Part B: Probability

Chapter 1: Combinatorial Analysis:

To calculate probabilities in multi-step random experiments, we first need to do a combinatorial analysis. This analysis depends on whether the order of the elements, replacement, and the size of the sets matter.

B.1.1 Arrangements ($\mathbf{A_n^p}$) :

The arrangements of a set of elements correspond to the ordered arrangements of certain elements of that set.

The arrangements of a set **are distinguished** by the order of the elements that compose them. For example, (A,C) and (C,A) are 2 different arrangements of the set {A,B,C}. If this experience has the following characteristics:

- The experience takes order into account.
- The experiment is done **with** or **without replacement** (sans remise).
- The experience involves **some** of the elements in the starting set.

In this case, the arrangements are calculated as follows:

With replacement: $Number\ of\ arrangements = A_n^p = n^p$

Without replacement: $Number\ of\ arrangements = A_n^p = \frac{n!}{(n-p)!}$

n: The number of elements in the starting set.

p: The number of elements selected in the starting set.

Example 1:

We choose randomly, without replacement and taking the order into account, 2 balls from a bag containing all a red ball (R), a blue ball (B), a yellow ball (Y) and a green ball (G). How many possible outcomes are there?

Solution:

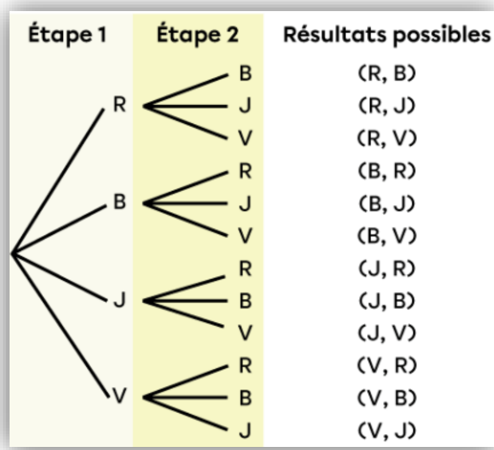
However, we notice that this random experiment has the 3 characteristics below.

- The experience <u>takes order into account</u>, <u>without replacement</u> and <u>involves some of the elements</u> in the starting set.

It is therefore more efficient to calculate the number of possible outcomes using the formula for the number of **arrangements** of the starting set {R,B,Y,G}. Since this set has 4 elements (n=4), and 2 of them are chosen (p=2). The following calculation is made:

$$Number\ of\ arrangements = \frac{n!}{(n-p)!} = \frac{4!}{(4-2)!} = \frac{4*3*2*1}{2*1} = 12$$

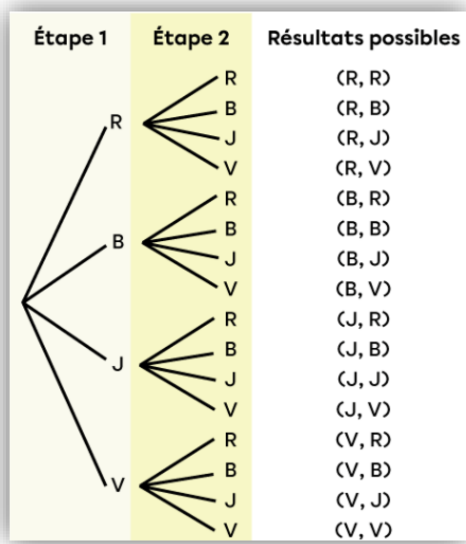
So, there are 12 possible outcomes in this random experiment. We can represent the possibilities using **a tree diagram**.



❖ The same example **with replacement**:

$$Number\ of\ arrangements = n^p = 4^2 = 16$$

So, there are 16 possible outcomes in this random experiment. We can represent the possibilities using **a tree diagram**.



B.1.2 Combinations C_n^p:

Combinations are unordered selections of elements from a set.

