The steady state of Ξ-type atoms in the squeezed vacuum reservoir

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We study the steady state of Ξ-type atoms coupled to the squeezed vacuum reservoir. The population inversion can occur for the steady state when the coupling strength of two transitions to the squeezed vacuum are properly set. We also discuss the possibility of the population inversion of a group of atoms in the squeezed vacuum.

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I. INTRODUCTION

In thermal equilibrium, a population inversion can never exist for a system. Traditionally, the achievements of population inversion requires pushing the system into a non-equilibrated state. The concept of population inversion is of fundamental importance in laser physics because the production of population inversion is a key step in the workings of a standard laser.

II. MASTER EQUATION OF THREE LEVEL ATOMS IN THE SQUEEZED VACUUM

In this section, we consider a scenario where N_a three-level atoms are located inside the waveguide with the squeezed vacuum injected from both ends, as shown in Fig. 1(a). The atomic electronic structure is shown in Fig. 1(b) where the atomic states are labeled $|a\rangle$, $|b\rangle$, $|c\rangle$ from the excited state to the ground state. We assume that $\omega_{ac} = 2\omega_0$ where ω_0 is the center frequency of the broad band squeed vacuum. ω_{ab} and ω_{bc} are not equal but they are still within the bandwith of the squeezed vacuum. We assume that the squeezed vacuum bandwidth is much larger than $|\omega_{ab} - \omega_{bc}|$ so it forms a squeezed vacuum reservoir.

The atom-field system is described by the Hamiltonian

$$H = H_A + H_F + H_{AF} \tag{1}$$

where $H_A = \sum_{e=a,b,c} \sum_{l=1}^{N_a} \hbar\omega_{e,l} |e_l\rangle \langle e_l|$ is the atomic Hamiltonian, and $|e_l\rangle$ is the energy state of the lth atom with energy $\hbar\omega_{e,l}$. The Hamiltonian of the EM field is $H_F = \sum_{\boldsymbol{ks}} \hbar\omega_{\boldsymbol{ks}}(\hat{a}^\dagger_{\boldsymbol{ks}}\hat{a}_{\boldsymbol{ks}} + \frac{1}{2})$ where $\hat{a}_{\boldsymbol{ks}}$ and $\hat{a}^\dagger_{\boldsymbol{ks}}$ are the annihilation and creation operators of the filed mode with wavevector $\boldsymbol{k}$, polarization s, and frequency $\omega_{\boldsymbol{k},s}$. The interaction Hamiltonian in electric-dipole approximation is $H_{AF} = -i\hbar \sum_{\boldsymbol{ks}} \sum_{i=1,2} \sum_{l=1}^{N_a} [\boldsymbol{\mu}_{l,i} \cdot \boldsymbol{u}_{\boldsymbol{ks}}(\boldsymbol{r}_{l,i}) S^+_{l,i} \hat{a}_{\boldsymbol{ks}} + \boldsymbol{\mu}^*_{l,i} \cdot \boldsymbol{u}_{\boldsymbol{ks}}(\boldsymbol{r}_{l,i}) S^-_{l,i} \hat{a}_{\boldsymbol{ks}} - H.c.]$ where $\boldsymbol{\mu}_{l,i}$ is the electric dipole moment for ith transition of the lth atom, where $i = 1$ denotes the transition from $|a\rangle$ to $|b\rangle$, and $i = 2$ denotes the transition from $|b\rangle$ to $|c\rangle$. $S^+_{l,i}$ and $S^-_{l,i}$ are the raising and lowering operator for the lth atom. The mode function of the squeezed vacuum is given by

$$\boldsymbol{u}_{\boldsymbol{ks}}(\boldsymbol{r}_i) = \sqrt{\frac{\omega_{\boldsymbol{ks}}}{2\epsilon_0\hbar V}} \boldsymbol{e}_{ks} e^{i\boldsymbol{k}\cdot(\boldsymbol{r}_i - \boldsymbol{o}_{\boldsymbol{k}s})} \tag{2}$$

where $\boldsymbol{o}_{\boldsymbol{ks}}$ is a phenomenological parameter which includes the effects of the initial phase and the position of the squeezing source with wavevector $\boldsymbol{k}s$[**?**]. The correlation functions for the squeezed vacuum are[**?**]:

$$\begin{aligned}
\left\langle a^\dagger_{\boldsymbol{k},s} a_{\boldsymbol{k}',s'} \right\rangle &= \sinh^2 r \delta_{\boldsymbol{k}'\boldsymbol{k}} \delta_{ss'} \\
\left\langle a_{\boldsymbol{k},s} a^\dagger_{\boldsymbol{k}',s'} \right\rangle &= \cosh^2 r \delta_{\boldsymbol{k}'\boldsymbol{k}} \delta_{ss'} \\
\left\langle a^\dagger_{\boldsymbol{k},s} a^\dagger_{\boldsymbol{k}',s'} \right\rangle &= -e^{-i\theta} \cosh(r) \sinh(r) \delta_{\boldsymbol{k}',2\boldsymbol{k}_0 - \boldsymbol{k}} \delta_{ss'} \\
\langle a_{\boldsymbol{k},s} a_{\boldsymbol{k}',s'} \rangle &= -e^{i\theta} \cosh(r) \sinh(r) \delta_{\boldsymbol{k}',2\boldsymbol{k}_0 - \boldsymbol{k}} \delta_{ss'}
\end{aligned} \tag{3}$$

For simplicity, we can set the squeezing parameter $\theta = 0$, and all atoms to align along the same direction. The dynamics of the atomic system can be described by the following master equation(See Appendix A for details of derivation):

$$\begin{aligned}
\frac{d\rho^S}{dt} =& -i\sum_{i\neq j} \Lambda_{ij} [S^+_i S^-_j, \rho^S] e^{i(\omega_i - \omega_j)t} - \frac{1}{2}\sum_{i,j} \gamma_{ij}(1+N)(\rho^S S^+_i S^-_j + S^+_i S^-_j \rho^S - 2S^-_j \rho^S S^+_i) e^{i(\omega_i - \omega_j)t} \\
& - \frac{1}{2}\sum_{i,j} \gamma_{ij} N (\rho^S S^-_i S^+_j + S^-_i S^+_j \rho^S - 2S^+_j \rho^S S^-_i) e^{-i(\omega_i - \omega_j)t} \\
& - \frac{1}{2}\sum_{\alpha=\pm} \sum_{i,j} \gamma'_{ij} M e^{2\alpha i k_{0z} R} e^{i\alpha(\omega_i + \omega_j - 2\omega_0)t} (\rho^S S^\alpha_i S^\alpha_j + S^\alpha_i S^\alpha_j \rho^S - 2S^\alpha_j \rho^S S^\alpha_i)
\end{aligned} \tag{4}$$

