HDU-ITMO Joint Institute

Electrical Engineering

Laboratory work 4

Research on DC transients

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Laboratory work № 4: «Research on DC transients»

Objectives

to research transients in first and second order electric circuits with DC voltage source.

Program of work

Part 1

Research on transients in the first order electric circuits with DC voltage source.

1. Transients in RC two-terminal load.
2. Transients in LR two-terminal load.

Part 2

Research on transients in the second order electric circuits with DC voltage source.

1. Aperiodic transients in RLC two-terminal load.
2. Oscillation transients in RLC two-terminal load.

Overview

This lab is dedicated to understanding the principles of the continuity laws for transient analysis of DC circuits.

Students are developing the skill of simulation circuits in the time domain.

After performing the lab task students can use the differential equation for the analysis of many real-world applications.

GUIDANCE

Part one

1. Assemble an equivalent circuit presented in Figure 4.1,a in the «LTspice» application. The square wave source E and load parameters **are defined by the instructor**.

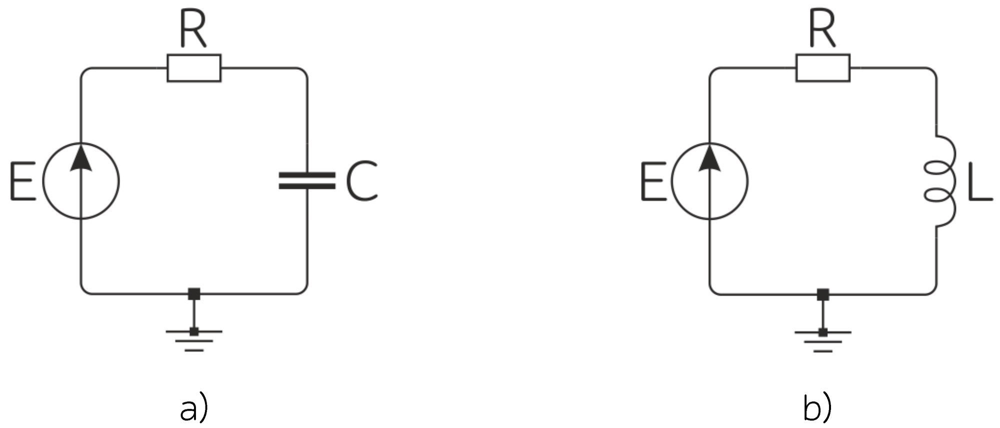


Figure 4.1 – Equivalent circuit of square wave generator with a) RC load, b) RL load.

2. Measure current value in the circuit and voltage value on the capacitor at the moment of switching (t = 0+) and in steady state (t = ∞). Transient plots will have the form of the functions presented in Figure 4.2.

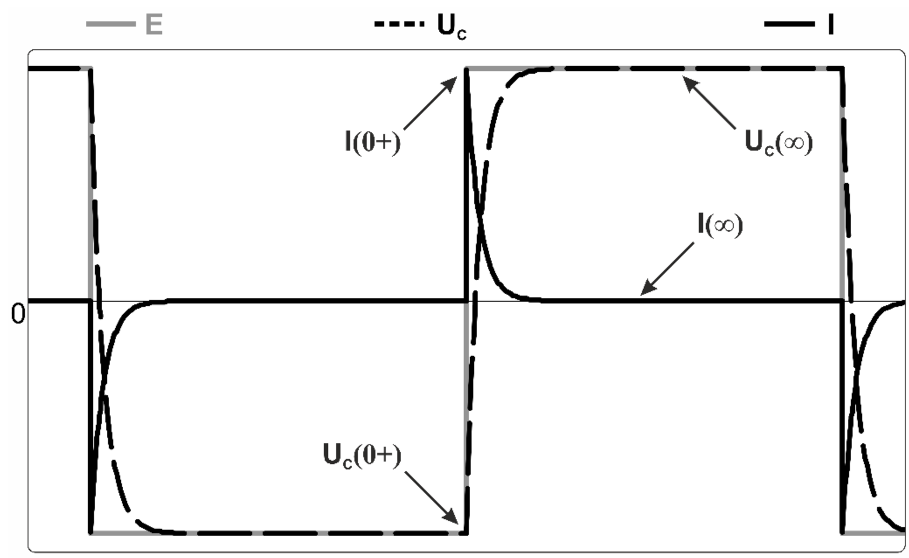


Figure 4.2 – Transient plots in circuit with RC load

3. Find time constant τ using experimental results. The time constant determines the duration of the transient. The duration of the transient in

practical cases considered as the time which the exponential function requires to reach a value that differs from the steady state value by no more than 5%. This condition corresponds to the value of 3τ. That means that defining the duration of the transient t_p it is possible to define a time constant as $t_p/3$.

