The steady state population inversion of multiple Ξ-type atoms by the squeezed vacuum in a waveguide

Jieyu You, Zeyang Liao, and M. Suhail Zubairy
Institute for Quantum Science and Engineering (IQSE) and Department of Physics and Astronomy, Texas A&M University, College Station, TX 77843-4242, USA

We study the steady state of Ξ-type atoms coupled to the squeezed vacuum reservoir in the quasi-one-dimensional waveguide. We show that the system ends up in a steady state which is a pure state, and the complete population inversion can occur when the direction of dipole moment is properly set. This result can be generalized to arbitrary number of atoms coupled to each other by dipole-dipole interaction.

PACS numbers: 42.30.-d, 42.50.Hz, 42.62.Fi

I. INTRODUCTION

The concept of population inversion is of fundamental importance in laser physics because the production of population inversion is a key step in the workings of a standard laser. However, a population inversion can never exist for a system at thermal equilibrium because of the spontaneous emission. The achievement of population inversion therefore requires pushing the system into a non-equilibrated state[1]. Thus, the spontaneous emission must be modified in order to maintain the population inversion in a steady state. In 1946, Purcell showed that the spontaneous decay rate of an emitter can be modified by engineering the electromagnetic bath environment with which the emitters interact[2]. One famous example of bath engineering is the squeezed vacuum which leads to many novel effects and techniques in quantum optics and atomic spectroscopy. The reduction of quantum fluctuations below vacuum level by the squeezed vacuum yields many intersting phenomenons, for example, the suppression of dephasing rate in one direction and enhancment in the other for a two-level emitter[3–10], and the subnatural linewidth of resonance fluorescence[11, 12]. The entanglement nature of the squeezed vacuum leads to pairwise excitation of atomic states[13–15]. In 1993, Ficek et. al. studied the dynamical properties of a three-level atom in the squeezed vacuum with the result that a three-level atom in the cascade configuration coupled only to squeezed modes can reach steady state level populations relative to ordinary laser spectroscopy[16, 17]. In their proposal, the drawback that not all modes are squeezed due to techniques is no longer an issue at all. Recently, photon transport in a one-dimensional (1D) waveguide coupled to quantum emitters (well known as "waveguide-QED") has attracted much attention due to its possible applications in quantum device and quantum information[18–22]. In contrast to the 3D case, squeezing in 1D is more experimentally feasible. Suppression of the radiative decay of atomic coherence and the linewidth of the resonance fluorescence have been experimentally demonstrated in a 1D microwave transmission line coupled to single artificial atom[12, 23–27]. However, there are still two issues in Ficek's proposal: first, only a single independent atom is considered to be interacting with the squeezed vacuum, while the dipole-dipole interaction between atoms may affect the steady state population distribution; second, the effect of squeezing source is not included as discussed in Ref.[15]. Our goal is to fix these three issues.

In our study, we consider the Ξ-type atoms coupled to the broadband squeezed vacuum in the quasi-one-dimensional waveguide. and we found that the population of steady state is completely different than that in the thermal reservoir. We also demonstrate that the complete population inversion can be achieved with proper parameters.

II. MASTER EQUATION OF THREE LEVEL ATOMS IN THE SQUEEZED VACUUM

In this section, we consider a scenario where N_a Ξ-type atoms are located inside the waveguide with the squeezed vacuum injected from both ends, as shown in Fig. 1(a). The atomic electronic structure is shown in Fig. 1(b) where the atomic states are labeled $|a\rangle$, $|b\rangle$, $|c\rangle$ from the excited state to the ground state. We assume that $\omega_{ac} = 2\omega_0$ where ω_0 is the center frequency of the broad band squeed vacuum. ω_{ab} and ω_{bc} are not equal but they are still within the bandwith of the squeezed vacuum. We assume that the squeezed vacuum bandwidth is much larger than $|\omega_{ab} - \omega_{bc}|$ so it forms a squeezed vacuum reservoir.

The atom-field system is described by the Hamiltonian

$$H = H_A + H_F + H_{AF} \tag{1}$$

where $H_A = \sum_{e=a,b,c}\sum_{l=1}^{N_a} \hbar\omega_{e,l}\,|e_l\rangle\langle e_l|$ is the atomic Hamiltonian, and $|e_l\rangle$ is the energy state of the lth atom with energy $\hbar\omega_{e,l}$. The Hamiltonian of the EM field is $H_F = \sum_{\boldsymbol{k}s}\hbar\omega_{\boldsymbol{k}s}(\hat{a}^\dagger_{\boldsymbol{k}s}\hat{a}_{\boldsymbol{k}s} + \frac{1}{2})$ where $\hat{a}_{\boldsymbol{k}s}$ and $\hat{a}^\dagger_{\boldsymbol{k}s}$ are the annihilation and creation operators of the filed mode with wavevector $\boldsymbol{k}$, polarization s, and frequency $\omega_{\boldsymbol{k},s}$. The interaction Hamiltonian in electric-dipole approximation is $H_{AF} = -i\hbar\sum_{\boldsymbol{k}s}\sum_{i=1,2}\sum_{l=1}^{N_a}[\boldsymbol{\mu}_{l,i}\cdot\boldsymbol{u}_{\boldsymbol{k}s}(\boldsymbol{r}_{l,i})S^+_{l,i}\hat{a}_{\boldsymbol{k}s} + \boldsymbol{\mu}^*_{l,i}\cdot\boldsymbol{u}_{\boldsymbol{k}s}(\boldsymbol{r}_{l,i})S^-_{l,i}\hat{a}_{\boldsymbol{k}s} - H.c.]$ where $\boldsymbol{\mu}_{l,i}$ is the electric dipole

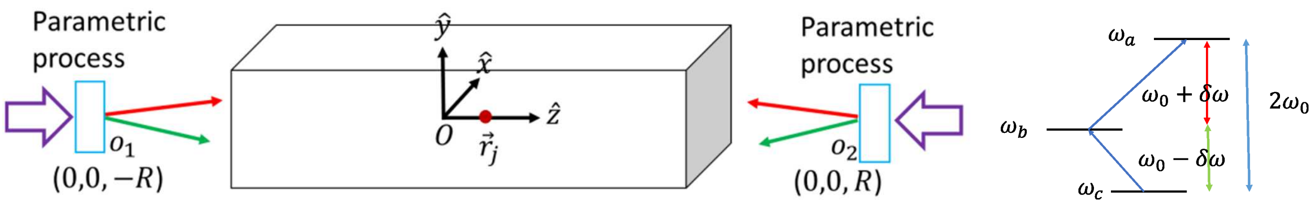

Fig. 1: (a) Schematic setup: A Ξ-type atom is located inside the waveguide with the broadband squeezed vacuum incident from both ends. (b)The energy structure of the three level atom. Transition $|a\rangle \to |c\rangle$ is forbidden and $\omega_{ac} = 2\omega_0$ where ω_0 is the center frequency of the squeezed vacuum. ω_{ab} and ω_{bc} differ by a small amount $2\delta\omega_0$ and they are within the bandwidth of the squeezed vacuum reservoir.

moment for ith transition of the lth atom, where $i = 1$ denotes the transition from $|a\rangle$ to $|b\rangle$, and $i = 2$ denotes the transition from $|b\rangle$ to $|c\rangle$. $S^+_{l,i}$ and $S^-_{l,i}$ are the raising and lowering operator for transition i of the lth atom. The mode function of the squeezed vacuum is given by

$$\boldsymbol{u}_{\boldsymbol{k}s}(\boldsymbol{r}_i) = \sqrt{\frac{\omega_{\boldsymbol{k}s}}{2\epsilon_0 \hbar V}}\boldsymbol{e}_{ks} e^{i\boldsymbol{k}\cdot(\boldsymbol{r}_i - \boldsymbol{o}_{\boldsymbol{k}s})} \tag{2}$$

where $\boldsymbol{o}_{\boldsymbol{k}s}$ is a phenomenological parameter which includes the effects of the initial phase and the position of the squeezing source[15]. The correlation functions for the squeezed vacuum are[28]:

$$\begin{aligned} \left\langle a^\dagger_{\boldsymbol{k},s} a_{\boldsymbol{k}',s'} \right\rangle &= \sinh^2 r \delta_{\boldsymbol{k}'\boldsymbol{k}} \delta_{ss'} \\ \left\langle a_{\boldsymbol{k},s} a^\dagger_{\boldsymbol{k}',s'} \right\rangle &= \cosh^2 r \delta_{\boldsymbol{k}'\boldsymbol{k}} \delta_{ss'} \\ \left\langle a^\dagger_{\boldsymbol{k},s} a^\dagger_{\boldsymbol{k}',s'} \right\rangle &= -e^{-i\theta} \cosh(r)\sinh(r)\delta_{\boldsymbol{k}',2\boldsymbol{k}_0-\boldsymbol{k}}\delta_{ss'} \\ \langle a_{\boldsymbol{k},s} a_{\boldsymbol{k}',s'} \rangle &= -e^{i\theta}\cosh(r)\sinh(r)\delta_{\boldsymbol{k}',2\boldsymbol{k}_0-\boldsymbol{k}}\delta_{ss'} \end{aligned} \tag{3}$$

