Magnet design for MedAustron

February 20, 2022

1 Analytical

1.1 Magnet type decision

The arguments for and against a H-type magnet are:

+ Mechanical rigid
+ Symmetrical
− Hard to get the beam pipe in and out

1.2 Aperture Height

The aperture height is given as the sum of the good field region $h_{\mathrm{GFR}} = 2 \cdot \mathrm{GFR}_y$, the thickness of the vacuum pipe d_{vacuum} and a tolerance for installation and thermal expansion $d_{\mathrm{tolerance}}$ as

$$\begin{aligned} h &= h_{\mathrm{GFR}} + 2 \cdot d_{\mathrm{vacuum}} + d_{\mathrm{tolerance}} \\ &= 2 \cdot 23\,\mathrm{mm} + 2 \cdot 2\,\mathrm{mm} + 2\,\mathrm{mm} \\ &= 52\,\mathrm{mm}. \end{aligned} \tag{1}$$

1.3 Flux Density

The total bending angle generated by all $m = 3$ magnets is

$$\theta_{\mathrm{tot}} = 3 \cdot \theta_{\mathrm{mag}} = 3 \cdot 36^\circ = 108^\circ. \tag{2}$$

The magnetic length of the (dipole) magnet can be approximated by

$$\begin{aligned} l_{\mathrm{mag}} &= l_{\mathrm{iron,\ max}} + 2hk \\ &= 0.340\,\mathrm{m} + 2 \cdot 0.55 \cdot 52\,\mathrm{mm} \\ &= 397.2\,\mathrm{mm}. \end{aligned} \tag{3}$$

With the definition of the radian ($\theta = s/\rho$) and $s = l_{\mathrm{mag}}$, the bending radius ρ is

$$\theta_{\mathrm{mag}} = \frac{l_{\mathrm{mag}}}{\rho} \tag{4}$$

$$\begin{aligned} \Rightarrow \rho &= \frac{l_{\mathrm{mag}}}{\theta_{\mathrm{mag}}} \\ &= \frac{397.2\,\mathrm{mm}}{0.6283\,\mathrm{rad}} \\ &= 0.642\,\mathrm{m}. \end{aligned} \tag{5}$$

With the given minimum and maximum $(B\rho)$ for a proton and a C^{6+} beam, the minimum and maximum needed flux densities are

$$B_{\mathrm{min}} = \frac{(B\rho)_{\mathrm{min}}}{\rho} = \frac{0.383\,\mathrm{T\,m}}{0.642\,\mathrm{m}} = 0.596\,\mathrm{T} \tag{6}$$

$$B_{\mathrm{max}} = \frac{(B\rho)_{\mathrm{max}}}{\rho} = \frac{0.766\,\mathrm{T\,m}}{0.642\,\mathrm{m}} = 1.19\,\mathrm{T} \tag{7}$$

1.4 Pole width and yoke thickness

With the given field quality inside the GFR ${}^{\Delta B}/_{B_0} = 1 \cdot 10^{-3}$, the optimized normalized pole overhang is[1]

$$\begin{aligned} x_{\text{optimized}} &= 2\frac{a_{\text{optimized}}}{h} \\ &= -0.14 \ln\left[\frac{\Delta B}{B_0}\right] - 0.25 \\ &= 0.7171. \end{aligned} \tag{8}$$

This equals a pole overhang of

$$\begin{aligned} a_{\text{optimized}} &= \frac{h \cdot x_{\text{optimized}}}{2} \\ &= \frac{52\,\text{mm} \cdot 0.7171}{2} \\ &= 18.645\,\text{mm}. \end{aligned} \tag{9}$$

Taking the sagitta s into account and using

$$\text{GFRx'} = \text{GFRx} + \underbrace{\rho(1 - \cos{}^{\theta}/_{2}}_{s} = 20\,\text{mm} + 31.42\,\text{mm} = 51.42\,\text{mm}, \tag{10}$$

the pole width w computes to

$$\begin{aligned} w &= 2 \cdot (\text{GFRx'} + a_{\text{optimized}}) \\ &= 2 \cdot (51.42\,\text{mm} + 18.645\,\text{mm}) \\ &= 140.13\,\text{mm}. \end{aligned} \tag{11}$$

