Magnet design for MedAustron

February 20, 2022

1 Analytical

1.1 Magnet type decision

The arguments for and against a H-type magnet are:

+ Mechanical rigid

+ Symmetrical

− Hard to get the beam pipe in and out

1.2 Aperture Height

The aperture height is given as the sum of the good field region $h_{\text{GFR}} = 2 \cdot \text{GFR}_y$, the thickness of the vacuum pipe d_{vacuum} and a tolerance for installation and thermal expansion $d_{\text{tolerance}}$ as

$$
\begin{aligned}
h &= h_{\text{GFR}} + 2 \cdot d_{\text{vacuum}} + d_{\text{tolerance}} \\
&= 2 \cdot 23\,\text{mm} + 2 \cdot 2\,\text{mm} + 2\,\text{mm} \\
&= 52\,\text{mm}.
\end{aligned}
\tag{1}
$$

1.3 Flux Density

The total bending angle generated by all $m = 3$ magnets is

$$
\theta_{\text{tot}} = 3 \cdot \theta_{\text{mag}} = 3 \cdot 36° = 108°.
\tag{2}
$$

The magnetic length of the (dipole) magnet can be approximated by

$$
\begin{aligned}
l_{\text{mag}} &= l_{\text{iron, max}} + 2hk \\
&= 0.340\,\text{m} + 2 \cdot 0.55 \cdot 52\,\text{mm} \\
&= 397.2\,\text{mm}.
\end{aligned}
\tag{3}
$$

With the definition of the radian $(\theta = {}^{s}\!/\!_{\rho})$ and $s = l_{\text{mag}}$, the bending radius ρ is

$$
\theta_{\text{mag}} = \frac{l_{\text{mag}}}{\rho}
\tag{4}
$$

$$
\begin{aligned}
\Rightarrow \rho &= \frac{l_{\text{mag}}}{\theta_{\text{mag}}} \\
&= \frac{397.2\,\text{mm}}{0.6283\,\text{rad}} \\
&= 0.642\,\text{m}.
\end{aligned}
\tag{5}
$$

With the given minimum and maximum $(B\rho)$ for a proton and a C^{6+} beam, the minimum and maximum needed flux densities are

$$
B_{\text{min}} = \frac{(B\rho)_{\text{min}}}{\rho} = \frac{0.383\,\text{T m}}{0.642\,\text{m}} = 0.596\,\text{T}
\tag{6}
$$

$$
B_{\text{max}} = \frac{(B\rho)_{\text{max}}}{\rho} = \frac{0.766\,\text{T m}}{0.642\,\text{m}} = 1.19\,\text{T}
\tag{7}
$$

1.4 Pole width and yoke thickness

With the given field quality inside the GFR $\Delta B / B_0 = 1 \cdot 10^{-3}$, the optimized normalized pole overhang is[1]

$$x_{\text{optimized}} = 2 \frac{a_{\text{optimized}}}{h} \tag{8}$$
$$= -0.14 \ln \left[\frac{\Delta B}{B_0} \right] - 0.25$$
$$= 0.7171.$$

This equals a pole overhang of

$$a_{\text{optimized}} = \frac{h \cdot x_{\text{optimized}}}{2} \tag{9}$$
$$= \frac{52 \, \text{mm} \cdot 0.7171}{2}$$
$$= 18.645 \, \text{mm}.$$

Taking the sagitta s into account and using

$$\text{GFRx'} = \text{GFRx} + \underbrace{\rho(1 - \cos \theta/2)}_{s} = 20 \, \text{mm} + 31.42 \, \text{mm} = 51.42 \, \text{mm}, \tag{10}$$

the pole width w computes to

$$w = 2 \cdot (\text{GFRx'} + a_{\text{optimized}}) \tag{11}$$
$$= 2 \cdot (51.42 \, \text{mm} + 18.645 \, \text{mm})$$
$$= 140.13 \, \text{mm}.$$