For simplicity, we can set the squeezing parameter $\theta = 0$, and all atoms to align along the same direction. The dynamics of the atomic system can be described by the following master equation(See Appendix A for details of derivation):

$$\begin{aligned} \frac{d\rho^S}{dt} = &- i\sum_{ijkl}\Lambda_{ijkl}[S^+_{i,j}S^-_{k.l},\rho^S]e^{i(\omega_j-\omega_l)t} - \frac{1}{2}\sum_{ijkl}\gamma_{ijkl}(1+N)(\rho^S S^+_{i,j}S^-_{k.l} + S^+_{i,j}S^-_{k.l}\rho^S - 2S^-_{k.l}\rho^S S^+_{i,j})e^{i(\omega_j-\omega_l)t} \\ &- \frac{1}{2}\sum_{ijkl}\gamma_{ijkl}N(\rho^S S^-_{i,j}S^+_{k.l} + S^-_{i,j}S^+_{k,l}\rho^S - 2S^+_{k,l}\rho^S S^-_{i,j})e^{-i(\omega_j-\omega_l)t} \\ &- \frac{1}{2}\sum_{\alpha=\pm}\sum_{ijkl}\gamma'_{ijkl}Me^{2\alpha i k_{0z}R}e^{i\alpha(\omega_j+\omega_l-2\omega_0)t}(\rho^S S^\alpha_{i,j}S^\alpha_{k.l} + S^\alpha_{i,j}S^\alpha_{k,l}\rho^S - 2S^\alpha_{k,l}\rho^S S^\alpha_{i,j}) \end{aligned} \tag{4}$$

where $N = \sinh(r)^2$, $M = \sinh(r)\cosh(r)$, and the coefficients are

$$\begin{aligned} \gamma_{ijkl} &= \sqrt{\gamma_j\gamma_l}\cos(k_{0z}r_{ik}) \\ \Lambda_{ijkl} &= \frac{\sqrt{\gamma_j\gamma_l}}{2}\sin(k_{0z}r_{ik}) \\ \gamma'_{ijkl} &= \sqrt{\gamma_j\gamma_l}\cos[k_{0z}(r_i + r_k)] \end{aligned} \tag{5}$$

where subscript i, k label the atoms, $j(l)$ labels the transitions of the $ith(jth)$ atom. γ_j is the decay rate for transition j in ordinary vacuum.

III. STEADY STATE OF A SINGLE ATOM

In this section, we will study the steady state of a single atom in the squeezed vacuum reservior. For a single three level atom, we have $r_i = r_k$, and for simplicity we set $R = r_i = 0$. Then the steady state of Eq.(4) can be derived by re-writing it as:

$$\dot{\rho}_{aa} = -\gamma_1 ch^2 \rho_{aa} + \gamma_1 sh^2 \rho_{bb} - \frac{1}{2}\sqrt{\gamma_1\gamma_2} chsh(e^{-i(\omega_1+\omega_2-2\omega_0)t}\rho_{ac} + e^{i(\omega_1+\omega_2-2\omega_0)t}\rho_{ca}) \tag{6a}$$

$$\dot{\rho}_{bb} = \gamma_1(ch^2\rho_{aa} - sh^2\rho_{bb}) + \gamma_2(sh^2\rho_{cc} - ch^2\rho_{bb}) + \sqrt{\gamma_1\gamma_2} chsh(e^{-i(\omega_1+\omega_2-2\omega_0)t}\rho_{ac} + e^{i(\omega_1+\omega_2-2\omega_0)t}\rho_{ca}) \tag{6b}$$

$$\dot{\rho}_{cc} = \gamma_2 ch^2 \rho_{bb} - \gamma_2 sh^2 \rho_{cc} - \frac{1}{2}\sqrt{\gamma_1\gamma_2} chsh(e^{-i(\omega_1+\omega_2-2\omega_0)t}\rho_{ac} + e^{i(\omega_1+\omega_2-2\omega_0)t}\rho_{ca}) \tag{6c}$$

$$\begin{aligned} e^{-i(\omega_1+\omega_2-2\omega_0)t}\dot{\rho}_{ac} + e^{i(\omega_1+\omega_2-2\omega_0)t}\dot{\rho}_{ca} &= -\frac{1}{2}(\gamma_1 ch^2 + \gamma_2 sh^2)(e^{-i(\omega_1+\omega_2-2\omega_0)t}\rho_{ac} + e^{i(\omega_1+\omega_2-2\omega_0)t}\rho_{ca}) \\ &- \sqrt{\gamma_1\gamma_2} shch(\rho_{aa} - 2\rho_{bb} + \rho_{cc}) \end{aligned} \tag{6d}$$

$$\begin{aligned} e^{i(\omega_0-\omega_1)t}\dot{\rho}_{ab} + e^{-i(\omega_0-\omega_1)t}\dot{\rho}_{ba} &= -\frac{1}{2}((\gamma_1+\gamma_2)ch^2 + \gamma_1 sh^2 - \gamma_1 chsh)(e^{i(\omega_0-\omega_1)t}\rho_{ab} + e^{-i(\omega_0-\omega_1)t}\rho_{ba}) \\ &- \frac{1}{2}\sqrt{\gamma_1\gamma_2}(ch - 2sh)sh(e^{-i(\omega_0-\omega_2)t}\rho_{cb} + e^{i(\omega_0-\omega_2)t}\rho_{bc}) \end{aligned} \tag{6e}$$

$$\begin{aligned} e^{-i(\omega_0-\omega_2)t}\dot{\rho}_{cb} + e^{i(\omega_0-\omega_2)t}\dot{\rho}_{bc} &= \frac{1}{2}\sqrt{\gamma_1\gamma_2}(2ch - sh)ch(e^{i(\omega_0-\omega_1)t}\rho_{ab} + e^{-i(\omega_0-\omega_1)t}\rho_{bc}) \\ &- \frac{1}{2}((\gamma_1+\gamma_2)sh^2 + \gamma_2 ch^2 - 2\gamma_2 chsh)(e^{-i(\omega_0-\omega_2)t}\rho_{cb} + e^{i(\omega_0-\omega_2)t}\rho_{bc}) \end{aligned} \tag{6f}$$

