Linear Regression

Multi-Feature Linear Regression

Residual Errors:

```
[[ 107059.90923762]
 [ 165469.52146816]
 [  52638.84308501]
 ...
 [  94715.49764775]
 [ 126820.75343685]
 [-119901.60815164]]
```

RMSE Values:

```
RMSE: 477644.54338617116
MSE: 228144309826.58392
MAE: 305309.9548087045
```

The Multi-Feature Regression model clearly performed better than the Gradient Descent model, according to the performance metrics. All the metrics but most importantly, the RMSE, are lower than the Gradient Descent ones. This is because the Multi-Feature Equation approach directly computes the optimal parameters using a closed-form solution, which can provide more accurate results.

Gradient Descent

Residual Errors:

```
[-18792.53383128  78557.32231529 487682.77347538 ...  16678.85452007
 763856.57744668 179729.55044688]
```

RMSE Values:

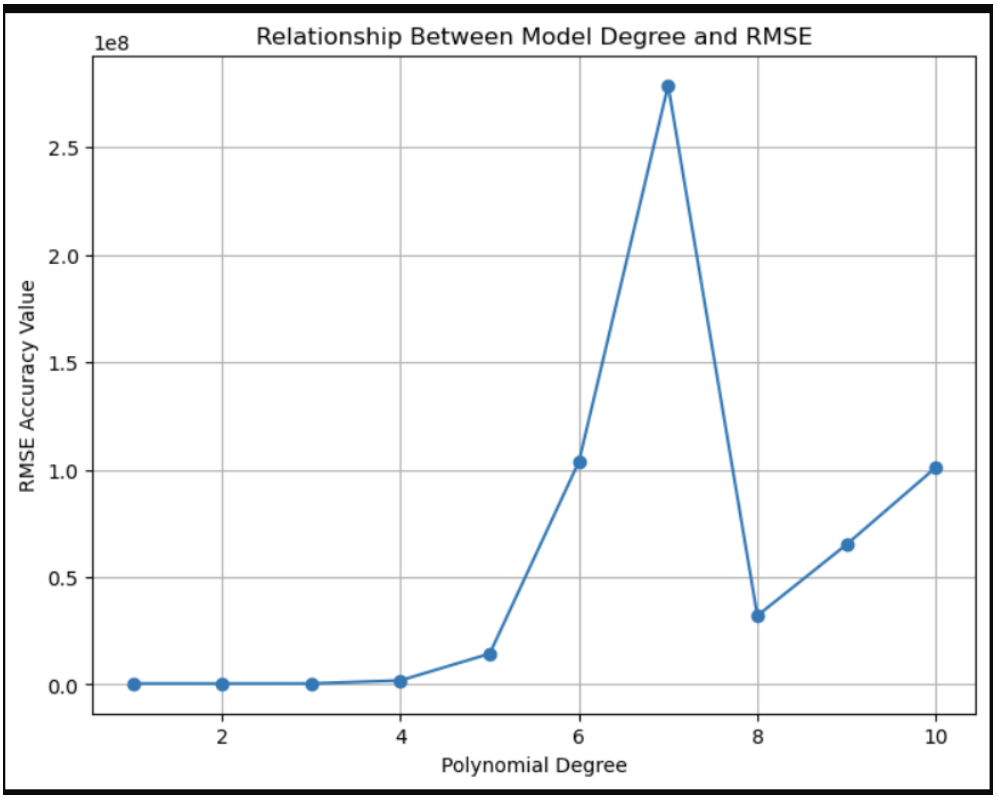
```
Gradient Descent Results:
RMSE: 730394.6159114572
MSE: 533476294952.44507
MAE: 435890.70235361584
```

The optimal degree is the one that results in the lowest RMSE because it signifies the best trade-off between model complexity and prediction accuracy. Degree 2 (quadratic polynomial) has the lowest RMSE of approximately 422,754, which is the smallest among the tested degrees. This degree provides the best fit for the data based on RMSE. The reason degree 2 performs well is probably that it captures some degree of non-linearity in the data without overfitting.

Polynomial Regression

Degree	***Residual Errors*** *(values are the mean of the residual error array)*	***RMSE Values***
1	-93858.7701969853	517944.6411044948
2	-53819.9903947973	422754.3750149545
3	-60218.635060806046	409671.8820524619
4	-53228.57727771867	675229.785650757
5	-395789.583590657	1123881.4083380792
6	1891374.5469480802	7696632.964348295
7	4835125.142791794	18406265.303462625
8	-702126.4094422878	29028029.340814497
9	952370.3318780735	2013191.8916789212
10	-1800296.0681096676	9361718.52750563

Model Degree and RMSE Plot:



Degree 2 is the most optimal degree:

Degree 2 has one of the lowest RMSE values (422754.3750149545), which means it provides a reasonably good fit to your test data.

The RMSE value for degree 2 is considerably lower than that of the other degrees, except for degree 3. However, degree 2 has a lower Residual Error mean than degree 3, indicating a slightly better performance. Hence, degree 2 polynomial regression provides a good balance between model complexity and fit to the data, or in other words, *between overfitting and underfitting*.