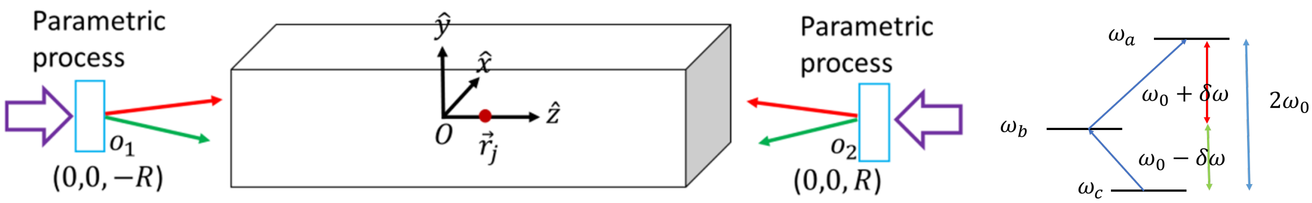

Fig. 1: (a) Schematic setup: A three-level atom is located inside the waveguide with the broadband squeezed vacuum incident from both ends. (b)The energy structure of the three level atom. Transition $|a\rangle \to |c\rangle$ is forbidden and $\omega_{ac} = 2\omega_0$ where ω_0 is the center frequency of the squeezed vacuum. ω_{ab} and ω_{bc} differ by a small amount $2\delta\omega_0$ and they are within the bandwidth of the squeezed vacuum reservoir.

where $N = \sinh(r)^2$, $M = \sinh(r)\cosh(r)$, and the coefficients are

$$\begin{aligned}\gamma_{ij} &= \sqrt{\gamma_i\gamma_j}\cos(k_{0z}r_{ij})\\ \Lambda_{ij} &= \frac{\sqrt{\gamma_i\gamma_j}}{2}\sin(k_{0z}r_{ij})\\ \gamma'_{ij} &= \sqrt{\gamma_i\gamma_j}\cos[k_{0z}(r_i+r_j)]\end{aligned} \tag{5}$$

where γ_i is the decay rate for transition i in ordinary vacuum.

III. STEADY STATE OF A SINGLE ATOM

In this section, we will study the steady state of a single atom in the squeezed vacuum reservior. For a single three level atom, we have $r_i = r_j$, and for simplicity we set $R = r_i = 0$. Then the steady state of Eq.(4) can be derived by re-writing it as:

$$\dot{\rho}_{aa} = -\gamma_1 ch^2\rho_{aa} + \gamma_1 sh^2\rho_{bb} - \frac{1}{2}\sqrt{\gamma_1\gamma_2}chsh(e^{-i(\omega_1+\omega_2-2\omega_0)t}\rho_{ac} + e^{i(\omega_1+\omega_2-2\omega_0)t}\rho_{ca}) \tag{6a}$$

$$\dot{\rho}_{bb} = \gamma_1(ch^2\rho_{aa} - sh^2\rho_{bb}) + \gamma_2(sh^2\rho_{cc} - ch^2\rho_{bb}) + \sqrt{\gamma_1\gamma_2}chsh(e^{-i(\omega_1+\omega_2-2\omega_0)t}\rho_{ac} + e^{i(\omega_1+\omega_2-2\omega_0)t}\rho_{ca}) \tag{6b}$$

$$\dot{\rho}_{cc} = \gamma_2 ch^2\rho_{bb} - \gamma_2 sh^2\rho_{cc} - \frac{1}{2}\sqrt{\gamma_1\gamma_2}chsh(e^{-i(\omega_1+\omega_2-2\omega_0)t}\rho_{ac} + e^{i(\omega_1+\omega_2-2\omega_0)t}\rho_{ca}) \tag{6c}$$

$$e^{-i(\omega_1+\omega_2-2\omega_0)t}\dot{\rho}_{ac} + e^{i(\omega_1+\omega_2-2\omega_0)t}\dot{\rho}_{ca} = -\frac{1}{2}(\gamma_1 ch^2 + \gamma_2 sh^2)(e^{-i(\omega_1+\omega_2-2\omega_0)t}\rho_{ac} + e^{i(\omega_1+\omega_2-2\omega_0)t}\rho_{ca}) \tag{6d}$$

$$-\sqrt{\gamma_1\gamma_2}shch(\rho_{aa} - 2\rho_{bb} + \rho_{cc}) \tag{6e}$$

$$e^{i(\omega_0-\omega_1)t}\dot{\rho}_{ab} + e^{-i(\omega_0-\omega_1)t}\dot{\rho}_{ba} = -\frac{1}{2}((\gamma_1+\gamma_2)ch^2 + \gamma_1 sh^2 - \gamma_1 chsh)(e^{i(\omega_0-\omega_1)t}\rho_{ab} + e^{-i(\omega_0-\omega_1)t}\rho_{ba}) \tag{6f}$$

$$-\frac{1}{2}\sqrt{\gamma_1\gamma_2}(ch - 2sh)sh(e^{-i(\omega_0-\omega_2)t}\rho_{cb} + e^{i(\omega_0-\omega_2)t}\rho_{bc}) \tag{6g}$$

$$e^{-i(\omega_0-\omega_2)t}\dot{\rho}_{cb} + e^{i(\omega_0-\omega_2)t}\dot{\rho}_{bc} = \frac{1}{2}\sqrt{\gamma_1\gamma_2}(2ch - sh)ch(e^{i(\omega_0-\omega_1)t}\rho_{ab} + e^{-i(\omega_0-\omega_1)t}\rho_{bc}) \tag{6h}$$

$$-\frac{1}{2}((\gamma_1+\gamma_2)sh^2 + \gamma_2 ch^2 - 2\gamma_2 chsh)(e^{-i(\omega_0-\omega_2)t}\rho_{cb} + e^{i(\omega_0-\omega_2)t}\rho_{bc}) \tag{6i}$$