where $ch = \cosh(r)$, $sh = \sinh(r)$, and $\gamma_1 = \gamma_{ab}(\gamma_2 = \gamma_{bc})$ indicates the decay rate from $|a\rangle$ to $|b\rangle(|b\rangle$ to $|c\rangle)$ in ordinary vacuum due to the waveguide modes. Equations (6e)-(6f) are for off diagonal elements ρ_{ab}, ρ_{bc}. The steady state of these two equations is $\rho_{ab} = \rho_{bc} = 0$ since they are homogeneous linear equations. The first four equations Eq.(6a)-(6d) also have a steady state solution when they are combined with the constraints $\rho_{aa}+\rho_{bb}+\rho_{cc} = 1$ and $\omega_1 + \omega_2 = 2\omega_0$. It is also worth noting that Eq.(6a)-(6d) are independent of $\delta\omega$, so the difference between ω_{ab} and ω_{bc} doesn't influence the steady state for the single atom case. When $\gamma_1 \neq \gamma_2$, the steady state solution is: $\frac{sh\sqrt{\gamma_2}}{\sqrt{ch^2\gamma_1+sh^2\gamma_2}}|a\rangle - \frac{ch\sqrt{\gamma_1}}{\sqrt{ch^2\gamma_1+sh^2\gamma_2}}|c\rangle$ which is a superposition state of $|a\rangle$ and $|c\rangle$. Thus, there is always population inversion between $|a\rangle$ and $|b\rangle$, and the population inversion between $|a\rangle$ and $|c\rangle$ occurs when $\tanh r > \sqrt{\frac{\gamma_1}{\gamma_2}}$. The population distribution for different ratios of $\frac{\gamma_{ab}}{\gamma_{bc}}$ is shown in Fig. 2(a). The mechanism of this population inversion can be interpreted with the help of Fig. 2(c). Figure 2(c) shows that the direct transition between $|a\rangle$, $|b\rangle$, and $|c\rangle$ are allowed just like the thermal reservoir case. However, in the squeezed vacuum, there is additional paths for population flow: electrons in any of these three states can evolve into the other two through an intermediate "state" ρ_{ac}. Although ρ_{ac} is actually an off-diagonal element rather than a state, it can be used to elucidate our idea. When $\gamma_{ab} \ll \gamma_{bc}$, the transition $|a\rangle \to |b\rangle$ is negligible compared with γ_{bc} and $\sqrt{\gamma_{ab}\gamma_{bc}}$. Thus electrons in the state $|c\rangle$ can be excited to $|a\rangle$ through $|c\rangle \to |b\rangle \to \rho_{ac} \to |a\rangle$, but $|a\rangle$ can not decay back to $|c\rangle$, which achieves the population trapping in $|a\rangle$. This phenomenon is similar to coherent trapping, but here we achieve the trapping for Ξ structure with the squeezed vacuum reservoir, which cannot be realized with coherent pump due to spontaneous emission. What's more, since only a few modes are allowed in the waveguide with particular polarizations[15], $\gamma_{ab} \ll \gamma_{bc}$ can be achieved if we can properly manipulate the orientation of the atom. Since it is hard to achieve perfect squeezing with $M = \sqrt{N(N+1)}$ in experiments, we also study the effect of different values of M on the steady state population with parameters $\gamma_{ab} = \frac{1}{4}\gamma_{bc}$ and $r = 1$, which is shown in Fig. 2(b). Although the steady state population distribution is very sensitive to the value of M, the population inversion between $|a\rangle$ and $|b\rangle$ still holds for $M = 0.8\sqrt{N(N+1)}$.

IV. STEADY STATE OF MULTIPLE ATOMS

In the last section, we demonstrate that a single atom can achieve steady state population inversion in the squeezed vacuum reservoir. However, with Eq.(5), this result can not be generated to the multi-atom case since $\gamma'_{ijij} = \sqrt{\gamma_j\gamma_j}\cos[2k_{0z}r_i]$. The squeezing term in Eq.(4) vanishes for atoms located around $r_i = \frac{\pi}{4k_{0z}} + \frac{n\pi}{2k_{0z}}$. Thus, for a group of randomly located atoms, if we want to achieve steady state population inversion in the squeezed vacuum, we need to modify our scheme. Here we consider the following correlation functions:

$$\begin{aligned} \left\langle a^\dagger_{\boldsymbol{k},s} a_{\boldsymbol{k}',s'} \right\rangle &= \sinh^2 r \delta_{\boldsymbol{k}'\boldsymbol{k}}\delta_{ss'} \\ \left\langle a_{\boldsymbol{k},s} a^\dagger_{\boldsymbol{k}',s'} \right\rangle &= \cosh^2 r \delta_{\boldsymbol{k}'\boldsymbol{k}}\delta_{ss'} \\ \left\langle a^\dagger_{\boldsymbol{k},s} a^\dagger_{\boldsymbol{k}',s'} \right\rangle &= -e^{-i\theta}\cosh(r)\sinh(r)\delta_{\boldsymbol{k}',-(2\boldsymbol{k}_0-\boldsymbol{k})}\delta_{ss'} \\ \langle a_{\boldsymbol{k},s} a_{\boldsymbol{k}',s'} \rangle &= -e^{i\theta}\cosh(r)\sinh(r)\delta_{\boldsymbol{k}',-(2\boldsymbol{k}_0-\boldsymbol{k})}\delta_{ss'} \end{aligned} \tag{7}$$

which indicates that photons are entangled with those from the opposite direction, then the coefficients in the

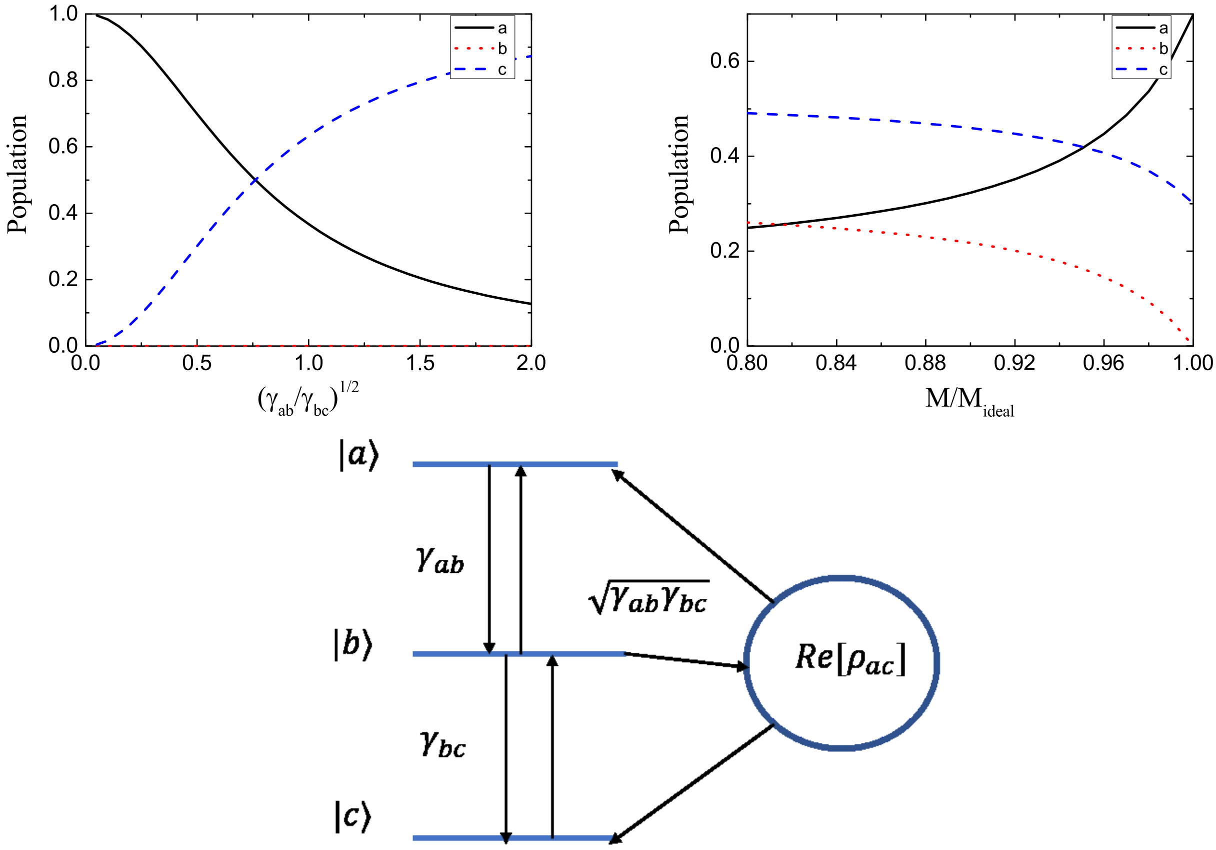

Fig. 2: (a) The steady state population distribution for different μ_{ab} andμ_{bc}. The squeezing parameter $r = 1$. (b) The steady state population distribution for non-ideal squeezed vacuum which is characterized by the ratio of M and $\sqrt{N(N+1)}$. (c) The allowed population flow in the squeezed vacuum. The squeezing parameter $r = 1$, and $\gamma_{ab} = \frac{1}{4}\gamma_{bc}$.

