε is a constant of the motion … it is independent of „s“*
* *parametric representation of an ellipse in the x x‘ space*
* *shape and orientation of ellipse are given by α, β, γ*

Phase space ellipse - II

$$\epsilon = \gamma(s)x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)x'^2(s)$$

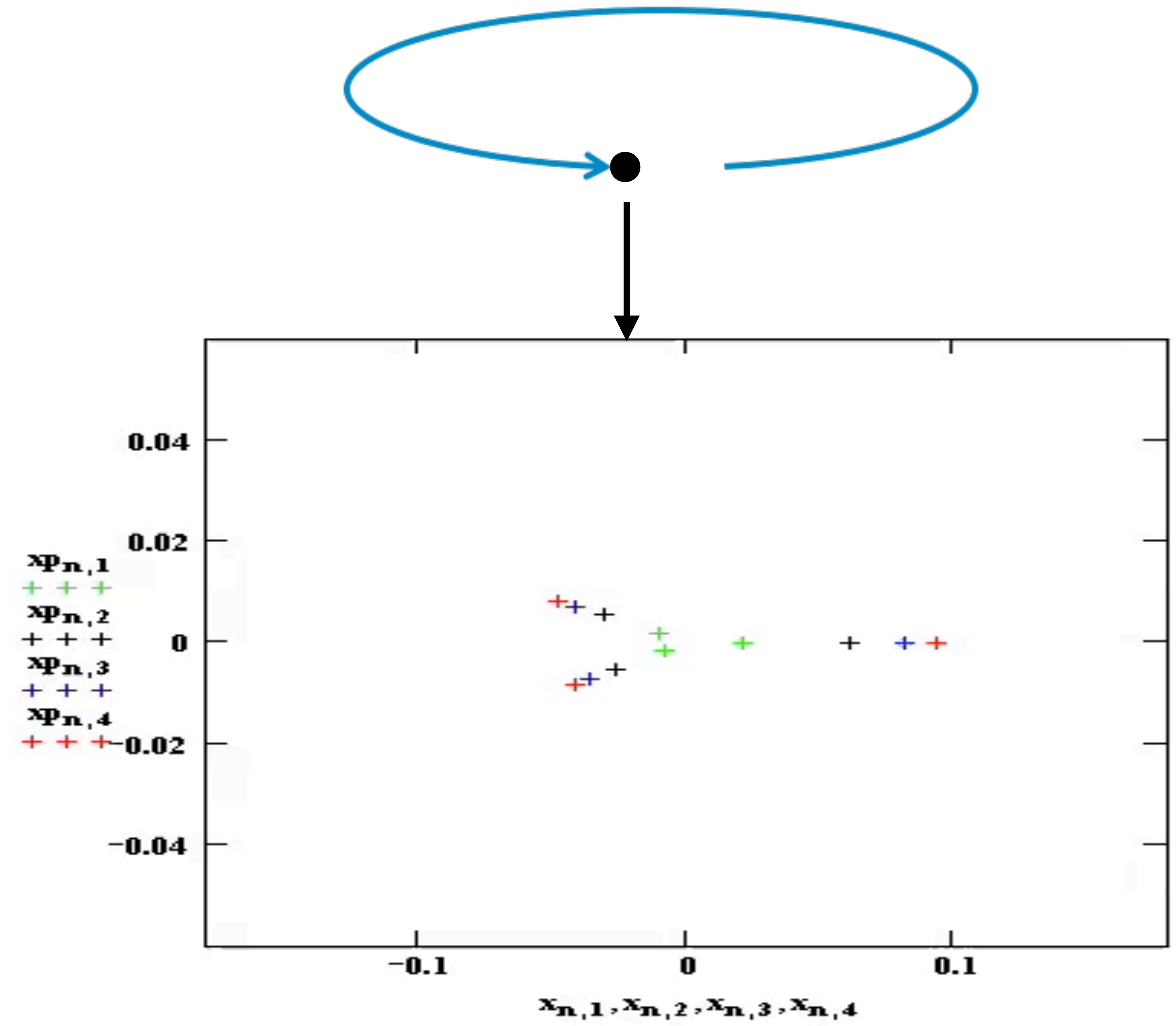


- Parametrisation describes ellipse in xx'-space.
- **Liouville's theorem:** Area $A = \pi\ \epsilon$ is constant as long as x- and y-motion are uncoupled and energy is conserved.
- Area cannot be changed by focussing properties (e.g. quadrupoles).
- Single solution of Hill's equation (x,x') is one point on this ellipse.
- This ellipse represents all solutions/states the particle can be in at this position *s*.

(Phil Bryant)

In the case of a ring or matched cell, the periodicity imposes equality on the input and output α and β values.

This means that the particle returns after each turn to the same ellipse but at phases $\mu_1 = b$, $\mu_2 = b+2\pi Q$, $\mu_3 = b+4\pi Q$, ….., $\mu_n = b + n2\pi Q$ and so on.



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Beam size and divergence

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particle trajectory: $x(s) = \sqrt{\varepsilon}\,\sqrt{\beta(s)}\,\cos\{\psi(s)+\phi\}$

max. Amplitude: $\boxed{\hat{x}(s) = \sqrt{\varepsilon\beta}}$ $\longrightarrow$ ***x´ at that position …?***

… put $\hat{x}(s)$ ***into*** $\varepsilon = \gamma(s)\,x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)\,x'^2(s)$ ***and solve for x´***

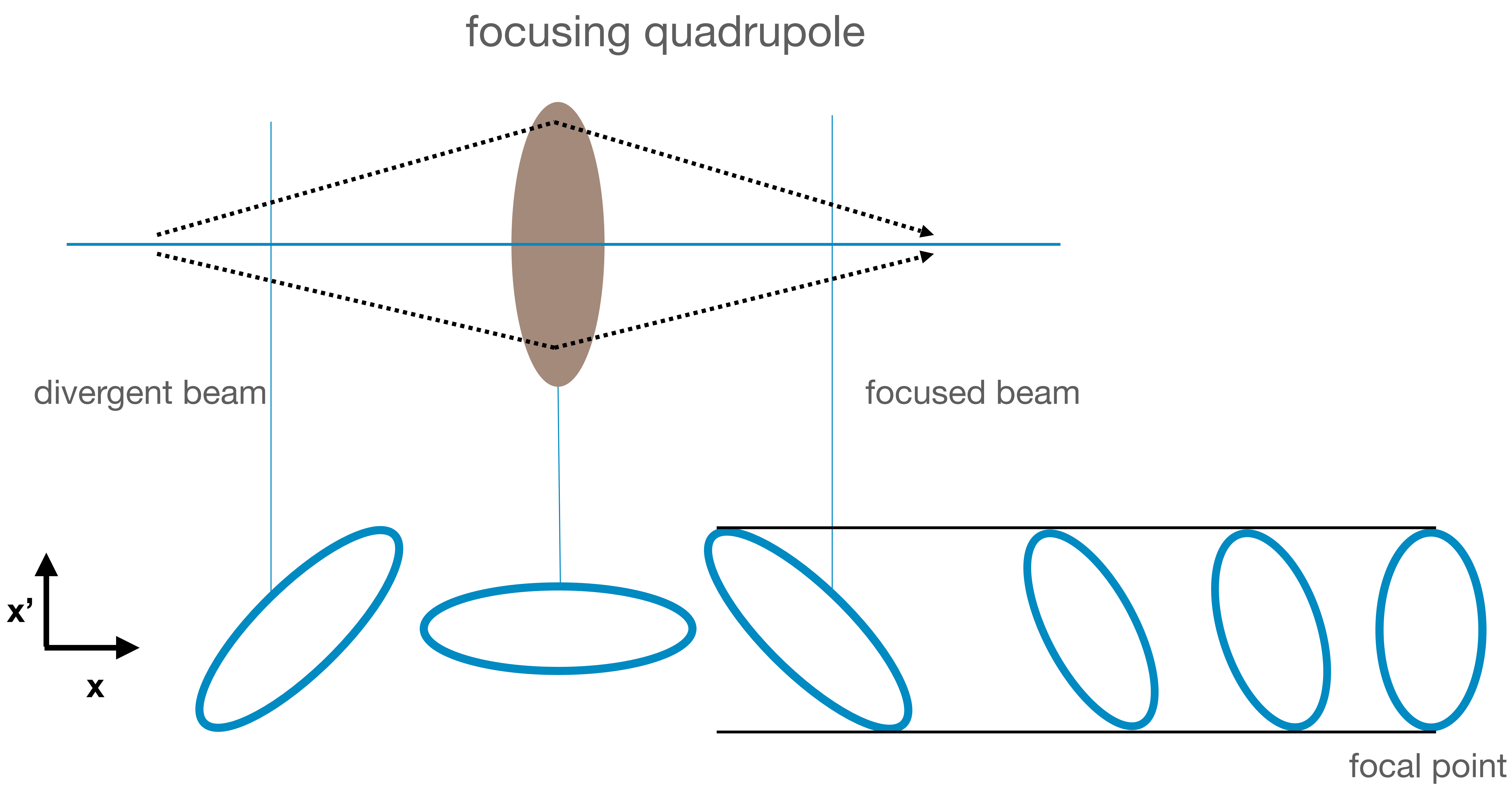
$$\varepsilon = \gamma\cdot\varepsilon\beta + 2\alpha\sqrt{\varepsilon\beta}\cdot x' + \beta x'^2$$

$$\longrightarrow \boxed{x' = -\alpha\cdot\sqrt{\varepsilon/\beta}}$$

* ***A high β-function means a large beam size and a small beam divergence.***
… et vice versa !!! !

* ***In the middle of a quadrupole β = maximum,***
α = zero $\Big\}$ $x' = 0$ ***… and the ellipse is flat***

Evolution of phase space ellipse along the lattice



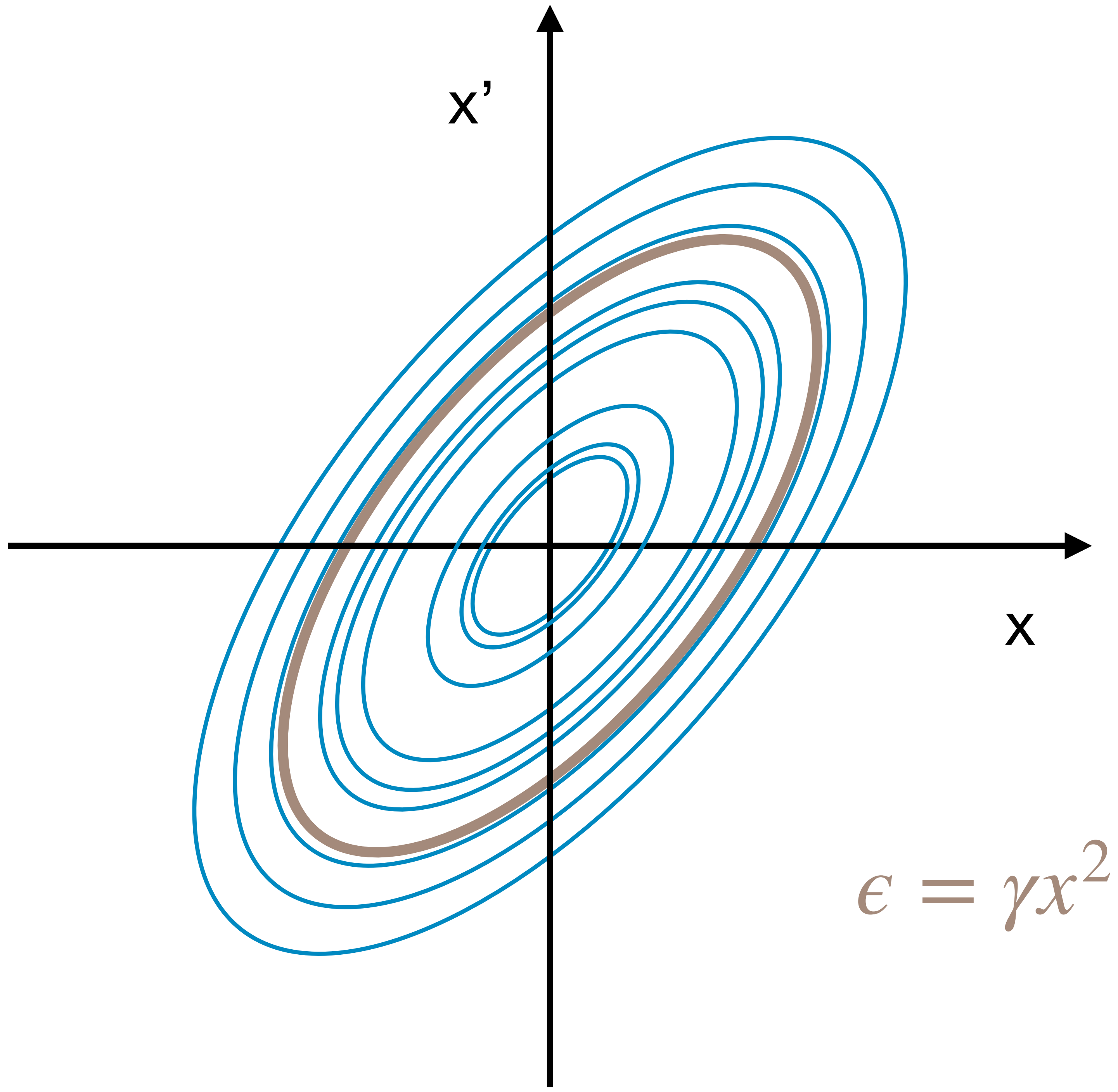
$\alpha > 0$
$\beta = \beta_1$

$\alpha = 0$

$\alpha < 0$
$\beta_2 = \beta_1$

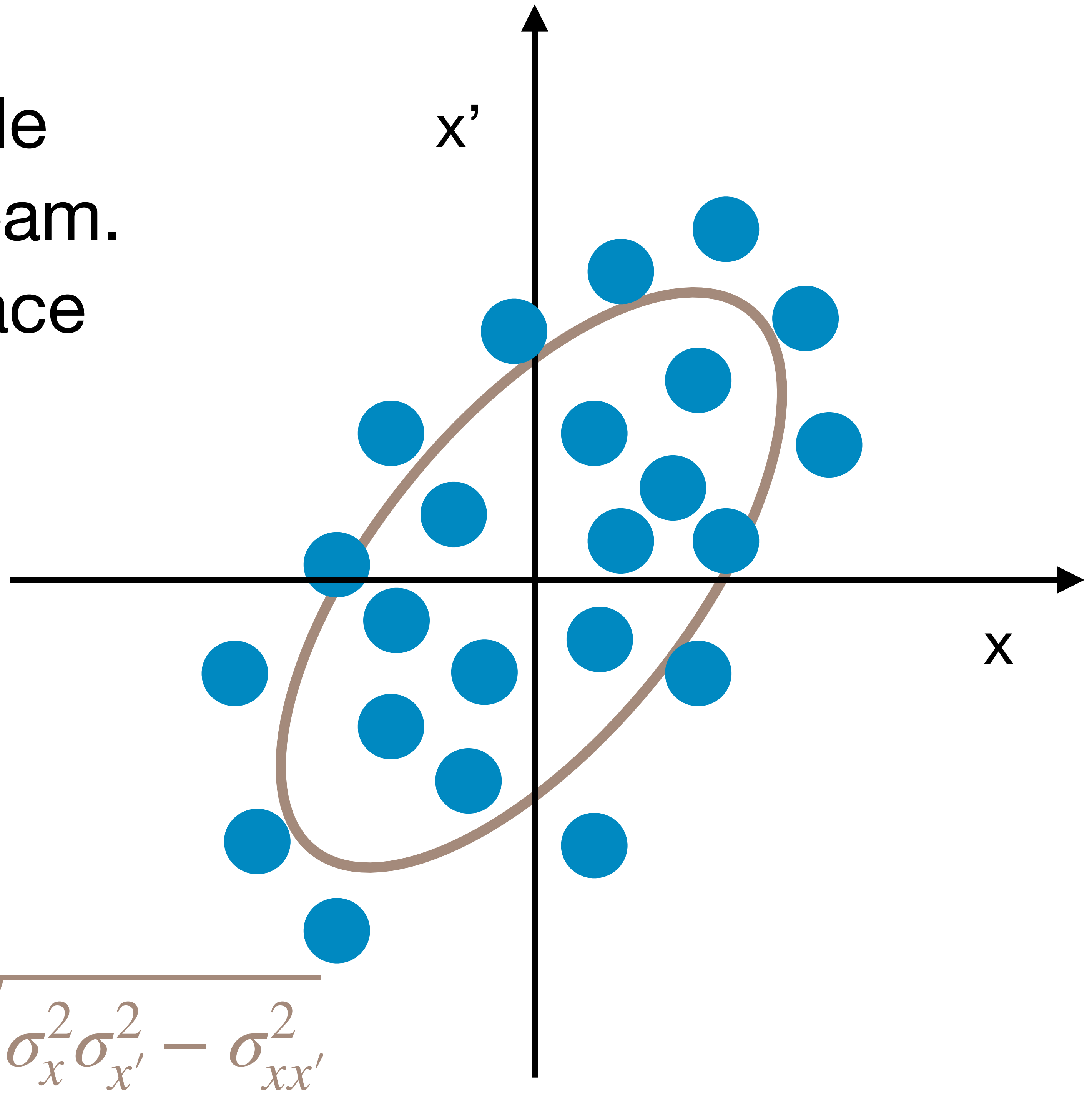
$\alpha = 0$

Single particle -> particles ensemble



- Optical functions are the same for all particles.
- Ellipses all have the same shape.
- Choose particle of specific amplitude as “representative” for the whole beam.
- The area of this particle’s phase space ellipse is the beam emittance.

