

# The dynamics of a $\Xi$ -type atom in the Squeezed Vacuum

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## 1 Three Level Atom

In this section, we consider a scenario where a three-level atom is located inside the waveguide with the squeezed vacuum injected from both ends, as shown in Fig. 1(a). The atomic electronic structure is shown in Fig. 1(b) where the atomic states are labeled  $|a\rangle$ ,  $|b\rangle$ ,  $|c\rangle$  from the excited state to the ground state. We assume that  $\omega_{ac} = 2\omega_0$  where  $\omega_0$  is the center frequency of the broad band squeezed vacuum.  $\omega_{ab}$  and  $\omega_{bc}$  are not equal but they are still within the bandwidth of the squeezed vacuum.

The interaction Hamiltonian is:

$$V(t) = -i\hbar \sum_{\mathbf{k}s} [D(t)a_{\mathbf{k}s}(t) - D^\dagger(t)a_{\mathbf{k}s}^\dagger(t)], \quad (\text{A1})$$

where

$$D(t) = \sum_i [\boldsymbol{\mu}_i \cdot \mathbf{u}_{\mathbf{k},s}(r_i)S_i^\dagger(t) + \boldsymbol{\mu}_i^* \cdot \mathbf{u}_{\mathbf{k},s}(r_i)S_i^-(t)]. \quad (\text{A2})$$

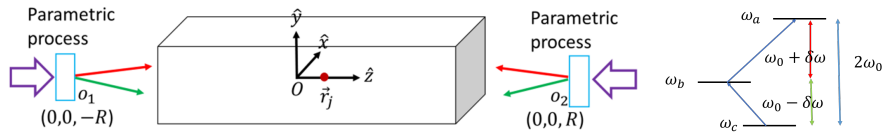


Fig. 1: (a) Schematic setup: A three-level atom is located inside the waveguide with the broadband squeezed vacuum incident from both ends. (b) The energy structure of the three level atom. Transition  $|a\rangle \rightarrow |c\rangle$  is forbidden and  $\omega_{ac} = 2\omega_0$  where  $\omega_0$  is the center frequency of the squeezed vacuum.  $\omega_{ab}$  and  $\omega_{bc}$  differ by a small amount  $2\delta\omega_0$  and they are within the bandwidth of the squeezed vacuum reservoir.

The reduced master equation of atoms in the reservoir is:

$$\begin{aligned}
\frac{d\rho^S}{dt} &= -\frac{1}{\hbar^2} \int_0^t d\tau \text{Tr}_F \{ [V(t), [V(t-\tau), \rho^S(t-\tau)\rho^F] \} \\
&= -\frac{1}{\hbar^2} \int_0^t d\tau \text{Tr}_F \{ V(t)V(t-\tau)\rho^S(t-\tau)\rho^F + \rho^S(t-\tau)\rho^F V(t-\tau)V(t) \\
&\quad - V(t)\rho^S(t-\tau)\rho^F V(t-\tau) - V(t-\tau)\rho^S(t-\tau)\rho^F V(t) \}.
\end{aligned} \tag{A3}$$

Here we just show how to deal with the first term in Eq.(A3), the remaining terms can be calculated in the same way. For the first term, we have

$$\begin{aligned}
& -\frac{1}{\hbar^2} \int_0^t d\tau \text{Tr}_F \{ V(t)V(t-\tau)\rho^S(t-\tau)\rho^F \} \\
&= \int_0^t d\tau \sum_{\mathbf{k}s, \mathbf{k}'s'} \{ D(t)D(t-\tau) \text{Tr}_F [\rho^F a_{\mathbf{k}s}(t) a_{\mathbf{k}'s'}(t-\tau)] - D(t)D^+(t-\tau) \text{Tr}_F [\rho^F a_{\mathbf{k}s}(t) a_{\mathbf{k}'s'}^\dagger(t-\tau)] \\
&\quad - D^+(t)D(t-\tau) \text{Tr}_F [\rho^F a_{\mathbf{k}s}^\dagger(t) a_{\mathbf{k}'s'}(t-\tau)] + D^+(t)D^+(t-\tau) \text{Tr}_F [\rho^F a_{\mathbf{k}s}^\dagger(t) a_{\mathbf{k}'s'}^\dagger(t-\tau)] \} \rho^S(t-\tau).
\end{aligned} \tag{A4}$$