the master equation Eq.(4) becomes

$$\begin{aligned}\gamma_{ijkl} &= \sqrt{\gamma_i\gamma_k}\cos(k_{0z}r_{jl})\\ \Lambda_{ijkl} &= \frac{\sqrt{\gamma_i\gamma_k}}{2}\sin(k_{0z}r_{jl})\\ \gamma'_{ijkl} &= \sqrt{\gamma_i\gamma_k}\cos[k_{0z}(r_{jl})]\end{aligned} \tag{8}$$

which is exactly the traditionally studied master equation[13] by Ficek. However, they didn't consider the effect of squeezing source, so here we give a detailed derivation on getting Eq. (B1) in Appendix B, where the effect of squeezing source is included. With this new master equation, a single atom can reach population inversion anywhere in the waveguide, as discussed in the Sec. III. When there are multiple atoms in the waveguide where the dipole-dipole interaction should be considered, the population inversion still holds true. In fact, the final state is just the direct product of the steady state of independent atoms, and this will be proved with the mathematical induction. Firstly, we need to apply the rotating wave approximation on Eq.(4):

$$\begin{aligned}\frac{d\rho^S}{dt} =& -i\sum_{i,k,j}\Lambda_{ijkl}[S^+_{i,j}S^-_{k.j},\rho^S] - \frac{1}{2}\sum_{i,j,k}\gamma_{ijkl}(1+N)(\rho^S S^+_{i,j}S^-_{k,j} + S^+_{i,j}S^-_{k,j}\rho^S - 2S^-_{k,j}\rho^S S^+_{i,j})\\ &-\frac{1}{2}\sum_{i,j,k}\gamma_{ijkl}N(\rho^S S^-_{i,j}S^+_{k,j} + S^-_{i,j}S^+_{k,j}\rho^S - 2S^+_{k,j}\rho^S S^-_{i,j})\\ &-\frac{1}{2}\sum_{\alpha=\pm}\sum_{i,k,j\neq l}\gamma'_{ijkl}M(\rho^S S^\alpha_{i,j}S^\alpha_{k,l} + S^\alpha_{i,j}S^\alpha_{k,l}\rho^S - 2S^\alpha_{k,l}\rho^S S^\alpha_{i,j})\end{aligned} \tag{9}$$

We assume the steady solution for N-atom case is: $\rho^S = \rho_1\rho_2 ... \rho_N$ where $\rho_i = (A|a_i\rangle + C|c_i\rangle)(A\langle a_i| + C\langle c_i|)$ and $A = \frac{sh\sqrt{\gamma_2}}{\sqrt{ch^2\gamma_1 + sh^2\gamma_2}}, C = -\frac{ch\sqrt{\gamma_1}}{\sqrt{ch^2\gamma_1 + sh^2\gamma_2}}$. Then for $N+1$-atom case, the extra terms induced by the $(N+1)th$ atom on the right hand side of Eq.(9) are composed of three parts: $i = k = N+1$ terms, $i = N+1, k = 1$ N terms, and $i = 1$ $N, k = N+1$ terms. The $i = k = N+1$ terms are the exact terms for the single independent atom of the $(N+1)th$ atom, so the result is 0. The $i = N+1, k = 1 \sim N$ terms are

$$
\begin{aligned}
(i = N+1, k = 1 \sim N) = & -i\sum_{j,k}\Lambda_{N+1,j,k,j}[S^+_{N+1,j}S^-_{k.j}, \rho^S] \\
& - \frac{1}{2}\sum_{j,k}\gamma_{N+1,j,k,j}(ch^2)(\rho^S S^+_{N+1,j}S^-_{k,j} + S^+_{N+1,j}S^-_{k,j}\rho^S - 2S^-_{k,j}\rho^S S^+_{N+1,j}) \\
& - \frac{1}{2}\sum_{j,k}\gamma_{N+1,j,k,j}sh^2(\rho^S S^-_{N+1,j}S^+_{k,j} + S^-_{N+1,j}S^+_{k,j}\rho^S - 2S^+_{k,j}\rho^S S^-_{N+1,j}) \\
& - \frac{1}{2}\sum_{\alpha=\pm}\sum_{i,k,j\neq l}\gamma'_{N+1,j,k,j}chsh(\rho^S S^\alpha_{N+1,j}S^\alpha_{k,l} + S^\alpha_{N+1,j}S^\alpha_{k,l}\rho^S - 2S^\alpha_{k,l}\rho^S S^\alpha_{N+1,j})
\end{aligned}
\tag{10}
$$

For the energy shift term (first term) of Eq.(10), we have

$$
\begin{aligned}
S^+_{N+1,j}S^-_{k.j}\rho^S &= \rho_1 ... (S^-_{k.j}\rho_k) ... \rho_N(S^+_{N+1,j}\rho_{N+1}) = 0 \\
\rho^S S^+_{N+1,j}S^-_{k.l} &= \rho_1 ... (\rho_k S^-_{k.l}) ... \rho_N(\rho_{N+1}S^+_{N+1,j}) = 0
\end{aligned}
\tag{11}
$$

For the thermal terms(second and third term) of Eq.(10), we have:

$$
\begin{aligned}
\rho^S S^+_{N+1,j}S^-_{k,j} =& S^+_{N+1,j}S^-_{k,j}\rho^S = 0 \\
\rho^S S^-_{N+1,j}S^+_{k,j} =& S^-_{N+1,j}S^+_{k,j}\rho^S = 0 \\
S^-_{k,j}\rho^S S^+_{N+1,j} =& \rho_1 ... (S^-_{k.j}\rho_k) ... \rho_N(\rho_{N+1}S^+_{N+1,j}) \\
=& \rho_1 ... (S^-_{k.1}\rho_k) ... \rho_N(\rho_{N+1}S^+_{N+1,1}) \\
=& \rho_1 ... (A|b_k\rangle)(A\langle a_k| + C\langle c_k|) .. \rho_N \\
& \times (A|a_{N+1}\rangle + C|c_{N+1}\rangle)(A\langle b_{N+1}|) \\
S^+_{k,j}\rho^S S^-_{N+1,j} =& \rho_1 ... (S^+_{k.j}\rho_k) ... \rho_N(\rho_{N+1}S^-_{N+1,j}) \\
=& \rho_1 ... (S^+_{k.2}\rho_k) ... \rho_N(\rho_{N+1}S^-_{N+1,2}) \\
=& \rho_1 ... (C|b_k\rangle)(A\langle a_k| + C\langle c_k|) ... \rho_N \\
& \times (A|a_{N+1}\rangle + C|c_{N+1}\rangle)(C\langle b_{N+1}|)
\end{aligned}
\tag{12}
$$

For the squeezed vacuum terms(forth term), we have:

$$
\begin{aligned}
\rho^S S^\alpha_{N+1,j}S^\alpha_{k,l} &= S^\alpha_{N+1,j}S^\alpha_{k,l}\rho^S = 0 \\
S^+_{k,1}\rho^S S^+_{N+1,2} &= \rho_1 ... (S^+_{k,1}\rho_k) ... \rho_N(\rho_{N+1}S^+_{N+1,2}) = 0 \\
S^+_{k,2}\rho^S S^+_{N+1,1} &= \rho_1 ... (S^+_{k,2}\rho_k) ... \rho_N(\rho_{N+1}S^+_{N+1,1}) \\
&= \rho_1 ... (C|b_k\rangle)(A\langle a_k| + C\langle c_k|) ... \rho_N \\
&\quad \times (A|a_{N+1}\rangle + C|c_{N+1}\rangle)(A\langle b_{N+1}|) \\
S^-_{k,1}\rho^S S^-_{N+1,2} &= \rho_1 ... (S^-_{k,1}\rho_k) ... \rho_N(\rho_{N+1}S^-_{N+1,2}) \\
&= \rho_1 ... (A|b_k\rangle)(A\langle a_k| + C\langle c_k|) ... \rho_N \\
&\quad \times (A|a_{N+1}\rangle + C|c_{N+1}\rangle)(C\langle b_{N+1}|) \\
S^-_{k,2}\rho^S S^-_{N+1,1} &= \rho_1 ... (S^-_{k,2}\rho_k) ... \rho_N(\rho_{N+1}S^-_{N+1,1}) = 0
\end{aligned}
\tag{13}
$$

