Task 1

FD1 $\{A\} \rightarrow \{B, C\}$

FD2 $\{C\} \rightarrow \{A, D\}$

FD3 $\{D, E\} \rightarrow \{F\}$

a) $\{C\} \rightarrow \{B\}$

we can derive $\{C\} \rightarrow \{B\}$ using transitivity (if $X \rightarrow Y$ and $Y \rightarrow Z$ then $X \rightarrow Y$)

FD2 says that $\{C\} \rightarrow \{A\}$, and from

FD1 gives us $\{A\} \rightarrow \{B\}$,

directly through trasitivity we derive $\{C\} \rightarrow \{B\}$

b) $\{A, E\} \rightarrow \{F\}$

From FD1 and FD2 through trasistivity

$A \rightarrow C \rightarrow D$ we get $A \rightarrow D$

using augmentation (if $X \rightarrow Y$, then $XZ \rightarrow YZ$ for any Z)

we get $\{A, E\} \rightarrow \{D, E\}$

using transistivity on the new FD1 and FD3

$A, E \rightarrow D, E \rightarrow F$ we get $\{A, E\} \rightarrow \{F\}$

Task 2

$FD1 \ \{A\} \to \{B,C\}$
$FD2 \ \{C\} \to \{A,D\}$
$FD3 \ \{D,E\} \to \{F\}$

a) $X = \{A\}$

Apply FD1 we get $X+ = \{A,B,C\}$

we have C in X^+, we can apply FD2

$\{A\}$ Already in $X+$, we add $\{D\}$ so $X+ = \{A,B,C,D\}$

we don't have $\{E\}$ in $X+$

the attribute closure is $X+ = \{A,B,C,D\}$

b) $X = \{C,E\}$

we have $\{C\}$, so we can apply FD2

we get $X+ = \{C,E,A,D\}$

we have $\{D,E\}$, we apply FD3

$X+ = \{C,E,A,D,F\}$

lastley apply FD1

So the attribute closure is $X+ = \{C,E,A,D,F,B\}$

Task 3

FD1 {A,B} → {C,D,E,F}
FD2 {E} → {F}
FD3 {D} → {B}

a) determine candidate key:

{A,B} attribute closure is

$X^+ = \{A, B, C, D, E, F, B\}$

so it functionally determines all attributes in R.

we need to check it for minimality

neither subsets {A} or {B} can determine R alone

. so {A,B} is minimal.

The other attributes {E} and {D} can't determine R alone.

So {A,B} is the candidate key.

b) we check that for $X \to Y$, X is a super key for R.

{A,B} is a super key, {E} and {D} are not superkeys, they don't determine all other attributes in R.

Both FD2 and FD3 violate BCNF.

c) FD2 and FD3 violates BCNF as mentioned in question b.

Decomposition of relation that violate BCNF

start with FD2, deompose into R_1, R_2

$R_1(E,F)$, $R_2(A,B,C,D,E)$,

decompose FD3 into R_3, R_4

$R_3(D,B)$, $R_4(A,C,D,E)$

check for candidate key in all new relations

$R_1(E,F)$ with FD2, candidate key: E

$R_3(D,B)$ with FD3, candidate key: D

R4, FD1 is superkey, but we need D to link it to B, so we choose candidate (A,D)

Task 4

FD1	$\{A,B,C\} \rightarrow \{D,E\}$
FD2	$\{B,C,D\} \rightarrow \{A,E\}$
FD3	$\{C\} \rightarrow \{D\}$

a) we need to check every FD, $X \rightarrow Y$ where X should be a superkey.

FD1: X is a superkey, it doesn't violate BCNF

FD2: X is also a superkey, doesn't violate BCNF

FD3: $\{C\}$ only determinen D, it's not a super key, violates BCNF

R is not in BCNF, because FD3 violates its condition

b) - we determind that FD3 violates BCNF

- Decompose R into R_1 and R_2, preserving FD3:
  - $R_1(C,D)$ from FD_3, C is key
  - $R_2(A,B,C,E)$ for the remaining attributes
- R_2 related new FDs: FD_1 $\{A,B,C\} \rightarrow \{E\}$
  - FD_2 $\{B,C\} \rightarrow \{A,E\}$ (D determind by C)
- check if X in FD_1 and FD_2 are superkeys:
  - FD_1 is a super key
  - FD_2 is not a superkey (missing D)
- decompose R_2 based on FD2:
  - $R_3(B,C,A,E)$ from FD_2
  - $R_4(B,C,E)$ for rest of attributes
- check candidate key and BCNF:
  - $R_1(C,D)$, $\{C\}$ is candidate key, $\{C\} \rightarrow \{D\}$
  - $R_3(B,C,A,E)$, $\{B,C\}$ is candidate key, $\{B,C\} \rightarrow \{A,E\}$
  - $R_4(B,C,E)$, $\{B,C\}$ is candidate key, it determines $\{A,E\}$ in R_3, $\{A,E\}$ determines all other attributes in R_2