Under the rotating wave approximation(RWA), we have

$$\begin{aligned}
& -\frac{1}{\hbar^2} \int_0^t d\tau \text{Tr}_F \{ V(t)V(t-\tau)\rho^S(t-\tau)\rho^F \} \\
&= \sum_{ij} \sum_{\mathbf{k}s, \mathbf{k}'s'} \int_0^t d\tau \{ \boldsymbol{\mu}_i \cdot \mathbf{u}_{\mathbf{k}s}(r_i) S_i^+ e^{i\omega_i t} \boldsymbol{\mu}_j \cdot \mathbf{u}_{\mathbf{k}'s'}(r_j) S_j^+ e^{i\omega_j(t-\tau)} e^{-i(\omega_{\mathbf{k}s} + \omega_{\mathbf{k}'s'})t + i\omega_{\mathbf{k}'s'}\tau} [-\sinh(r) \cosh(r) \delta_{\mathbf{k}', 2\mathbf{k}_0 - \mathbf{k}}] \\
&\quad - \boldsymbol{\mu}_i \cdot \mathbf{u}_{\mathbf{k}s}(r_i) S_i^+ e^{i\omega_i t} \boldsymbol{\mu}_j^* \cdot \mathbf{u}_{\mathbf{k}'s'}^*(r_j) S_j^- e^{-i\omega_j(t-\tau)} e^{-i\omega_{\mathbf{k}'s'}\tau} \cosh^2 r \delta_{\mathbf{k}\mathbf{k}'} \delta_{ss'} \\
&\quad - \boldsymbol{\mu}_i^* \cdot \mathbf{u}_{\mathbf{k}s}(r_i) S_i^- e^{-i\omega_i t} \boldsymbol{\mu}_j \cdot \mathbf{u}_{\mathbf{k}'s'}(r_j) S_j^+ e^{i\omega_j(t-\tau)} e^{-i\omega_{\mathbf{k}'s'}\tau} \cosh^2 r \delta_{\mathbf{k}\mathbf{k}'} \delta_{ss'} \\
&\quad - \boldsymbol{\mu}_i^* \cdot \mathbf{u}_{\mathbf{k}s}(r_i) S_i^- e^{-i\omega_i t} \boldsymbol{\mu}_j^* \cdot \mathbf{u}_{\mathbf{k}'s'}^*(r_j) S_j^+ e^{i\omega_j(t-\tau)} e^{i\omega_{\mathbf{k}'s'}\tau} \sinh^2 r \delta_{\mathbf{k}\mathbf{k}'} \delta_{ss'} \\
&\quad - \boldsymbol{\mu}_i \cdot \mathbf{u}_{\mathbf{k}s}(r_i) S_i^+ e^{i\omega_i t} \boldsymbol{\mu}_j^* \cdot \mathbf{u}_{\mathbf{k}'s'}^*(r_j) S_j^- e^{-i\omega_j(t-\tau)} e^{i\omega_{\mathbf{k}'s'}\tau} \sinh^2 r \delta_{\mathbf{k}\mathbf{k}'} \delta_{ss'} \\
&\quad + \boldsymbol{\mu}_i^* \cdot \mathbf{u}_{\mathbf{k}s}(r_i) S_i^- e^{-i\omega_i t} \boldsymbol{\mu}_j \cdot \mathbf{u}_{\mathbf{k}'s'}(r_j) S_j^- e^{-i\omega_j(t-\tau)} e^{i(\omega_{\mathbf{k}s} + \omega_{\mathbf{k}'s'})t - i\omega_{\mathbf{k}'s'}\tau} [-\sinh(r) \cosh(r) \delta_{\mathbf{k}', 2\mathbf{k}_0 - \mathbf{k}} \delta_{ss'}] \} \rho^S(t-\tau)
\end{aligned} \tag{A5}$$