1.5 Excitation Current

The ampereturns are given as

$$(NI)_{\text{dipole, min}} = \frac{B_{\min} h}{2 \, \mu_0} = \frac{0.596 \, \text{T} \cdot 52 \, \text{mm}}{2 \cdot \mu_0} = 12.331 \, \text{kA} \tag{12}$$

$$(NI)_{\text{dipole, max}} = \frac{B_{\max} h}{2 \, \mu_0} = \frac{1.19 \, \text{T} \cdot 52 \, \text{mm}}{2 \cdot \mu_0} = 24.621 \, \text{kA} \tag{13}$$
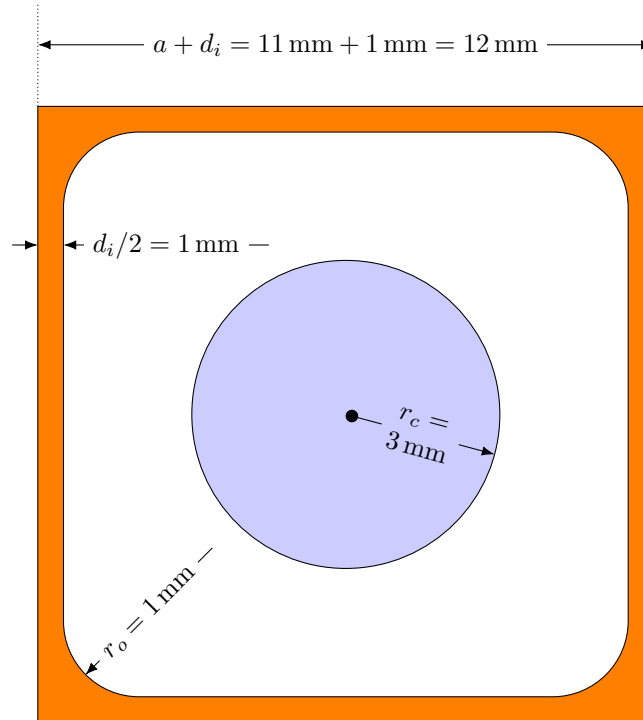

Figure 1: Cross section of the copper winding showing insulation(■) and water channel(■)

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[1]Compared to the unoptimized $x_{\text{unoptimized}} = 1.5898$

With the given dimensions of the copper winding (see Figure 1) and using

$$A_{\text{rounded edges}} = 4 \cdot A_{\text{rounded edge}} = 4\frac{r_o^2(4-\pi)}{4} = r_o^2(4-\pi) \tag{14}$$

$$A_{\text{water channel}} = \pi r_c^2 \tag{15}$$

the copper cross section area is

$$A_{\text{copper}} = a^2 - \underbrace{r_o^2(4-\pi)}_{A_{\text{rounded edges}}} - \underbrace{\pi r_c^2}_{A_{\text{water channel}}} \tag{16}$$

$$= 121\,\text{mm}^2 - 0.8584\,\text{mm}^2 - 28.274\,\text{mm}^2$$

$$= 91.867\,\text{mm}^2$$

With the given maximum current density of $j_{\text{max}} = 6\,\text{A}\,\text{mm}^{-2}$, the maximum possible current in the winding is

$$I_{\text{max}} = j_{\text{max}} \cdot A_{\text{copper}} \tag{17}$$

$$= 6\,\text{A}\,\text{mm}^{-2} \cdot 91.867\,\text{mm}^2$$

$$= 551.202\,\text{A}.$$

With the calculated ampereturns, the necessary minimum number of turns are then

$$N_{B_{\text{min}}} = \left\lceil \frac{(NI)_{\text{min}}}{I_{\text{max}}} \right\rceil = 23 \tag{18}$$

$$N_{B_{\text{max}}} = \left\lceil \frac{(NI)_{\text{max}}}{I_{\text{max}}} \right\rceil = 45 \tag{19}$$

As it is easier to lower the current then to change the number of turns, the minimum number of turns should be $N_{B_{\text{max}}}$.
To full fill the empirical good practice $N_{\text{horizontal}}/N_{\text{vertical}} = 2$, the number of turn is set to

$$N = N_{\text{horizontal}} \times N_{\text{vertical}} = 10 \times 5 = 50. \tag{20}$$

With the new $N = 50$, the winding current is

$$I = \frac{(NI)_{\text{max}}}{50} = 492.42\,\text{A}, \tag{21}$$

which is well below the maximum current of the power supply ($I_{\text{max}} = 600\,\text{A}$).