$$\epsilon = \gamma x^2 + 2\alpha x x' + \beta x'^2$$



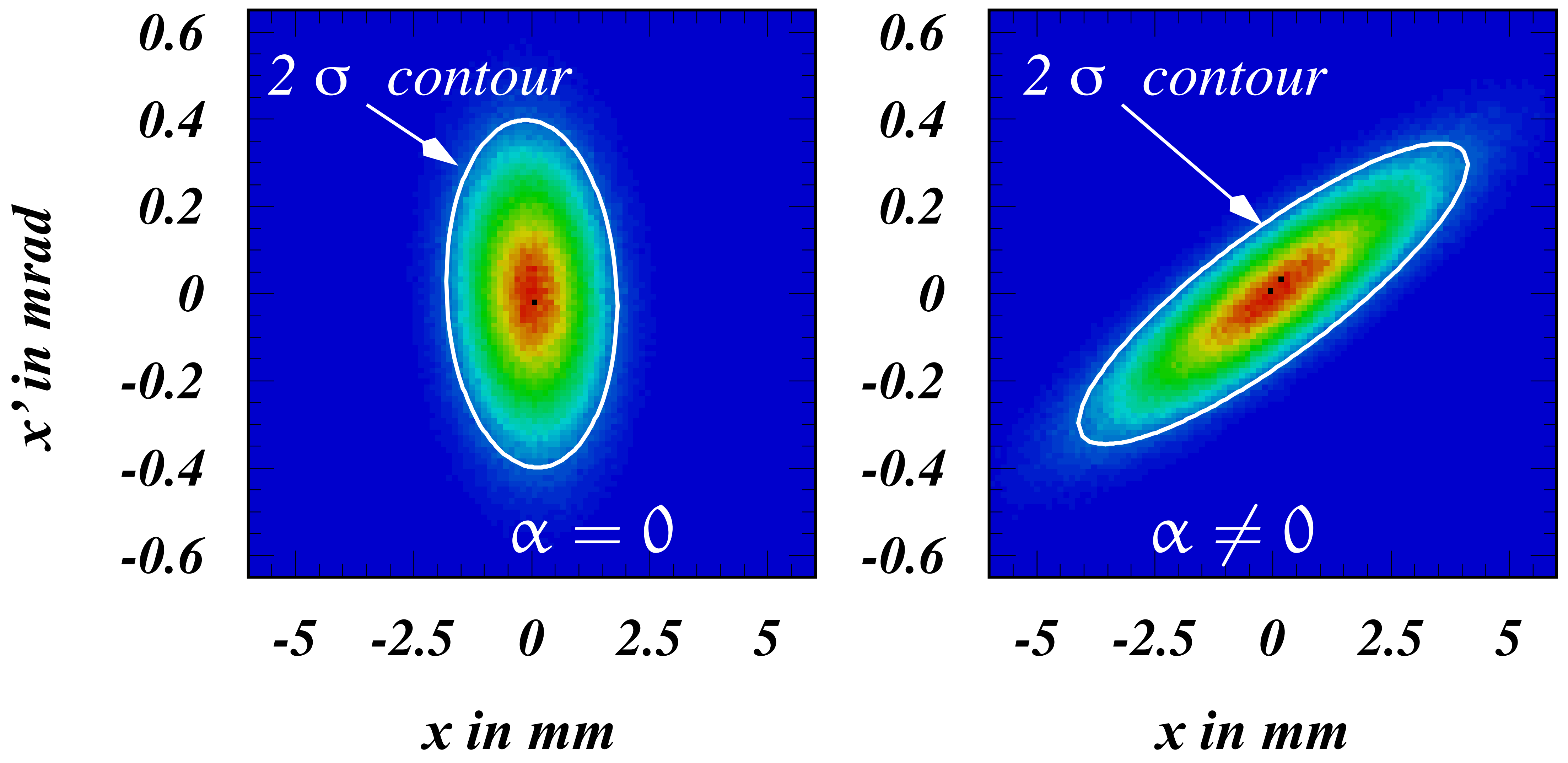
Statistical definition: $\epsilon_{\mathrm{RMS}} = \sqrt{\sigma_x^2 \sigma_{x'}^2 - \sigma_{xx'}^2}$

(Phil Bryant)

The emittance of a beam is related to the phase-space area that it occupies and is therefore related to the motion invariants of the constituent particles.

A practical definition of emittance requires a choice for the limiting ellipse that defines the phase-space area of the beam.

Usually this is related to some number of standard deviations of the beam distribution, for example "the 1-sigma emittance is … " .



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Liouville states that phase space is conserved.

Primarily, this refers to 6-dimensional phase space (x-x´, y-y´ and s-dp/p).

When the component phase spaces are uncoupled, the phase space is conserved within the 2- dimensional and/or 4-dimensional spaces.

The invariant of the motion in the uncoupled x-x´or y-y´ spaces is another way of saying the phase space is conserved.

Phase space is not conserved if ions change, e.g. by stripping or nuclear fragmentation, or if non-Hamiltonian forces appear e.g. scattering or photon emission.

(Phil Bryant)

Phase advance and tune

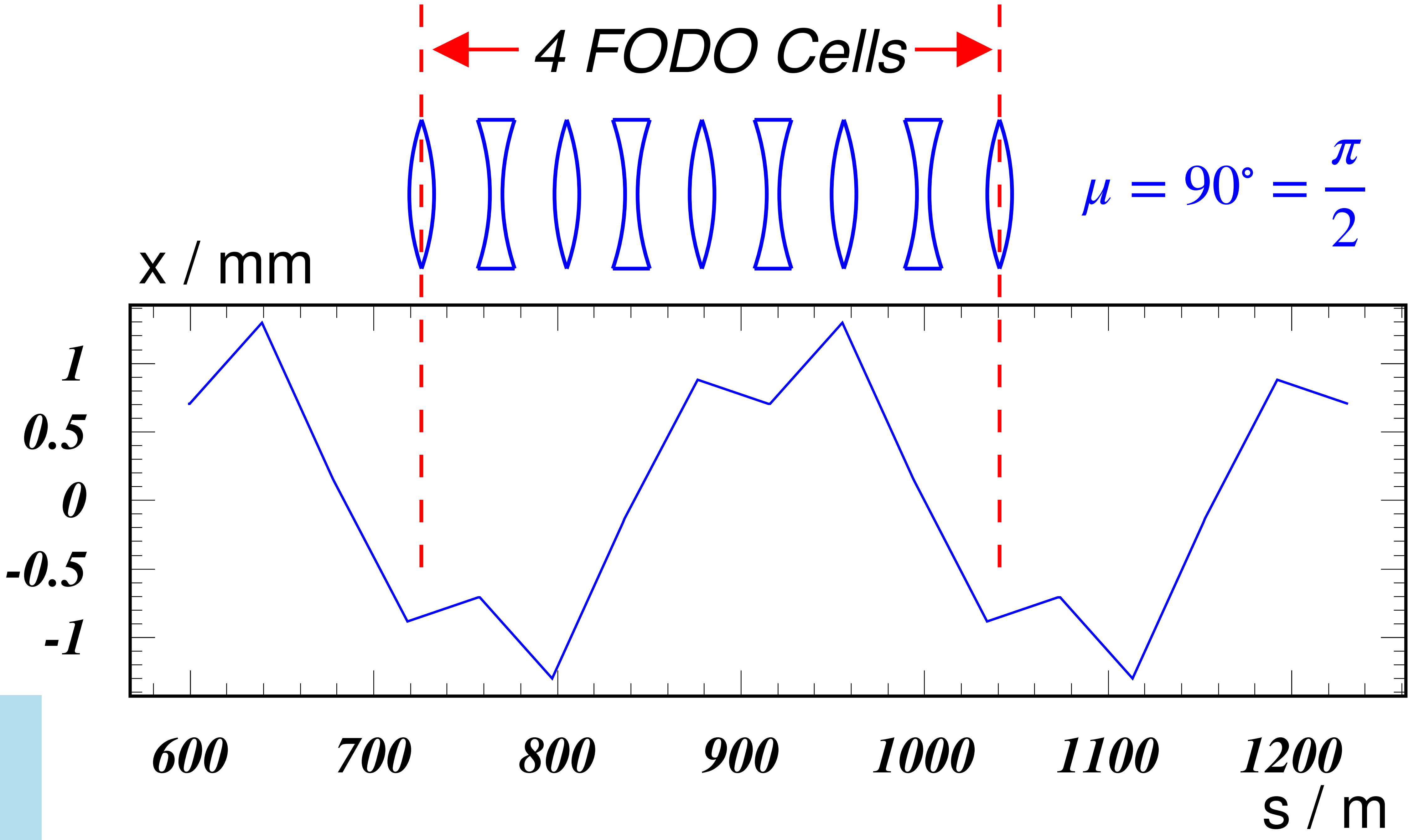
- Focussing of quadrupoles creates transverse oscillation around the design orbit ("betatron oscillation"):

$$x(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}\cos\{\psi(s)+\phi\}$$

- The difference of the phase functions is called the phase advance:

$$\mu = \psi(s_2) - \psi(s_1) = \int_{s_1}^{s_2} \frac{1}{\beta(s)}\,\mathrm{ds}$$

- The phase advance of one revolution is called the "tune" and gives the number of transverse oscillations per turn:



$$\boxed{Q = \frac{1}{2\pi}\oint \frac{1}{\beta(s)}\,\mathrm{ds}}$$

Chromaticity

- Focal length of a quadrupole depends on the particle energy:

$$k_1 = = \frac{e}{p_0}\frac{dB}{dy}$$

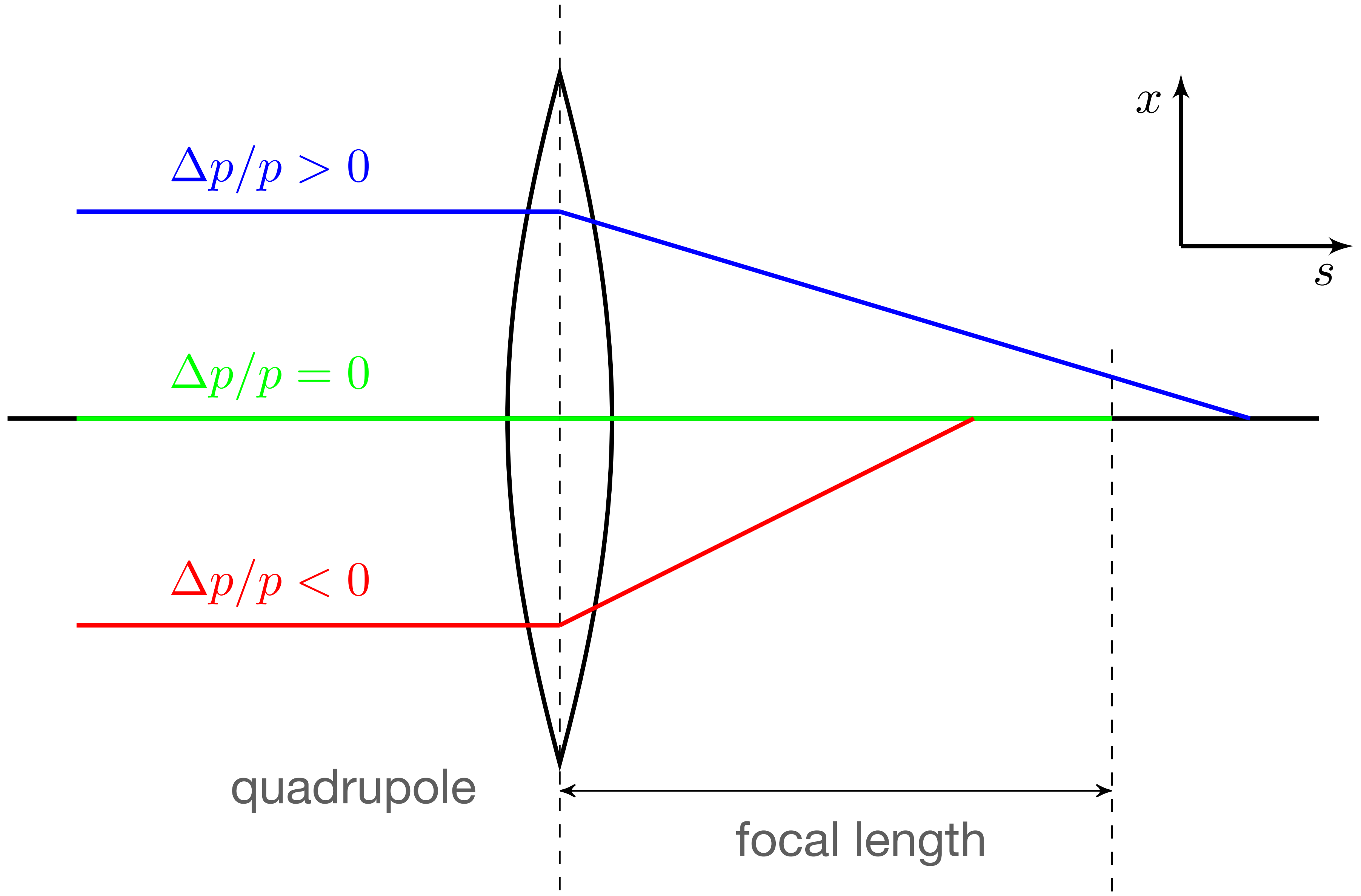
- As a consequence, the tune also depends on the particle energy:

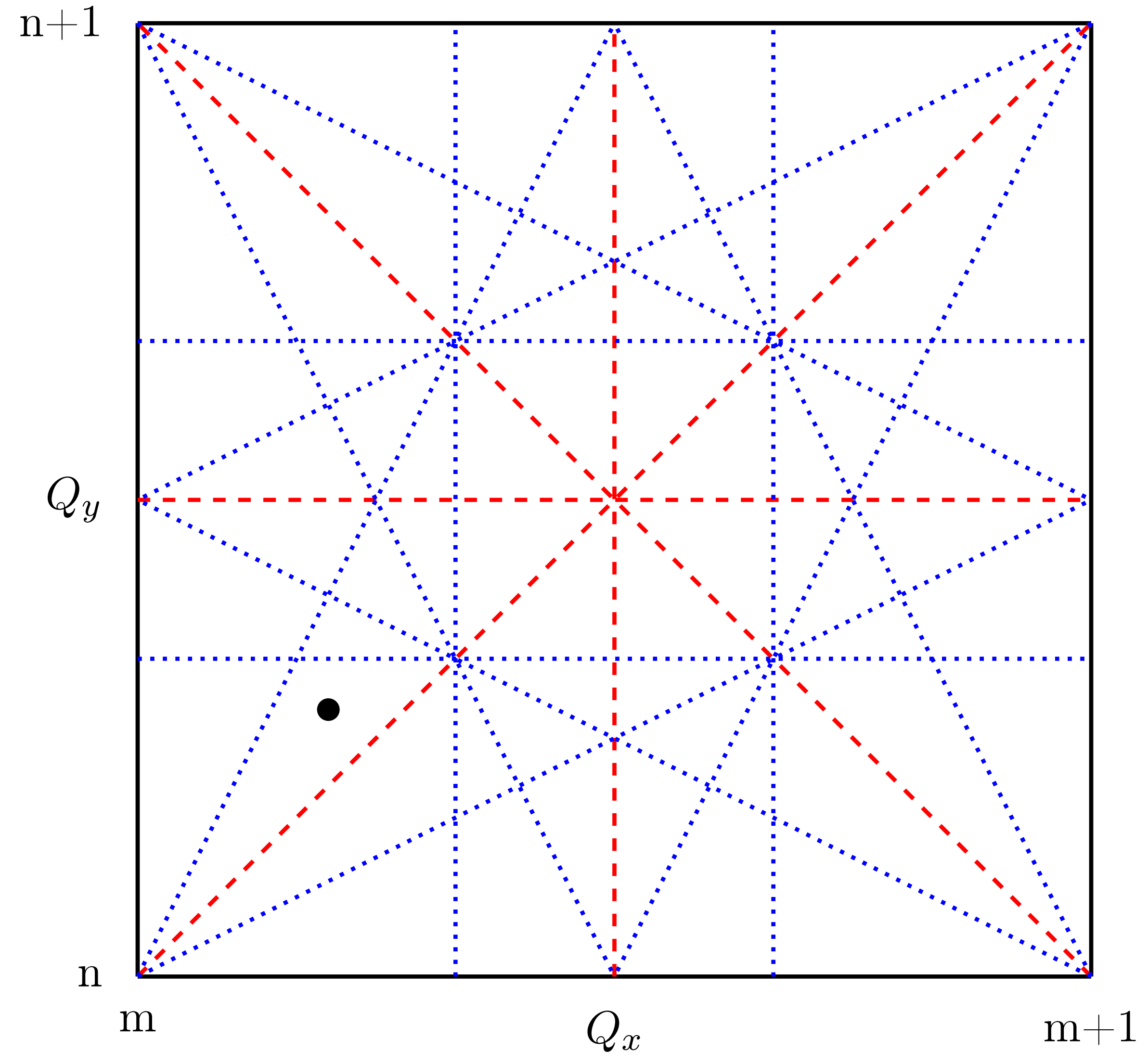
$$Q' = p_0\frac{\mathrm{d}Q}{\mathrm{d}p} \approx \frac{\Delta Q}{\Delta p/p_0}$$

- This so-called “chromaticity” is for a linear lattice:

$$Q' = -\frac{1}{4\pi}\oint \mathrm{d}s\, \beta(s) k_1(s)$$

- **Tuneshift needs to be corrected, otherwise particles might hit resonances and get lost.**





Chromaticity correction

- … with sextupole magnets:

$$\frac{e}{p_0}B_x = k_2 \cdot x \cdot y \quad \text{and} \quad \frac{e}{p_0}B_y = \frac{1}{2}k_2(x^2 - y^2)$$

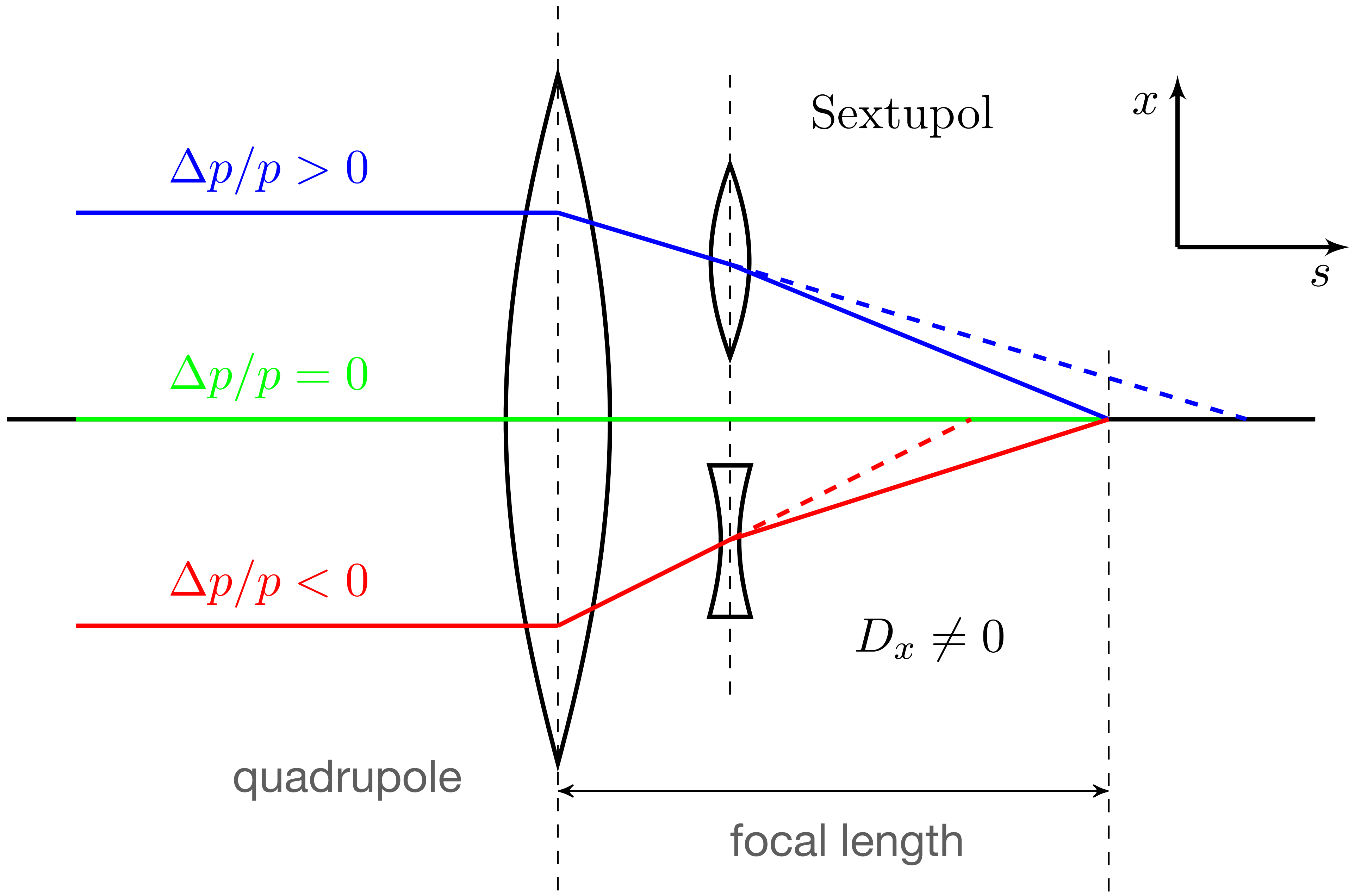
- Gradient (= focussing strength) proportional to particle amplitude:

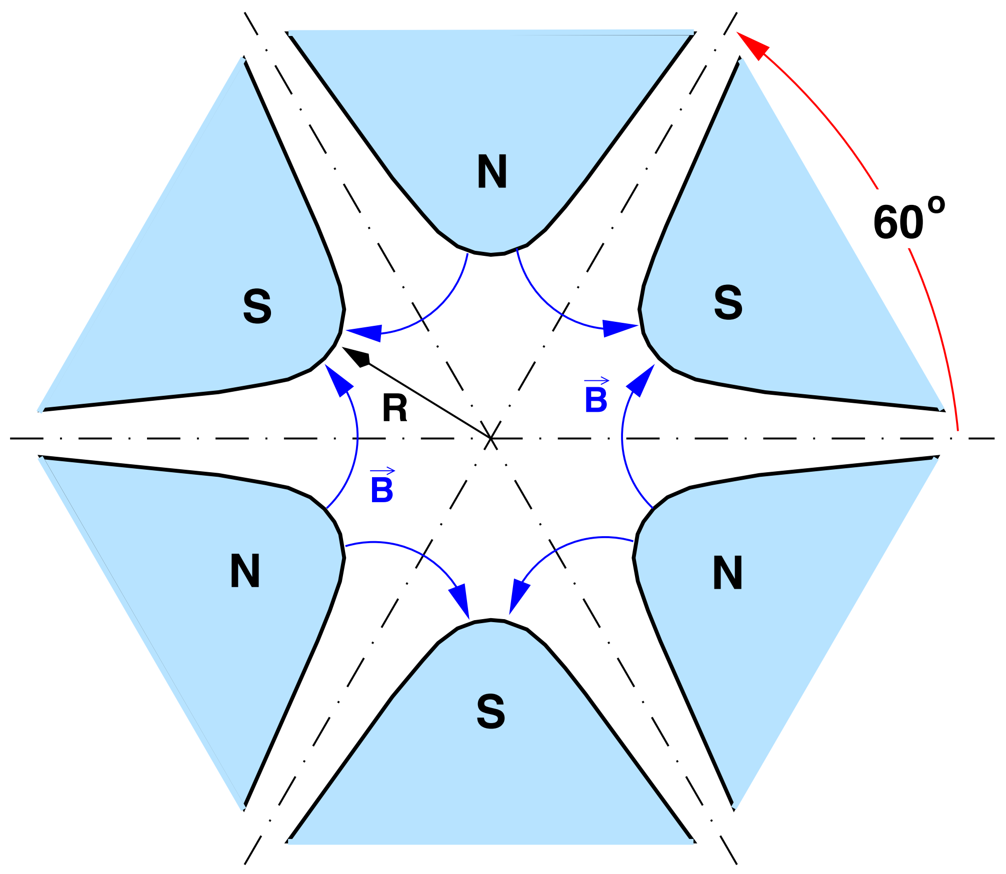
$$\frac{\partial B_x}{\partial y} = k_2\, x \quad \text{and} \quad \frac{\partial B_y}{\partial x} = k_2\, x$$

- Tune shift including sextupoles:

$$Q' = -\frac{1}{4\pi}\oint \beta(s)\left[k_1(s) + D(s)k_2(s)\right]\,\mathrm{d}s$$

Sextupoles for chromaticity correction must be installed in dispersive regions!





Hadron and electron storage rings

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Hadron storage rings

- Heavy particles require strong B fields
- Push for highest B fields up to technical limit
- Energy limit given by maximum acceptable circumference

$$2\pi \frac{p_0}{e} = \int B \, \mathrm{d}l$$

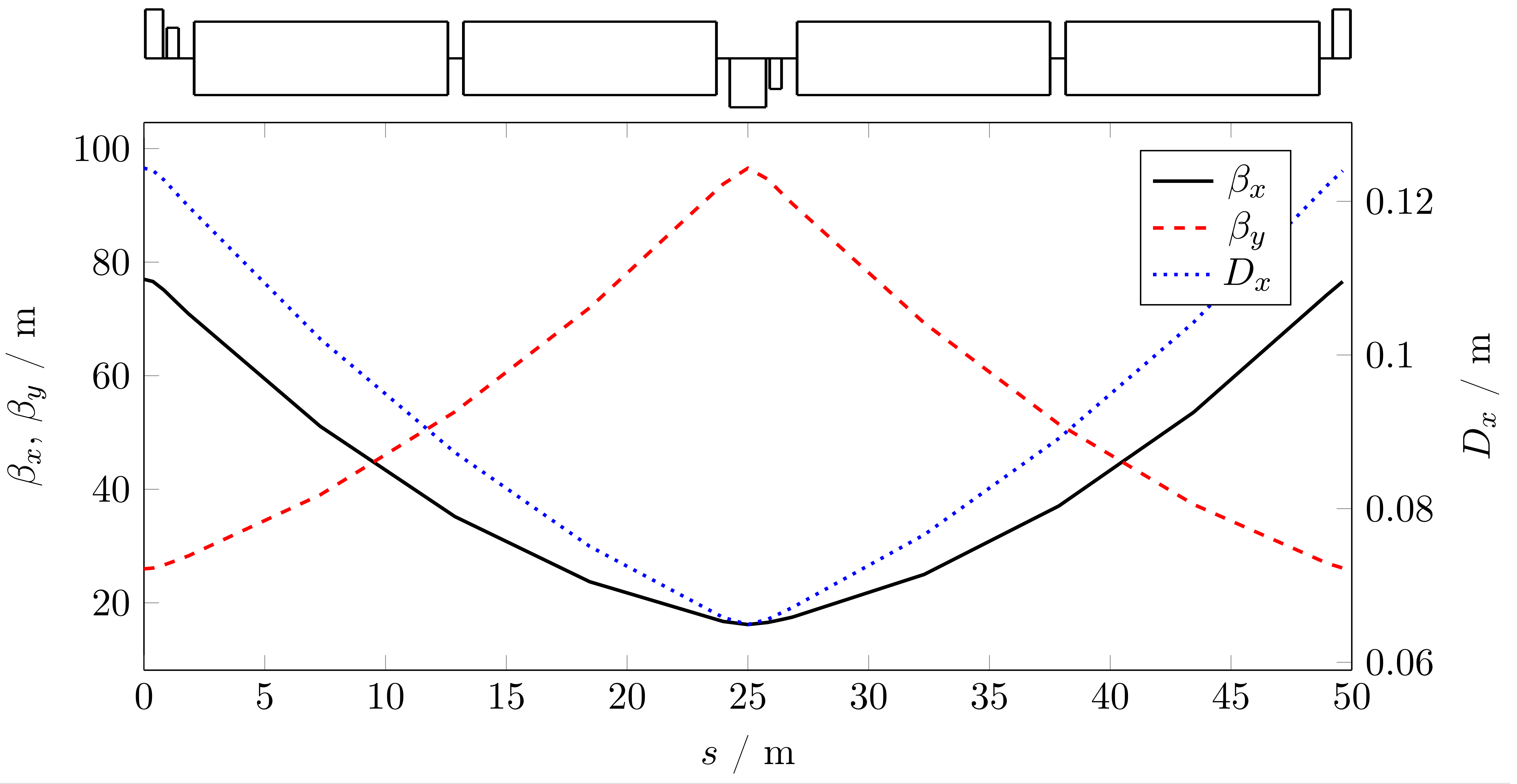
Electron storage rings

- Synchrotron light dominated
- Push for small B fields thus large bending radius

$$P_\gamma = \propto \frac{\gamma^4}{\rho^2}$$

Common feature: For high beam energies -> Push for highest possible dipole filling factor

FODO structure



Arc cell that has been proposed for FCC-ee

$$L_{\text{cell}} = 50\,m$$

$$L_{\text{bend}} = 11\,m$$

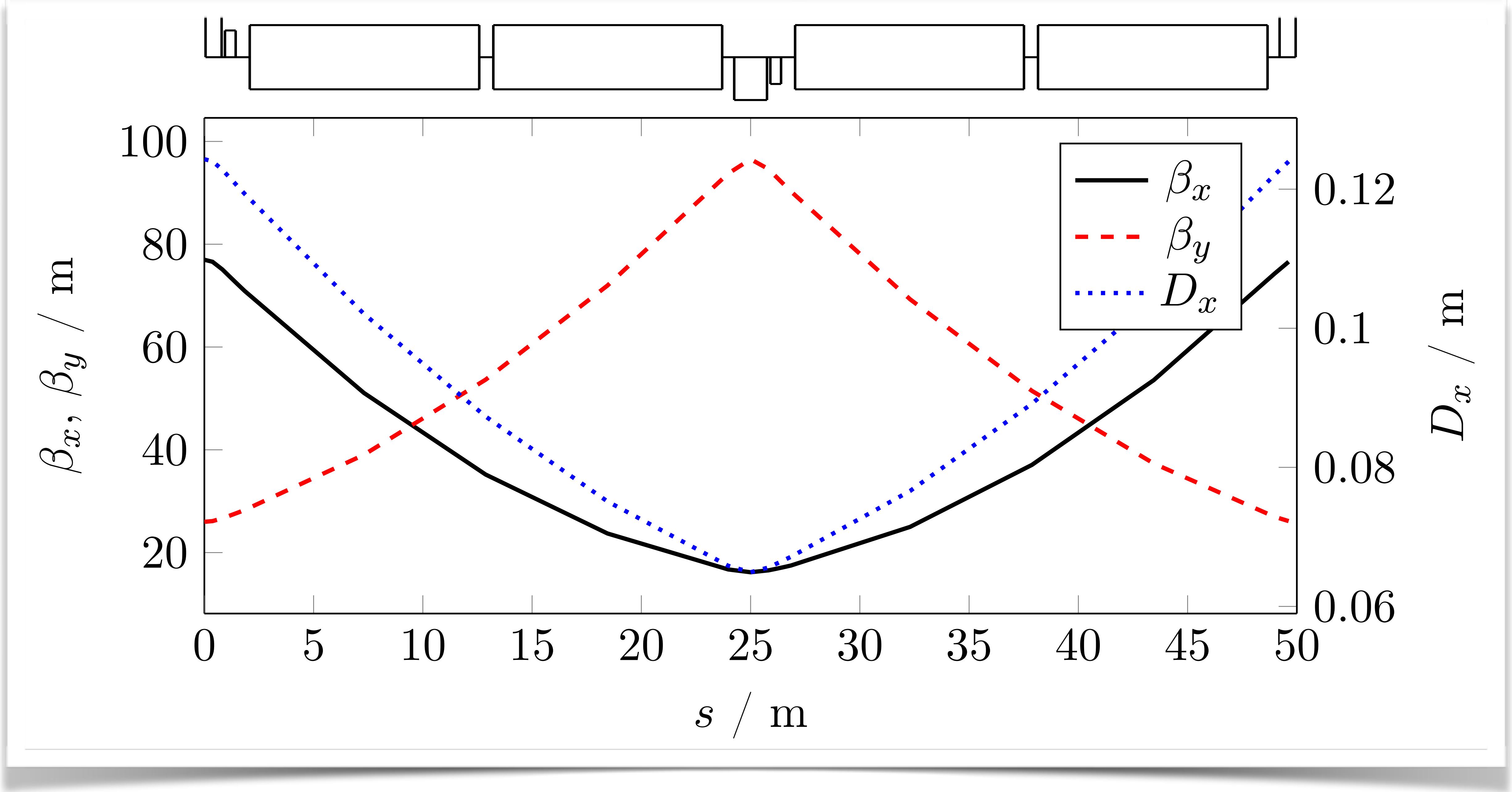
$$\Rightarrow \frac{L_{\text{bend}}}{L_{\text{cell}}} = 0.84$$

Characteristics of the FODO structure

- Low number of quadrupoles
- Easy to calculate analytically
- Long drift spaces

  -> lots of free space or

  -> high filling factor

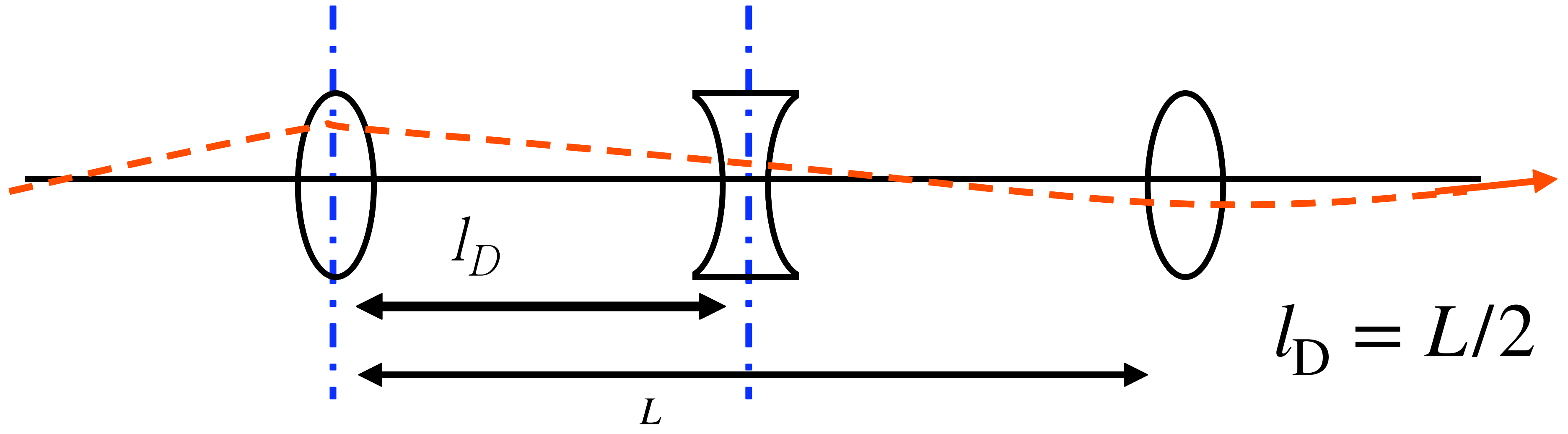


Applications:

- In transfer lines that have to cover long distances with few hardware
- In Linacs or FELs that require lots of space for RF cavities or undulators
- **Storage ring colliders that require high dipole filling factor**

Analytical calculations

FODO in thin lens approximation



$l_\mathrm{D} = L/2$

$\tilde{f} = 2f$ focal length of a half quadrupole

transfer matrix from the centre of the first to the centre of the second quadrupole:

$$M_\mathrm{halfcell} = \begin{pmatrix} 1 - \frac{l_\mathrm{D}}{\tilde{f}} & l_\mathrm{D} \\ -\frac{l_\mathrm{D}}{\tilde{f}^2} & 1 + \frac{l_\mathrm{D}}{\tilde{f}} \end{pmatrix}$$

- **Goal of this calculation:** maximum and minimum value of the betafunction depending on cell length and phase advance
- Transport matrix s_1 -> s_2 based on optics functions:

$$M = \begin{pmatrix} \sqrt{\frac{\beta_2}{\beta_1}} \cos\mu_{12} + \alpha_1 \sin\mu_{12} & \sqrt{\beta_1\beta_2}\sin\mu_{12} \\ \frac{(\alpha_1 - \alpha_2)\cos\mu_{12} - (1+\alpha_1\alpha_2)\sin\mu_{12}}{\sqrt{\beta_1\beta_2}} & \sqrt{\frac{\beta_1}{\beta_2}}\cos\mu_{12} - \alpha_1 \sin\mu_{12} \end{pmatrix}$$

- For the half FODO cell applies in the centre of the quadrupole:

$\alpha_1 = \alpha_2 = 0$ and $\beta_1 = \hat{\beta}, \quad \beta_2 = \check{\beta}$

Analytical calculation II

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$$M_{\text{halfcell}} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} = \underbrace{\begin{pmatrix} 1-\frac{l_\text{D}}{\tilde{f}} & l_\text{D} \\ -\frac{l_\text{D}}{\tilde{f}^2} & 1+\frac{l_\text{D}}{\tilde{f}} \end{pmatrix}}_{\text{transfer matrix from single matrices}} = \underbrace{\begin{pmatrix} \sqrt{\frac{\check{\beta}}{\hat{\beta}}}\cos(\mu_\text{cell}/2) & \sqrt{\hat{\beta}\check{\beta}}\sin(\mu_\text{cell}/2) \\ -\frac{1}{\sqrt{\hat{\beta}\check{\beta}}}\sin(\mu_\text{cell}/2) & \sqrt{\frac{\hat{\beta}}{\check{\beta}}}\cos(\mu_\text{cell}/2) \end{pmatrix}}_{\text{transfer matrix based on optics functions}}$$

- Solution of the system of equations:

$$m_{12}m_{21} = \sqrt{\hat{\beta}\check{\beta}}\sin(\mu/2)\frac{-1}{\sqrt{\hat{\beta}\check{\beta}}}\sin(\mu/2) = -\sin^2(\mu/2)$$
$$= l_\text{D}\cdot\left(-\frac{l_\text{D}}{\tilde{f}^2}\right) = -\frac{l_\text{D}^2}{\tilde{f}^2}$$

$$\Rightarrow \sin(\mu/2) = \frac{l_\text{D}}{\tilde{f}} = \frac{L}{4f} \qquad \left(\frac{1}{f} = KL_\text{Q}\right)$$