where $ch = \cosh(r)$, $sh = \sinh(r)$, and $\gamma_1 = \gamma_{ab}(\gamma_2 = \gamma_{bc})$ indicates the decay rate from $|a\rangle$ to $|b\rangle(|b\rangle$ to $|c\rangle)$ in ordinary vacuum. We noticed that the equations for off diagonal elements Eq.(7e)-(7f) result in $\rho_{ab} = \rho_{bc} = 0$. Since $\omega_1 + \omega_2 = 2\omega_0$, it has a steady state solution. When $\gamma_1 = \gamma_2$, the steady state solution is: $\rho_{aa} = \frac{sh^2}{sh^2+ch^2}$, $\rho_{bb} = 0$, $\rho_{cc} = \frac{ch^2}{sh^2+ch^2}$, $\rho_{ac} = -\frac{shch}{sh^2+ch^2}$. Thus, the steady state is actually a superposition state of $|a\rangle$ and $|c\rangle$: $\frac{sh}{\sqrt{sh^2+ch^2}}|a\rangle - \frac{ch}{\sqrt{sh^2+ch^2}}|c\rangle$. This phenomenon is similar to coherent trapping, but here we achieve the trapping for Ξ structure with the squeezed vacuum reservoir, which cannot be realized with coherent pump due to spontaneous emission. For the general case where $\gamma_1 \neq \gamma_2$, although the analytical solution is quite lengthy, we can easily get the numerical solution for different μ_{ab} and μ_{bc}. In Fig. 2(a), population inversion is achieved with $\mu_{ab} < \mu_{bc}$. This can be interpreted with the help of Fig. 2(c). Figure 2(c) shows that the direct transition between $|a\rangle$, $|b\rangle$, and $|c\rangle$ are allowed just like the thermal reservoir case. However, in the squeezed vacuum, there is additional paths for population flow: electrons in any of these three states can evolve into the other two through an intermediate "state" ρ_{ac}. Although ρ_{ac} is actually not a state, and $\rho_{ac} < 0$ in our convention, it can be used to elucidate our idea. When $\mu_{ab} \ll \mu_{bc}$, the transition $|a\rangle \rightarrow |b\rangle$ is negligible. Thus state $|c\rangle$ can be excited to $|a\rangle$ through $|c\rangle \rightarrow |b\rangle \rightarrow \rho_{ac} \rightarrow |a\rangle$, but $|a\rangle$ can not decay back to $|c\rangle$. Thus, the population is trapped from $|c\rangle$ to $|a\rangle$. Although the population inversion between $|a\rangle$ and $|c\rangle$ is very sensitive to the value of M, the population inversion between $|a\rangle$ and $|b\rangle$ still holds for $M = 0.8\sqrt{N(N+1)}$, which is shown in Fig. 2(b).

IV. STEADY STATE OF MULTIPLE ATOMS

In the last section, we showed that a single atom can achieve steady state population inversion in the squeezed vacuum reservoir. However, with Eq.(5), this result can not be generated to the multi-atom case since $\gamma'_{ij} = \sqrt{\gamma_i\gamma_j}\cos[k_{0z}(r_i+r_j)]$. The squeezing term in Eq.(4) vanishes for atoms located around $r_i = \frac{\pi}{4k_{0z}} + \frac{n\pi}{2k_{0z}}$. Thus, for a group of randomly located atoms, if we want to achieve steady state population inversion in the squeezed vacuum, we need to modify our scheme. Consider the following correlation functions:

$$\begin{aligned}
\left\langle a^\dagger_{\boldsymbol{k},s} a_{\boldsymbol{k}',s'} \right\rangle &= \sinh^2 r \delta_{\boldsymbol{k}'\boldsymbol{k}}\delta_{ss'} \\
\left\langle a_{\boldsymbol{k},s} a^\dagger_{\boldsymbol{k}',s'} \right\rangle &= \cosh^2 r \delta_{\boldsymbol{k}'\boldsymbol{k}}\delta_{ss'} \\
\left\langle a^\dagger_{\boldsymbol{k},s} a^\dagger_{\boldsymbol{k}',s'} \right\rangle &= -e^{-i\theta}\cosh(r)\sinh(r)\delta_{\boldsymbol{k}',-(2\boldsymbol{k}_0-\boldsymbol{k})}\delta_{ss'} \\
\langle a_{\boldsymbol{k},s} a_{\boldsymbol{k}',s'} \rangle &= -e^{i\theta}\cosh(r)\sinh(r)\delta_{\boldsymbol{k}',-(2\boldsymbol{k}_0-\boldsymbol{k})}\delta_{ss'}
\end{aligned} \tag{7}$$

which indicates that photons are entangled with those from the opposite direction, then the coefficients in the the master equation Eq.(4) becomes

$$\begin{aligned}
\gamma_{ij} &= \sqrt{\gamma_i\gamma_j}\cos(k_{0z}r_{ij}) \\
\Lambda_{ij} &= \frac{\sqrt{\gamma_i\gamma_j}}{2}\sin(k_{0z}r_{ij}) \\
\gamma'_{ij} &= \sqrt{\gamma_i\gamma_j}\cos[k_{0z}(r_{ij})]
\end{aligned} \tag{8}$$

which is exactly the traditionally studied master equation[**?**] by Ficek. However, they didn't consider the effect of squeezing source, and here we give a detailed derivation on getting Eq. (2) in Appendix B, where the effect of squeezing source is included. With this

APPENDIX A: DERIVATION OF EQ.(??)