Here we just calculate the first and second term to show how to get the master

equation Eq.(1). For the second term, we have

$$\begin{aligned}
& - \sum_{k_z} \int_0^t d\tau \boldsymbol{\mu}_i \cdot \mathbf{u}_{\mathbf{k}s}(r_i) S_i^+ e^{i\omega_i t} \boldsymbol{\mu}_j^* \cdot \mathbf{u}_{\mathbf{k}'s'}^*(r_j) S_j^- e^{-i\omega_j(t-\tau)} e^{-i\omega_{\mathbf{k}'s'}\tau} \cosh^2 r \rho^S(t-\tau) \delta_{\mathbf{k}\mathbf{k}'} \delta_{ss'} \\
&= - \frac{L}{2\pi} e^{i(\omega_i - \omega_j)t} \int_{-\infty}^{\infty} dk_z \int_0^t d\tau e^{i\omega_j \tau} e^{-i\omega_{k_z} \tau} \frac{\omega_k \mu_i \mu_j}{\epsilon_0 L S \hbar} e^{ik_z(r_i - r_j)} \cosh^2 r S_i^+ S_j^- \rho^S(t-\tau) \\
&\approx - \frac{L}{2\pi} e^{i(\omega_i - \omega_j)t} \int_0^{\infty} dk_z \int_0^t d\tau e^{i\omega_j \tau} e^{-i[\omega_j + c^2 k_{jz}(k_z - k_{jz})/\omega_j]\tau} \frac{\omega_k \mu_i \mu_j}{\epsilon_0 L S \hbar} [e^{ik_z(r_i - r_j)} + e^{-ik_z(r_i - r_j)}] \cosh^2 r S_i^+ S_j^- \rho^S(t-\tau) \\
&\approx - \frac{L}{2\pi} e^{i(\omega_i - \omega_j)t} \int_{-k_{0z}}^{\infty} d\delta k_z \int_0^t d\tau e^{-i\tau c^2 k_{jz} \delta k_z / \omega_j} \frac{\omega_k \mu_i \mu_j}{\epsilon_0 L S \hbar} [e^{i(k_{jz} + \delta k_z)(r_i - r_j)} + e^{-i(k_{jz} + \delta k_z)(r_i - r_j)}] \cosh^2 r S_i^+ S_j^- \rho^S(t-\tau) \\
&\approx - \frac{L}{2\pi} e^{i(\omega_i - \omega_j)t} \int_{-\infty}^{\infty} d\delta k_z \int_0^t d\tau e^{-i(c^2 k_{jz} \delta k_z / \omega_j)\tau} \frac{\omega_k \mu_i \mu_j}{\epsilon_0 L S \hbar} [e^{i(k_{jz} + \delta k_z)(r_i - r_j)} + e^{-i(k_{jz} + \delta k_z)(r_i - r_j)}] \cosh^2 r S_i^+ S_j^- \rho^S(t-\tau) \\
&\approx - \frac{L}{2\pi} e^{i(\omega_i - \omega_j)t} \int_0^t d\tau \frac{\omega_j \mu_i \mu_j}{\epsilon_0 L S \hbar} 2\pi [e^{ik_{jz}(r_i - r_j)} \delta((r_i - r_j) - \frac{c^2 k_{jz}}{\omega_0} \tau) + e^{-ik_{jz}(r_i - r_j)} \delta((r_i - r_j) + \frac{c^2 k_{jz}}{\omega_0} \tau)] \cosh^2 r S_i^+ S_j^- \rho^S(t-\tau) \\
&\approx - \frac{L}{2\pi} e^{ik_{jz} r_{ij}} \frac{\omega_j \mu_i \mu_j}{\epsilon_0 L S \hbar} 2\pi \frac{\omega_j}{c^2 k_{0z}} \cosh^2 r S_i^+ S_j^- \rho^S(t) e^{i(\omega_i - \omega_j)t} \\
&\approx - [\frac{\sqrt{\gamma_i \gamma_j}}{2} \cos(k_{0z} r_{ij}) + i \frac{\sqrt{\gamma_i \gamma_j}}{2} \sin(k_{0z} r_{ij})] \cosh^2 r S_i^+ S_j^- \rho^S(t) e^{i(\omega_i - \omega_j)t} \\
&\equiv - (\frac{\sqrt{\gamma_i \gamma_j}}{2} + i \Lambda_{ij}) \cosh^2 r S_i^+ S_j^- \rho^S(t) e^{i(\omega_i - \omega_j)t}
\end{aligned} \tag{A6}$$