Plugging Eq.(11)(12)(13) into expression (10), we have

$$
\begin{aligned}
(i=N+1,k=1\sim N)=&\sum_k \gamma_{N+1,1,k,1}(ch^2)S^-_{k,1}\rho^S S^+_{N+1,1}+\sum_{j,k}\gamma_{N+1,2,k,2}sh^2 S^+_{k,2}\rho^S S^-_{N+1,2}\\
&+\sum_{\alpha=\pm}\sum_{i,k,j\neq l}\gamma'_{N+1,j,k,l}chshS^\alpha_{k,l}\rho^S S^\alpha_{N+1,j}\\
=&\sum_k \gamma_{N+1,1,k,1}(ch^2)\rho_1...(A|b_k\rangle)(A\langle a_k|+C\langle c_k|)..\rho_N(A|a_{N+1}\rangle+C|c_{N+1}\rangle)(A\langle b_{N+1}|)\\
&+\sum_k \gamma_{N+1,2,k,2}sh^2\rho_1...(C|b_k\rangle)(A\langle a_k|+C\langle c_k|)...\rho_N(A|a_{N+1}\rangle+C|c_{N+1}\rangle)(C\langle b_{N+1}|)\\
&+\sum_k \gamma'_{N+1,j,k,l}chsh[\rho_1...(C|b_k\rangle)(A\langle a_k|+C\langle c_k|)...\rho_N(A|a_{N+1}\rangle+C|c_{N+1}\rangle)(A\langle b_{N+1}|)\\
&+\rho_1...(A|b_k\rangle)(A\langle a_k|+C\langle c_k|)...\rho_N(A|a_{N+1}\rangle+C|c_{N+1}\rangle)(C\langle b_{N+1}|)]\\
=&\sum_k(\gamma_{N+1,1,k,1}ch^2A^2+\gamma_{N+1,2,k,2}sh^2C^2+2\gamma'_{N+1,j,k,l}chshCA)\\
&\times\rho_1...(|b_k\rangle)(A\langle a_k|+C\langle c_k|)..\rho_N(A|a_{N+1}\rangle+C|c_{N+1}\rangle)(\langle b_{N+1}|)
\end{aligned}
\tag{14}
$$

since $ch^2A^2\gamma_{N+1,1,k,1} + sh^2C^2\gamma_{N+1,2,k,2} + 2chshCA\gamma'_{N+1,j,k,l} = 0$, the extra terms induced by $i = N+1, k = 1 \sim N$ are 0. The same goes for $i = 1 \sim N, k = N+1$. Thus, the steady state of multiple atoms is merely the direct product of that in the single atom case, which also means that the dipole-dipole interaction doesn't have any effect on the final steady state.

V. SUMMARY

We study the Ξ-type atoms coupled to the broadband squeezed vacuum reservoir in a quasi-one-dimensional waveguide, with the overall transition frequency $\omega_{ac} = 2\omega_0$. The atoms will evolve into a steady state where most atoms are in the excited state and almost no atoms are in the intemediate state. We also study how the non-perfect squeezing will influence our result. Our main conclusions still hold true when dipole-dipole interaction is considered.

APPENDIX A: DERIVATION OF EQ.(??)

Here we will show how to derive the master equation Eq. (4). The interaction Hamiltonian is:

$$V(t) = -i\hbar\sum_{\boldsymbol{k}s}[D(t)a_{\boldsymbol{k}s}(t) - D^+(t)a^\dagger_{\boldsymbol{k}s}(t)], \tag{A1}$$

where

$$D(t) = \sum_{l,i}[\boldsymbol{\mu}_{l,i}\cdot\boldsymbol{u}_{\boldsymbol{k},s}(r_{l,i})S^\dagger_{l,i}(t) + \boldsymbol{\mu}^*_{l,i}\cdot\boldsymbol{u}_{\boldsymbol{k},s}(r_{l,i})S^-_{l,i}(t)] \tag{A2}$$

The reduced master equation of atoms in the reservoir is:

$$
\begin{aligned}
\frac{d\rho^S}{dt}=&-\frac{1}{\hbar^2}\int_0^t d\tau Tr_F\{[V(t),[V(t-\tau),\rho^S(t-\tau)\rho^F\}\\
=&-\frac{1}{\hbar^2}\int_0^t d\tau Tr_F\{V(t)V(t-\tau)\rho^S(t-\tau)\rho^F+\rho^S(t-\tau)\rho^F V(t-\tau)V(t)\\
&-V(t)\rho^S(t-\tau)\rho^F V(t-\tau)-V(t-\tau)\rho^S(t-\tau)\rho^F V(t)\}
\end{aligned}
\tag{A3}
$$

Here we just show how to deal with the first term in Eq.(A3), the remaining terms can be calculated in the same way. For the first term, we have

$$\begin{aligned}
&-\frac{1}{\hbar^2}\int_0^t d\tau Tr_F\{V(t)V(t-\tau)\rho^S(t-\tau)\rho^F\} \\
&=\int_0^t d\tau \sum_{\boldsymbol{k}s,\boldsymbol{k}'s'}\{D(t)D(t-\tau)Tr_F[\rho^F a_{ks}(t)a_{k's'}(t-\tau)]-D(t)D^{+}(t-\tau)Tr_F[\rho^F a_{ks}(t)a^{\dagger}_{k's'}(t-\tau)] \\
&-D^{+}(t)D(t-\tau)Tr_F[\rho^F a^{\dagger}_{ks}(t)a_{k's'}(t-\tau)]+D^{+}(t)D^{+}(t-\tau)Tr_F[\rho^F a^{\dagger}_{ks}(t)a^{\dagger}_{k's'}(t-\tau)]\}\rho^S(t-\tau)\}.
\end{aligned} \tag{A4}$$

Under the rotating wave approximation(RWA), we have

$$\begin{aligned}
&-\frac{1}{\hbar^2}\int_0^t d\tau Tr_F\{V(t)V(t-\tau)\rho^S(t-\tau)\rho^F\} \\
=&\sum_{ijlm}\sum_{\boldsymbol{k}s,\boldsymbol{k}'s'}\int_0^t d\tau\{\boldsymbol{\mu}_{l,i}\cdot\boldsymbol{u}_{\boldsymbol{k}s}(r_{l,i})S^{+}_{l,i}e^{i\omega_i t}\boldsymbol{\mu}_{m,j}\cdot\boldsymbol{u}_{\boldsymbol{k}'s'}(r_{m,j})S^{+}_{m,j}e^{i\omega_j(t-\tau)}e^{-i(\omega_{\boldsymbol{k}s}+\omega_{\boldsymbol{k}'s'})t+i\omega_{\boldsymbol{k}'s'}\tau}[-\sinh(r)\cosh(r)\delta_{\boldsymbol{k}',2\boldsymbol{k}_0-\boldsymbol{k}}\delta_{ss'}] \\
&-\boldsymbol{\mu}_{l,i}\cdot\boldsymbol{u}_{\boldsymbol{k}s}(r_{l,i})S^{+}_{l,i}e^{i\omega_i t}\boldsymbol{\mu}^{*}_{m,j}\cdot\boldsymbol{u}^{*}_{\boldsymbol{k}'s'}(r_{m,j})S^{-}_{m,j}e^{-i\omega_j(t-\tau)}e^{-i\omega_{\boldsymbol{k}'s'}\tau}\cosh^2 r\delta_{\boldsymbol{k}\boldsymbol{k}'}\delta_{ss'} \\
&-\boldsymbol{\mu}^{*}_{l,i}\cdot\boldsymbol{u}_{\boldsymbol{k}s}(r_{l,i})S^{-}_{l,i}e^{-i\omega_i t}\boldsymbol{\mu}_{m,j}\cdot\boldsymbol{u}^{*}_{\boldsymbol{k}'s'}(r_{m,j})S^{+}_{m,j}e^{i\omega_j(t-\tau)}e^{-i\omega_{\boldsymbol{k}'s'}\tau}\cosh^2 r\delta_{\boldsymbol{k}\boldsymbol{k}'}\delta_{ss'} \\
&-\boldsymbol{\mu}^{*}_{l,i}\cdot\boldsymbol{u}^{*}_{\boldsymbol{k}s}(r_{l,i})S^{-}_{l,i}e^{-i\omega_i t}\boldsymbol{\mu}_{m,j}\cdot\boldsymbol{u}_{\boldsymbol{k}'s'}(r_{m,j})S^{+}_{m,j}e^{i\omega_j(t-\tau)}e^{i\omega_{\boldsymbol{k}'s'}\tau}\sinh^2 r\delta_{\boldsymbol{k}\boldsymbol{k}'}\delta_{ss'} \\
&-\boldsymbol{\mu}_{l,i}\cdot\boldsymbol{u}^{*}_{\boldsymbol{k}s}(r_{l,i})S^{+}_{l,i}e^{i\omega_i t}\boldsymbol{\mu}^{*}_{m,j}\cdot\boldsymbol{u}_{\boldsymbol{k}'s'}(r_{m,j})S^{-}_{m,j}e^{-i\omega_j(t-\tau)}e^{i\omega_{\boldsymbol{k}'s'}\tau}\sinh^2 r\delta_{\boldsymbol{k}\boldsymbol{k}'}\delta_{ss'} \\
&+\boldsymbol{\mu}^{*}_{l,i}\cdot\boldsymbol{u}^{*}_{\boldsymbol{k}s}(r_{l,i})S^{-}_{l,i}e^{-i\omega_i t}\boldsymbol{\mu}^{*}_{m,j}\cdot\boldsymbol{u}^{*}_{\boldsymbol{k}'s'}(r_{m,j})S^{-}_{m,j}e^{-i\omega_j(t-\tau)}e^{i(\omega_{\boldsymbol{k}s}+\omega_{\boldsymbol{k}'s'})t-i\omega_{\boldsymbol{k}'s'}\tau}[-\sinh(r)\cosh(r)\delta_{\boldsymbol{k}',2\boldsymbol{k}_0-\boldsymbol{k}}\delta_{ss'}]\}\rho^S(t-\tau)
\end{aligned} \tag{A5}$$