Here we will show how to derive the master equation Eq. (4). The interaction Hamiltonian is:

$$V(t) = -i\hbar\sum_{\boldsymbol{k}s}[D(t)a_{\boldsymbol{k}s}(t) - D^+(t)a^\dagger_{\boldsymbol{k}s}(t)], \tag{A1}$$

where

$$D(t) = \sum_{l,i}[\boldsymbol{\mu}_{l,i}\cdot\boldsymbol{u}_{\boldsymbol{k},s}(r_{l,i})S^\dagger_{l,i}(t) + \boldsymbol{\mu}^*_{l,i}\cdot\boldsymbol{u}_{\boldsymbol{k},s}(r_{l,i})S^-_{l,i}(t)] \tag{A2}$$

The reduced master equation of atoms in the reservoir is:

$$\begin{aligned}
\frac{d\rho^S}{dt} &= -\frac{1}{\hbar^2}\int_0^t d\tau Tr_F\{[V(t),[V(t-\tau),\rho^S(t-\tau)\rho^F\} \\
&= -\frac{1}{\hbar^2}\int_0^t d\tau Tr_F\{V(t)V(t-\tau)\rho^S(t-\tau)\rho^F + \rho^S(t-\tau)\rho^F V(t-\tau)V(t) \\
&\quad - V(t)\rho^S(t-\tau)\rho^F V(t-\tau) - V(t-\tau)\rho^S(t-\tau)\rho^F V(t)\}
\end{aligned} \tag{A3}$$

Here we just show how to deal with the first term in Eq.(A3), the remaining terms can be calculated in the same way. For the first term, we have

$$\begin{aligned}
&-\frac{1}{\hbar^2}\int_0^t d\tau Tr_F\{V(t)V(t-\tau)\rho^S(t-\tau)\rho^F\} \\
=&\int_0^t d\tau\sum_{\boldsymbol{k}s,\boldsymbol{k}'s'}\{D(t)D(t-\tau)Tr_F[\rho^F a_{ks}(t)a_{k's'}(t-\tau)] - D(t)D^+(t-\tau)Tr_F[\rho^F a_{ks}(t)a^\dagger_{k's'}(t-\tau)] \\
&- D^+(t)D(t-\tau)Tr_F[\rho^F a^\dagger_{ks}(t)a_{k's'}(t-\tau)] + D^+(t)D^+(t-\tau)Tr_F[\rho^F a^\dagger_{ks}(t)a^\dagger_{k's'}(t-\tau)]\}\rho^S(t-\tau)\}.
\end{aligned} \tag{A4}$$

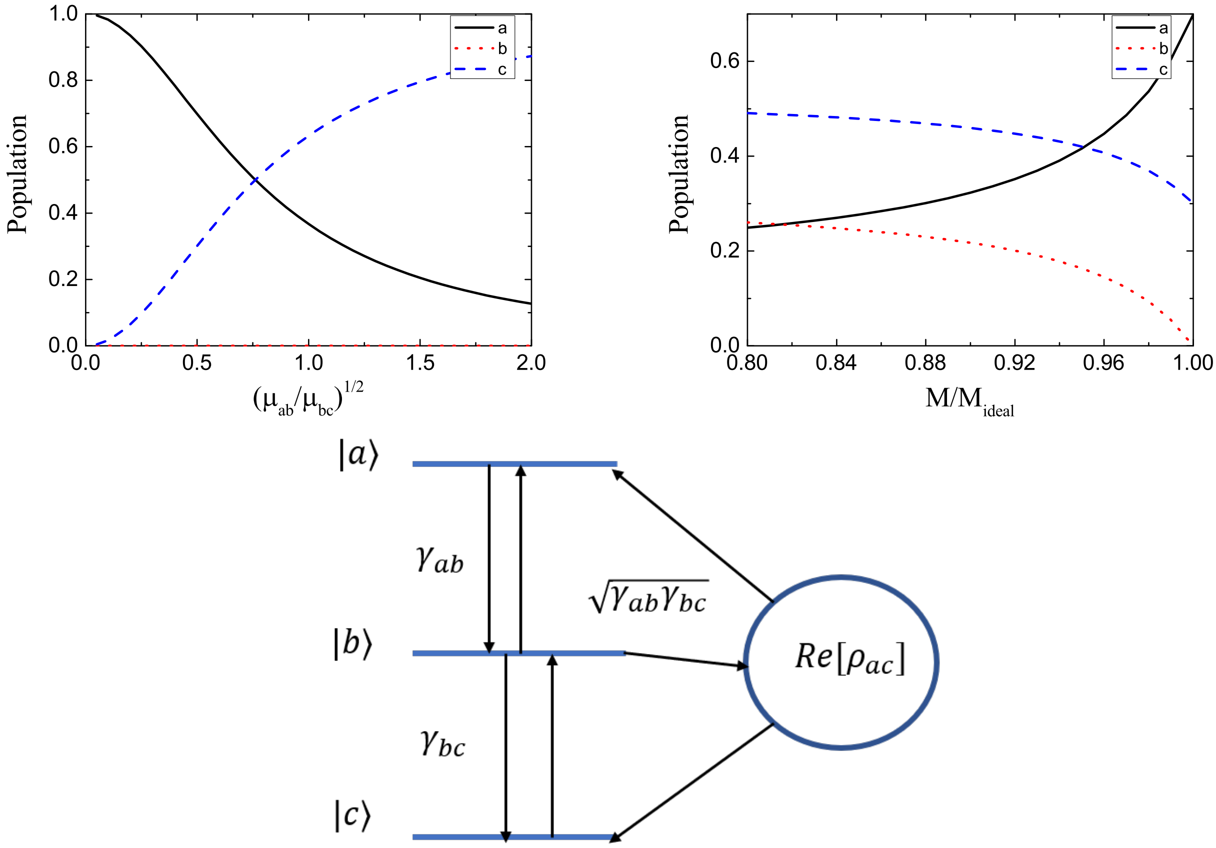

Fig. 2: (a) The steady state population distribution for different μ_{ab} andμ_{bc}. The squeezing parameter $r = 1$. (b) The steady state population distribution for non-ideal squeezed vacuum which is characterized by the ratio of M and $\sqrt{N(N+1)}$. (c) The allowed population flow in the squeezed vacuum. The squeezing parameter $r = 1$, and $\mu_{ab} = \frac{1}{4}\mu_{bc}$.