where emitter separation  $r_{ij} = |r_i - r_j|$ ,  $\gamma_i = 2\mu_i^2 \omega_i^2 / \hbar \epsilon_0 S c^2 k_{iz}$  which is the collective decay rate when  $i = j$ , and  $\Lambda_{ij} = \gamma_{1d} \sin(k_{0z} r_{ij}) / 2$  is the collective energy shift. In the third line we expand  $\omega_k = c\sqrt{(\frac{\pi}{a})^2 + (k_z)^2}$  around  $k_z = k_{0z}$  since resonant modes provide dominant contributions. In the fifth line we extend the integration  $\int_{-k_{0z}}^{\infty} dk_z \rightarrow \int_{-\infty}^{\infty} dk_z$  because the main contribution comes from the components around  $\delta k_z = 0$ . In the next line, Weisskopf-Wigner approximation is used. Thus, we have obtained  $\gamma_{ij}$  and  $\Lambda_{ij}$  as is shown in Eq.(2).

Next we need to calculate the first term (squeezing term) in Eq.(A5):

$$\begin{aligned}
& e^{i(\omega_i + \omega_j - 2\omega_0)t} \sum_{k_z} \int_0^t d\tau \{ \boldsymbol{\mu}_i \cdot \mathbf{u}_{2\mathbf{k}_0 - \mathbf{k}}(r_i) S_i^+ \boldsymbol{\mu}_j \cdot \mathbf{u}_{\mathbf{k}}(r_j) S_j^+ e^{i(\omega_{\mathbf{k}} - \omega_j)\tau} [-\sinh(r) \cosh(r)] \rho^S(t-\tau) \\
&= - \frac{L}{2\pi} e^{i(\omega_i + \omega_j - 2\omega_0)t} \int_0^{2k_{0z}} dk_z \int_0^t d\tau e^{i(\omega_{k_z} - \omega_j)\tau} e^{i(2k_{jz} - k_z)(r_i - o_1)} e^{ik_z(r_j - o_1)} \frac{\sqrt{\omega_{k_z} \omega_{2k_{0z} - k_z}} \mu^2}{\epsilon_0 L S \hbar} \sinh(r) \cosh(r) S_i^+ \\
&- \frac{L}{2\pi} e^{i(\omega_i + \omega_j - 2\omega_0)t} \int_{-2k_{0z}}^0 dk_z \int_0^t d\tau e^{i(\omega_{k_z} - \omega_j)\tau} e^{i(-2k_{jz} - k_z)(r_i - o_2)} e^{ik_z(r_j - o_2)} \frac{\sqrt{\omega_{k_z} \omega_{-2k_{0z} - k_z}} \mu^2}{\epsilon_0 L S \hbar} \sinh(r) \cosh(r) S_i^+
\end{aligned} \tag{A7}$$

Putting the overall factor  $e^{i(\omega_i+\omega_j-2\omega_0)t}$  aside, for  $r_i = r_j$ , Eq.(A7) reduces to