1.6 Coil Parameters

The coil width and height are given by (see Figure 2)

$$w_{\text{coil}} = N_{\text{horizontal}} \cdot (a + d_{\text{insulation}}) + 2 \cdot (d_{\text{epoxy}} + d_{\text{air}}) = 128\,\text{mm} \tag{22}$$

and

$$h_{\text{coil}} = N_{\text{vertical}} \cdot (a + d_{\text{insulation}}) + 2 \cdot (d_{\text{epoxy}} + d_{\text{air}}) = 68\,\text{mm} \tag{23}$$

using $N_{\text{horizontal}} = 10$, $N_{\text{vertical}} = 5$, $a = 11\,\text{mm}$, $d_{\text{insulation}} = 1\,\text{mm}$, $d_{\text{epoxy}} = 2\,\text{mm}$ and $d_{\text{air}} = 2\,\text{mm}$.
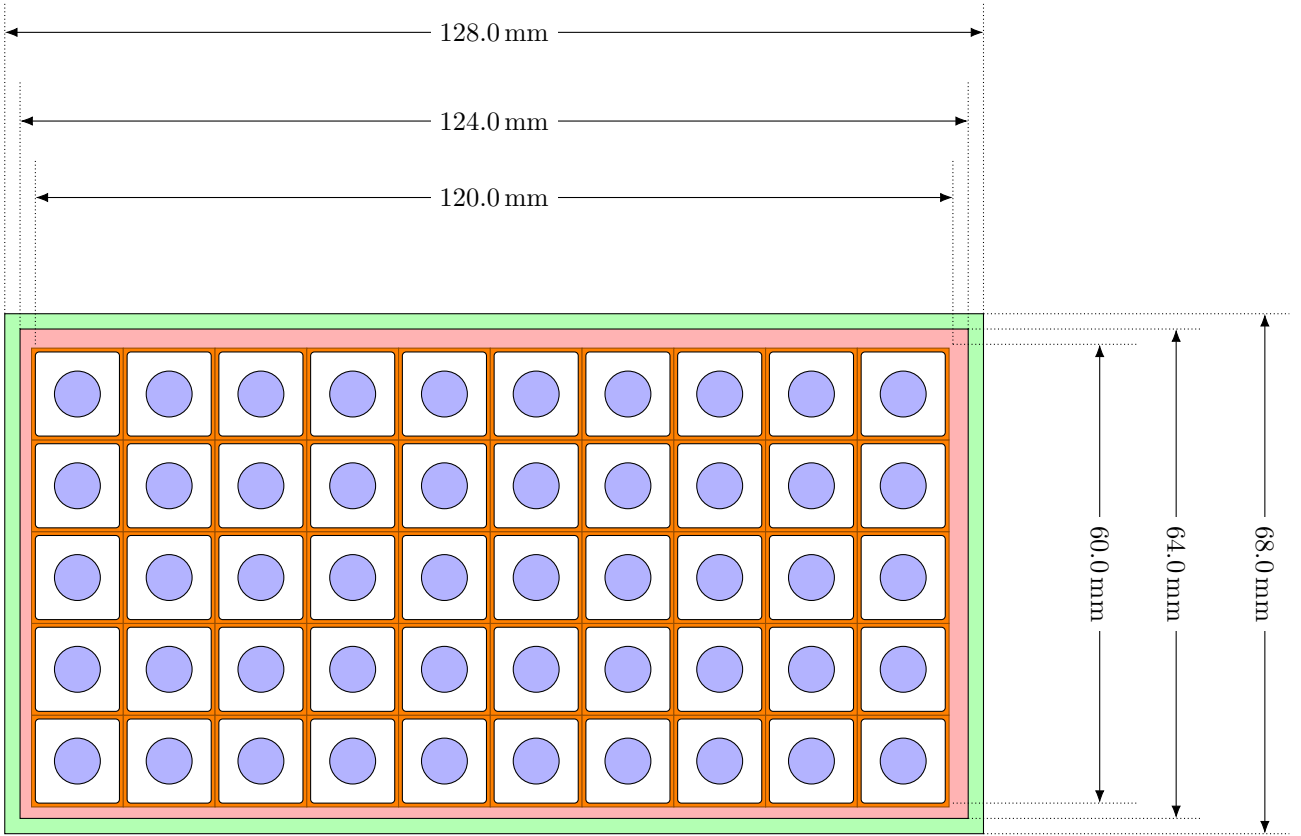
Figure 2: Cross section of one coil showing copper(□), insulation(■), the water channels(■), the coil epoxy coating(■) and the air gap around the coil(■)