Under the rotating wave approximation(RWA), we have

$$
\begin{aligned}
&-\frac{1}{\hbar^2}\int_0^t d\tau Tr_F\{V(t)V(t-\tau)\rho^S(t-\tau)\rho^F\} \\
=&\sum_{ijlm}\sum_{\boldsymbol{k}s,\boldsymbol{k'}s'}\int_0^t d\tau\{\boldsymbol{\mu}_{l,i}\cdot\boldsymbol{u}_{\boldsymbol{k}s}(r_{l,i})S^+_{l,i}e^{i\omega_i t}\boldsymbol{\mu}_{m,j}\cdot\boldsymbol{u}_{\boldsymbol{k'}s'}(r_{m,j})S^+_{m,j}e^{i\omega_j(t-\tau)}e^{-i(\omega_{\boldsymbol{k}s}+\omega_{\boldsymbol{k'}s'})t+i\omega_{\boldsymbol{k'}s'}\tau}[-\sinh(r)\cosh(r)\delta_{\boldsymbol{k'},2\boldsymbol{k}_0-\boldsymbol{k}}\delta_{ss'}] \\
&-\boldsymbol{\mu}_{l,i}\cdot\boldsymbol{u}_{\boldsymbol{k}s}(r_{l,i})S^+_{l,i}e^{i\omega_i t}\boldsymbol{\mu}^*_{m,j}\cdot\boldsymbol{u}^*_{\boldsymbol{k'}s'}(r_{m,j})S^-_{m,j}e^{-i\omega_j(t-\tau)}e^{-i\omega_{\boldsymbol{k'}s'}\tau}\cosh^2 r\delta_{\boldsymbol{k}\boldsymbol{k'}}\delta_{ss'} \\
&-\boldsymbol{\mu}^*_{l,i}\cdot\boldsymbol{u}_{\boldsymbol{k}s}(r_{l,i})S^-_{l,i}e^{-i\omega_i t}\boldsymbol{\mu}_{m,j}\cdot\boldsymbol{u}^*_{\boldsymbol{k'}s'}(r_{m,j})S^+_{m,j}e^{i\omega_j(t-\tau)}e^{-i\omega_{\boldsymbol{k'}s'}\tau}\cosh^2 r\delta_{\boldsymbol{k}\boldsymbol{k'}}\delta_{ss'} \\
&-\boldsymbol{\mu}^*_{l,i}\cdot\boldsymbol{u}^*_{\boldsymbol{k}s}(r_{l,i})S^-_{l,i}e^{-i\omega_i t}\boldsymbol{\mu}_{m,j}\cdot\boldsymbol{u}_{\boldsymbol{k'}s'}(r_{m,j})S^+_{m,j}e^{i\omega_j(t-\tau)}e^{i\omega_{\boldsymbol{k'}s'}\tau}\sinh^2 r\delta_{\boldsymbol{k}\boldsymbol{k'}}\delta_{ss'} \\
&-\boldsymbol{\mu}_{l,i}\cdot\boldsymbol{u}^*_{\boldsymbol{k}s}(r_{l,i})S^+_{l,i}e^{i\omega_i t}\boldsymbol{\mu}^*_{m,j}\cdot\boldsymbol{u}_{\boldsymbol{k'}s'}(r_{m,j})S^-_{m,j}e^{-i\omega_j(t-\tau)}e^{i\omega_{\boldsymbol{k'}s'}\tau}\sinh^2 r\delta_{\boldsymbol{k}\boldsymbol{k'}}\delta_{ss'} \\
&+\boldsymbol{\mu}^*_{l,i}\cdot\boldsymbol{u}^*_{\boldsymbol{k}s}(r_{l,i})S^-_{l,i}e^{-i\omega_i t}\boldsymbol{\mu}^*_{m,j}\cdot\boldsymbol{u}^*_{\boldsymbol{k'}s'}(r_{m,j})S^-_{m,j}e^{-i\omega_j(t-\tau)}e^{i(\omega_{\boldsymbol{k}s}+\omega_{\boldsymbol{k'}s'})t-i\omega_{\boldsymbol{k'}s'}\tau}[-\sinh(r)\cosh(r)\delta_{\boldsymbol{k'},2\boldsymbol{k}_0-\boldsymbol{k}}\delta_{ss'}]\}\rho^S(t-\tau)
\end{aligned}
\tag{A5}
$$

where l, m are used for labeling different atoms, and i, j are used for transitions within an atom. Here we just calculate the first and second term to show how to get the master equation Eq.(4). Since all atoms are identical, $\omega_{l,i} = \omega_i$,

$|\boldsymbol{\mu}_{l,i}| = |\boldsymbol{\mu}_i|$, and $r_{l,i} = r_l$ can be used to simplify Eq.(A5). For the second term(thermal term), we have

$$\begin{aligned}
&-\sum_{k_z}\int_0^t d\tau \boldsymbol{\mu}_{l,i}\cdot \boldsymbol{u}_{\boldsymbol{k}s}(r_l) S^+_{l,i} e^{i\omega_i t}\boldsymbol{\mu}^*_{m,j}\cdot \boldsymbol{u}^*_{\boldsymbol{k}'s'}(r_m) S^-_{m,j} e^{-i\omega_j(t-\tau)} e^{-i\omega_{\boldsymbol{k}'s'}\tau}\cosh^2 r\rho^S(t-\tau)\delta_{\boldsymbol{k}\boldsymbol{k}'}\delta_{ss'} \\
=& -\frac{L}{2\pi} e^{i(\omega_i-\omega_j)t}\int_{-\infty}^{\infty} dk_z \int_0^t d\tau e^{i\omega_j\tau} e^{-i\omega_{k_z}\tau}\frac{\omega_k\mu_i\mu_j}{\epsilon_0 LS\hbar} e^{ik_z(r_l-r_m)}\cosh^2 r S^+_{l,i}S^-_{m,j}\rho^S(t-\tau) \\
\approx& -\frac{L}{2\pi} e^{i(\omega_i-\omega_j)t}\int_0^{\infty} dk_z \int_0^t d\tau e^{i\omega_j\tau} e^{-i[\omega_j + c^2k_{jz}(k_z-k_{jz})/\omega_j]\tau}\frac{\omega_k\mu_i\mu_j}{\epsilon_0 LS\hbar}[e^{ik_z(r_l-r_m)} + e^{-ik_z(r_l-r_m)}]\cosh^2 r S^+_{l,i}S^-_{m,j}\rho^S(t-\tau) \\
\approx& -\frac{L}{2\pi} e^{i(\omega_i-\omega_j)t}\int_{-k_{0z}}^{\infty} d\delta k_z \int_0^t d\tau e^{-i\tau c^2 k_{jz}\delta k_z/\omega_j}\frac{\omega_k\mu_i\mu_j}{\epsilon_0 LS\hbar}[e^{i(k_{jz}+\delta k_z)(r_l-r_m)} + e^{-i(k_{jz}+\delta k_z)(r_l-r_m)}]\cosh^2 r S^+_{l,i}S^-_{m,j}\rho^S(t-\tau) \\
\approx& -\frac{L}{2\pi} e^{i(\omega_i-\omega_j)t}\int_{-\infty}^{\infty} d\delta k_z \int_0^t d\tau e^{-i(c^2 k_{jz}\delta k_z/\omega_j)\tau}\frac{\omega_k\mu_i\mu_j}{\epsilon_0 LS\hbar}[e^{i(k_{jz}+\delta k_z)(r_l-r_m)} + e^{-i(k_{jz}+\delta k_z)(r_l-r_m)}]\cosh^2 r S^+_{l,i}S^-_{m,j}\rho^S(t-\tau) \\
\approx& -\frac{L}{2\pi} e^{i(\omega_i-\omega_j)t}\int_0^t d\tau \frac{\omega_j\mu_i\mu_j}{\epsilon_0 LS\hbar} 2\pi[e^{ik_{jz}(r_l-r_m)}\delta((r_l-r_m)-\frac{c^2k_{jz}}{\omega_0}\tau) + e^{-ik_{jz}(r_l-r_m)}\delta((r_l-r_m)+\frac{c^2k_{jz}}{\omega_0}\tau)]\cosh^2 r S^+_{l,i}S^-_{m,j}\rho^S(t \\
\approx& -\frac{L}{2\pi} e^{ik_{jz}r_{lm}}\frac{\omega_j\mu_i\mu_j}{\epsilon_0 LS\hbar} 2\pi \frac{\omega_j}{c^2 k_{0z}}\cosh^2 r S^+_{l,i}S^-_{m,j}\rho^S(t) e^{i(\omega_i-\omega_j)t} \\
\approx& -[\frac{\sqrt{\gamma_i\gamma_j}}{2}\cos(k_{0z}r_{lm}) + i\frac{\sqrt{\gamma_i\gamma_j}}{2} sin(k_{0z}r_{lm})]\cosh^2 r S^+_{l,i}S^-_{m,j}\rho^S(t) e^{i(\omega_i-\omega_j)t}
\end{aligned} \tag{A6}$$