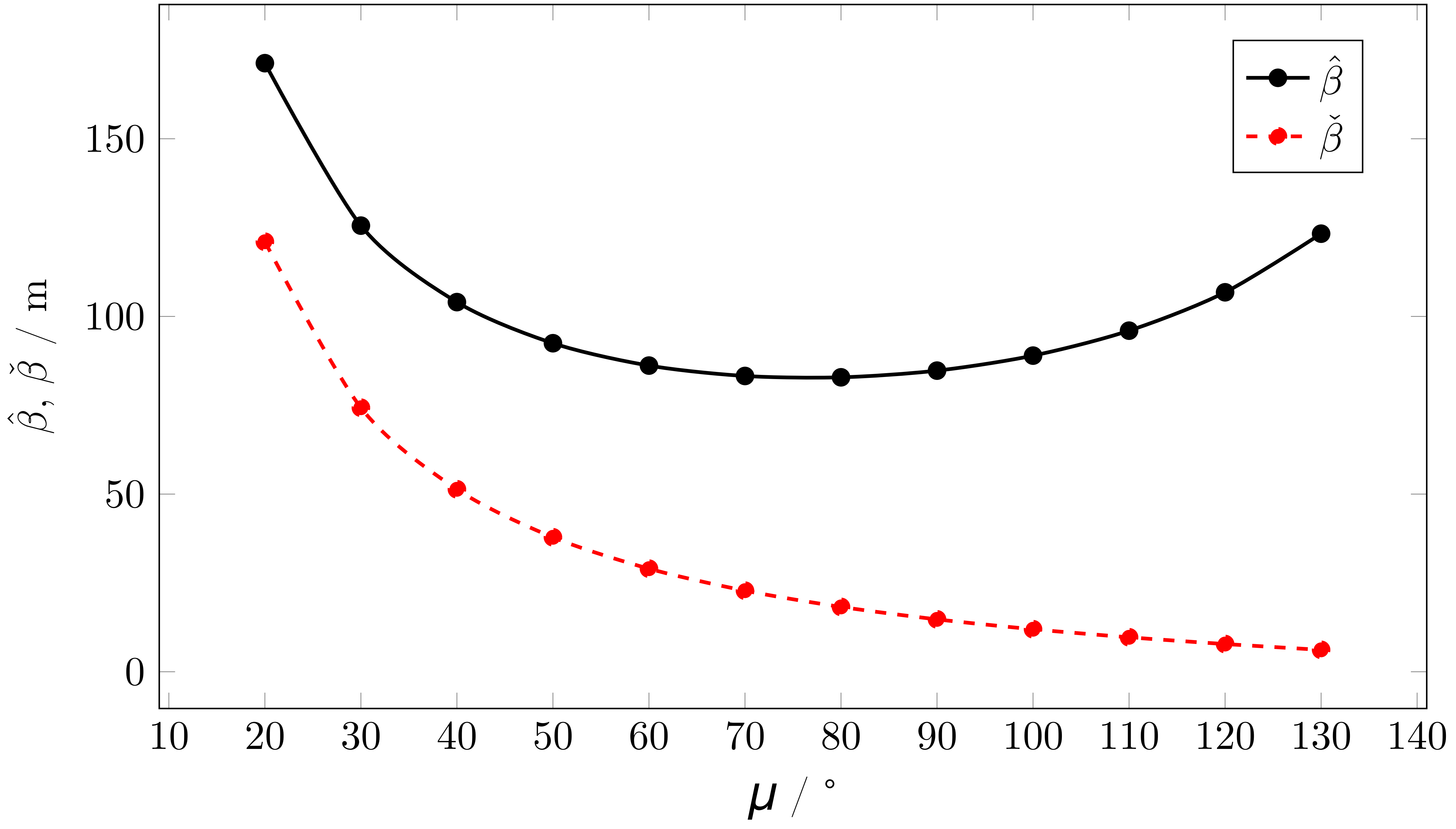
$$\frac{m_{22}}{m_{11}} = \frac{\hat{\beta}}{\check{\beta}} = \frac{1+l_\text{D}/\tilde{f}}{1-l_\text{D}/\tilde{f}} = \frac{1+\sin(\mu/2)}{1-\sin(\mu/2)} \quad (1)$$

$$\frac{m_{12}}{m_{21}} = \hat{\beta}\check{\beta} = \tilde{f}^2 = \frac{l_\text{D}^2}{\sin^2(\mu/2)} \quad (2)$$

- Multiplication and division respectively of Eqs. (1) and (2) in combination with some addition theorems yield:

$$\boxed{\hat{\beta} = L\frac{1+\sin(\mu/2)}{\sin\mu}} \quad \text{and} \quad \boxed{\check{\beta} = L\frac{1-\sin(\mu/2)}{\sin\mu}}$$

Betafunction in a FODO cell



Maximum and minimum values of the betafunction in an arc FODO cell designed for FCC-ee

- The minimum value of $\hat{\beta}$ is obtained for a phase advance of μ = 76°.
- $\check{\beta}$ decreases for increasing phase advance.

-> What phase advance should we choose?

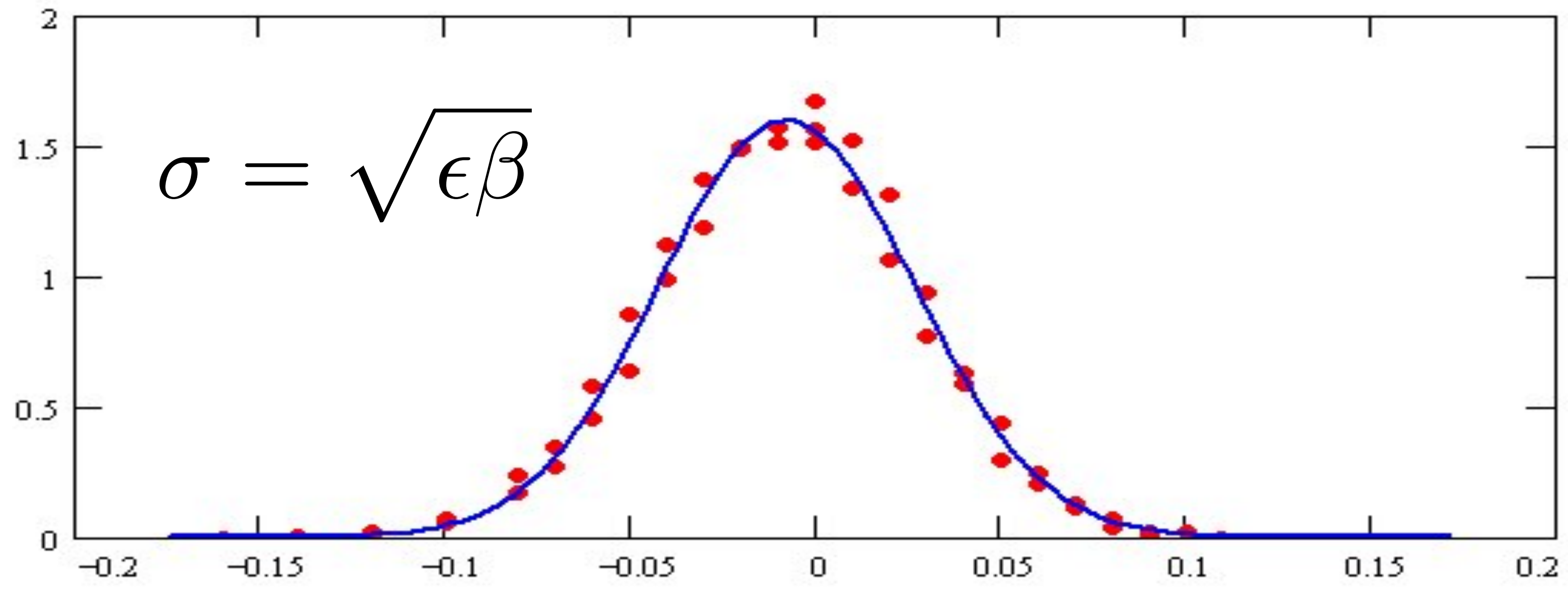
Hadron rings: Choice of phase advance per cell

- **Beam size**

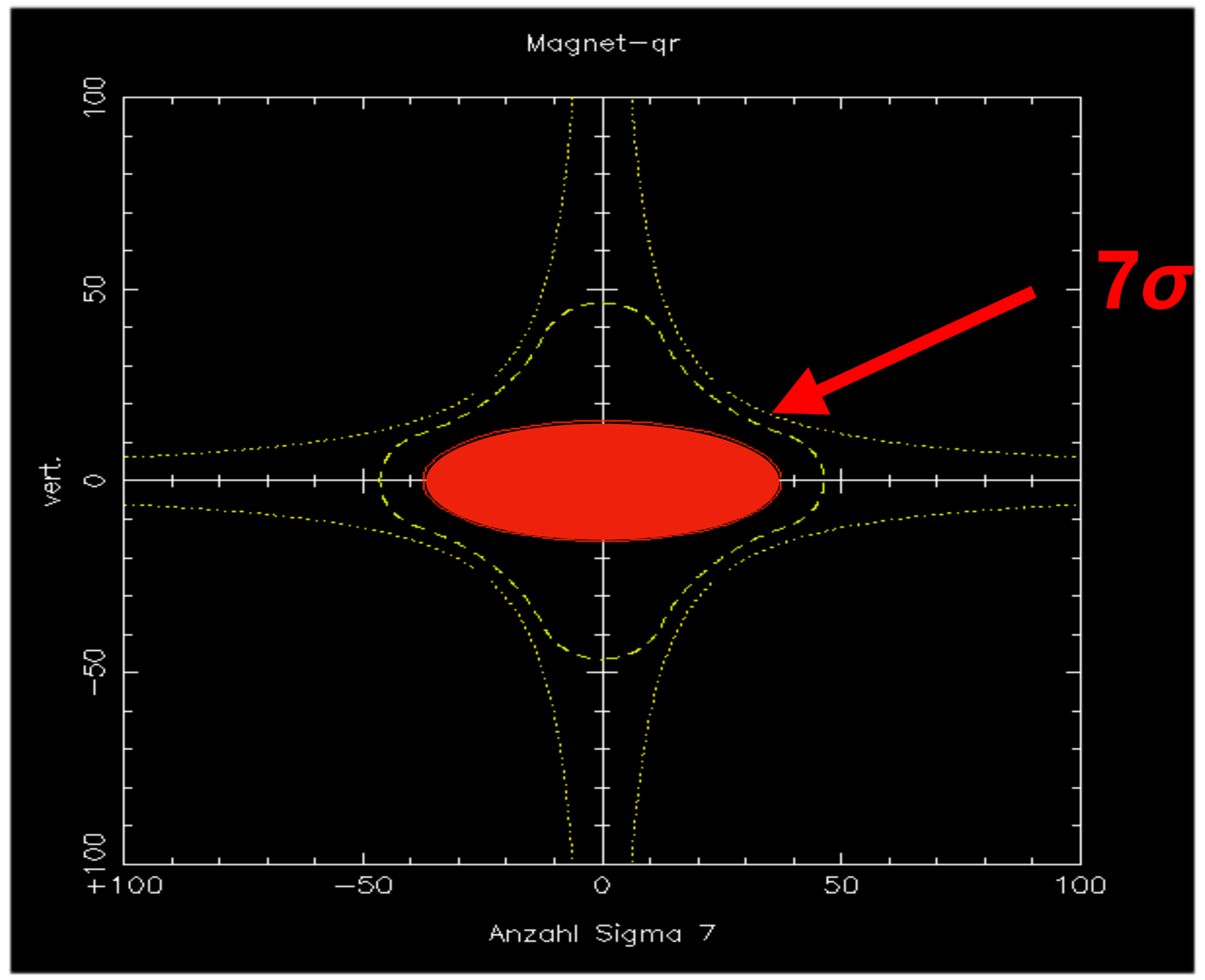
Aperture requirement: 10-20 σ

Protons: Remember adiabatic damping $\epsilon_u \propto \frac{1}{\beta\gamma}$

-> Choose phase advance that obtains low values of β



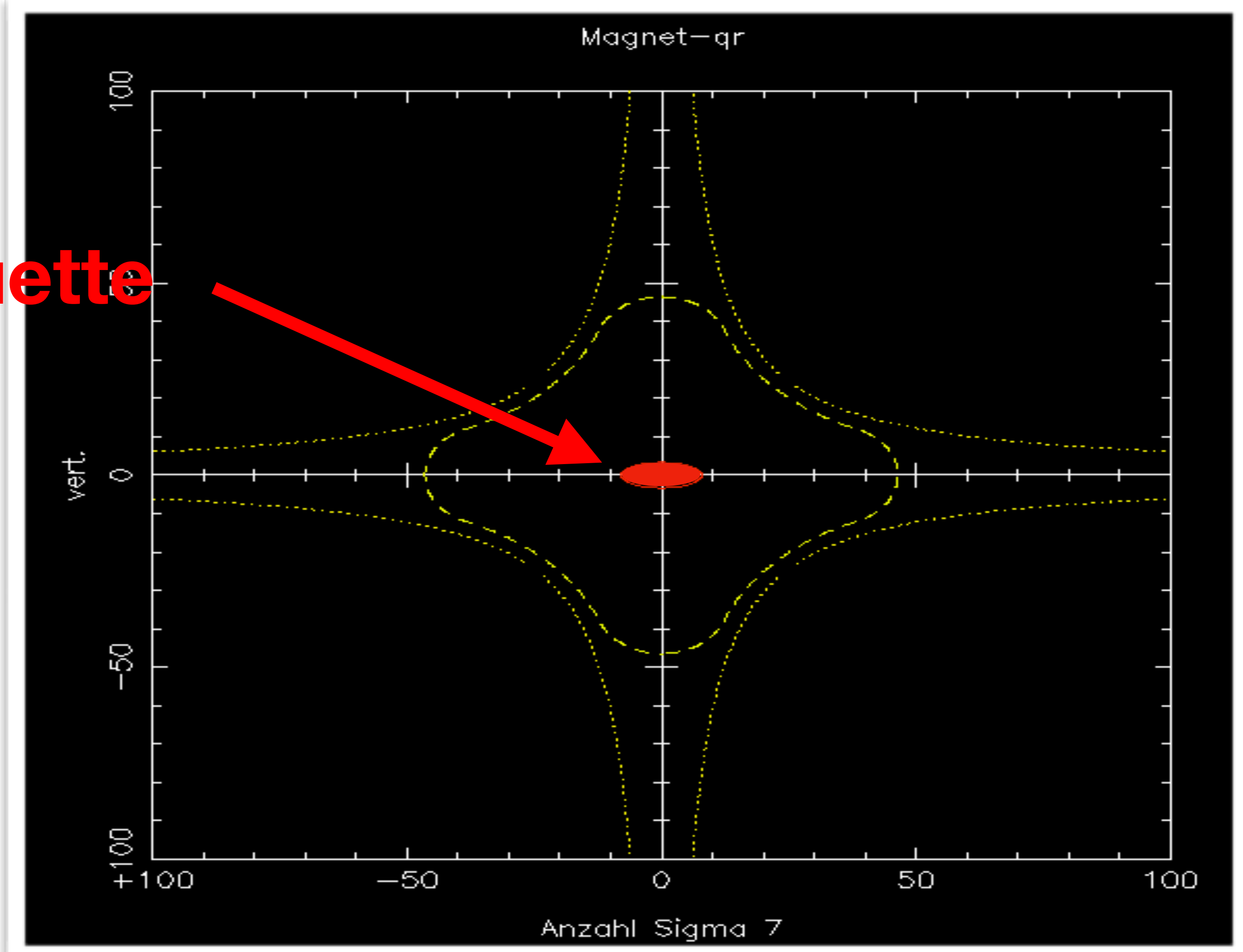
y



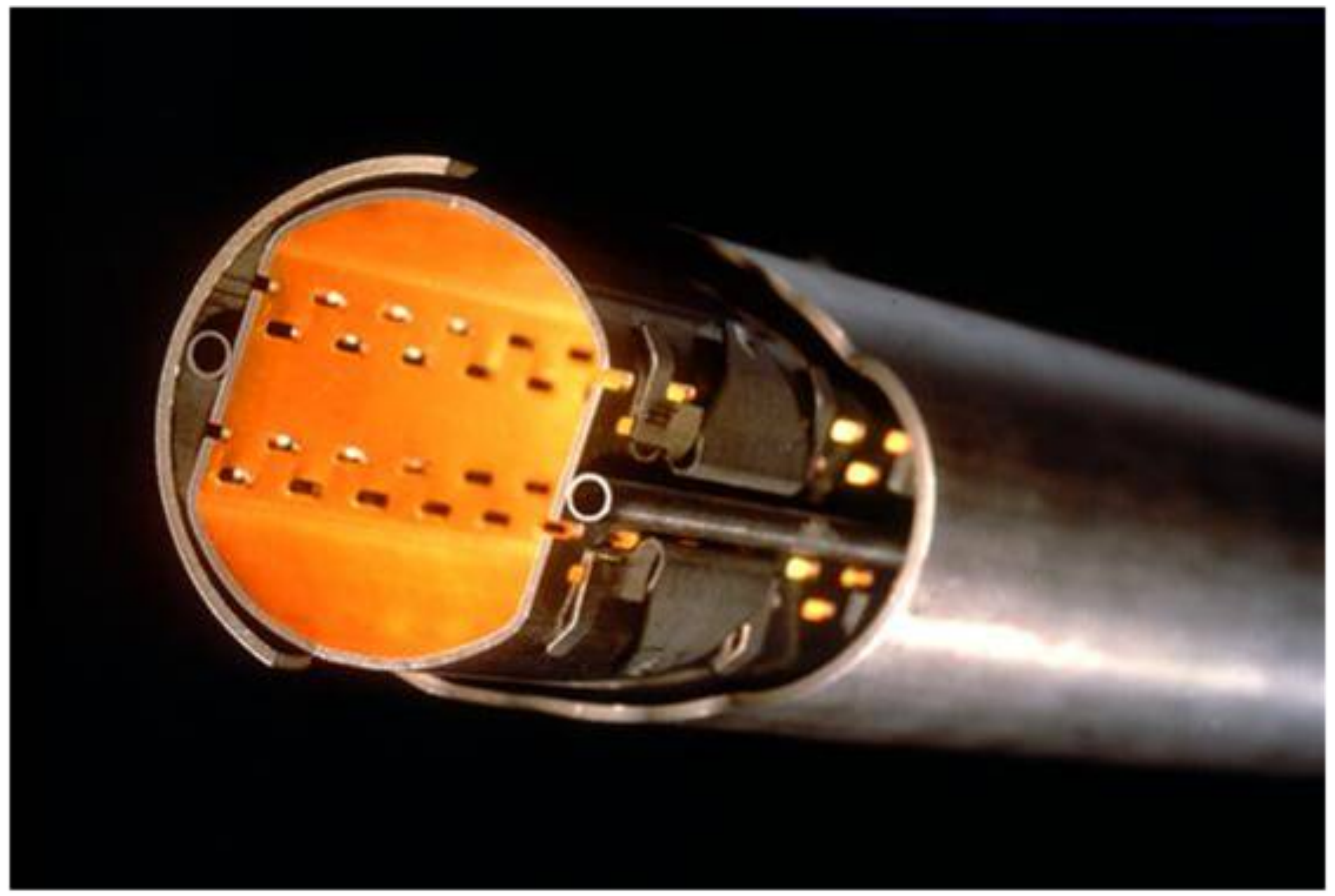
x

7σ - Silhouette

y



x



Injection energy: 40 GeV (γ = 43)