With the pole perimeter

$$p = 2 \cdot l\text{iron} + 2 \cdot w = 960 \, \text{mm}, \tag{24}$$

the average turn length is ($l\text{iron} = 340 \, \text{mm}$, $w = 140.13 \, \text{mm}$ and $w_{\text{coil}} = 128 \, \text{mm}$)

$$l_{\text{avg}} = p + 4 \cdot w_{\text{coil}} = 1472 \, \text{mm}. \tag{25}$$

The resistance of one coil winding is (with $1/\sigma_{\text{Cu}} = 1.72 \, \mu\Omega \, \text{cm}$)

$$\begin{aligned} R_c &= \frac{N \cdot l_{\text{avg}}}{A \cdot \sigma_{\text{Cu}}} \\ &= \frac{50 \cdot 1472 \, \text{mm}}{91.867 \, \text{mm}^2 \cdot 1/1.72 \, \mu\Omega \, \text{cm}} \\ &= 13.442 \, \text{m}\Omega \end{aligned} \tag{26}$$

With the number of coils per magnet m, the DC steady-state voltage per magnet is

$$\begin{aligned} V_m &= I \cdot m \, R_c \\ &= 492.42 \, \text{A} \cdot 2 \cdot 0.016 \, \Omega \\ &= 13.197 \, \text{V}. \end{aligned} \tag{27}$$

The total voltage over all magnets is (number of magnets M)

$$\begin{aligned} V_{\text{total}} &= M \cdot V_m \\ &= 3 \cdot 16.197 \, \text{V} \\ &= 39.591 \, \text{V}. \end{aligned} \tag{28}$$

This is well below the maximum output voltage of the power supply ($V_{\text{total, max}} = 80 \, \text{V}$).
The dissipated power in one magnet is calculated to

$$\begin{aligned} P_m &= V_m \cdot I = I^2 \cdot m \, R_c \\ &= 13.197 \, \text{V} \cdot 492.42 \, \text{A} \\ &= 6.498 \, \text{kW}. \end{aligned} \tag{29}$$

1.7 Cooling

With the given $\Delta T = 15\,\mathrm{K}$ and the dissipated power $P_m = 6.498\,\mathrm{kW}$, the total required water flow in one magnet is

$$Q = 14.3 \frac{P_m}{\Delta T} \cdot 1 \cdot 10^{-3} = 6.2\,\mathrm{L\,min}^{-1} \tag{30}$$

Using the length of cooling circuit for one coil is ($K_c = 1$ and $K_w = 1$)

$$
\begin{aligned}
l &= \frac{K_c\, N\, l_{\mathrm{avg}}}{K_w} \\
&= \frac{1 \cdot 50 \cdot 1472\,\mathrm{mm}}{1} \\
&= 73.6\,\mathrm{m}.
\end{aligned}
\tag{31}
$$