where emitter separation $r_{lm} = |r_l - r_m|$, collective decay rate $\gamma_i = 2\mu_i^2\omega_i^2/\hbar\epsilon_0 Sc^2 k_{iz}$, and collective energy shift $\Lambda_{ij} = \sqrt{\gamma_i\gamma_j}\sin(k_{0z}r_{ij})/2$. In the third line we expand $\omega_k = c\sqrt{(\frac{\pi}{a})^2 + (k_z)^2}$ around $k_z = k_{0z}$ since resonant modes provide dominant contributions. In the fifth line we extend the integration $\int_{-k_{jz}}^{\infty} dk_z \to \int_{-\infty}^{\infty} dk_z$ because the main contribution comes from the components around $\delta k_z = 0$. In the next line, Weisskopf-Wigner approximation is used. Since index i and j are always associated with index l and m respectively, we can combine l, i as i, and m, j as j for simplicity. Thus, we have obtained γ_{ij} and Λ_{ij} as is shown in Eq.(5).

Next we need to calculate the first term (squeezing term) in Eq.(A5):

$$\begin{aligned}
& e^{i(\omega_i+\omega_j-2\omega_0)t}\sum_{k_z}\int_0^t d\tau\{\boldsymbol{\mu}_{l,i}\cdot \boldsymbol{u}_{2\boldsymbol{k}_0-\boldsymbol{k}}(r_l) S^+_{l,i}\boldsymbol{\mu}_{m,j}\cdot \boldsymbol{u}_{\boldsymbol{k}}(r_m) S^+_{m,j} e^{i(\omega_{\boldsymbol{k}}-\omega_j)\tau}[-\sinh(r)\cosh(r)]\rho^S(t-\tau) \\
=& -\frac{L}{2\pi} e^{i(\omega_i+\omega_j-2\omega_0)t}\int_0^{2k_{0z}} dk_z \int_0^t d\tau e^{i(\omega_{k_z}-\omega_j)\tau} e^{i(2k_{jz}-k_z)(r_l-o_1)} e^{ik_z(r_m-o_1)}\frac{\sqrt{\omega_{k_z}\omega_{2k_{0z}-k_z}}\mu_i\mu_j}{\epsilon_0 LS\hbar}\sinh(r)\cosh(r) S^+_{l,i}S^+_{m,j}\rho^S(t- \\
& -\frac{L}{2\pi} e^{i(\omega_i+\omega_j-2\omega_0)t}\int_{-2k_{0z}}^{0} dk_z \int_0^t d\tau e^{i(\omega_{k_z}-\omega_j)\tau} e^{i(-2k_{jz}-k_z)(r_l-o_2)} e^{ik_z(r_m-o_2)}\frac{\sqrt{\omega_{k_z}\omega_{-2k_{0z}-k_z}}\mu_i\mu_j}{\epsilon_0 LS\hbar}\sinh(r)\cosh(r) S^+_{l,i}S^+_{m,j}\rho^S(t
\end{aligned} \tag{A7}$$

Putting aside the overall factor $e^{i(\omega_i+\omega_j-2\omega_0)t}$, for $r_l = r_j$, Eq.(A7) reduces to