where l,m are used for labeling different atoms, and i,j are used for transitions within an atom. Here we just calculate the first and second term to show how to get the master equation Eq.(4). Since all atoms are identical, $\omega_{l,i}=\omega_i$, $|\boldsymbol{\mu}_{l,i}|=|\boldsymbol{\mu}_i|$, and $r_{l,i}=r_l$ can be used to simplify Eq.(A5). For the second term(thermal term), we have

$$\begin{aligned}
&-\sum_{k_z}\int_0^t d\tau\boldsymbol{\mu}_{l,i}\cdot\boldsymbol{u}_{\boldsymbol{k}s}(r_l)S^{+}_{l,i}e^{i\omega_i t}\boldsymbol{\mu}^{*}_{m,j}\cdot\boldsymbol{u}^{*}_{\boldsymbol{k}'s'}(r_m)S^{-}_{m,j}e^{-i\omega_j(t-\tau)}e^{-i\omega_{\boldsymbol{k}'s'}\tau}\cosh^2 r\rho^S(t-\tau)\delta_{\boldsymbol{k}\boldsymbol{k}'}\delta_{ss'} \\
=&-\frac{L}{2\pi}e^{i(\omega_i-\omega_j)t}\int_{-\infty}^{\infty}dk_z\int_0^t d\tau e^{i\omega_j\tau}e^{-i\omega_{k_z}\tau}\frac{\omega_k\mu_i\mu_j}{\epsilon_0 LS\hbar}e^{ik_z(r_l-r_m)}\cosh^2 rS^{+}_{l,i}S^{-}_{m,j}\rho^S(t-\tau) \\
\approx&-\frac{L}{2\pi}e^{i(\omega_i-\omega_j)t}\int_0^{\infty}dk_z\int_0^t d\tau e^{i\omega_j\tau}e^{-i[\omega_j+c^2k_{jz}(k_z-k_{jz})/\omega_j]\tau}\frac{\omega_k\mu_i\mu_j}{\epsilon_0 LS\hbar}[e^{ik_z(r_l-r_m)}+e^{-ik_z(r_l-r_m)}]\cosh^2 rS^{+}_{l,i}S^{-}_{m,j}\rho^S(t-\tau) \\
\approx&-\frac{L}{2\pi}e^{i(\omega_i-\omega_j)t}\int_{-k_{0z}}^{\infty}d\delta k_z\int_0^t d\tau e^{-i\tau c^2k_{jz}\delta k_z/\omega_j}\frac{\omega_k\mu_i\mu_j}{\epsilon_0 LS\hbar}[e^{i(k_{jz}+\delta k_z)(r_l-r_m)}+e^{-i(k_{jz}+\delta k_z)(r_l-r_m)}]\cosh^2 rS^{+}_{l,i}S^{-}_{m,j}\rho^S(t-\tau) \\
\approx&-\frac{L}{2\pi}e^{i(\omega_i-\omega_j)t}\int_{-\infty}^{\infty}d\delta k_z\int_0^t d\tau e^{-i(c^2k_{jz}\delta k_z/\omega_j)\tau}\frac{\omega_k\mu_i\mu_j}{\epsilon_0 LS\hbar}[e^{i(k_{jz}+\delta k_z)(r_l-r_m)}+e^{-i(k_{jz}+\delta k_z)(r_l-r_m)}]\cosh^2 rS^{+}_{l,i}S^{-}_{m,j}\rho^S(t-\tau) \\
\approx&-\frac{L}{2\pi}e^{i(\omega_i-\omega_j)t}\int_0^t d\tau\frac{\omega_j\mu_i\mu_j}{\epsilon_0 LS\hbar}2\pi[e^{ik_{jz}(r_l-r_m)}\delta((r_l-r_m)-\frac{c^2k_{jz}}{\omega_0}\tau)+e^{-ik_{jz}(r_l-r_m)}\delta((r_l-r_m)+\frac{c^2k_{jz}}{\omega_0}\tau)]\cosh^2 rS^{+}_{l,i}S^{-}_{m,j}\rho^S(t \\
\approx&-\frac{L}{2\pi}e^{ik_{jz}r_{lm}}\frac{\omega_j\mu_i\mu_j}{\epsilon_0 LS\hbar}2\pi\frac{\omega_j}{c^2k_{0z}}\cosh^2 rS^{+}_{l,i}S^{-}_{m,j}\rho^S(t)e^{i(\omega_i-\omega_j)t} \\
\approx&-[\frac{\sqrt{\gamma_i\gamma_j}}{2}\cos(k_{0z}r_{lm})+i\frac{\sqrt{\gamma_i\gamma_j}}{2}sin(k_{0z}r_{lm})]\cosh^2 rS^{+}_{l,i}S^{-}_{m,j}\rho^S(t)e^{i(\omega_i-\omega_j)t}
\end{aligned} \tag{A6}$$

where emitter separation $r_{lm}=|r_l-r_m|$, collective decay rate $\gamma_i=2\mu_i^2\omega_i^2/\hbar\epsilon_0 Sc^2k_{iz}$, and collective energy shift $\Lambda_{ij}=\sqrt{\gamma_i\gamma_j}\sin(k_{0z}r_{ij})/2$. In the third line we expand $\omega_k=c\sqrt{(\frac{\pi}{a})^2+(k_z)^2}$ around $k_z=k_{0z}$ since resonant modes provide dominant contributions. In the fifth line we extend the integration $\int_{-k_{jz}}^{\infty}dk_z\to\int_{-\infty}^{\infty}dk_z$ because the main contribution comes from the components around $\delta k_z=0$. In the next line, Weisskopf-Wigner approximation is used. Thus, we have obtained γ_{ij} and Λ_{ij} as is shown in Eq.(5).

Next we need to calculate the first term (squeezing term) in Eq.(A5):

$$
\begin{aligned}
&e^{i(\omega_i+\omega_j-2\omega_0)t}\sum_{k_z}\int_0^t d\tau\{\boldsymbol{\mu}_{l,i}\cdot\boldsymbol{u}_{2\boldsymbol{k}_0-\boldsymbol{k}}(r_l)S^+_{l,i}\boldsymbol{\mu}_{m,j}\cdot\boldsymbol{u}_{\boldsymbol{k}}(r_m)S^+_{m,j}e^{i(\omega_{\boldsymbol{k}}-\omega_j)\tau}[-\sinh(r)\cosh(r)]\rho^S(t-\tau)\\
=&-\frac{L}{2\pi}e^{i(\omega_i+\omega_j-2\omega_0)t}\int_0^{2k_{0z}}dk_z\int_0^t d\tau e^{i(\omega_{k_z}-\omega_j)\tau}e^{i(2k_{jz}-k_z)(r_l-o_1)}e^{ik_z(r_m-o_1)}\frac{\sqrt{\omega_{k_z}\omega_{2k_{0z}-k_z}}\mu_i\mu_j}{\epsilon_0 LS\hbar}\sinh(r)\cosh(r)S^+_{l,i}S^+_{m,j}\rho^S(t-\tau)\\
&-\frac{L}{2\pi}e^{i(\omega_i+\omega_j-2\omega_0)t}\int_{-2k_{0z}}^{0}dk_z\int_0^t d\tau e^{i(\omega_{k_z}-\omega_j)\tau}e^{i(-2k_{jz}-k_z)(r_l-o_2)}e^{ik_z(r_m-o_2)}\frac{\sqrt{\omega_{k_z}\omega_{-2k_{0z}-k_z}}\mu_i\mu_j}{\epsilon_0 LS\hbar}\sinh(r)\cosh(r)S^+_{l,i}S^+_{m,j}\rho^S(t-\tau)
\end{aligned}
\tag{A7}
$$