1.5 Excitation Current

The ampereturns are given as

$$(NI)_{\text{dipole, min}} = \frac{B_{\min}\,h}{2\,\mu_0} = \frac{0.596\,\text{T} \cdot 52\,\text{mm}}{2 \cdot \mu_0} = 12.331\,\text{kA} \tag{12}$$

$$(NI)_{\text{dipole, max}} = \frac{B_{\max}\,h}{2\,\mu_0} = \frac{1.19\,\text{T} \cdot 52\,\text{mm}}{2 \cdot \mu_0} = 24.621\,\text{kA} \tag{13}$$

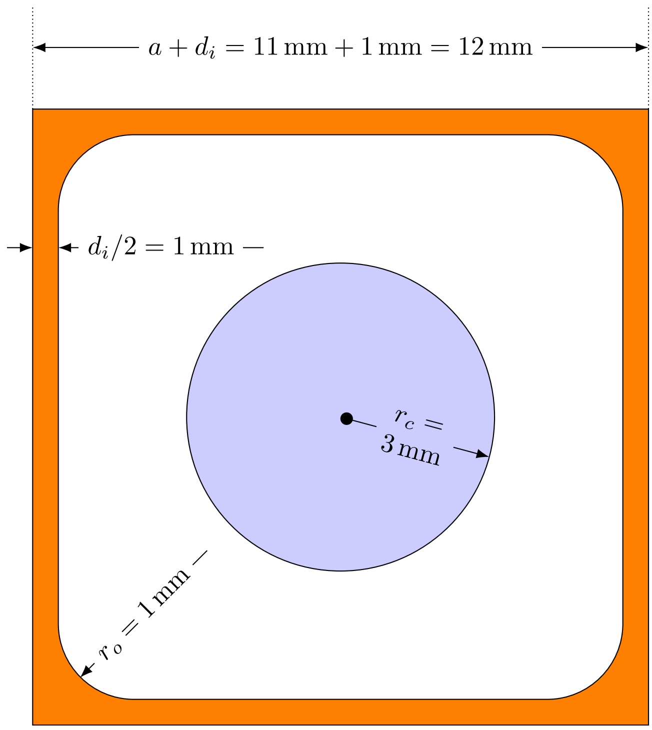


Figure 1: Cross section of the copper winding showing insulation(■) and water channel(■)

[1] Compared to the unoptimized $x_{\text{unoptimized}} = 1.5898$

With the given dimensions of the copper winding (see Figure 1) and using

$$A_{\text{rounded edges}} = 4 \cdot A_{\text{rounded edge}} = 4\frac{r_o^2(4-\pi)}{4} = r_o^2(4-\pi) \tag{14}$$

$$A_{\text{water channel}} = \pi r_c^2 \tag{15}$$

the copper cross section area is

$$\begin{aligned} A_{\text{copper}} &= a^2 - \underbrace{r_o^2(4-\pi)}_{A_{\text{rounded edges}}} - \underbrace{\pi r_c^2}_{A_{\text{water channel}}} \\ &= 121\,\text{mm}^2 - 0.8584\,\text{mm}^2 - 28.274\,\text{mm}^2 \\ &= 91.867\,\text{mm}^2 \end{aligned} \tag{16}$$

With the given maximum current density of $j_{\text{max}} = 6\,\text{A}\,\text{mm}^{-2}$, the maximum possible current in the winding is

$$\begin{aligned} I_{\text{max}} &= j_{\text{max}} \cdot A_{\text{copper}} \\ &= 6\,\text{A}\,\text{mm}^{-2} \cdot 91.867\,\text{mm}^2 \\ &= 551.202\,\text{A}. \end{aligned} \tag{17}$$