Flat-top energy: 920 GeV (γ = 980)

LHC vacuum chamber and beam screen

Phase advance for highest aperture

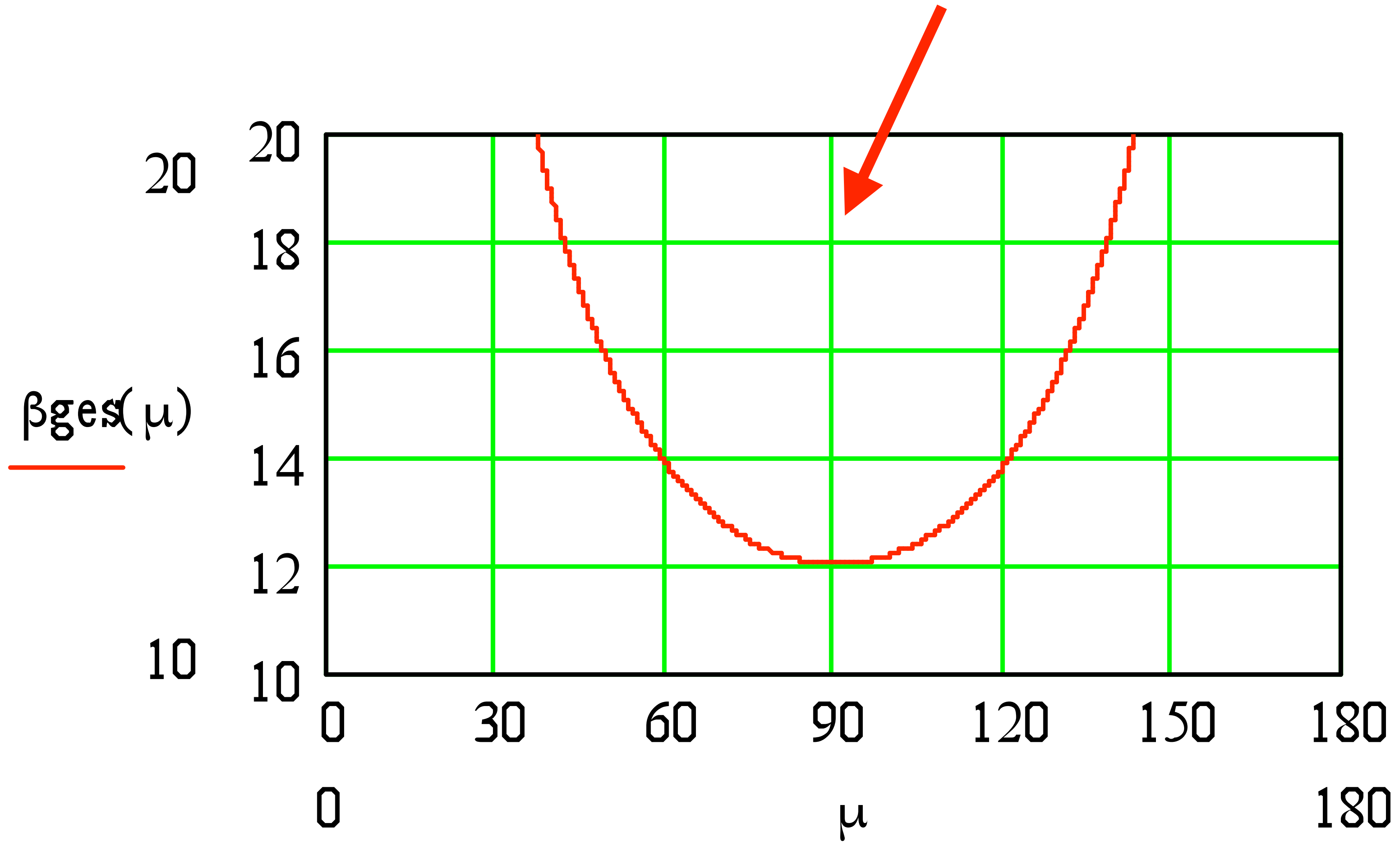
- For highest aperture we have to minimise the β-function in both planes:

$$r^2 = \epsilon_x \beta_x + \epsilon_y \beta_y$$

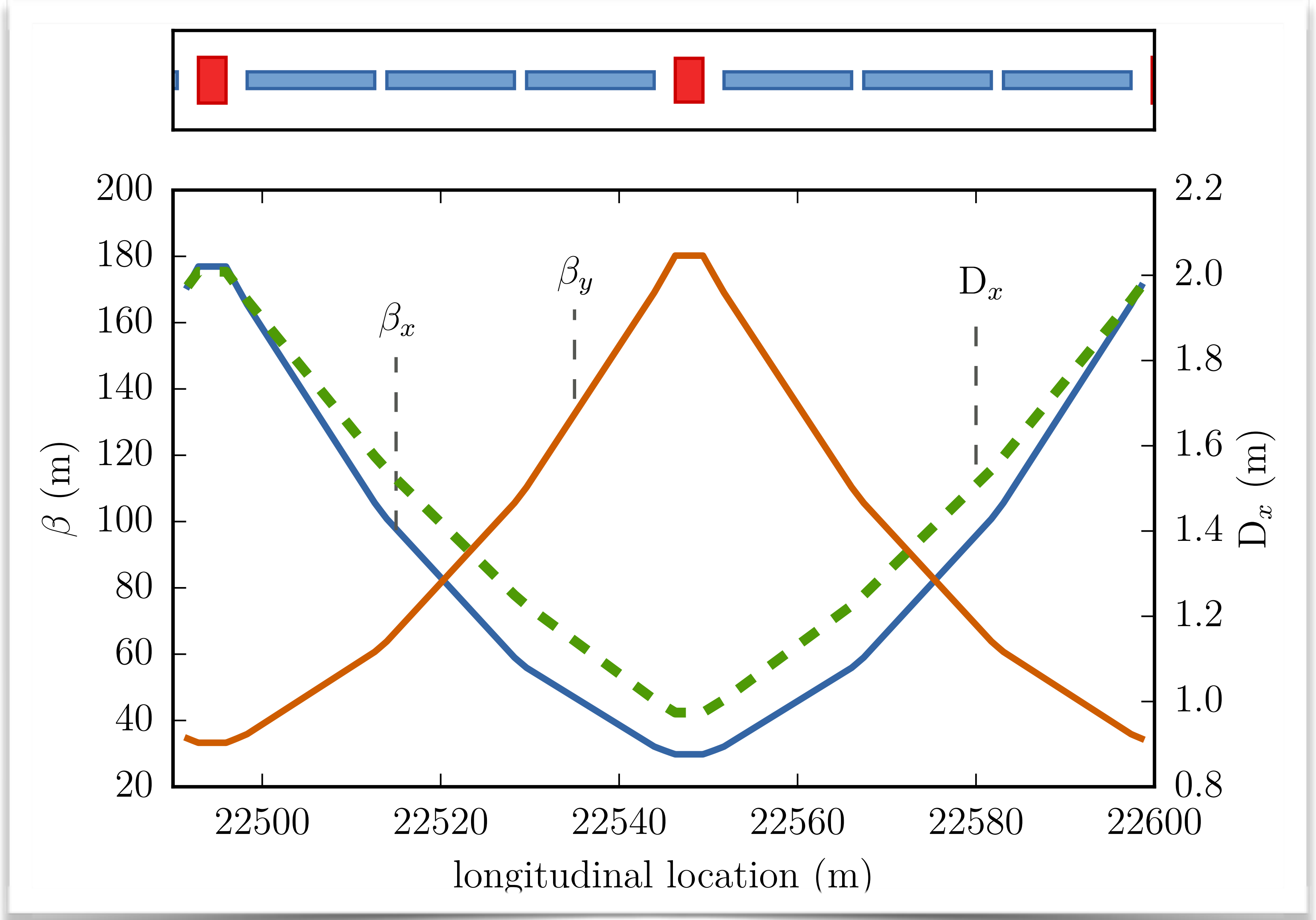
- Proton beams are "round" in the sense of: $\epsilon_x \approx \epsilon_y \quad \Rightarrow r^2/\epsilon = \beta_x + \beta_y$

$$\Rightarrow \frac{\mathrm{d}}{\mathrm{d}\mu}(\hat{\beta} + \check{\beta}) = \frac{\mathrm{d}}{\mathrm{d}\mu}\frac{2L}{\sin\mu} = -2L\frac{\cos\mu}{\sin^2\mu} \overset{!}{=} 0$$

$$\boxed{\mu = 90^\circ}$$



LHC FODO cell



Andy Langner, PhD thesis

$$L = 106.9\,\mathrm{m}$$

$$\mu_x = \mu_y = 90^\circ$$

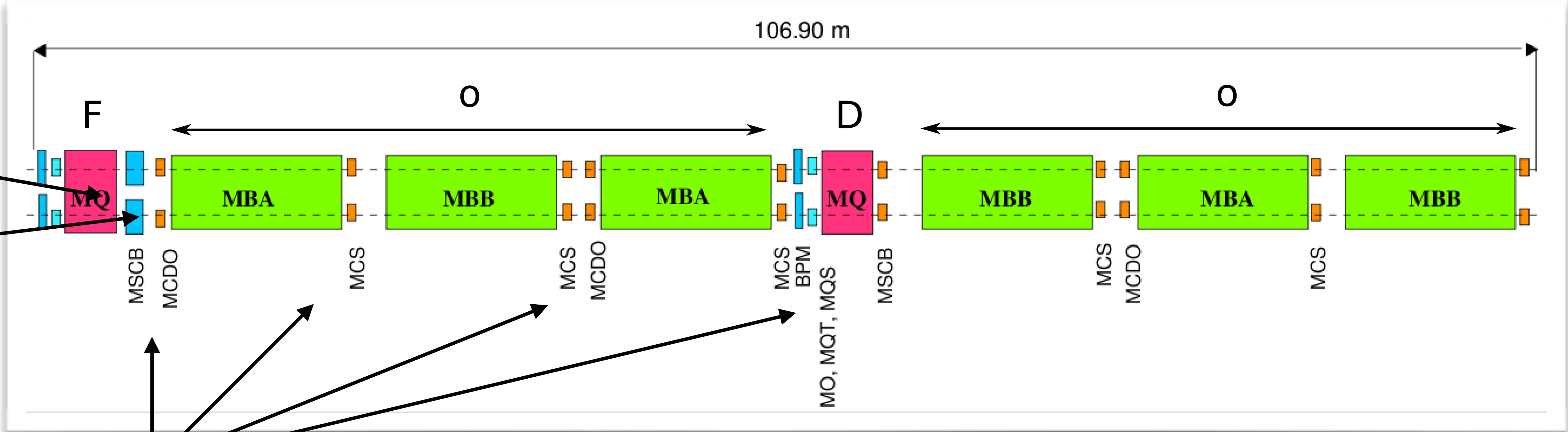
$$\hat{\beta} \approx 180\,\mathrm{m}, \qquad \check{\beta} \approx 30\,\mathrm{m}$$

$$\hat{D}_x = 2\,\mathrm{m}$$

dipole filling factor: 80% $(L_{\mathrm{Bend}} = 14.3\,\mathrm{m})$

quadrupole

sextupole



Correctors: dipole, quadrupole, sextupole, octupole, decapole
for orbit correction, coupling correction, eddy currents, instabilities, …

Summary for hadron rings

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- Emittance is defined by beam quality delivered by injectors.
- Hadron storage rings feature round beams: $\epsilon_x \approx \epsilon_y$
- Emittance shrinks during acceleration: $\epsilon_x \propto \frac{1}{\beta\gamma}$
- Aperture requirements call for smallest sum of beta functions: $\mu = 90°$

  -> Maximum beta function defined via cell length $\hat{\beta} = L\frac{1+\sin(\mu/2)}{\sin\mu}$
- Beam energy defined by integrated B field

  -> Highest dipole fields $\int B\,\mathrm{d}l = 2\pi\frac{p_0}{e}$

  -> Maximum dipole filling factor

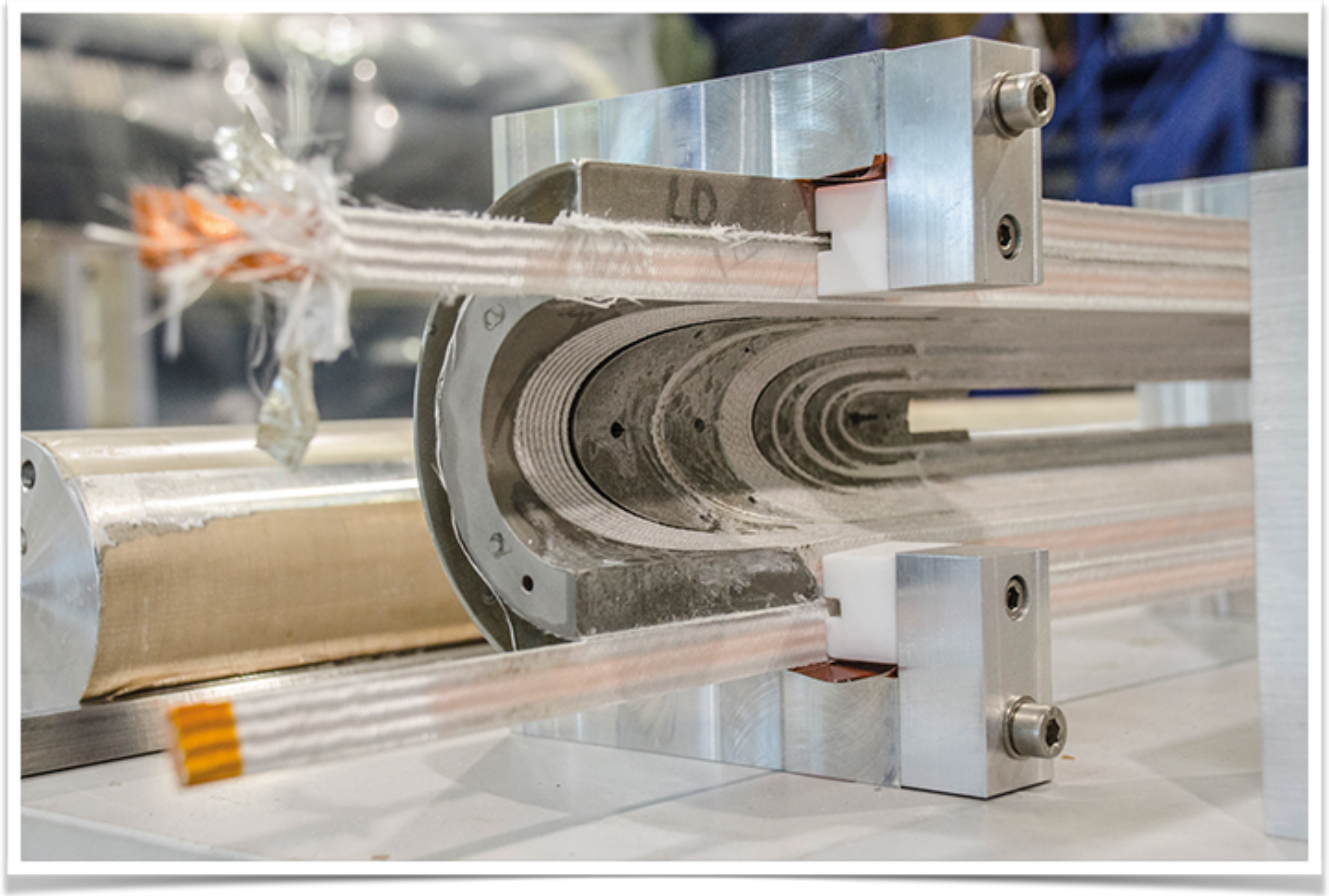
Hadron colliders and the quest for highest dipole fields

juas
Joint Universities Accelerator School

The two key players in SC magnet technology:

NbTi LHC standard dipoles
8.3 T

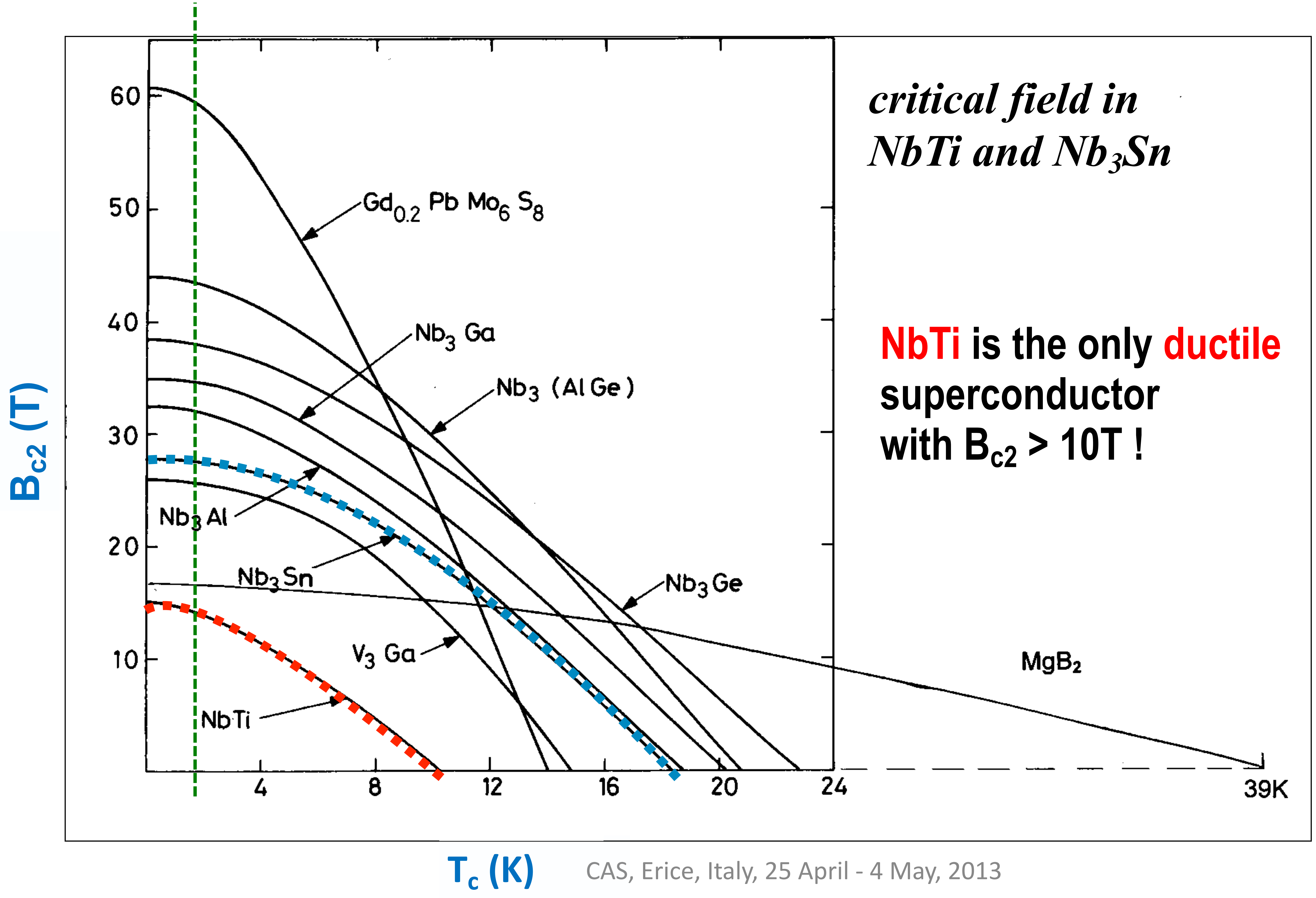
Nb_3Sn FCC type dipole coils
11 T – 16 T