Putting aside the overall factor $e^{i(\omega_i+\omega_j-2\omega_0)t}$, for $r_l=r_j$, Eq.(A7) reduces to

$$
\begin{aligned}
&\sum_{k_z}\int_0^t d\tau\{\boldsymbol{\mu}_{l,i}\cdot\boldsymbol{u}_{2\boldsymbol{k}_0-\boldsymbol{k}}(r_l)S^+_{l,i}\boldsymbol{\mu}_{l,j}\cdot\boldsymbol{u}_{\boldsymbol{k}}(r_l)S^+_{l,j}e^{i(\omega_{\boldsymbol{k}}-\omega_j)\tau}[-\sinh(r)\cosh(r)]\rho^S(t-\tau)\\
=&-\frac{L}{2\pi}\int_0^{2k_{0z}}dk_z\int_0^t d\tau e^{i\frac{c^2k_{jz}}{\omega_j}(k_z-k_{jz})\tau}e^{i2k_{0z}(r_l-o_1)}\frac{\sqrt{\omega_{k_z}\omega_{2k_{0z}-k_z}}\mu_i\mu_j}{\epsilon_0 LS\hbar}\sinh(r)\cosh(r)S^+_{l,i}S^+_{l,j}\rho^S(t-\tau)\\
&-\frac{L}{2\pi}\int_{-2k_{0z}}^{0}dk_z\int_0^t d\tau e^{i\frac{c^2k_{jz}}{\omega_j}(k_z-k_{jz})\tau}e^{-i2k_{0z}(r_l-o_2)}\frac{\sqrt{\omega_{k_z}\omega_{-2k_{0z}-k_z}}\mu_i\mu_j}{\epsilon_0 LS\hbar}\sinh(r)\cosh(r)S^+_{l,i}S^+_{l,j}\rho^S(t-\tau)\\
=&-\frac{L}{2\pi}[e^{i2k_{0z}(r_l-o_1)}+e^{-i2k_{0z}(r_l-o_2)}]\frac{\sqrt{\omega_i\omega_j}\mu_i\mu_j}{\epsilon_0 LS\hbar}\int_0^t d\tau 2\pi\delta(\frac{c^2k_{jz}}{\omega_j}\tau)\sinh(r)\cosh(r)S^+_{l,i}S^+_{l,j}\rho^S(t-\tau)\\
=&-e^{i2k_{jz}R}\frac{\omega_0^2\mu_i\mu_j}{\epsilon_0\hbar Sc^2k_{0z}}\cos(2k_{0z}r_l)\sinh(r)\cosh(r)S^+_{l,i}S^+_{l,j}\rho^S(t)\\
=&-e^{i2k_{0z}R}\frac{\sqrt{\gamma_i\gamma_j}}{2}\cos(2k_{0z}r_l)\sinh(r)\cosh(r)S^+_{l,i}S^+_{l,j}\rho^S(t)
\end{aligned}
\tag{A8}
$$

where we have used the fact that the origin of coordinate system is at equal distant from two sources(i.e., $o_2=-o_1=R$) in the second last line. Incorporating index l into i, we have $\gamma'_{ij}=\sqrt{\gamma_i\gamma_j}\cos(2k_{0z}r_i)$. For $r_i\neq r_j$, Eq. (A7) reduces to

$$
\begin{aligned}
&\sum_{k_z}\int_0^t d\tau\{\boldsymbol{\mu}_{l,i}\cdot\boldsymbol{u}_{2\boldsymbol{k}_0-\boldsymbol{k}}(r_l)S^+_{l,i}\boldsymbol{\mu}_{m,j}\cdot\boldsymbol{u}_{\boldsymbol{k}}(r_m)S^+_{m,j}e^{i(\omega_{\boldsymbol{k}}-\omega_j)\tau}[-\sinh(r)\cosh(r)]\rho^S(t-\tau)\\
=&-\frac{L}{2\pi}\int_0^{2k_{0z}}dk_z\int_0^t d\tau e^{i\frac{c^2k_{jz}}{\omega_j}(k_z-k_{jz})\tau}e^{i2k_{0z}(r_c-o_1)}e^{-i(k_z-k_{0z})(r_l-r_m)}\frac{\sqrt{\omega_{k_z}\omega_{2k_{0z}-k_z}}\mu_i\mu_j}{\epsilon_0 LS\hbar}\sinh(r)\cosh(r)S^+_{l,i}S^+_{m,j}\rho^S(t-\tau)\\
&-\frac{L}{2\pi}\int_{-2k_{0z}}^{0}dk_z\int_0^t d\tau e^{i\frac{c^2k_{jz}}{\omega_j}(-k_z-k_{jz})\tau}e^{-i2k_{0z}(r_c-o_2)}e^{-i(k_z+k_{0z})(r_l-r_m)}\frac{\sqrt{\omega_{k_z}\omega_{-2k_{0z}-k_z}}\mu_i\mu_j}{\epsilon_0 LS\hbar}\sinh(r)\cosh(r)S^+_{l,i}S^+_{m,j}\rho^S(t-\tau)\\
=&-\frac{L}{2\pi}e^{i2k_{0z}(r_c-o_1)}\frac{\sqrt{\omega_i\omega_j}\mu_i\mu_j}{\epsilon_0 LS\hbar}\int_{-\infty}^{\infty}dk_z\int_0^t d\tau e^{i\frac{c^2k_{jz}}{\omega_j}(k_z-k_{jz})\tau}e^{-i(k_z-k_{0z})(r_l-r_m)}\sinh(r)\cosh(r)S^+_{l,i}S^+_{m,j}\rho^S(t-\tau)\\
&-\frac{L}{2\pi}e^{-i2k_{0z}(r_c-o_2)}\frac{\sqrt{\omega_i\omega_j}\mu_i\mu_j}{\epsilon_0 LS\hbar}\int_{-\infty}^{\infty}dk_z\int_0^t d\tau e^{i\frac{c^2k_{jz}}{\omega_j}(k_z-k_{jz})\tau}e^{i(k_z-k_{0z})(r_l-r_m)}\sinh(r)\cosh(r)S^+_{l,i}S^+_{m,j}\rho^S(t-\tau)\\
\approx&-\frac{L}{2\pi}e^{i2k_{0z}R}\frac{\omega_0^2\mu_i\mu_j}{\epsilon_0 LS\hbar}\int_0^t d\tau 2\pi[e^{i2k_{0z}r_c}\delta(r_l-r_m-\frac{c^2k_{0z}}{\omega_0}\tau)+e^{-i2k_{0z}r_c}\delta(r_l-r_m+\frac{c^2k_{0z}}{\omega_0}\tau)]\sinh(r)\cosh(r)S^+_{l,i}S^+_{m,j}\rho^S(t-\tau)\\
\approx&-e^{i2k_{0z}R}\frac{\omega_0^2\mu_i\mu_j}{\epsilon_0\hbar Sc^2k_{0z}}e^{i2k_{0z}r_c sgn(r_l-r_m)}S^+_{l,i}S^+_{m,j}\rho^S(t)\to-\frac{\sqrt{\gamma_i\gamma_j}}{2}e^{i2k_{0z}R}\cos(k_{0z}(r_l+r_m))S^+_{l,i}S^+_{m,j}\rho^S(t)
\end{aligned}
\tag{A9}
$$

where $sgn(r_l-r_m)$ is the sign function. The last arrow is because we need to sum over i,j, so the imaginary part of $e^{i2k_{0z}r_c sgn(i-j)}$ vanishes, so the neat result is that $\gamma'_{ijkl}=e^{i2k_{0z}R}\sqrt{\gamma_j\gamma_l}\cos(k_{0z}(r_i+r_k))$. As for $S_i^+\rho^S(t)S_j^+$ terms, the combination of the last two terms in Eq.(A3) will make the imaginary part of $e^{i2k_{0z}r_c sgn(r_l-r_m)}$ vanish. Thus, we have γ'_{ijkl} in Eq.(5).

APPENDIX B: DERIVATION OF COEFFICIENTS EQ.(??)

