Project Title: Adaptive Energy Compaction & Spatial Entropy Mapping

DSP applications project - 2 students per group - deadline Feb 13th - there will be a face to face discussion - use AI tools to write the code, but understand each line of it and focus on the DSP concepts

1. Objectives

- **Demonstrate Energy Compaction:** Quantify how the 2D-DCT concentrates signal energy into a minority of coefficients.
- **Visualize Information Density:** Map the spatial "cost" of image features using coefficient heatmaps.
- **Analyze Rate-Distortion:** Characterize the mathematical tradeoff between Compression Ratio (CR) and Peak Signal-to-Noise Ratio (PSNR).

2. The Implementation Pipeline

Step A: Image Partitioning

Download the images from here

https://drive.google.com/file/d/1qePHSR6BxIHYiZUQvlx63mMC3n6CfMAW/view?usp=sharing

You are provided with **4 distinct images**. For each image, you must partition it into non-overlapping blocks of size $n \times n$.

- **System Parameter:** You must repeat the experiment for $n \in \{8,16,32,64\}$.
- **Preprocessing:** If an image's dimensions are not divisible by n, you may either crop the image or pad it with zeros to ensure full blocks.

Step B: Frequency Domain Transformation

For every $n \times n$ block in the image:

1. Apply the **2D-Discrete Cosine Transform (2D-DCT)**.
2. Convert the 2D matrix of coefficients into a 1D vector using a **zigzag scan**. This ensures the vector is ordered from the lowest frequency (DC) to the highest frequencies (AC).

Step C: Adaptive Energy-Targeted Truncation

Instead of keeping a fixed number of coefficients, you will implement a "Keep-Energy" strategy:

1. **Calculate Total Energy:** For the current block, compute $E_{total}=\sum|C_{i,j}|^2$ (the sum of squares of all coefficients).
2. **Accumulate coefficients:** Following the zigzag order, keep the minimum number of coefficients, k, such that their cumulative energy first reaches or exceeds T% of Etotal.
3. **Variable Parameter:** T should vary from **75% to 100%** (e.g., in steps of 3%).
4. **Zeroing:** Set all coefficients beyond the kth position to zero.
5. **Record Metadata:** Store the value of k for each block to generate the heatmap later.

Step D: Reconstruction

1. Apply the **2D-Inverse DCT (2D-IDCT)** to the truncated coefficient blocks.
2. Reassemble the blocks into the full reconstructed image.

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3. Required Metrics & Visualizations

Mathematical Metrics

For every image, for every n, and for every T, calculate:

- **Compression Ratio (CR):**
  CR=Total Pixels in Image / ∑coefficients kept (k) across all blocks
- **Peak Signal-to-Noise Ratio (PSNR):**
  $PSNR=10\cdot\log_{10}(255^2/MSE)$
  Where MSE is the Mean Squared Error between the original and reconstructed pixels.

The Coefficient Heatmap

For a selected T and n, generate a **Heatmap** where each n×n spatial region is colored/shaded according to the value of k used for that block.

- **Insight:** Observe how edges and textures (high-frequency areas) appear "brighter" (more coefficients) than smooth gradients.

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4. Final Analysis & Master Plots

Plot A: Content Sensitivity (Fixed n=8)

Create one plot of **PSNR (y-axis) vs. Compression Ratio (x-axis)**. This plot should contain **10 lines**, one for each image.

- **Goal:** Identify which images are "compressible" and which are "complex."

Plot B: Block Size Sensitivity (Fixed Image)

Select one representative image. Create a plot of **PSNR vs. Compression Ratio** containing **4 lines**, one for each $n \in \{8,16,32,64\}$.

- **Goal:** Determine the effect of the transform window size on coding efficiency.

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5. Deliverables

1. **Python Script:** A clean, functional implementation
2. **Visual Results:** * Comparison of the original vs. reconstructed image for a high and low T.
    - Heatmaps for at least two images showing the impact of changing T.
3. **Technical Report:**
    - Explain the "blocking artifacts" observed at n=32 compared to n=4.
    - Discuss why the PSNR drops significantly as T decreases, and correlate this with the visual loss of high-frequency detail.