… and we do NOT talk about $YBa_2Cu_3O_7$ and friends

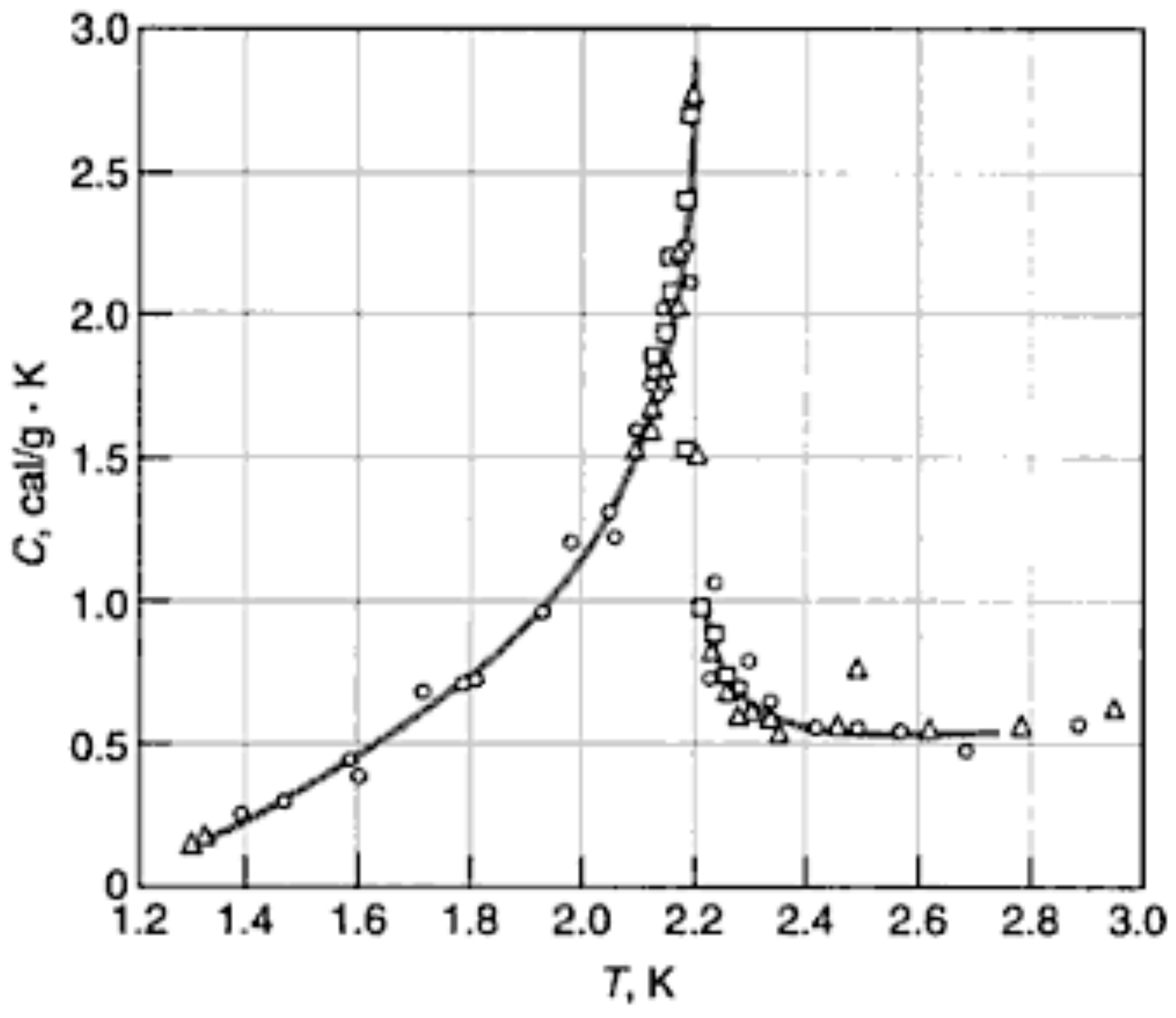
Upper critical fields of metallic (LTS) superconductors

… the top ten of the charts



CAS, Erice, Italy, 25 April - 4 May, 2013

Heat capacity of liquid helium: transition to superfluidity



Electron storage rings

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…. are different! Electrons radiate!!!

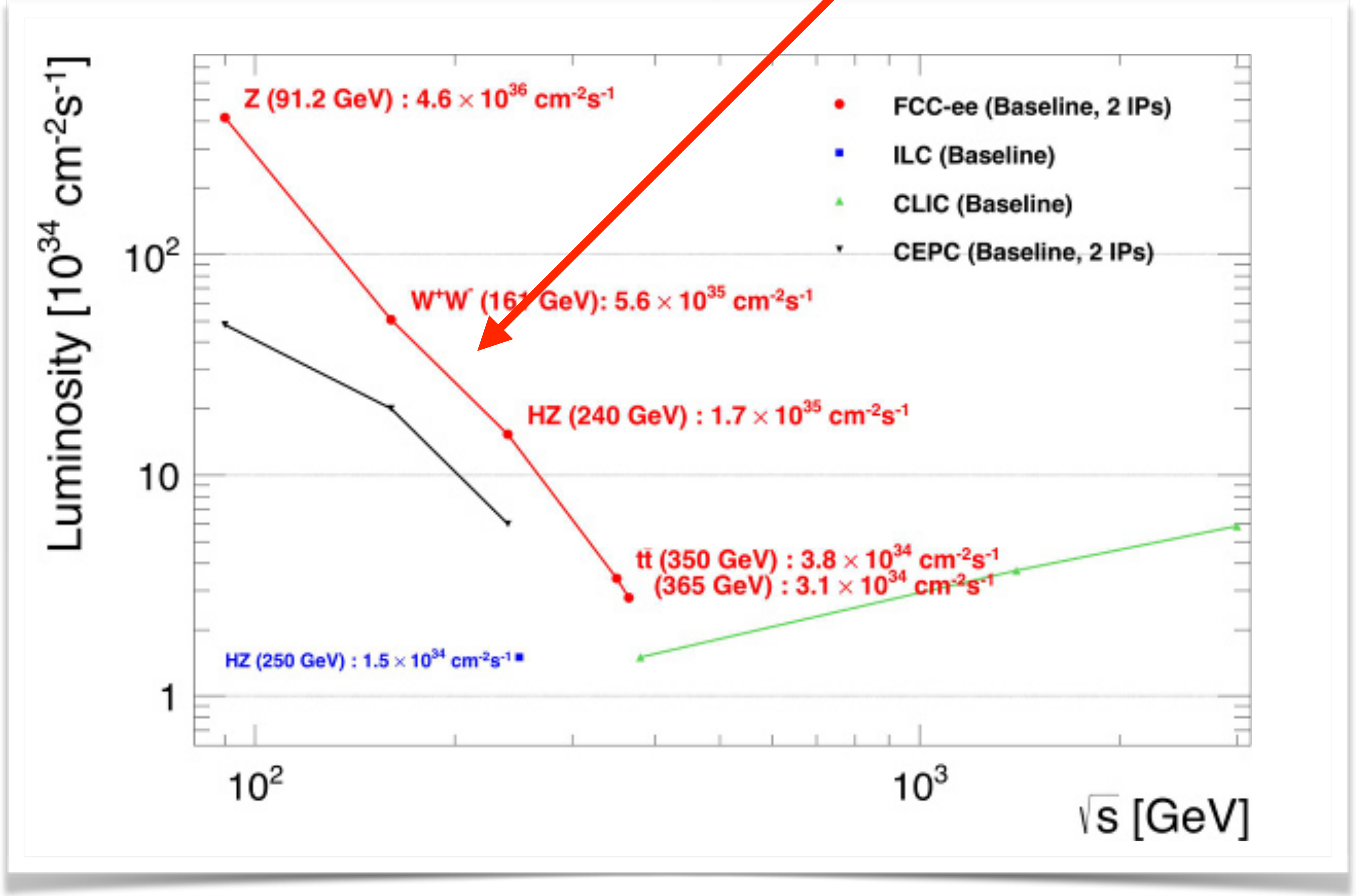
- **Beam current and thus luminosity are limited by maximum acceptable synchrotron radiation power**

Example: FCC-ee: $\boldsymbol{P_{max} = 50\ \mathrm{MW}}$

- Beam dynamics determined by emission of synchrotron radiation.
- The emission of synchrotron radiation ist determined by the lattice.

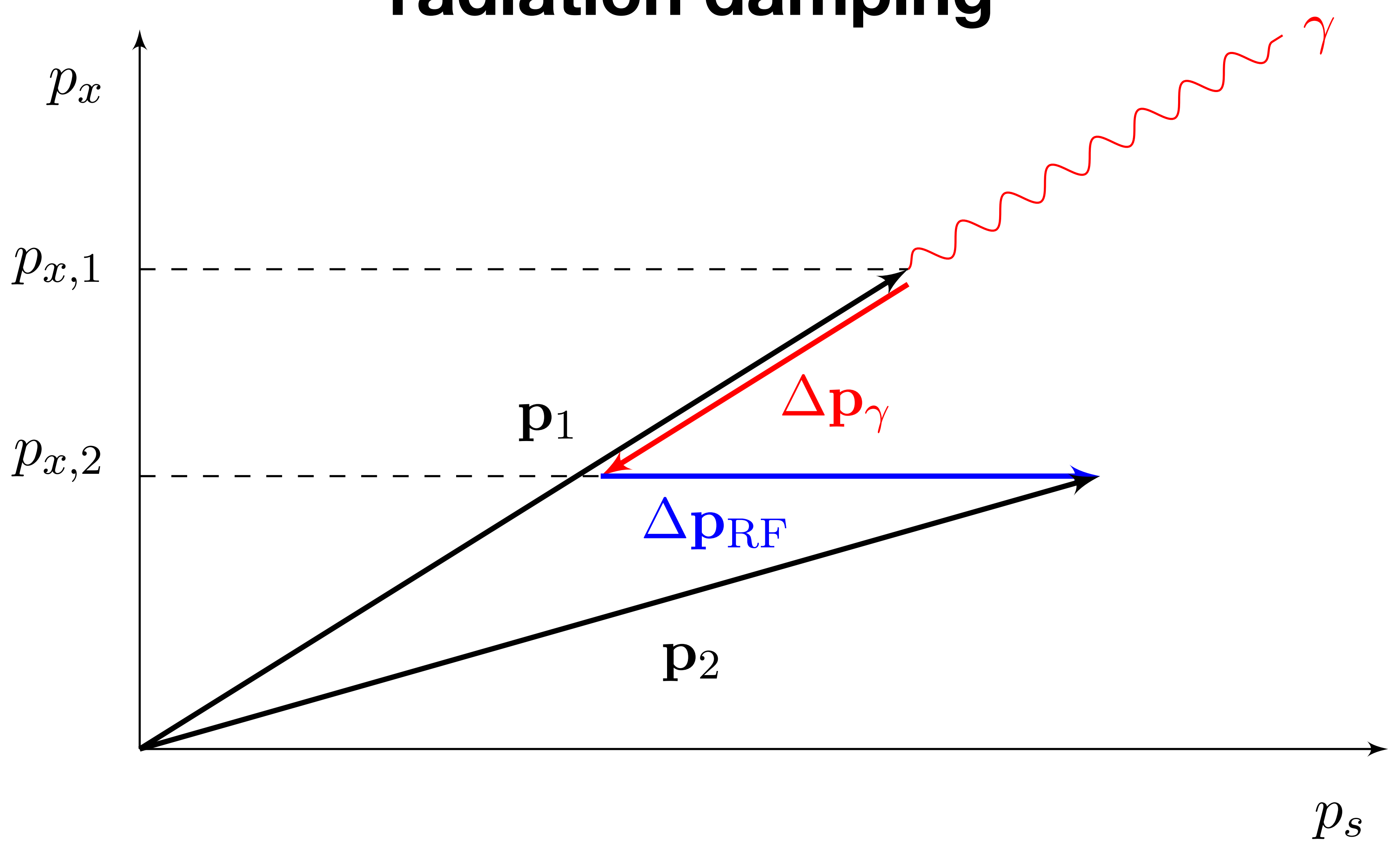
=> Lattice design allows to tailor beam parameters!!!

Energy (GeV)	# bunches	# particles per bunch (10^{11})	Luminosity ($10^{34}/cm^2s$)
45.6	16640	1.7	230
80.0	2000	1.5	28
120.0	328	1.8	8.5
182.5	48	2.3	1.55



Radiation effects in electron storage rings

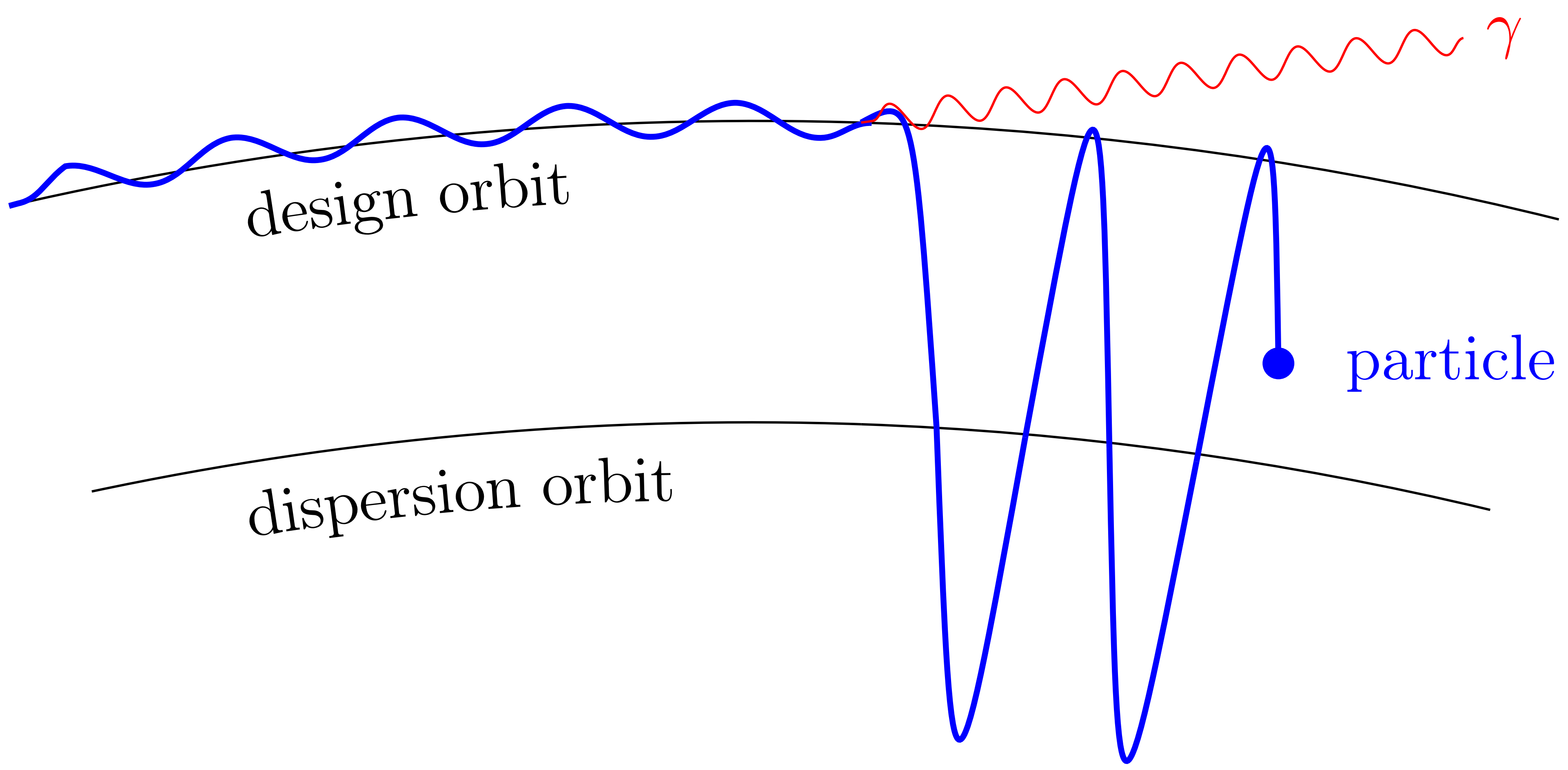
radiation damping



- Photon emission in current direction of movement
- Loss of both transverse and longitudinal momentum
- Energy gain in cavities in longitudinal direction only

→ **Decrease of transverse momentum**

quantum excitation



- Transverse oscillation of electron around design orbit
- Photon emission creates energy loss
- Electron starts oscillations around dispersion orbit