With the calculated ampereturns, the necessary minimum number of turns are then

$$N_{B_{\text{min}}} = \left\lceil \frac{(NI)_{\text{min}}}{I_{\text{max}}} \right\rceil = 23 \tag{18}$$

$$N_{B_{\text{max}}} = \left\lceil \frac{(NI)_{\text{max}}}{I_{\text{max}}} \right\rceil = 45 \tag{19}$$

As it is easier to lower the current then to change the number of turns, the minimum number of turns should be $N_{B_{\text{max}}}$.
To full fill the empirical good practice ${}^{N_{\text{horizontal}}}/_{N_{\text{vertical}}} = 2$, the number of turn is set to

$$N = N_{\text{horizontal}} \times N_{\text{vertical}} = 10 \times 5 = 50. \tag{20}$$

With the new $N = 50$, the winding current is

$$I = \frac{(NI)_{\text{max}}}{50} = 492.42\,\text{A} \tag{21}$$

1.6 Coil Parameters

The coil width and height are given by (see Figure 2)

$$w_{\text{coil}} = N_{\text{horizontal}} \cdot (a + d_{\text{insulation}}) + 2 \cdot (d_{\text{epoxy}} + d_{\text{air}}) = 128\,\text{mm} \tag{22}$$

and

$$h_{\text{coil}} = N_{\text{vertical}} \cdot (a + d_{\text{insulation}}) + 2 \cdot (d_{\text{epoxy}} + d_{\text{air}}) = 68\,\text{mm} \tag{23}$$

using $N_{\text{horizontal}} = 10$, $N_{\text{vertical}} = 5$, $a = 11\,\text{mm}$, $d_{\text{insulation}} = 1\,\text{mm}$, $d_{\text{epoxy}} = 2\,\text{mm}$ and $d_{\text{air}} = 2\,\text{mm}$.