A more accurate definition of the time constant can be done as follows. The first order transient processes in circuit can be described as $a(t) = A \cdot e^{-t/\tau}$ and $a(t) = A \cdot (1 - e^{-t/\tau})$, their relative levels equal

$$d = a(t_d)/A = e^{-t_d/\tau} \text{ и } d = 1 - a(t_d)/A = e^{-t_d/\tau}, \tag{4.1}$$

where $0 \le d \le 1$ is relative signal level at time t_d. Thus, it is possible to determine the value τ, for example, at time $t_{0.5}$, when the current or voltage on any element of the circuit reaches half of its steady state magnitude value

$$0.5 = e^{-\frac{t_{0.5}}{\tau}} \Rightarrow \ln(0.5) = -\frac{t_{0.5}}{\tau} \Rightarrow \boxed{\tau = \frac{t_{0.5}}{\ln 2} \approx 1.44 \cdot t_{0.5}}. \tag{4.2}$$

4. Calculate the current in the circuit and voltage on the capacitor at the moment of switching (t = 0+) and in steady state (t = ∞) according to the formulas presented in the Table 4.1. Time constant should be calculated as τ = RC. Add measurement and calculation results in Table 4.2.
5. Assemble an equivalent circuit presented in Figure 4.1,b in the «LTspice» application. The square wave source E and load parameters **are defined by the instructor**.
6. Measure current value in the circuit and voltage on the inductor at the moment of switching (t = 0+) and in steady state (t = ∞). Transient plots will have the form of the functions presented in Figure 4.3.
7. Find time constant τ from experimental results.
8. Calculate the current in the circuit and voltage on the inductor at the moment of switching (t = 0+) and in steady state (t = ∞) according to the formulas presented in the Table 4.1. Time constant should be calculated as

$\tau = L/(R + R_k)$, where R_k is active resistance of the inductance coil. Add measurement and calculation results in Table 4.3.

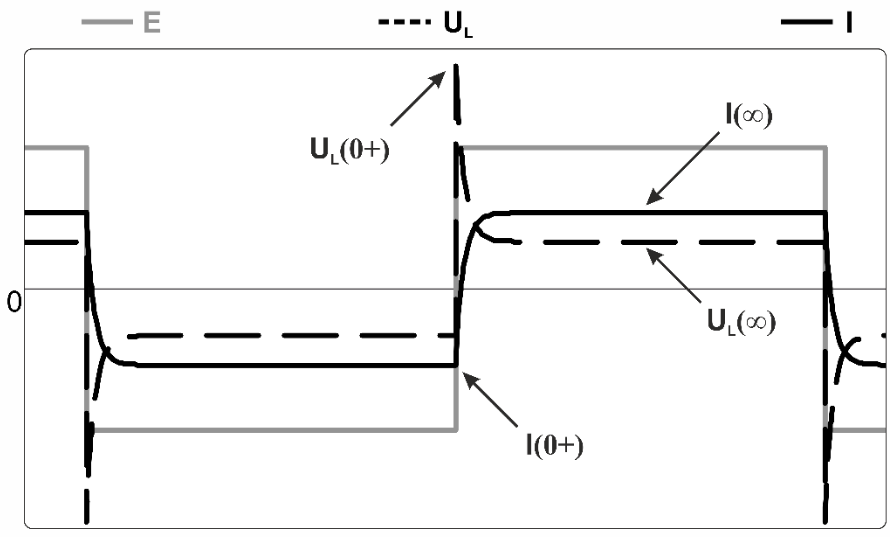


Figure 4.3 - Transient plots in circuit with RC load.

Table 4.1

Load Type	Initial values	Steady state values
RC	$U_C(0+) = U_C(0-) = E(0-)$, $I(0+) = (E(0+) - E(0-))/R$;	$U_C(\infty) = E(0+)$, $I(\infty) = I(0-) = 0$;
RL	$I(0+) = I(0-) = E(0-)/(R + R_k)$, $U_L(0+) = E(0+) - I(0-) \cdot R$;	$I(\infty) = E(0+)/(R + R_k)$, $U_L(\infty) = I(\infty) \cdot R_k$.

U_L – voltage across an inductance coil.

Table 4.2

R, [Ω]	C, [μF]	Data Type	I(0+), [mA]	I(∞), [mA]	$U_C(0+)$, [V]	$U_C(\infty)$, [V]	τ, [ms]
		exp.					
		calc.					

Table 4.3

R, [Ω]	L, [mH]	R_k, [Ω]	Data Type	I(0+), [mA]	I(∞), [mA]	$U_L(0+)$, [V]	$U_L(\infty)$, [V]	τ, [ms]
			exp.					
			calc.					