$$\begin{aligned}
& \sum_{k_z} \int_0^t d\tau \{ \boldsymbol{\mu}_i \cdot \mathbf{u}_{2\mathbf{k}_0 - \mathbf{k}}(r_i) S_i^+ \boldsymbol{\mu}_j \cdot \mathbf{u}_{\mathbf{k}}(r_j) S_j^+ e^{i(\omega_{\mathbf{k}} - \omega_j)\tau} [-\sinh(r) \cosh(r)] \rho^S(t - \tau) \\
&= -\frac{L}{2\pi} \int_0^{2k_{0z}} dk_z \int_0^t d\tau e^{i\frac{c^2 k_{jz}}{\omega_j}(k_z - k_{jz})\tau} e^{i2k_{0z}(r_i - o_1)} \frac{\sqrt{\omega_{k_z} \omega_{2k_{0z} - k_z}} \mu_i \mu_j}{\epsilon_0 L S \hbar} \sinh(r) \cosh(r) S_i^+ S_j^+ \rho^S(t - \tau) \\
&- \frac{L}{2\pi} \int_{-2k_{0z}}^0 dk_z \int_0^t d\tau e^{i\frac{c^2 k_{jz}}{\omega_j}(k_z - k_{jz})\tau} e^{-i2k_{0z}(r_i - o_2)} \frac{\sqrt{\omega_{k_z} \omega_{-2k_{0z} - k_z}} \mu_i \mu_j}{\epsilon_0 L S \hbar} \sinh(r) \cosh(r) S_i^+ S_j^+ \rho^S(t - \tau) \\
&= -\frac{L}{2\pi} [e^{i2k_{0z}(r_i - o_1)} + e^{-i2k_{0z}(r_i - o_2)}] \frac{\sqrt{\omega_i \omega_j} \mu_i \mu_j}{\epsilon_0 L S \hbar} \int_0^t d\tau 2\pi \delta\left(\frac{c^2 k_{jz}}{\omega_j} \tau\right) \sinh(r) \cosh(r) S_i^+ S_j^+ \rho^S(t - \tau) \\
&= -e^{i2k_{jz}R} \frac{\omega_0^2 \mu_i \mu_j}{\epsilon_0 \hbar S c^2 k_{0z}} \cos(2k_{0z}r_i) \sinh(r) \cosh(r) S_i^+ S_j^+ \rho^S(t) \\
&= -e^{i2k_{0z}R} \frac{\sqrt{\gamma_i \gamma_j}}{2} \cos(2k_{0z}r_i) \sinh(r) \cosh(r) S_i^+ S_j^+ \rho^S(t)
\end{aligned} \tag{A8}$$

where we have used the fact that the origin of coordinate system is at equal distant from two sources(i.e.,  $o_2 = -o_1 = R$ ) in the second last line. Thus, we have  $\gamma'_{ij} = \sqrt{\gamma_i \gamma_j} \cos(2k_{0z}r_i)$ . For  $r_i \neq r_j$ , Eq. (A7) reduces to