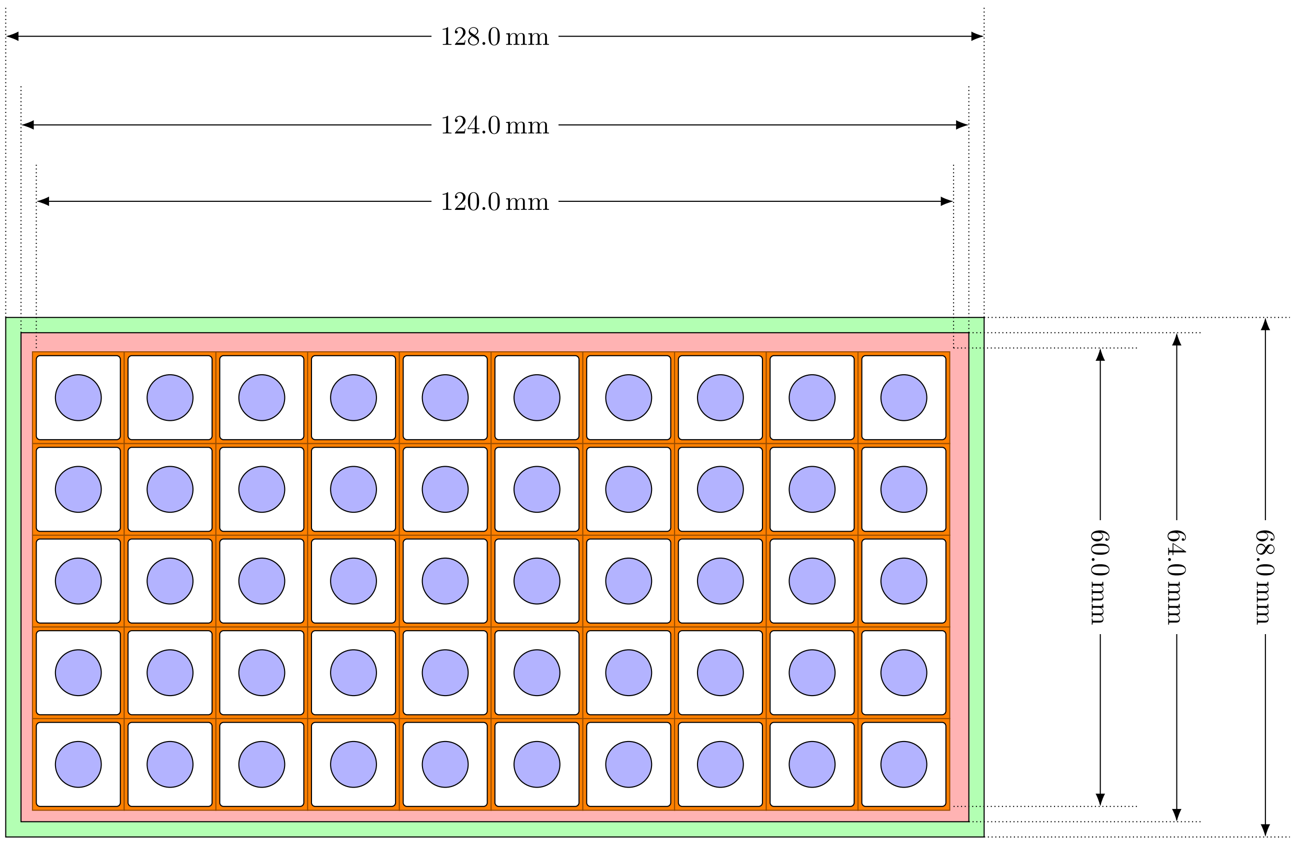


Figure 2: Cross section of one coil showing copper(□), insulation(■), the water channels(■), the coil epoxy coating(■) and the air gap around the coil(■)

With the pole perimeter

$$p = 2 \cdot liron + 2 \cdot w = 960\,\mathrm{mm}, \tag{24}$$

the average turn length is ($liron = 340\,\mathrm{mm}$, $w = 140.13\,\mathrm{mm}$ and $w_{\mathrm{coil}} = 128\,\mathrm{mm}$)

$$l_{\mathrm{avg}} = p + 4 \cdot w_{\mathrm{coil}} = 1472\,\mathrm{mm}. \tag{25}$$

The resistance of one coil winding is (with $1/\sigma_{\mathrm{Cu}} = 1.72\,\mu\Omega\,\mathrm{cm}$)

$$\begin{aligned} R_c &= \frac{N \cdot l_{\mathrm{avg}}}{A \cdot \sigma_{\mathrm{Cu}}} \\ &= \frac{50 \cdot 1472\,\mathrm{mm}}{91.867\,\mathrm{mm}^2 \cdot {}^{1}/_{1.72\,\mu\Omega\,\mathrm{cm}}} \\ &= 13.442\,\mathrm{m}\Omega \end{aligned} \tag{26}$$

With the number of coils per magnet m, the DC steady-state voltage per magnet is

$$\begin{aligned} V_m &= I \cdot m\,R_c \\ &= 492.42\,\mathrm{A} \cdot 2 \cdot 0.016\,\Omega \\ &= 13.197\,\mathrm{V}. \end{aligned} \tag{27}$$

The total voltage over all magnets is (number of magnets M)

$$\begin{aligned} V_{\mathrm{total}} &= M \cdot V_m \\ &= 3 \cdot 16.197\,\mathrm{V} \\ &= 39.591\,\mathrm{V}. \end{aligned} \tag{28}$$

The dissipated power in one magnet is calculated to

$$\begin{aligned} P_m &= V_m \cdot I = I^2 \cdot m\,R_c \\ &= 13.197\,\mathrm{V} \cdot 492.42\,\mathrm{A} \\ &= 6.498\,\mathrm{kW}. \end{aligned} \tag{29}$$

1.7 Cooling

With the given $\Delta T = 15\,\mathrm{K}$ and the dissipated power $P_m = 6.498\,\mathrm{kW}$, the total required water flow in one magnet is

$$Q = 14.3\frac{P_m}{\Delta T} \cdot 1 \cdot 10^{-3} = 6.2\,\mathrm{L\,min^{-1}} \tag{30}$$