The pressure drop over a cooling circuit is given by ($d = 2r_c = 6\,\mathrm{mm}$)

$$\Delta p = 60 \cdot l \cdot \frac{Q^{1.75}}{d^{4.75}} \tag{32}$$

This can be interpreted a (non-linear) hydraulic equivalent to Ohm's law in the electrical domain:

$$\Delta p = \underbrace{\frac{60 \cdot l}{d^{4.75}}}_{R} \cdot Q^{1.75} = R \cdot Q^{1.75} \tag{33}$$

The hydraulic resistance of one coil is given (in AU) as

$$R = \frac{60 \cdot l}{d^{4.75}} = 0.888 \tag{34}$$

To cool one magnet, the cooling circuits can either be connected in series or in parallel (see Figure 3).
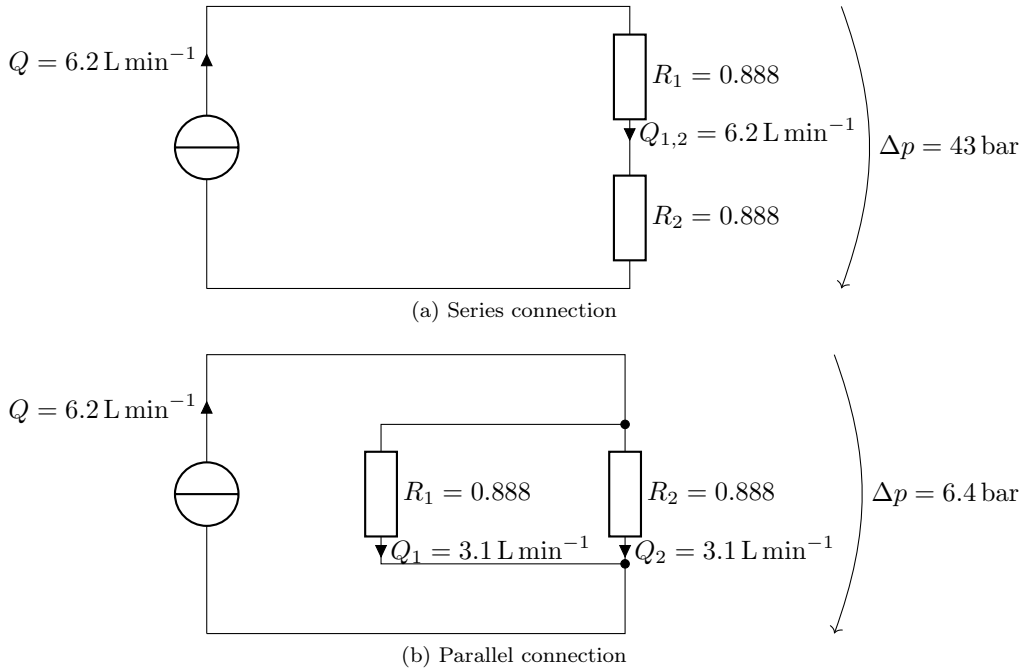

(a) Series connection

(b) Parallel connection

Figure 3: Possible cooling circuit configurations

For both cases, the pressure drops are

$$\Delta p_{\mathrm{Series}} = 2 \cdot 0.888 \cdot 6.2^{1.75} = 43.26\,\mathrm{bar} \tag{35}$$
$$\Delta p_{\mathrm{Parallel}} = 0.888 \cdot 3.1^{1.75} = 6.43\,\mathrm{bar} \tag{36}$$

As the series connection pressure drop exceeds the limit of the pump ($\Delta p_{\mathrm{max}} = 7\,\mathrm{bar}$), the parallel connection is used.