$$\begin{aligned}
&\sum_{k_z}\int_0^t d\tau\{\boldsymbol{\mu}_{l,i}\cdot \boldsymbol{u}_{2\boldsymbol{k}_0-\boldsymbol{k}}(r_l) S^+_{l,i}\boldsymbol{\mu}_{l,j}\cdot \boldsymbol{u}_{\boldsymbol{k}}(r_l) S^+_{l,j} e^{i(\omega_{\boldsymbol{k}}-\omega_j)\tau}[-\sinh(r)\cosh(r)]\rho^S(t-\tau) \\
=& -\frac{L}{2\pi}\int_0^{2k_{0z}} dk_z \int_0^t d\tau e^{i\frac{c^2k_{jz}}{\omega_j}(k_z-k_{jz})\tau} e^{i2k_{0z}(r_l-o_1)}\frac{\sqrt{\omega_{k_z}\omega_{2k_{0z}-k_z}}\mu_i\mu_j}{\epsilon_0 LS\hbar}\sinh(r)\cosh(r) S^+_{l,i}S^+_{l,j}\rho^S(t-\tau) \\
& -\frac{L}{2\pi}\int_{-2k_{0z}}^{0} dk_z \int_0^t d\tau e^{i\frac{c^2k_{jz}}{\omega_j}(k_z-k_{jz})\tau} e^{-i2k_{0z}(r_l-o_2)}\frac{\sqrt{\omega_{k_z}\omega_{-2k_{0z}-k_z}}\mu_i\mu_j}{\epsilon_0 LS\hbar}\sinh(r)\cosh(r) S^+_{l,i}S^+_{l,j}\rho^S(t-\tau) \\
=& -\frac{L}{2\pi}[e^{i2k_{0z}(r_l-o_1)} + e^{-i2k_{0z}(r_l-o_2)}]\frac{\sqrt{\omega_i\omega_j}\mu_i\mu_j}{\epsilon_0 LS\hbar}\int_0^t d\tau 2\pi\delta(\frac{c^2k_{jz}}{\omega_j}\tau)\sinh(r)\cosh(r) S^+_{l,i}S^+_{l,j}\rho^S(t-\tau) \\
=& -e^{i2k_{jz}R}\frac{\omega_0^2\mu_i\mu_j}{\epsilon_0\hbar Sc^2k_{0z}}\cos(2k_{0z}r_l)\sinh(r)\cosh(r) S^+_{l,i}S^+_{l,j}\rho^S(t) \\
=& -e^{i2k_{0z}R}\frac{\sqrt{\gamma_i\gamma_j}}{2}\cos(2k_{0z}r_l)\sinh(r)\cosh(r) S^+_{l,i}S^+_{l,j}\rho^S(t)
\end{aligned} \tag{A8}$$

where we have used the fact that the origin of coordinate system is at equal distant from two sources(i.e., $o_2 = -o_1 = R$) in the second last line. Incorporating index l into i, we have $\gamma'_{ij} = \sqrt{\gamma_i\gamma_j}\cos(2k_{0z}r_i)$. For $r_i \neq r_j$, Eq. (A7) reduces to

$$\begin{aligned}
&\sum_{k_z}\int_0^t d\tau\{\boldsymbol{\mu}_{l,i}\cdot\boldsymbol{u}_{2\boldsymbol{k}_0-\boldsymbol{k}}(r_l)S^+_{l,i}\boldsymbol{\mu}_{m,j}\cdot\boldsymbol{u}_{\boldsymbol{k}}(r_m)S^+_{m,j}e^{i(\omega_{\boldsymbol{k}}-\omega_j)\tau}[-\sinh(r)\cosh(r)]\rho^S(t-\tau)\\
=&-\frac{L}{2\pi}\int_0^{2k_{0z}}dk_z\int_0^t d\tau e^{i\frac{c^2k_{jz}}{\omega_j}(k_z-k_{jz})\tau}e^{i2k_{0z}(r_c-o_1)}e^{-i(k_z-k_{0z})(r_l-r_m)}\frac{\sqrt{\omega_{k_z}\omega_{2k_{0z}-k_z}}\mu_i\mu_j}{\epsilon_0 LS\hbar}\sinh(r)\cosh(r)S^+_{l,i}S^+_{m,j}\rho^S(t-\tau)\\
&-\frac{L}{2\pi}\int_{-2k_{0z}}^0 dk_z\int_0^t d\tau e^{i\frac{c^2k_{jz}}{\omega_j}(-k_z-k_{jz})\tau}e^{-i2k_{0z}(r_c-o_2)}e^{-i(k_z+k_{0z})(r_l-r_m)}\frac{\sqrt{\omega_{k_z}\omega_{-2k_{0z}-k_z}}\mu_i\mu_j}{\epsilon_0 LS\hbar}\sinh(r)\cosh(r)S^+_{l,i}S^+_{m,j}\rho^S(t-\tau)\\
=&-\frac{L}{2\pi}e^{i2k_{0z}(r_c-o_1)}\frac{\sqrt{\omega_i\omega_j}\mu_i\mu_j}{\epsilon_0 LS\hbar}\int_{-\infty}^{\infty}dk_z\int_0^t d\tau e^{i\frac{c^2k_{jz}}{\omega_j}(k_z-k_{jz})\tau}e^{-i(k_z-k_{0z})(r_l-r_m)}\sinh(r)\cosh(r)S^+_{l,i}S^+_{m,j}\rho^S(t-\tau)\\
&-\frac{L}{2\pi}e^{-i2k_{0z}(r_c-o_2)}\frac{\sqrt{\omega_i\omega_j}\mu_i\mu_j}{\epsilon_0 LS\hbar}\int_{-\infty}^{\infty}dk_z\int_0^t d\tau e^{i\frac{c^2k_{jz}}{\omega_j}(k_z-k_{jz})\tau}e^{i(k_z-k_{0z})(r_l-r_m)}\sinh(r)\cosh(r)S^+_{l,i}S^+_{m,j}\rho^S(t-\tau)\\
\approx&-\frac{L}{2\pi}e^{i2k_{0z}R}\frac{\omega_0^2\mu_i\mu_j}{\epsilon_0 LS\hbar}\int_0^t d\tau 2\pi[e^{i2k_{0z}r_c}\delta(r_l-r_m-\frac{c^2k_{0z}}{\omega_0}\tau)+e^{-i2k_{0z}r_c}\delta(r_l-r_m+\frac{c^2k_{0z}}{\omega_0}\tau)]\sinh(r)\cosh(r)S^+_{l,i}S^+_{m,j}\rho^S(t-\tau)\\
\approx&-e^{i2k_{0z}R}\frac{\omega_0^2\mu_i\mu_j}{\epsilon_0\hbar Sc^2k_{0z}}e^{i2k_{0z}r_c sgn(r_l-r_m)}S^+_{l,i}S^+_{m,j}\rho^S(t)\rightarrow-\frac{\sqrt{\gamma_i\gamma_j}}{2}e^{i2k_{0z}R}\cos(k_{0z}(r_l+r_m))S^+_{l,i}S^+_{m,j}\rho^S(t)
\end{aligned}\tag{A9}$$

where $sgn(r_l-r_m)$ is the sign function. The last arrow is because we need to sum over i, j, so the imaginary part of $e^{i2k_{0z}r_c sgn(i-j)}$ vanishes, so the neat result is that $\gamma'_{ij} = e^{i2k_{0z}R}\sqrt{\gamma_i\gamma_j}\cos(k_{0z}(r_i+r_j))$ after combining index l, i and m, j as i and j. As for $S^+_i\rho^S(t)S^+_j$ terms, the combination of the last two terms in Eq.(A3) will make the imaginary part of $e^{i2k_{0z}r_c sgn(r_l-r_m)}$ vanish. Thus, we have $\gamma'_{ij} = e^{i2k_{0z}R}\sqrt{\gamma_i\gamma_j}\cos(k_{0z}(r_i+r_j))$ in Eq.(5).