$$\begin{aligned}
& \sum_{k_z} \int_0^t d\tau \{ \boldsymbol{\mu}_i \cdot \mathbf{u}_{2\mathbf{k}_0 - \mathbf{k}}(r_i) S_i^+ \boldsymbol{\mu}_j \cdot \mathbf{u}_{\mathbf{k}}(r_j) S_j^+ e^{i(\omega_{\mathbf{k}} - \omega_j)\tau} [-\sinh(r) \cosh(r)] \rho^S(t - \tau) \\
&= -\frac{L}{2\pi} \int_0^{2k_{0z}} dk_z \int_0^t d\tau e^{i\frac{c^2 k_{jz}}{\omega_j}(k_z - k_{jz})\tau} e^{i2k_{0z}(r_c - o_1)} e^{-i(k_z - k_{0z})(r_i - r_j)} \frac{\sqrt{\omega_{k_z} \omega_{2k_{0z} - k_z}} \mu_i \mu_j}{\epsilon_0 L S \hbar} \sinh(r) \cosh(r) S_i^+ S_j^+ \rho^S(t - \tau) \\
&- \frac{L}{2\pi} \int_{-2k_{0z}}^0 dk_z \int_0^t d\tau e^{i\frac{c^2 k_{jz}}{\omega_j}(-k_z - k_{jz})\tau} e^{-i2k_{0z}(r_c - o_2)} e^{-i(k_z + k_{0z})(r_i - r_j)} \frac{\sqrt{\omega_{k_z} \omega_{-2k_{0z} - k_z}} \mu_i \mu_j}{\epsilon_0 L S \hbar} \sinh(r) \cosh(r) S_i^+ S_j^+ \rho^S(t - \tau) \\
&= -\frac{L}{2\pi} e^{i2k_{0z}(r_c - o_1)} \frac{\sqrt{\omega_i \omega_j} \mu_i \mu_j}{\epsilon_0 L S \hbar} \int_{-\infty}^{\infty} dk_z \int_0^t d\tau e^{i\frac{c^2 k_{jz}}{\omega_j}(k_z - k_{jz})\tau} e^{-i(k_z - k_{0z})(r_i - r_j)} \sinh(r) \cosh(r) S_i^+ S_j^+ \rho^S(t - \tau) \\
&- \frac{L}{2\pi} e^{-i2k_{0z}(r_c - o_2)} \frac{\sqrt{\omega_i \omega_j} \mu_i \mu_j}{\epsilon_0 L S \hbar} \int_{-\infty}^{\infty} dk_z \int_0^t d\tau e^{i\frac{c^2 k_{jz}}{\omega_j}(k_z - k_{jz})\tau} e^{i(k_z - k_{0z})(r_i - r_j)} \sinh(r) \cosh(r) S_i^+ S_j^+ \rho^S(t - \tau) \\
&\approx -\frac{L}{2\pi} e^{i2k_{0z}R} \frac{\omega_0^2 \mu_i \mu_j}{\epsilon_0 L S \hbar} \int_0^t d\tau 2\pi [e^{i2k_{0z}r_c} \delta(r_i - r_j - \frac{c^2 k_{0z}}{\omega_0} \tau) + e^{-i2k_{0z}r_c} \delta(r_i - r_j + \frac{c^2 k_{0z}}{\omega_0} \tau)] \sinh(r) \cosh(r) S_i^+ S_j^+ \rho^S(t) \\
&\approx -e^{i2k_{0z}R} \frac{\omega_0^2 \mu_i \mu_j}{\epsilon_0 \hbar S c^2 k_{0z}} e^{i2k_{0z}r_c \text{sgn}(i-j)} S_i^+ S_j^+ \rho^S(t) \rightarrow -\frac{\sqrt{\gamma_i \gamma_j}}{2} e^{i2k_{0z}R} \cos(k_{0z}(r_i + r_j)) S_i^+ S_j^+ \rho^S(t)
\end{aligned} \tag{A9}$$

where  $\text{sgn}(i - j)$  is the sign function. The last arrow is because we need to sum over  $i, j$ , so the imaginary part of  $e^{i2k_{0z}r_c \text{sgn}(i-j)}$  vanishes and the neat result is that  $\gamma'_{ij} = e^{i2k_{0z}R} \sqrt{\gamma_i \gamma_j} \cos(k_{0z}(r_i + r_j))$ . As for  $S_i^+ \rho^S(t) S_j^+$  terms, the combination of the last two terms in Eq.(A3) will make the imaginary part of  $e^{i2k_{0z}r_c \text{sgn}(i-j)}$  vanish. Thus, we have  $\gamma'_{ij} = e^{i2k_{0z}R} \sqrt{\gamma_i \gamma_j} \cos(k_{0z}(r_i + r_j))$ . If

one needs to get  $\gamma_{ij}, \gamma'_{ij}$  and  $\Lambda_{ij}$  in the unidirectional waveguide case, we just need to discard the second terms in the parenthesis of Eq.(A6) and Eq.(A9).

$$\begin{aligned}
\frac{d\rho^S}{dt} = & -i \sum_{i \neq j} \Lambda_{ij} [S_i^+ S_j^-, \rho^S] e^{i(\omega_i - \omega_j)t} \\
& - \frac{1}{2} \sum_{i,j} \gamma_{ij} (1+N) (\rho^S S_i^+ S_j^- + S_i^+ S_j^- \rho^S - 2S_j^- \rho^S S_i^+) e^{i(\omega_i - \omega_j)t} \\
& - \frac{1}{2} \sum_{i,j} \gamma_{ij} N (\rho^S S_i^- S_j^+ + S_i^- S_j^+ \rho^S - 2S_j^+ \rho^S S_i^-) e^{-i(\omega_i - \omega_j)t} \\
& - \frac{1}{2} \sum_{\alpha=\pm} \sum_{i,j} \gamma'_{ij} M e^{2\alpha i k_{0z} R} e^{i\alpha(\omega_i + \omega_j - 2\omega_0)t} (\rho^S S_i^\alpha S_j^\alpha + S_i^\alpha S_j^\alpha \rho^S - 2S_j^\alpha \rho^S S_i^\alpha)
\end{aligned} \tag{1}$$