→ **Increase of transverse momentum**

Equilibrium beam parameters

juas
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- After a few damping times an equilibrium of radiation damping and quantum excitation is established.
- Five characteristic integrals that depend on the lattice:
  "Synchrotron radiation integrals"

$$\mathcal{I}_1 = \oint \frac{D(s)}{\rho}\,\mathrm{d}s$$

$$\mathcal{I}_2 = \oint \frac{1}{\rho^2}\,\mathrm{d}s$$

$$\mathcal{I}_3 = \oint \frac{1}{|\rho^3|}\,\mathrm{d}s$$

$$\mathcal{I}_{4u} = \oint \frac{D_u}{\rho_u}\left(\frac{1}{\rho_u^2} + 2k_1\right)\mathrm{d}s$$

$$\mathcal{I}_{5u} = \oint \frac{1}{|\rho_u|^3}\mathcal{H}_u\,\mathrm{d}s$$

with $\mathcal{H}_u(s) = \beta_u D_u'^2 + 2\alpha_u D_u D_u' + \gamma_u D_u^2$

Energy loss per turn: $U_0 = \frac{C_\gamma}{2\pi} E^4 \mathcal{I}_2$

Equilibrium beam emittance: $\epsilon_u = C_\mathrm{q} \frac{\gamma^2}{J_u} \frac{\mathcal{I}_{5u}}{\mathcal{I}_2}$

$$C_\gamma = \frac{e^2}{3\epsilon_0} \frac{1}{(m_\mathrm{e}c^2)^4} = 8.8460 \times 10^{-5} \frac{\mathrm{m}}{\mathrm{GeV}^3}$$

$$C_\mathrm{q} = \frac{55}{32\sqrt{3}} \frac{\hbar c}{m_0 c^2} = 3.832 \times 10^{-13}\,\mathrm{m}$$

Choice of phase advance per cell

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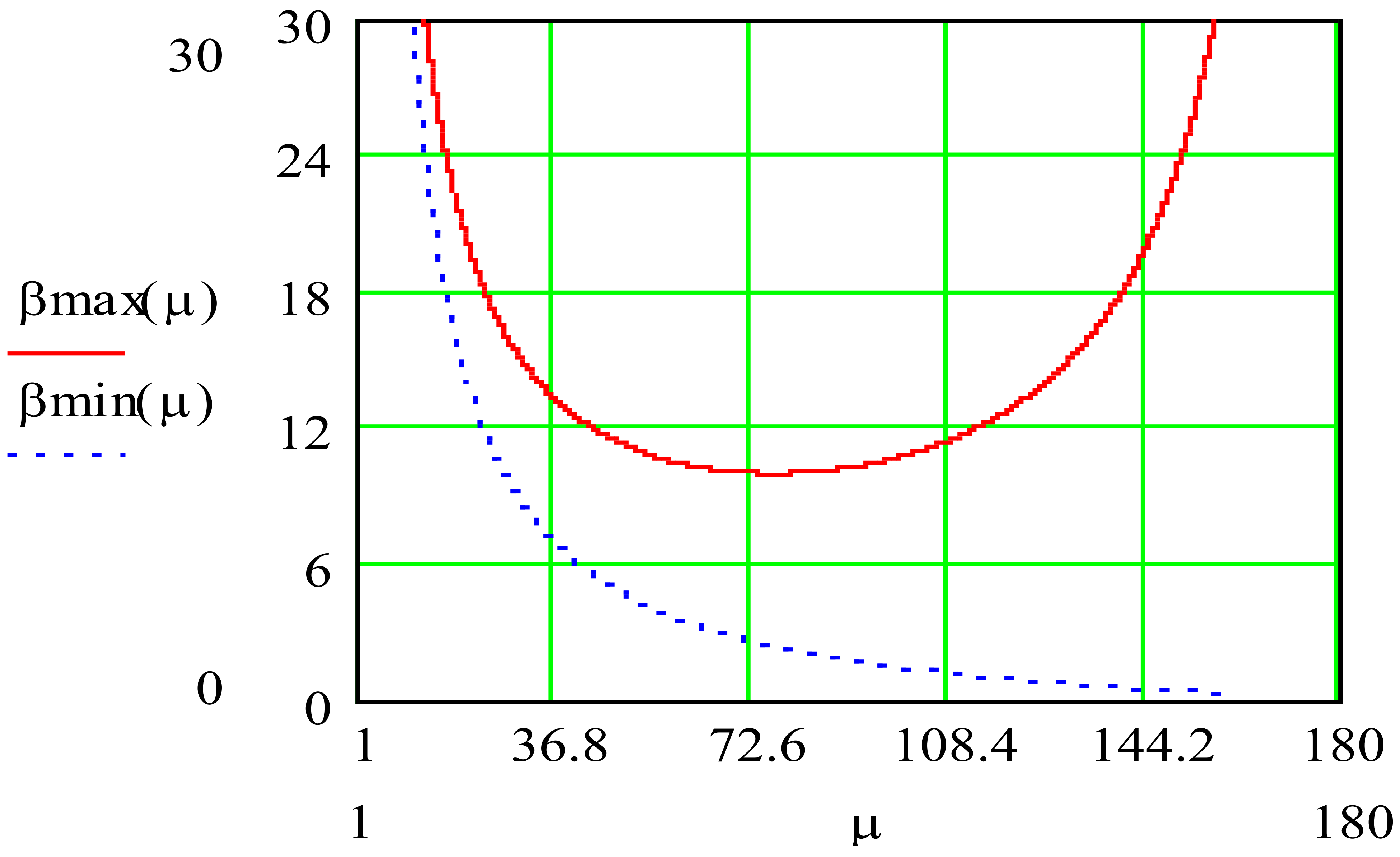
- Quantum excitation only in deflection plane
- Equilibrium emittance in vertical plane determined by coupling (imperfections, sextupoles, …)

$$\rightarrow \epsilon_y \approx 0.1 - 1\,\%\,\epsilon_x$$

- Electron beams in storage rings feature "flat" beams

-> Only optimise β_x:

$$\frac{\mathrm{d}}{\mathrm{d}\mu}(\hat{\beta}) = \frac{\mathrm{d}}{\mathrm{d}\mu}\frac{L(1+\sin(\mu/2))}{\sin(\mu)} = 0 \qquad \rightarrow \mu = 76^{\circ}$$



Choice of phase advance per cell - II

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Advanced level: Sextupole scheme

$$\frac{e}{p} B_x = k_2 \boxed{x\ y}$$

$$\frac{e}{p} B_y = \frac{1}{2} k_2 \boxed{(x^2 - y^2)}$$

- Sextupoles are non-linear elements

  -> disturb harmonic transverse oscillation

- Geometric aberrations can be canceled, if sextupoles are installed at positions with

$$\Delta\mu = \pi$$

"-I transformation"

-> Multiples of the phase advance should give 180°

$$\mu = 90^\circ \quad \Rightarrow 2 \times \mu = \pi$$

$$\mu = 60^\circ \quad \Rightarrow 3 \times \mu = \pi$$

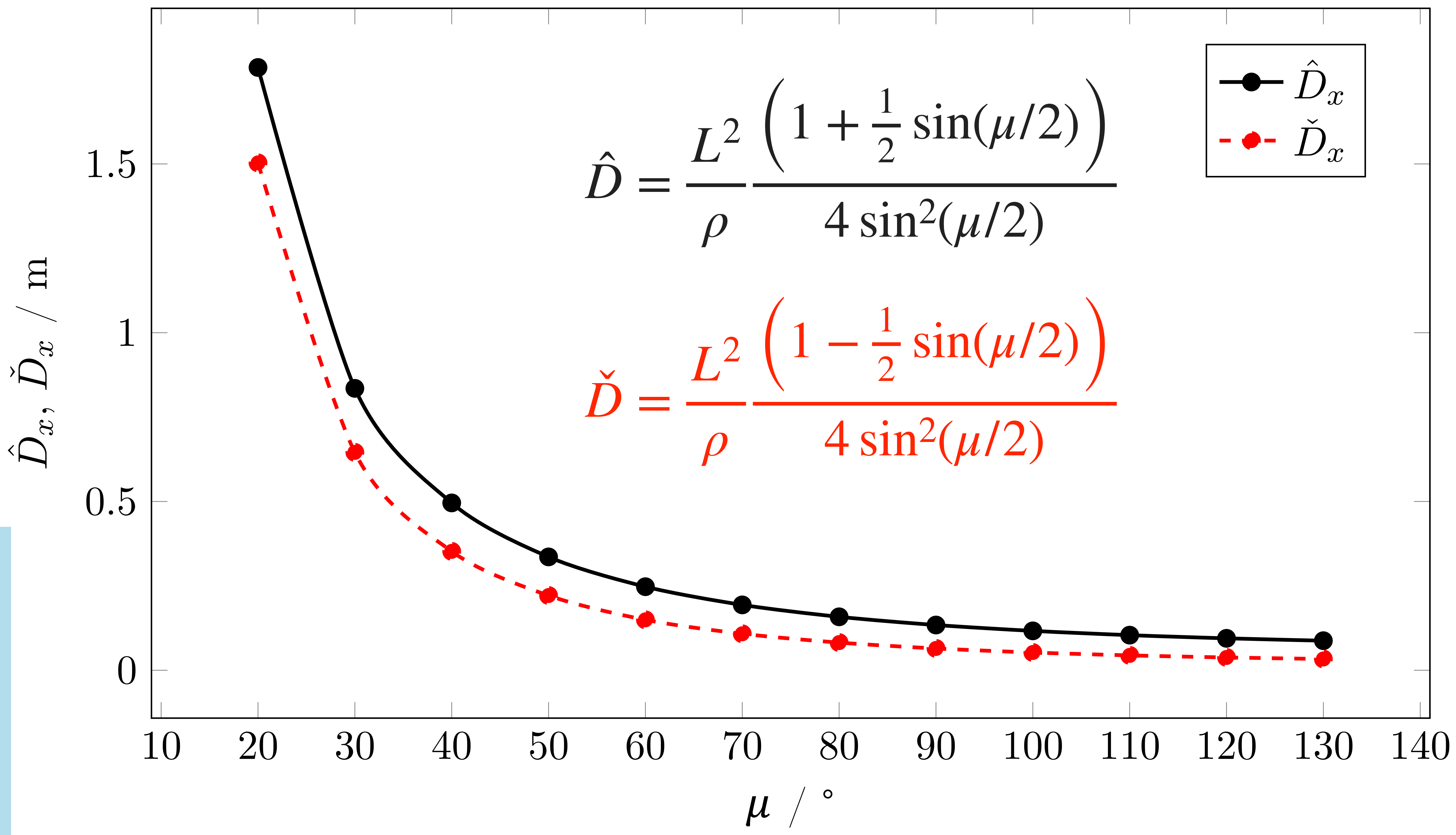
Emittance and dispersion function

$$\epsilon_u = C_\mathrm{q} \frac{\gamma^2}{J_u} \frac{\mathcal{I}_{5u}}{\mathcal{I}_2}$$

$$\mathcal{I}_2 = \oint \frac{1}{\rho^2}\,\mathrm{d}s \qquad \mathcal{I}_{5u} = \oint \frac{1}{|\rho_u|^3} \mathcal{H}_u\,\mathrm{d}s$$

$$\mathcal{H}_u(s) = \beta_u D_u'^2 + 2\alpha_u D_u D_u' + \gamma_u D_u^2$$

- Value of D and D' highly affect emittance.
- In a FODO lattice the emittance can be tuned via cell length, bending radius, and phase advance.



Maximum and minimum values of the dispersion function in an arc FODO cell designed for FCC-ee

Emittance of a FODO lattice

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Fermilab

TM-1269
0102.000

Minimizing the Emittance in Designing the Lattice of an Electron Storage Ring

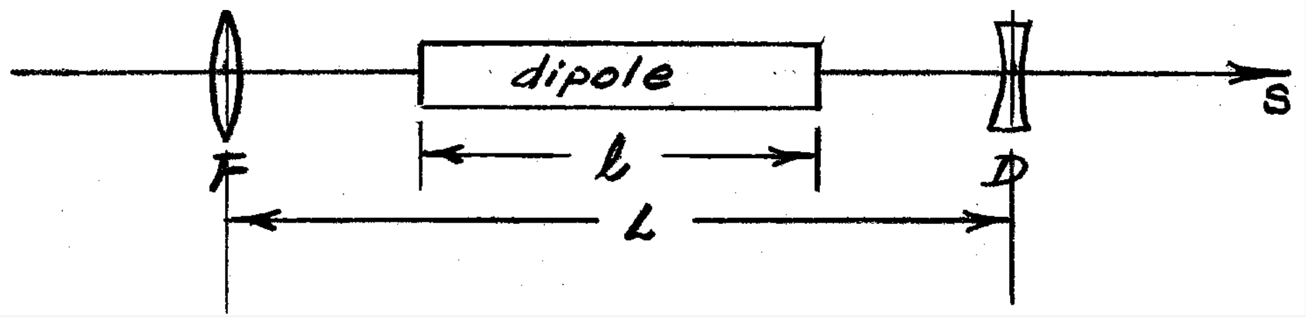
L.C. Teng

June 1984

A. Formulation

For a synchrotron radiation facility to get high spectral brilliance it is desirable to have a small emittance of the electron beam in the storage ring. It is well known that the horizontal emittance (the predominant emittance) of an electron beam in a storage ring is given by

$$\epsilon_x \equiv \frac{\sigma_x^2}{\beta_x} = \frac{C_q}{J_x}\frac{\gamma^2}{\rho}\langle\mathcal{H}\rangle_{dipole}$$



- Develops form factors to calculate emittance of electron storage rings:

$$\epsilon_x = \frac{C_q}{J_x}\gamma^2\frac{\mathcal{I}_5}{\mathcal{I}_2} \longrightarrow \epsilon_x = \frac{C_q}{J_x}\gamma^2\theta^3 F\,, \qquad F \equiv \frac{\rho^2}{l^3}\langle\mathcal{H}\rangle_{dipole}$$

(θ bending angle, ρ bending radius)

- Treats FODO as a bad example, but gives handy formula:

$$F_{\mathrm{FODO}} = \frac{1}{2\sin\mu}\,\frac{5+3\cos\mu}{1-\cos\mu}\,\frac{L}{l_{\mathrm{b}}}$$

$$\mu = 90^\circ : F = 2.50\,\frac{L}{l_{\mathrm{b}}}, \qquad \mu = 72^\circ : F = 4.51\,\frac{L}{l_{\mathrm{b}}}, \qquad \mu = 60^\circ : F = 7.51\,\frac{L}{l_{\mathrm{b}}}$$

- **Example:** FODO cell with 90° phase advance needs dipoles with bending angle:

$$\theta^3 = \frac{1}{2.50}\,\frac{\epsilon_x J_x}{C_q\gamma^2}\,\frac{l}{L}$$

requirement

dipole filling factor

defined by beam energy

e^+e^- colliders vs. synchrotron light sources

Joint Universities Accelerator School

Collider

- High dipole filling factor -> FODO structure
- High energy -> large circumference

  -> Naturally small emittance

$$\mathscr{L} = \frac{N_1 N_2 n_b f}{4\pi\sigma_x^* \sigma_y^*}$$

N particles per bunch
n_b number of bunches
f revolution frequency

Synchrotron light source

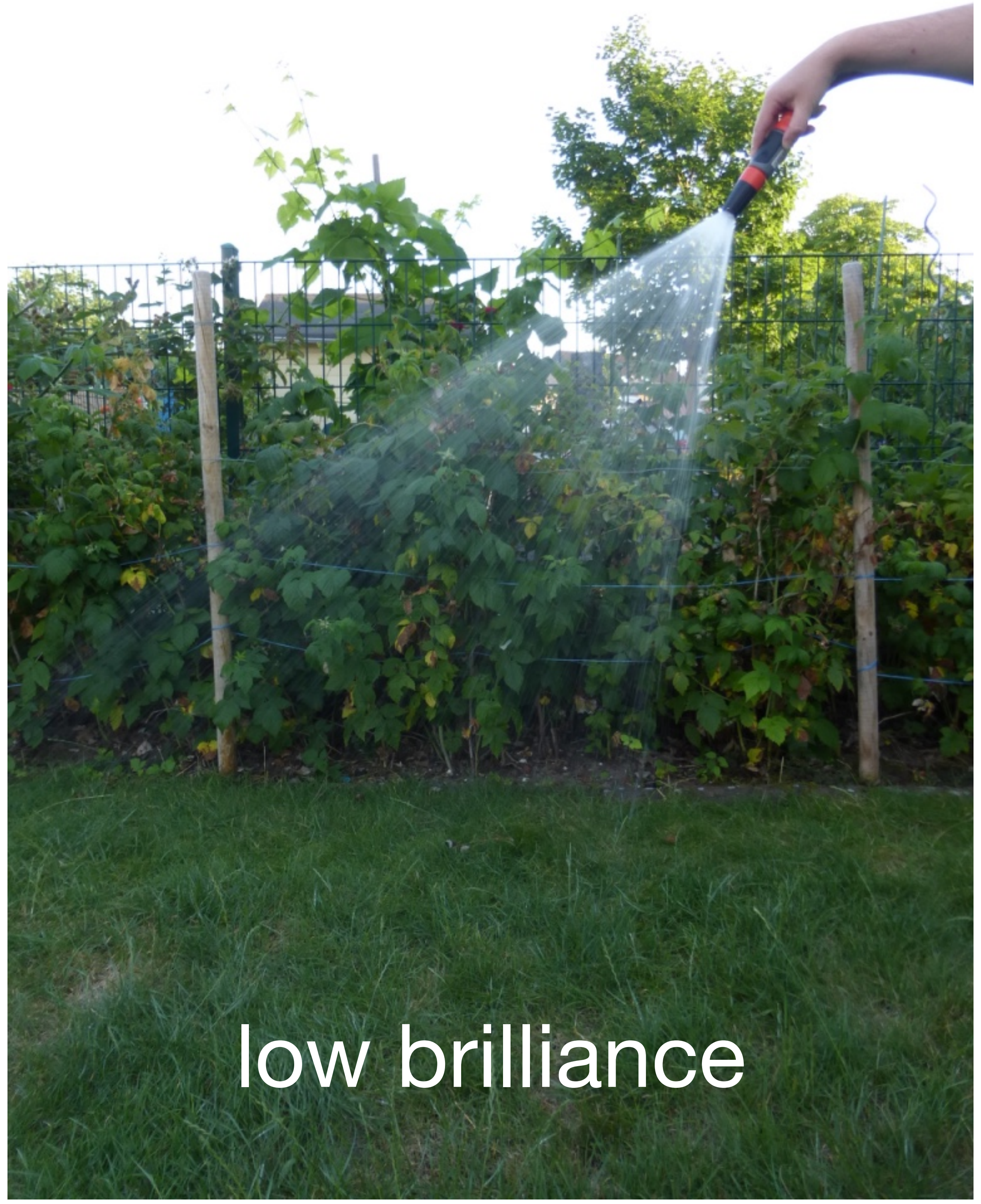
- Small footprint desired
- Low emittance beams for high brilliance

$$B(\lambda) = \frac{F(\lambda)}{(2\pi)^2 \sigma_x \sigma_{x'} \sigma_y \sigma_{y'}} \propto \frac{1}{\epsilon_x \epsilon_y}$$

with photon flux *F(λ)*

Jean-Luc Revol: ESRF - The European Synchrotron, JUAS 2020

Brillance



$$B(\lambda) = \frac{F(\lambda)}{(2\pi)^2 \sigma_x \sigma_{x'} \sigma_y \sigma_{y'}} \propto \frac{1}{\epsilon_x \epsilon_y}$$

with photon flux *F(λ)*

Courtesy M. Schuh

High brilliant beams require small emittances!