APPENDIX B: DERIVATION OF COEFFICIENTS EQ.(??)

Here we will show how to derive the master equation with coefficients Eq.(2) from the correlation function Eq. (9). Since the only difference is the squeezing terms $\left\langle a^\dagger_{\boldsymbol{k},s}a^\dagger_{\boldsymbol{k}',s'}\right\rangle$, $\langle a_{\boldsymbol{k},s}a_{\boldsymbol{k}',s'}\rangle$, we will just start from Eq. (A7). Apart from the factor $e^{i(\omega_i+\omega_j-2\omega_0)t}$, When $r_i \neq r_j$, Eq. (A7) becomes:

$$
\begin{aligned}
&\sum_{k_z}\int_0^t d\tau\{\boldsymbol{\mu}_{l,i}\cdot\boldsymbol{u}_{-2\boldsymbol{k}_0+\boldsymbol{k}}(r_l)S^+_{l,i}\boldsymbol{\mu}_{m,j}\cdot\boldsymbol{u}_{\boldsymbol{k}}(r_m)S^+_{m,j}e^{i(\omega_{\boldsymbol{k}}-\omega_0)\tau}[-\sinh(r)\cosh(r)]\rho^S(t-\tau)\\
\approx&-\frac{L}{2\pi}\int_0^{2k_0}dk\int_0^t d\tau e^{i\frac{c^2k_0}{\omega_0}(k-k_0)\tau}e^{-i(2k_0-k)(r_l-o_2)+ik(r_m-o_1)}\frac{\sqrt{\omega_{k_z}\omega_{2k_{0z}-k_z}}\mu_i\mu_j}{\epsilon_0 LS\hbar}\sinh(r)\cosh(r)S^+_{l,i}S^+_{m,j}\rho^S(t-\tau)\\
&-\frac{L}{2\pi}\int_{-2k_0}^{0}dk\int_0^t d\tau e^{i\frac{c^2k_0}{\omega_0}(-k-k_0)\tau}e^{i(2k_0+k)(r_l-o_2)+ik(r_m-o_1)}\frac{\sqrt{\omega_{k_z}\omega_{-2k_{0z}-k_z}}\mu_i\mu_j}{\epsilon_0 LS\hbar}\sinh(r)\cosh(r)S^+_{l,i}S^+_{m,j}\rho^S(t-\tau)\\
=&-\frac{L}{2\pi}\int_0^{2k_0}dk\int_0^t d\tau e^{i\frac{c^2k_0}{\omega_0}(-k_0)\tau}e^{-i(2k_0)(r_l-o_2)}e^{ik(\frac{c^2k_0}{\omega_0}\tau+r_l-o_1+r_m-o_2)}\frac{\sqrt{\omega_{k_z}\omega_{2k_{0z}-k_z}}\mu_i\mu_j}{\epsilon_0 LS\hbar}\sinh(r)\cosh(r)S^+_{l,i}S^+_{m,j}\rho^S(t-\tau)\\
&-\frac{L}{2\pi}\int_0^{2k_0}dk\int_0^t d\tau e^{i\frac{c^2k_0}{\omega_0}(-k_0)\tau}e^{i(2k_0)(r_l-o_2)}e^{ik(\frac{c^2k_0}{\omega_0}\tau-r_l+o_1-r_m+o_2)}\frac{\sqrt{\omega_{k_z}\omega_{-2k_{0z}-k_z}}\mu_i\mu_j}{\epsilon_0 LS\hbar}\sinh(r)\cosh(r)S^+_{l,i}S^+_{m,j}\rho^S(t-\tau)\\
\approx&-\frac{L}{2\pi}\int_0^t d\tau e^{i\frac{c^2k_0}{\omega_0}(-k_0)\tau}e^{-i(2k_0)(r_l)}e^{2ik_0o_2}2\pi\delta(\frac{c^2k_0}{\omega_0}\tau+2r_c-o_1-o_2)\frac{\sqrt{\omega_i\omega_j}\mu_i\mu_j}{\epsilon_0 LS\hbar}\sinh(r)\cosh(r)S^+_{l,i}S^+_{m,j}\rho^S(t-\tau)\\
&-\frac{L}{2\pi}\int_0^t d\tau e^{i\frac{c^2k_0}{\omega_0}(-k_0)\tau}e^{i(2k_0)(r_l)}e^{-2ik_0o_2}2\pi\delta(\frac{c^2k_0}{\omega_0}\tau-2r_c+o_1+o_2)\frac{\sqrt{\omega_i\omega_j}\mu_i\mu_j}{\epsilon_0 LS\hbar}\sinh(r)\cosh(r)S^+_{l,i}S^+_{m,j}\rho^S(t-\tau)\\
\approx&-\frac{L}{2\pi}e^{i(-k_0)|2r_c-o_1-o_2|}e^{sgn(r_c-o_1-o_2)i(2k_0)(r_l)}e^{-2ik_0o_2}2\pi\frac{\omega_0^2\mu_i\mu_j}{\epsilon_0 LS\hbar c^2k_{0z}}\sinh(r)\cosh(r)S^+_{l,i}S^+_{m,j}\rho^S(t)\\
=&-\frac{L}{2\pi}e^{sgn(2r_c-o_1-o_2)ik_0(r_l-r_m)}e^{ik_0(o_1-o_2)}2\pi\frac{\omega_0^2\mu_i\mu_j}{\epsilon_0 LS\hbar c^2k_{0z}}\sinh(r)\cosh(r)S^+_{l,i}S^+_{m,j}\rho^S(t)
\end{aligned}
\tag{B1}
$$

Since we need to sum over l, m, i, j, the imaginary part of $e^{sgn(2r_c-o_1-o_2)ik_0(r_l-r_m)}$ gets canceled.