Using the length of cooling circuit for one coil is ($K_c = 1$ and $K_w = 1$)

$$\begin{aligned} l &= \frac{K_c\, N\, l_{\mathrm{avg}}}{K_w} \\ &= \frac{1 \cdot 50 \cdot 1472\,\mathrm{mm}}{1} \\ &= 73.6\,\mathrm{m}. \end{aligned} \tag{31}$$

The pressure drop over a cooling circuit is given by ($d = 2r_c = 6\,\mathrm{mm}$)

$$\Delta p = 60 \cdot l \cdot \frac{Q^{1.75}}{d^{4.75}} \tag{32}$$

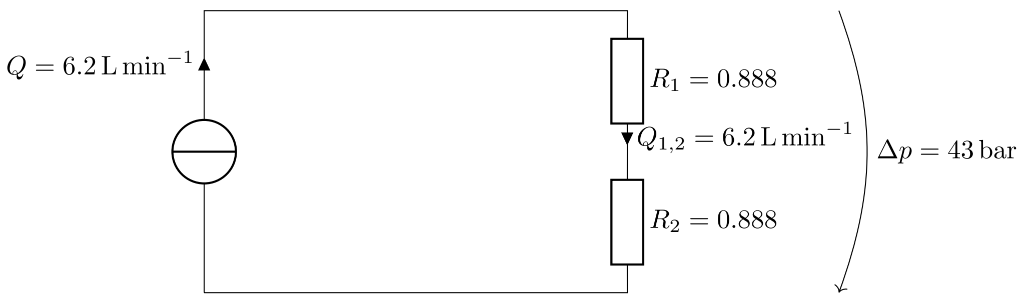
This can be interpreted a (non-linear) hydraulic equivalent to Ohm's law in the electrical domain:

$$\Delta p = \underbrace{\frac{60 \cdot l}{d^{4.75}}}_{R} \cdot Q^{1.75} = R \cdot Q^{1.75} \tag{33}$$

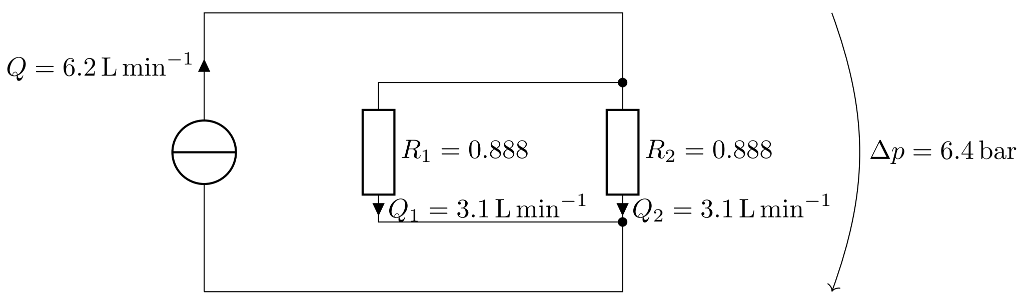
The hydraulic resistance of one coil is given (in AU) as

$$R = \frac{60 \cdot l}{d^{4.75}} = 0.888 \tag{34}$$

To cool one magnet, the cooling circuits can either be connected in series or in parallel (see Figure 3).



(a) Series connection



(b) Parallel connection

Figure 3: Possible cooling circuit configurations

For both cases, the pressure drops are

$$\Delta p_{\mathrm{Series}} = 2 \cdot 0.888 \cdot 6.2^{1.75} = 43.26\,\mathrm{bar} \tag{35}$$

$$\Delta p_{\mathrm{Parallel}} = 0.888 \cdot 3.1^{1.75} = 6.43\,\mathrm{bar} \tag{36}$$

As the series connection pressure drop exceeds the limit of the pump ($\Delta p_{\max} = 7\,\mathrm{bar}$), the parallel connection is used.