Part two

In the second-order circuits, transients can be damped (non-periodic) or underdamped depending on the parameters of the elements. In other words, transient functions can have one or more extremes. Mathematically, the nature of the transition process in the circuit represented in Figure 4.4 is determined by the roots of the characteristic equation $LC \cdot s^2 + RC \cdot s + 1 = 0$ where «s» is Laplace operator. Thus, the nature of the transition process can be changed by variation of one of the parameters R, L or C, but the easiest way to do this is to change the resistance R.

The damped transient will take place if all the roots of the characteristic equation are real, and the process will be described by a function representing the sum of two exponents with constant time $\tau_1 = |1/s_1|$ и $\tau_2 = |1/s_2|$, where $s_{1,2}$ – are roots of the characteristic equation. In case of $\tau_1 \gg \tau_2$ it is possible to estimate transient duration quite accurately as

$$t_p = 3\tau_1 = \frac{3}{\delta - \sqrt{\delta^2 - \omega_0^2}}, \tag{4.3}$$

where $\delta = R/2L$ is damping ratio and $\omega_0 = \sqrt{1/LC}$ resonance frequency.

The oscillation transient in the second and above orders circuits appears because of the presence of complex-conjugate roots of the characteristic equation. At the same time in the circuit of the second order damping ratio δ and the frequency of self or damped oscillations ω can be determined as

$$\delta = \frac{R}{2L} \text{ и } \omega = \sqrt{\frac{1}{LC} - \delta^2}. \tag{4.4}$$

Experimentally these parameters can be determined by two neighboring extremes of current fluctuations i_{m1}, i_{m2} and the oscillation period (Figure 4.4):

$$\delta^* = \frac{\ln\left(\frac{I_{m1}}{I_{m2}}\right)}{T} \text{ и } \omega^* = \frac{2\pi}{T}. \tag{4.5}$$

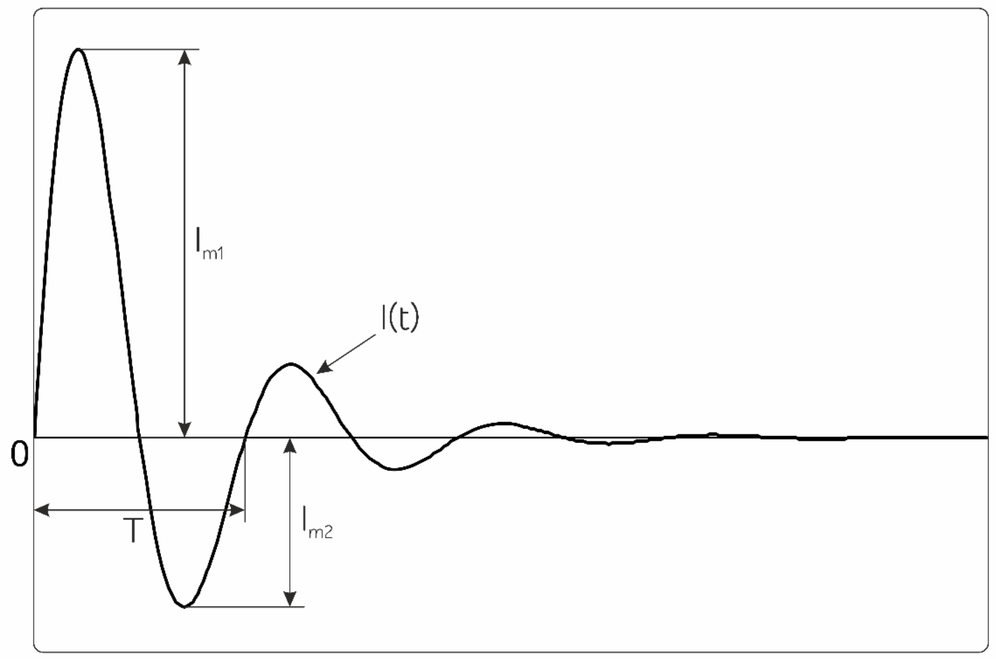


Figure 4.4 - Current transient in circuit with RLC load;
Experimental definition of parameters.

1. Assemble an equivalent circuit presented in Figure 4.5,a in the «LTspice» application. The square wave source E and load parameters **are defined by the instructor**.
2. Calculate resistance which correspond the condition $R = 4 \cdot \rho$ (where ρ is characteristic resistance) and simulate the circuit.

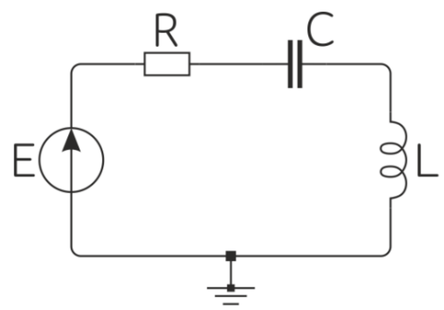


Figure 4.5 - The equivalent circuit with RLC load.