The average flow velocity in this case is

$$u_{\mathrm{avg}} = 16.67 \cdot \frac{Q}{A} = 16.67 \cdot \frac{4 \cdot Q}{\pi d^2} = 16.67 \cdot \frac{4 \cdot 3.1\,\mathrm{L\,min}^{-1}}{\pi \cdot (6\,\mathrm{mm})^2} = 1.827\,\mathrm{m\,s}^{-1}. \tag{37}$$

Reynolds number (with $v = 6.58 \cdot 10^{-7}\,\mathrm{m^2\,s^{-1}}$) is computed as

$$R_e = d \cdot \frac{u_{\mathrm{avg}}}{v} \cdot 1 \cdot 10^{-3} = 16\,660. \tag{38}$$

From $R_e > 4000$ turbulent flow can be assumed for the flow in the water channels, which is needed for equal heat distribution.
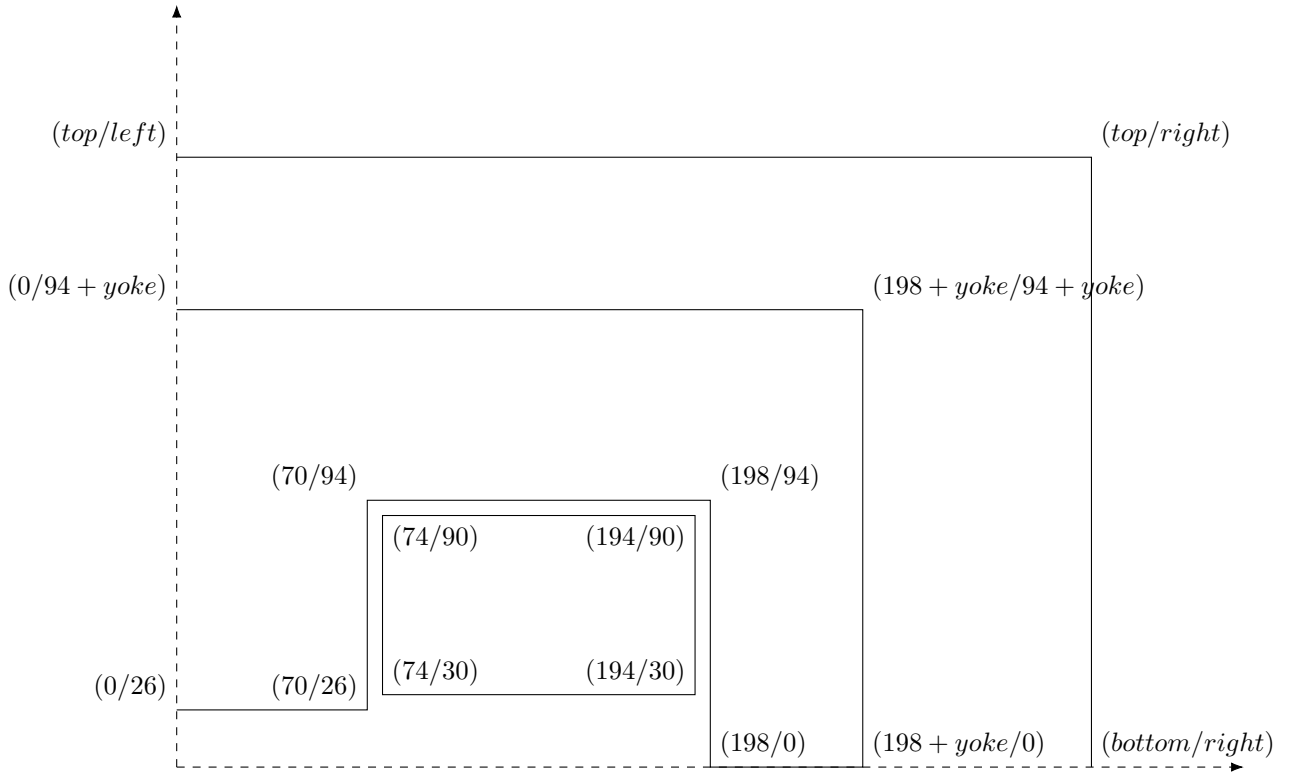
2 Numerical

Figure 4: Input data for numerical simulation; all measurements in mm; drawing not to scale

Table 1: Relevant magnet parameters

Name	Value
Flux density B	999 T
Gap height h	999 mm
Pole width w	999 mm
Ampereturns NI	999 A