The combinations of a set **are not distinguished** by the order of the elements that compose them. For example, (A,C) and (C,A) are 2 equivalent combinations of the set {A,B,C}. If this experience has the following characteristics:

- The experience **does not take the order into account.**
- The experiment is done **with** or **without replacement**.
- The experience involves **some** of the elements in the starting set.

In this case, the permutations are calculated as follows:

❖ **With replacement**:

$$Number\ of\ combinations = \boldsymbol{C_n^p} = \frac{(n+p-1)!}{p!\,(n-p)!}$$

❖ **Without replacement**:

$$Number\ of\ combinations = \boldsymbol{C_n^p} = \frac{n!}{p!\,(n-p)!}$$

Example 2:

We choose randomly, without replacement and don't taking the order into account, 2 balls from a bag containing all a red ball (R), a blue ball (B), a yellow ball (Y) and a green ball (G). How many possible outcomes are there?

Solution:

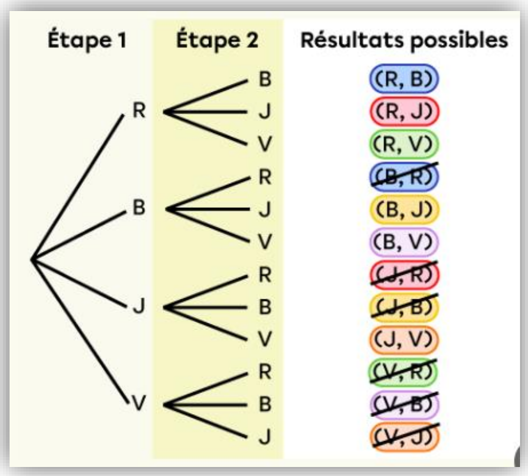
However, we notice that this random experiment has the 3 characteristics below.

- The experience does not take order into account, without replacement and involves some of the elements in the starting set.

It is therefore more efficient to calculate the number of possible outcomes using the formula for the number of **combinations** of the starting set {R,B,Y,G}. Since this set has 4 elements (n=4), and 2 of them are chosen (p=2). The following calculation is made:

$$\boldsymbol{C_n^p} = \frac{n!}{p!\,(n-p)!} = \frac{4!}{2!\,(4-2)!} = \frac{4*3*2*1}{2*1(2*1)} = 6$$

So, there are 6 possible outcomes in this random experiment. We can represent the possibilities using **a tree diagram**.



❖ The same example **with replacement**:

$$Number\ of\ arrangements = n^p = 4^2 = 16$$

So, there are 16 possible outcomes in this random experiment.

B.1.3 Permutations (P_n):

Permutations of a set of elements correspond to the ordered arrangements of all the elements in that set.

The permutations (Pn) of a set **are distinguished** by the order of the elements that compose them. For example, (C,A,B) and (B,A,C) are 2 different permutations of {A,B,C}.

If this experience has the following characteristics:

- The experience **takes order into account.**
- The experiment is done **without replacement**.
- **All** elements of the original set are used in the experiment.

In this case, the permutations are calculated as follows:

$$Number\ of\ permutations = P_n = n * (n-1) * \ldots * 2 * 1 = n!$$

n: the number of elements in the starting set.

Example 1:

We choose randomly, without replacement and taking the order into account, all the balls in a bag containing a red ball (R), a blue ball (B), a yellow ball (Y) and a green ball (G). How many possible outcomes are there?

Solution :

However, we notice that this random experiment has the 3 characteristics below.

- The experience <u>takes order into account</u>, <u>without replacement</u> and <u>all elements</u> of the original set are used in the experiment.

It is therefore more efficient to calculate the number of possible outcomes using the formula for the number of **permutations** of the starting set {R,B,Y,G}. Since this set has 4 elements (n=4), we perform the following calculation:

$$P_n = \text{n}! = 4! = 4 * 3 * 2 * 1 = 24$$

So, there are 24 possible outcomes in this random experiment. We can represent the possibilities using **a tree diagram**.