$\boxed{1}$

$$A = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \quad \& \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \& \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$(SI - A) = \begin{bmatrix} s & -1 \\ 6 & s+5 \end{bmatrix}$$

$$|(SI - A)| = s^2 + 5s + 6$$

$$|(SI - A)^{-1}| = C\,(sI - A)^{-1} B$$

$$(SI - A)^{-1} B = \frac{1}{s^2+5s+6} \begin{bmatrix} s+5 & 1 \\ -6 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ s \end{bmatrix} \cdot \frac{1}{s^2+5s+6}$$

$$\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ s \end{bmatrix} \cdot \frac{1}{s^2+5s+6} = \boxed{\frac{1}{s^2+5s+6}}$$

$\boxed{3}$

$$\phi(S) = \mathcal{L}^{-1}\left\{ (SI - A)^{-1} \right\}$$

$$(SI - A)^{-1} = \begin{bmatrix} \dfrac{s+5}{(s+2)(s+3)} & \dfrac{1}{(s+2)(s+3)} \\ \dfrac{-6}{(s+2)(s+3)} & \dfrac{s}{(s+2)(s+3)} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{3}{s+2} + \dfrac{-2}{s+3} & \dfrac{1}{s+2} + \dfrac{-1}{s+3} \\ \dfrac{-6}{s+2} + \dfrac{6}{s+3} & \dfrac{-2}{s+2} + \dfrac{3}{s+3} \end{bmatrix} \xrightarrow{L.T} \phi(t) = \begin{bmatrix} 3e^{-2t} - 2e^{-3t} & e^{-2t} - e^{-3t} \\ -6e^{-2t} + 6e^{-3t} & -2e^{-2t} + 3e^{-3t} \end{bmatrix}$$

5

$$A = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \quad \& \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \; \& \; C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} B & AB \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 0 & 1 \\ 1 & -5 \end{bmatrix} \rightarrow \det(Q_c) = -1 \neq 0 \text{ Then contralable}$$

$$Q_o = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow \det(Q_o) = 1 \neq 0$$

$\therefore$ observable.

6

Homogeneous solution;

$$\cdot\ X(t) = \phi(t) \cdot X(0)$$

$$= \begin{bmatrix} 3e^{-2t} - 2e^{-3t} & e^{-2t} - e^{-3t} \\ -6e^{-2t} + 6e^{-2t} & -2e^{-2t} + 3e^{-3t} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} e^{-2t} - e^{-3t} \\ -2e^{-2t} + 3e^{-3t} \end{bmatrix}$$

$$\cdot\ y(t) = C\,\phi(t)\,X(0)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} e^{-2t} - e^{-3t} \\ -2e^{-2t} + 3e^{-3t} \end{bmatrix} = \begin{bmatrix} e^{-2t} - e^{-3t} \end{bmatrix}$$

$\boxed{7}$

$$x(t) = \phi(t)\, x(0) + \int_0^t \phi(\tau) B\, u(t-\tau)\, d\tau$$

$$\phi(\tau)\cdot B = \begin{bmatrix} e^{-2t} - e^{-3t} \\ -2e^{-2t} + 3e^{-3t} \end{bmatrix}$$

$$\int \phi(\tau) B\, u(t-\tau)\, d\tau = \begin{bmatrix} -\frac{1}{2}e^{-2t} - \frac{1}{3}e^{-3t}\Big|_0^t \\ +e^{-2t} - e^{-3t}\Big|_0^t \end{bmatrix} = \begin{bmatrix} \frac{1}{2}e^{-2t} - \frac{1}{3}e^{-3t} + \frac{5}{6} \\ e^{-2t} - e^{-3t} \end{bmatrix}$$

$$\therefore x(t) = \begin{bmatrix} e^{-2t} - e^{-3t} \\ -2e^{-2t} + 3e^{-3t} \end{bmatrix} + \begin{bmatrix} \frac{1}{2}e^{-2t} - \frac{1}{3}e^{-3t} + \frac{5}{6} \\ e^{-2t} - e^{-3t} \end{bmatrix} = \begin{bmatrix} \frac{3}{2}e^{-2t} - \frac{4}{3}e^{-3t} + \frac{5}{6} \\ -e^{-2t} + 2e^{-3t} \end{bmatrix}$$

$$y(t) = C\,\phi(t)\, x(0) + \int_0^t C\,\phi(\tau) B\, u(t-\tau)\, d\tau$$

$$C\,\phi(\tau)\cdot B = [e^{-2t} - e^{-3t}]$$

$$\int_0^t [e^{-2t} - e^{-3t}]\, d\tau = \left[-\frac{1}{2}e^{-2t} + \frac{1}{3}e^{-3t} + \frac{1}{6}\right]$$

$$\therefore y(t) = [e^{-2t} - e^{-3t}] + \left[-\frac{1}{2}e^{-2t} + \frac{1}{3}e^{-3t} + \frac{1}{6}\right] =$$

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$\boxed{8}$

$T_s = 1$ & $\zeta = 0.7$ $\rightarrow \omega_n = \frac{40}{7}$

$s^2 + 8s + 32.653 = 0$

$|SI - A + BK| = 0$

$$BK = \underset{2\times 1}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} \underset{1\times 2}{\begin{bmatrix} K_1 & K_2 \end{bmatrix}} = \begin{bmatrix} 0 & 0 \\ K_1 & K_2 \end{bmatrix}$$

$$SI - A + BK = \begin{bmatrix} s & -1 \\ 6+K_1 & s+5+K_2 \end{bmatrix}$$

$$|SI - A + BK| = s\left[s+5+K_2\right] + 6 + K_1$$

$$= s^2 + s(5+K_2) + 6 + K_1$$

$5 + K_2 = 8 \longrightarrow K_2 = 3$

$6 + K_1 = 32.653 \longrightarrow K_1 = 26.65$

$$\underset{\text{matrix}}{K} = \begin{bmatrix} 26.65 & 3 \end{bmatrix}$$