FODO not adequate because $D_x \neq 0$

Double bend achromat lattice

juas
Joint Universities Accelerator School

Chasman-Green-Lattice

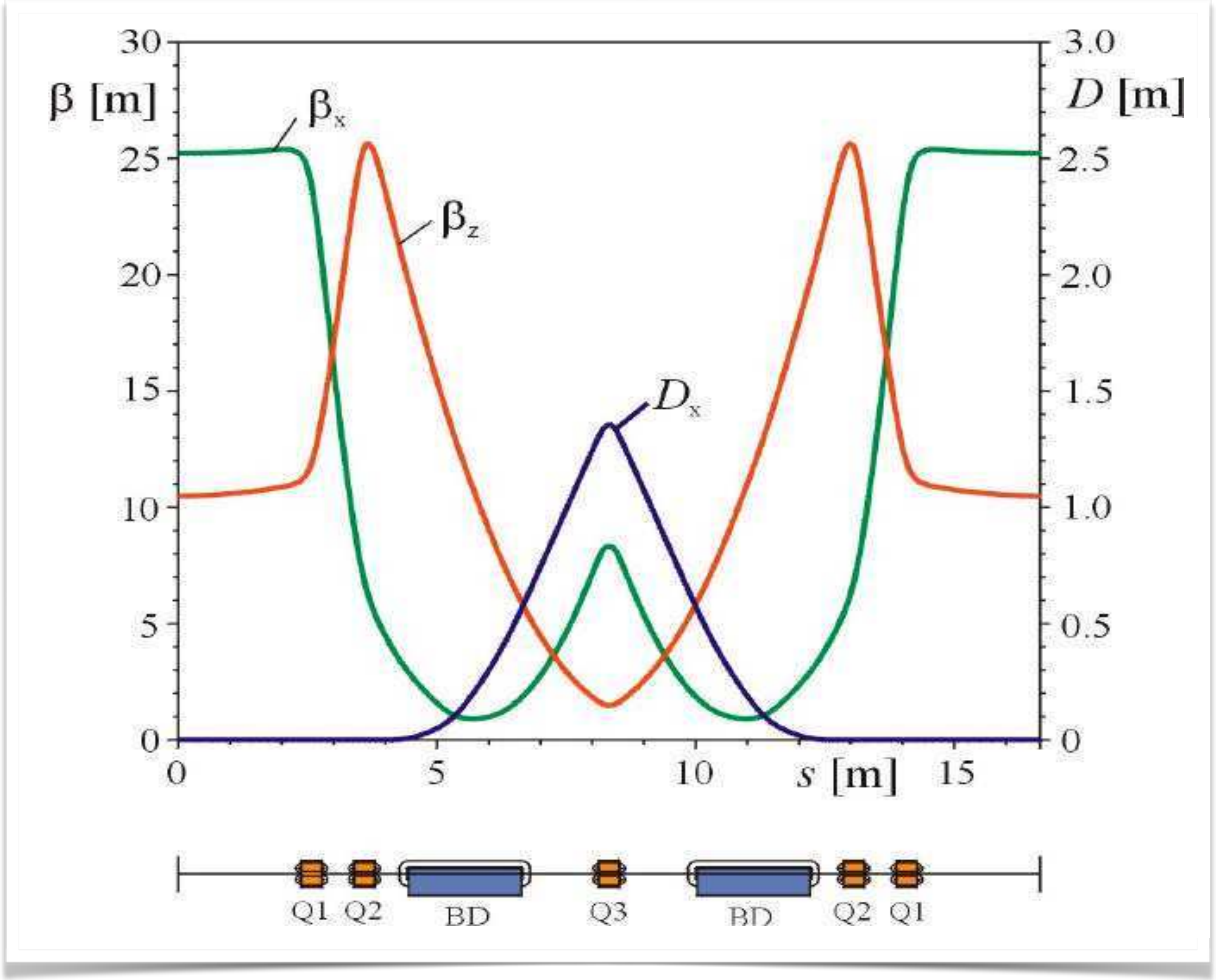
- Achromat means: Dispersion and its derivative vanish at start and end of the cell

- Dispersion is created by the first dipole. The quadrupole switches the sign of D' and the dispersion vanishes again in the second dipole.

- Long drift spaces without dispersion allow

  -> installation of insertion devices
  -> small integrated dispersion thus low values
    of $\mathcal{I}_5$ and ϵ_x

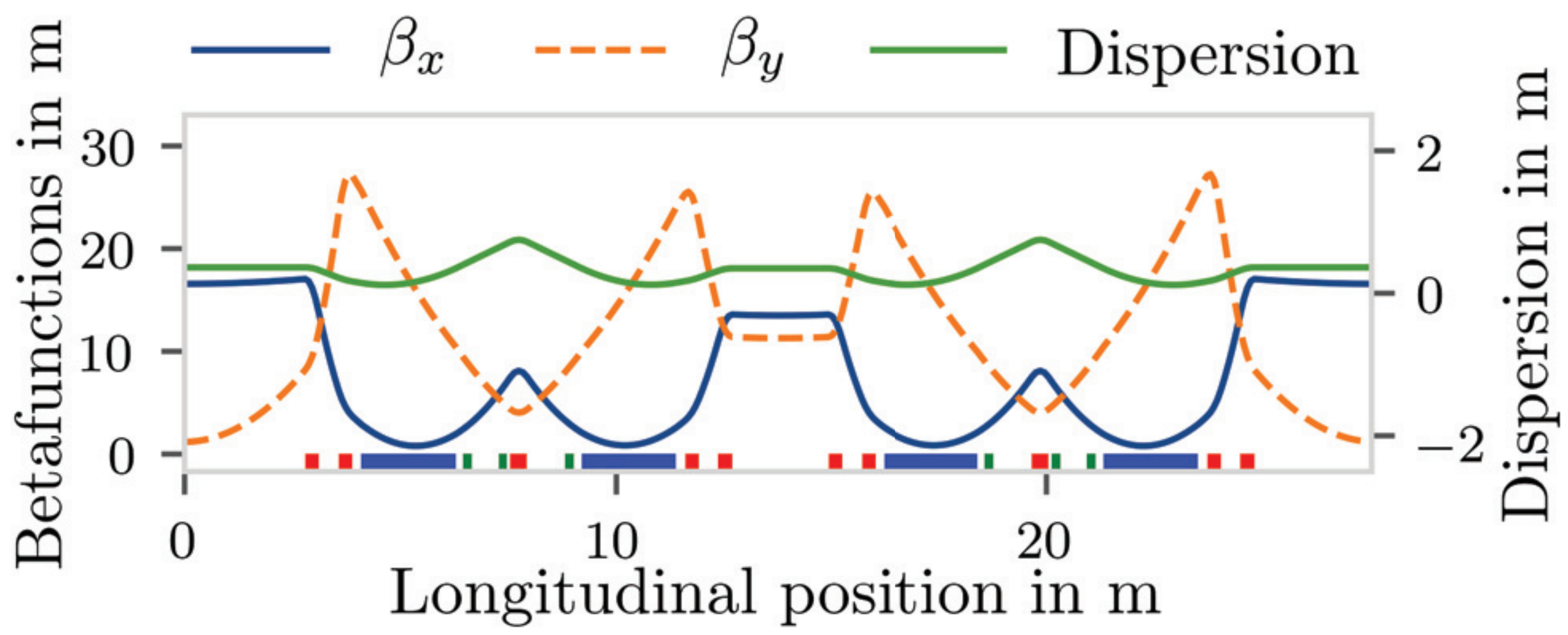
- Characteristic lattice for 3rd generation synchrotron light souces

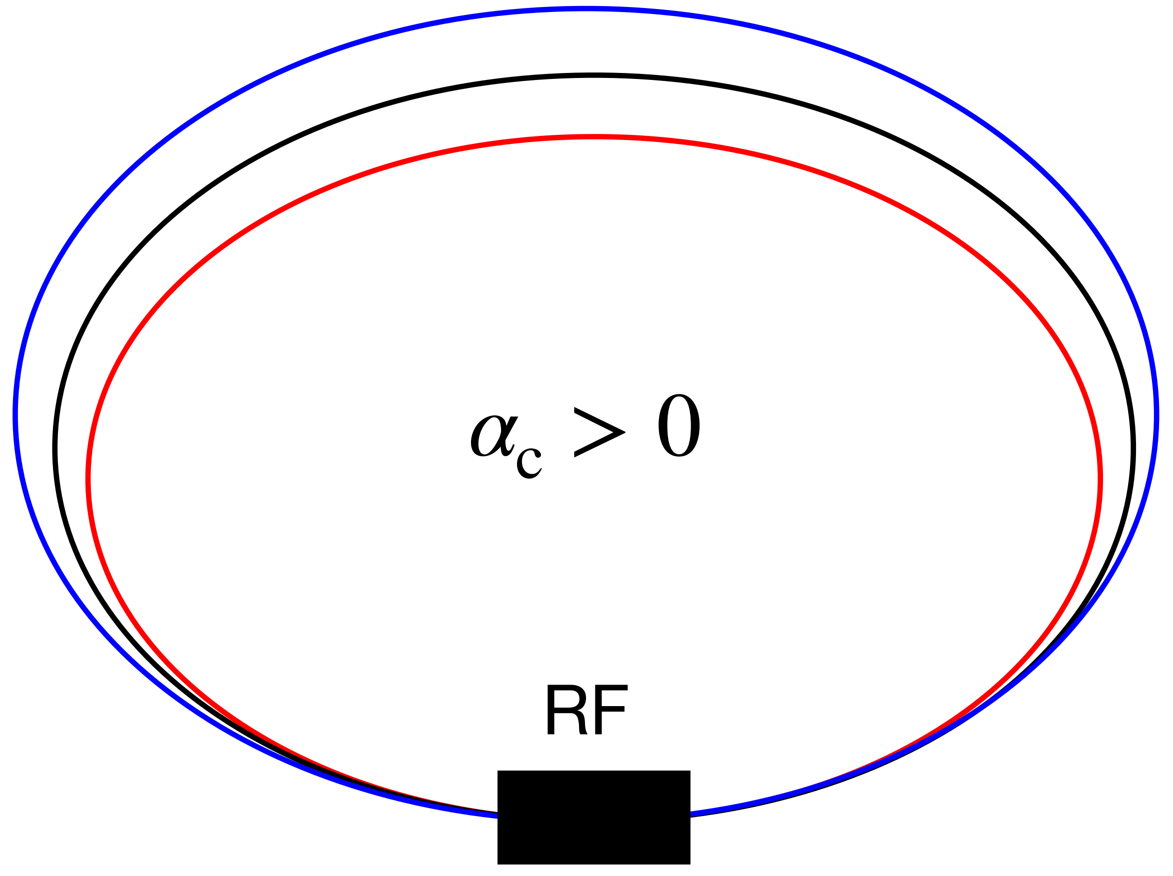


Negative momentum compaction factor

KARA - Karlsruhe Research Accelerator

Optics for user operation

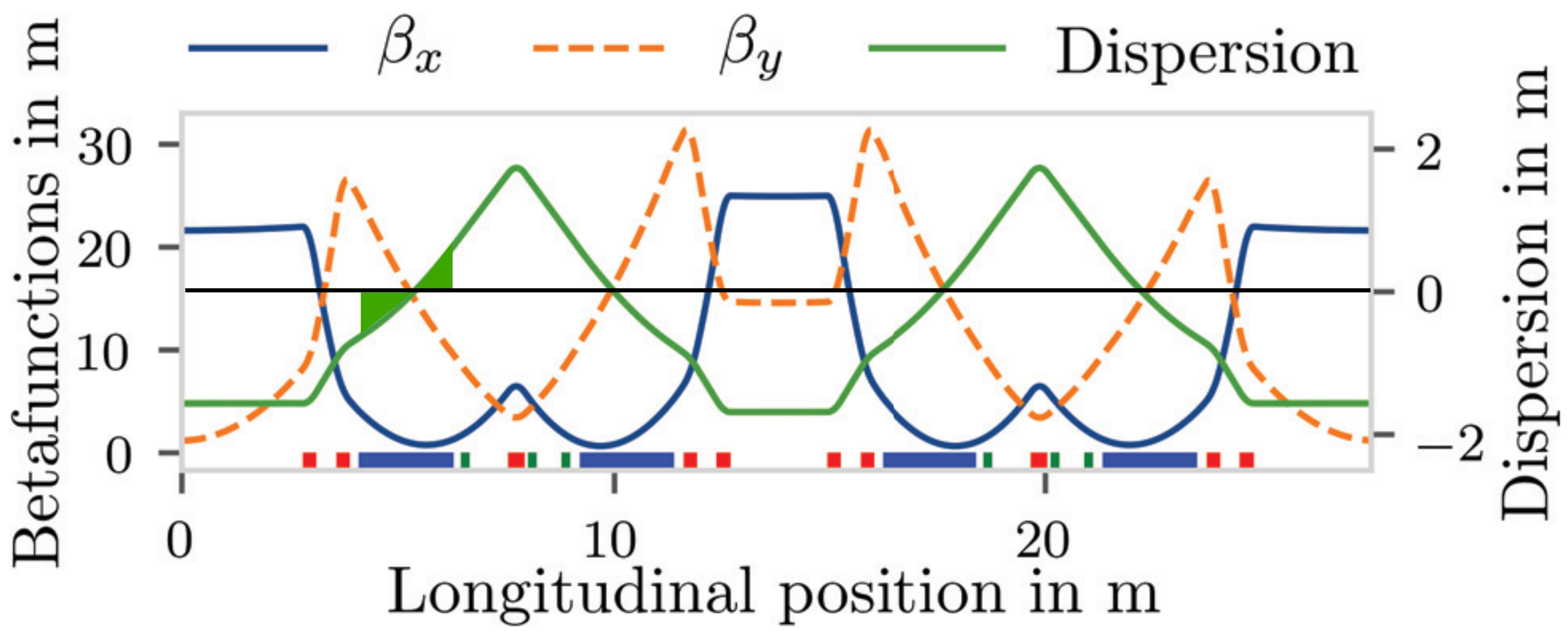




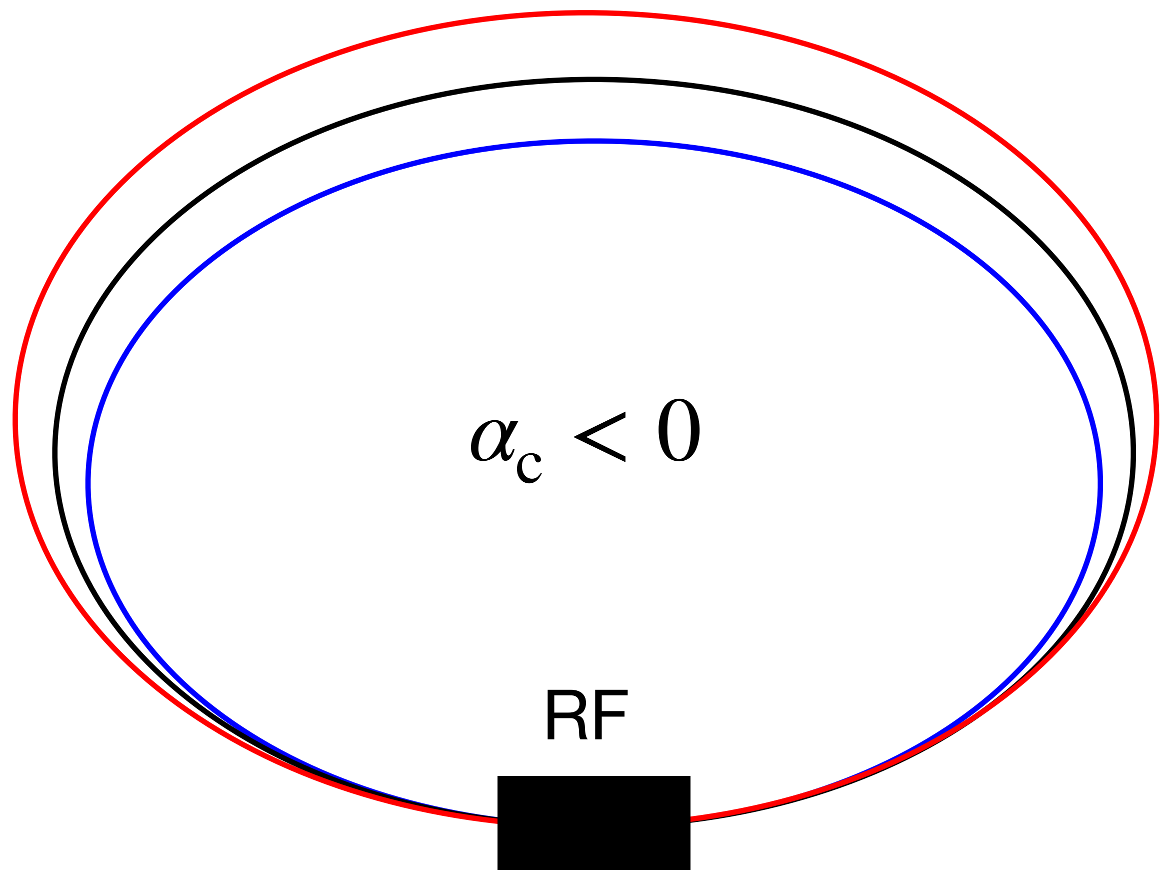
$\delta > 0$ $\delta = 0$ $\delta < 0$

Optics with negative momentum compaction factor

$$\alpha_c = \frac{1}{L}\oint \mathrm{d}s \frac{D(s)}{\rho(s)}$$



Courtesy P. Schreiber



Tomorrow:

- Achromat lattices for synchrotron light sources
- Dispersion suppressor
- Insertions
  -> RF sections
  -> mini-beta insertion
- How to build an accelerator model: Step-by-step

- Adrian: Details to groups, exercises and examination

Recommended literature

Joint Universities Accelerator School

- **J. Bryant, K. Johnson:**
  The Principles of Circular Accelerators and Storage Rings

- Proceedings of CAS Advanced Accelerator Physics:
  B. Holzer: Lattice Design in High-energy Particle Accelerators
  - 18 August 2013 - 29 August 2013, Trondheim, Norway
  - 15 September 2003 - 26 September 2003, Zeuthen, Germany

Big thanks to Bernhard Holzer and Phil Bryant who gave this lecture before me and provided me with their slides!!!