3. Measure capacitor voltage $U_C(0+)$, inductor voltage $U_L(0+)$ and current $I(0+)$ at the moment of switching. Determine transient time t_p from the capacitor voltage plot.
4. Calculate the voltages on the capacitor and the inductance coil and the current in the circuit at the moment of switching by the formulas presented in the Table 4.6. Calculate the transient time by formula (4.3). Add measurement and calculation results in Table 4.4.
5. Calculate resistance which correspond the condition $R = \rho/2$ and simulate the circuit.
6. According to plots of voltages and current transients and using formulas (4.5) determine damping ratio and self-oscillation frequency. Calculate this parameters values using formulas (4.4) and add the results in Table 4.5.

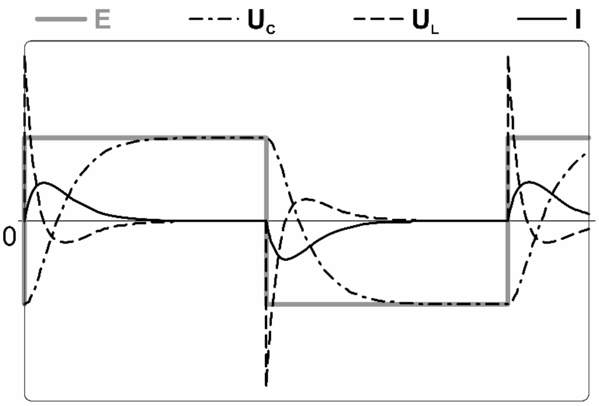
Table 4.4

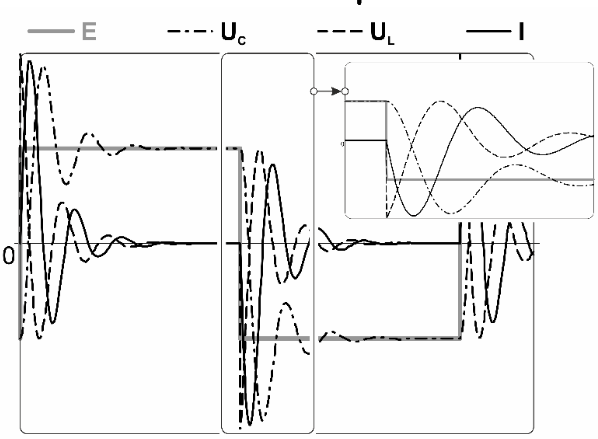
Elements parameters			$U_C(0+)$		$U_L(0+)$		$I(0+)$		t_p	
R [Ω]	L [mH]	C [μF]	calc [V]	exp [V]	calc [V]	exp [V]	calc [mA]	exp [mA]	calc [ms]	exp [ms]

Table 4.5

Elements parameters			δ		ω	
R [Ω]	L [mH]	C [μF]	calc [s⁻¹]	exp [s⁻¹]	calc [s⁻¹]	exp [s⁻¹]

Table 4.6

Transients types	Formulas
damped 	$s_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2}$ - real roots, $i(t) = \dfrac{E_\Sigma}{L(s_1 - s_2)}(e^{s_1 t} - e^{s_2 t})$, $E_\Sigma = \left(\lvert E(0-)\rvert + \lvert E(0+)\rvert\right)\cdot \mathrm{sign}\left(E(0+)\right)$, $U_R(t) = R \cdot i(t)$, $U_L(t) = \dfrac{E_\Sigma}{s_1 - s_2}(s_1 e^{s_1 t} - s_2 e^{s_2 t})$,

	$U_C(t) = E(0+) - \dfrac{E_\Sigma}{s_1 - s_2}\left(s_1 e^{s_2 t} - s_2 e^{s_1 t}\right)$
underdamped  	$s_{1,2} = -\delta \pm j\omega$ - complex-conjugate roots, $\omega = \sqrt{\dfrac{1}{LC} - \delta^2}$, $i(t) = \dfrac{E_\Sigma}{\omega L} e^{-\delta t} \sin\omega t$, $U_R(t) = R \cdot i(t)$, $U_L(t) = E_\Sigma \varpi e^{-\delta t} \cos(\omega t + \beta)$, $U_C(t) = E(0+) - E_\Sigma \varpi e^{-\delta t} \cos(\omega t - \beta)$ $\varpi = \omega_0/\omega$, $\beta = \mathrm{arctg}(\delta/\omega)$

CONTENTS OF THE REPORT

Part 1

1. Experimental plant scheme.
2. Equivalent scheme of investigated electric circuit.
3. Evaluation equations and calculation results. Diagrams of transient responses (as in Figure 4.2 and 4.3).
4. Filled Tables 4.2 and 4.3.

Part 2

1. Scheme of investigated electric circuit.
2. Evaluation equations and calculation results. Diagrams of transient responses (as in Figure 4.6).
3. Filled Tables 4.4 and 4.5.