The average flow velocity in this case is

$$u_{\text{avg}} = 16.67 \cdot \frac{Q}{A} = 16.67 \cdot \frac{4 \cdot Q}{\pi d^2} = 16.67 \cdot \frac{4 \cdot 3.1\,\mathrm{L\,min^{-1}}}{\pi \cdot (6\,\mathrm{mm})^2} = 1.827\,\mathrm{m\,s^{-1}}. \tag{37}$$

Reynolds number (with $v = 6.58 \cdot 10^{-7}\,\mathrm{m^2\,s^{-1}}$) is computed as

$$R_e = d \cdot \frac{u_{\text{avg}}}{v} \cdot 1 \cdot 10^{-3} = 16\,660. \tag{38}$$

From $R_e > 4000$ turbulent flow can be assumed for the flow in the water channels, which is needed for equal heat distribution.

2 Numerical

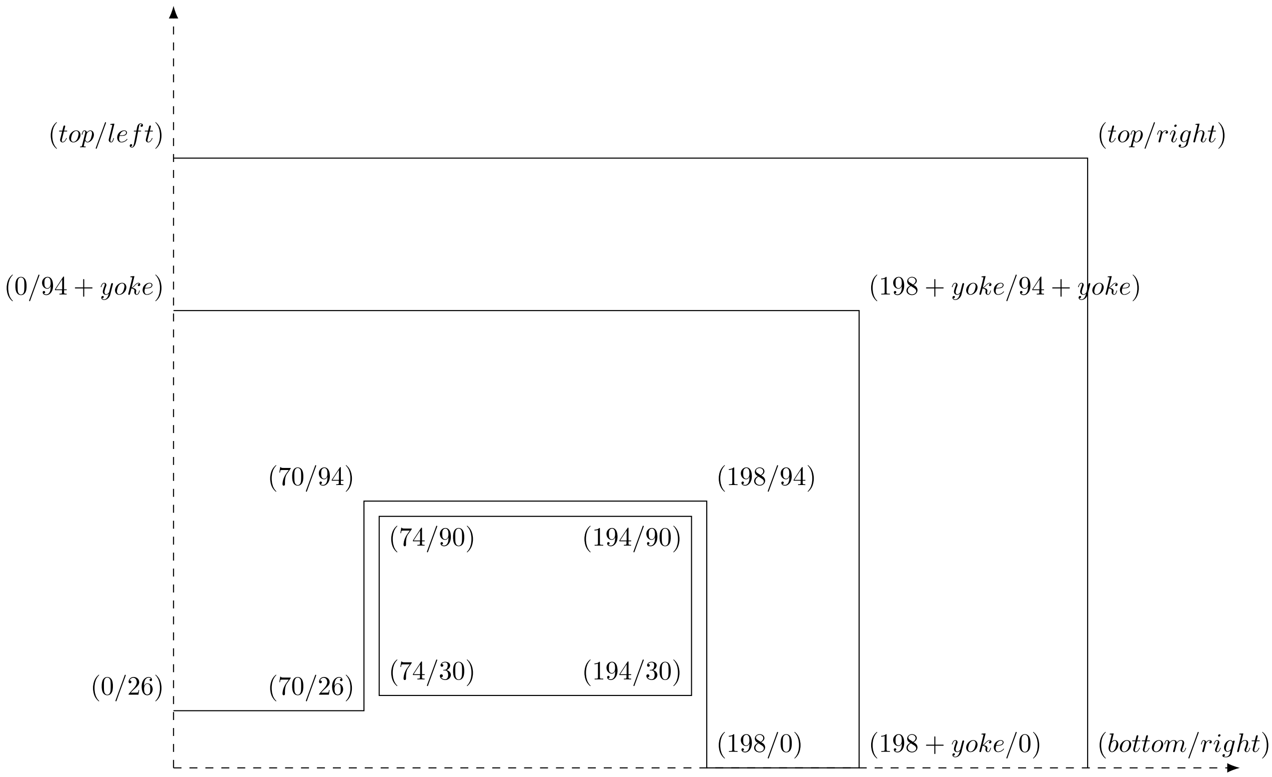


Figure 4: Input data for numerical simulation; all measurements in mm; drawing not to scale

Table 1: Relevant magnet parameters

Name	Value
Flux density B	999 T
Gap height h	999 mm
Pole width w	999 mm
Ampereturns NI	999 A