where the coefficients are

$$\begin{aligned}
\gamma_{ij} &= \sqrt{\gamma_i \gamma_j} \cos(k_{0z} r_{ij}) \\
\Lambda_{ij} &= \frac{\sqrt{\gamma_i \gamma_j}}{2} \sin(k_{0z} r_{ij}) \\
\gamma'_{ij} &= \sqrt{\gamma_i \gamma_j} \cos[k_{0z}(r_i + r_j)]
\end{aligned} \tag{2}$$

where  $\gamma_i$  is the decay rate for transition  $i$  in ordinary vacuum. For a single three level atom, we have  $r_i = r_j$ , for simplicity we set  $R = r_i = 0$  and  $\gamma_1 = \gamma_2 = \gamma$ . After applying the rotating wave approximation(RWA), the master equation Eq.(1) becomes (see Appendix A)

$$\begin{aligned}
\frac{d\rho^S}{dt} = & -\frac{1}{2} \sum_i \gamma (1+N) (\rho^S S_i^+ S_i^- + S_i^+ S_i^- \rho^S - 2S_i^- \rho^S S_i^+) \\
& - \frac{1}{2} \sum_i \gamma N (\rho^S S_i^- S_i^+ + S_i^- S_i^+ \rho^S - 2S_i^+ \rho^S S_i^-) \\
& - \frac{1}{2} \sum_{\alpha=\pm} \sum_{i \neq j} \gamma M (\rho^S S_i^\alpha S_j^\alpha + S_i^\alpha S_j^\alpha \rho^S - 2S_j^\alpha \rho^S S_i^\alpha)
\end{aligned} \tag{3}$$

where  $N = \sinh(r)^2$  and  $M = \sinh(r) \cosh(r)$  The steady state of Eq.(3) can be

derived by re-writing Eq.(3) as:

$$\dot{\rho}_{aa}/\gamma = -ch^2\rho_{aa} + sh^2\rho_{bb} - \frac{1}{2}chsh(\rho_{ac} + \rho_{ca}) \quad (4a)$$

$$\dot{\rho}_{bb}/\gamma = (ch^2\rho_{aa} - sh^2\rho_{bb}) + (sh^2\rho_{cc} - ch^2\rho_{bb}) + chsh(\rho_{ac} + \rho_{ca}) \quad (4b)$$

$$\dot{\rho}_{cc}/\gamma = ch^2\rho_{bb} - sh^2\rho_{cc} - \frac{1}{2}chsh(\rho_{ac} + \rho_{ca}) \quad (4c)$$

$$(\dot{\rho}_{ac} + \dot{\rho}_{ca})/\gamma = -\frac{1}{2}(ch^2 + sh^2)(\rho_{ac} + \rho_{ca}) - shch(\rho_{aa} - 2\rho_{bb} + \rho_{cc}) \quad (4d)$$

$$\dot{\rho}_{ab}/\gamma = -(1 + \frac{3}{2}sh^2)\rho_{ab} - \frac{1}{2}chsh\rho_{cb} \quad (4e)$$

$$\dot{\rho}_{cb}/\gamma = -\frac{1}{2}chsh\rho_{ab} - (\frac{1}{2} + \frac{3}{2}sh^2)\rho_{cb} \quad (4f)$$

where  $sh = \sinh(r)$ ,  $ch = \cosh(r)$ . Eq.(4e)(4f) yield  $\rho_{ab} = \rho_{cb} = 0$  for the steady state, and Eq.(4a)-(4d) yield  $\rho_{aa} = \frac{sh^2}{sh^2+ch^2}$ ,  $\rho_{cc} = \frac{ch^2}{sh^2+ch^2}$ ,  $\rho_{ac} = -\frac{shch}{sh^2+ch^2}$ . Thus, the steady state is actually a superposition state of  $|a\rangle$  and  $|c\rangle$ :  $\frac{sh}{\sqrt{sh^2+ch^2}}|a\rangle - \frac{ch}{\sqrt{sh^2+ch^2}}|c\rangle$ . This phenomenon is similar to coherent trapping, but here we achieve the trapping for  $\Xi$  structure with the squeezed vacuum reservoir, which cannot be realized with coherent pump due to spontaneous emission.

In general we can study the steady state population distribution when the dipole moments of transition  $|a\rangle \rightarrow |b\rangle$  and  $|b\rangle \rightarrow |c\rangle$  are different, i.e.,  $\mu_{ab} \neq \mu_{bc}$ . Although the analytical solution is quite lengthy, we can easily get the numerical solution for different  $\mu_{ab}$  and  $\mu_{bc}$ . In Fig. 2(a), population inversion is achieved with  $\mu_{ab} < \mu_{bc}$ . This can be interpreted with the help of Fig. 2(c). Figure 2(c) shows that the direct transition between  $|a\rangle$ ,  $|b\rangle$ , and  $|c\rangle$  are allowed just like the thermal reservoir case. However, in the squeezed vacuum, there is additional paths for population flow: electrons in any of these three states can evolve into the other two through an intermediate "state"  $\rho_{ac}$ . Although  $\rho_{ac}$  is actually not a state, and  $\rho_{ac} < 0$  in our convention, it can be used to elucidate our idea. When  $\mu_{ab} \ll \mu_{bc}$ , the transition  $|a\rangle \rightarrow |b\rangle$  can be removed. Thus state  $|c\rangle$  can be excited to  $|a\rangle$  through  $|c\rangle \rightarrow |b\rangle \rightarrow \rho_{ac} \rightarrow |a\rangle$ , but  $|a\rangle$  can not decay back to  $|c\rangle$ . Thus, the population is trapped from  $|c\rangle$  to  $|a\rangle$ . Although the population inversion between  $|a\rangle$  and  $|c\rangle$  is very sensitive to the value of  $M$ , the population inversion between  $|a\rangle$  and  $|b\rangle$  still holds for  $M = 0.8\sqrt{N(N+1)}$ , which is shown in Fig. 2(b).

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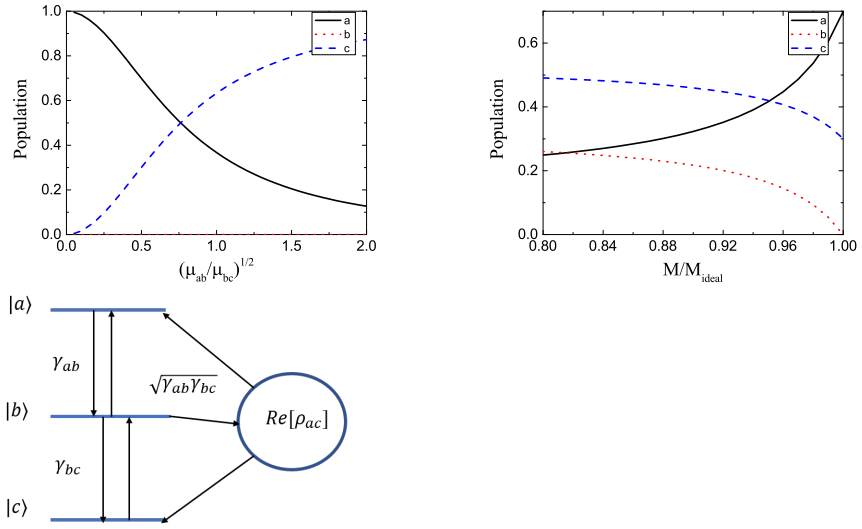


Fig. 2: (a) The steady state population distribution for different  $\mu_{ab}$  and  $\mu_{bc}$ . The squeezing parameter  $r = 1$ . (b) The steady state population distribution for non-ideal squeezed vacuum which is characterized by the ratio of  $M$  and  $\sqrt{N(N+1)}$ . (c) The allowed population flow in the squeezed vacuum. The squeezing parameter  $r = 1$ , and  $\mu_{ab} = \frac{1}{4}\mu_{bc}$ .

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