Here we will show how to derive the master equation with coefficients Eq.(B1). The mode function of the squeezed vacuum is given by

$$\boldsymbol{u}_{\boldsymbol{k}s}(\boldsymbol{r}_i) = \sqrt{\frac{\omega_{\boldsymbol{k}s}}{2\epsilon_0\hbar V}}\boldsymbol{e}_{ks}e^{i\boldsymbol{k}\cdot(\boldsymbol{r}_i-\boldsymbol{o}_{\boldsymbol{k}s})} \tag{B1}$$

where $\boldsymbol{o}_{\boldsymbol{k}s}$ is a phenomenological parameter which includes the effects of the initial phase and the position of the squeezing source[15]. The correlation functions for the squeezed vacuum are[28]:

$$\begin{aligned}
&\left\langle a^\dagger_{\boldsymbol{k},s}a_{\boldsymbol{k}',s'}\right\rangle = \sinh^2 r\delta_{\boldsymbol{k}'\boldsymbol{k}}\delta_{ss'}\\
&\left\langle a_{\boldsymbol{k},s}a^\dagger_{\boldsymbol{k}',s'}\right\rangle = \cosh^2 r\delta_{\boldsymbol{k}'\boldsymbol{k}}\delta_{ss'}\\
&\left\langle a^\dagger_{\boldsymbol{k},s}a^\dagger_{\boldsymbol{k}',s'}\right\rangle = -e^{-i\theta}\cosh(r)\sinh(r)\delta_{\boldsymbol{k}',2\boldsymbol{k}_0-\boldsymbol{k}}\delta_{ss'}\\
&\left\langle a_{\boldsymbol{k},s}a_{\boldsymbol{k}',s'}\right\rangle = -e^{i\theta}\cosh(r)\sinh(r)\delta_{\boldsymbol{k}',2\boldsymbol{k}_0-\boldsymbol{k}}\delta_{ss'}
\end{aligned} \tag{B2}$$

For simplicity, we can set the squeezing parameter $\theta = 0$, and all atoms to align along the same direction.

Since the only difference is the squeezing terms $\left\langle a^\dagger_{\boldsymbol{k},s}a^\dagger_{\boldsymbol{k}',s'}\right\rangle$, $\left\langle a_{\boldsymbol{k},s}a_{\boldsymbol{k}',s'}\right\rangle$, we will just start from Eq. (A7). Apart from the factor $e^{i(\omega_i+\omega_j-2\omega_0)t}$, When $r_i \neq r_j$, Eq. (A7) becomes:

$$\begin{aligned}
&\sum_{k_z}\int_0^t d\tau\{\boldsymbol{\mu}_{l,i}\cdot\boldsymbol{u}_{-2\boldsymbol{k}_0+\boldsymbol{k}}(r_l)S^+_{l,i}\boldsymbol{\mu}_{m,j}\cdot\boldsymbol{u}_{\boldsymbol{k}}(r_m)S^+_{m,j}e^{i(\omega_{\boldsymbol{k}}-\omega_0)\tau}[-\sinh(r)\cosh(r)]\rho^S(t-\tau)\\
\approx&-\frac{L}{2\pi}\int_0^{2k_0}dk\int_0^t d\tau e^{i\frac{c^2k_0}{\omega_0}(k-k_0)\tau}e^{-i(2k_0-k)(r_l-o_2)+ik(r_m-o_1)}\frac{\sqrt{\omega_{k_z}\omega_{2k_{0z}-k_z}}\mu_i\mu_j}{\epsilon_0 LS\hbar}\sinh(r)\cosh(r)S^+_{l,i}S^+_{m,j}\rho^S(t-\tau)\\
&-\frac{L}{2\pi}\int_{-2k_0}^{0}dk\int_0^t d\tau e^{i\frac{c^2k_0}{\omega_0}(-k-k_0)\tau}e^{i(2k_0+k)(r_l-o_2)+ik(r_m-o_1)}\frac{\sqrt{\omega_{k_z}\omega_{-2k_{0z}-k_z}}\mu_i\mu_j}{\epsilon_0 LS\hbar}\sinh(r)\cosh(r)S^+_{l,i}S^+_{m,j}\rho^S(t-\tau)\\
=&-\frac{L}{2\pi}\int_0^{2k_0}dk\int_0^t d\tau e^{i\frac{c^2k_0}{\omega_0}(-k_0)\tau}e^{-i(2k_0)(r_l-o_2)}e^{ik(\frac{c^2k_0}{\omega_0}\tau+r_l-o_1+r_m-o_2)}\frac{\sqrt{\omega_{k_z}\omega_{2k_{0z}-k_z}}\mu_i\mu_j}{\epsilon_0 LS\hbar}\sinh(r)\cosh(r)S^+_{l,i}S^+_{m,j}\rho^S(t-\tau)\\
&-\frac{L}{2\pi}\int_0^{2k_0}dk\int_0^t d\tau e^{i\frac{c^2k_0}{\omega_0}(-k_0)\tau}e^{i(2k_0)(r_l-o_2)}e^{ik(\frac{c^2k_0}{\omega_0}\tau-r_l+o_1-r_m+o_2)}\frac{\sqrt{\omega_{k_z}\omega_{-2k_{0z}-k_z}}\mu_i\mu_j}{\epsilon_0 LS\hbar}\sinh(r)\cosh(r)S^+_{l,i}S^+_{m,j}\rho^S(t-\tau)\\
\approx&-\frac{L}{2\pi}\int_0^t d\tau e^{i\frac{c^2k_0}{\omega_0}(-k_0)\tau}e^{-i(2k_0)(r_l)}e^{2ik_0o_2}2\pi\delta(\frac{c^2k_0}{\omega_0}\tau+2r_c-o_1-o_2)\frac{\sqrt{\omega_i\omega_j}\mu_i\mu_j}{\epsilon_0 LS\hbar}\sinh(r)\cosh(r)S^+_{l,i}S^+_{m,j}\rho^S(t-\tau)\\
&-\frac{L}{2\pi}\int_0^t d\tau e^{i\frac{c^2k_0}{\omega_0}(-k_0)\tau}e^{i(2k_0)(r_l)}e^{-2ik_0o_2}2\pi\delta(\frac{c^2k_0}{\omega_0}\tau-2r_c+o_1+o_2)\frac{\sqrt{\omega_i\omega_j}\mu_i\mu_j}{\epsilon_0 LS\hbar}\sinh(r)\cosh(r)S^+_{l,i}S^+_{m,j}\rho^S(t-\tau)\\
\approx&-\frac{L}{2\pi}e^{i(-k_0)|2r_c-o_1-o_2|}e^{sgn(r_c-o_1-o_2)i(2k_0)(r_l)}e^{-2ik_0o_2}2\pi\frac{\omega_0^2\mu_i\mu_j}{\epsilon_0 LS\hbar c^2k_{0z}}\sinh(r)\cosh(r)S^+_{l,i}S^+_{m,j}\rho^S(t)\\
=&-\frac{L}{2\pi}e^{sgn(2r_c-o_1-o_2)ik_0(r_l-r_m)}e^{ik_0(o_1-o_2)}2\pi\frac{\omega_0^2\mu_i\mu_j}{\epsilon_0 LS\hbar c^2k_{0z}}\sinh(r)\cosh(r)S^+_{l,i}S^+_{m,j}\rho^S(t)
\end{aligned} \tag{B1}$$

Since we need to sum over l, m, i, j, the imaginary part of $e^{sgn(2r_c-o_1-o_2)ik_0(r_l-r_m)}$ gets canceled, which yields $\gamma'_{ijkl} = \sqrt{\gamma_i\gamma_k}\cos[k_{0z}(r_{jl})]$. The above calculation is also valid when $r_i = r_j$.

[1] O. Svelto and D. C. Hanna, *Principles of lasers*, vol. 4 (Springer, 1998).

[2] E. Purcell, Phys. Rev. **69**, 681 (1946).

[3] C. Gardiner, Phys. Rev. Lett. **56**, 1917 (1986).

[4] M. Collett, Phys. Rev. A **30**, 1386 (1984).
[5] J. Gea-Banacloche, M. Scully, and M. Zubairy, Physica Scripta **1988**, 81 (1988).
[6] G. Palma, Opt. Commun. **73**, 131 (1989).
[7] G. Agarwal and R. Puri, Physical Review A **41**, 3782 (1990).
[8] Z. Ficek, Physical Review A **42**, 611 (1990).
[9] Z. Ficek, Phys. Rev. A **44**, 7759 (1991).
[10] E. Goldstein, Phys. Rev. A **53**, 3573 (1996).
[11] H. Carmichael, A. Lane, and D. Walls, Physical review letters **58**, 2539 (1987).
[12] D. Toyli, A. Eddins, S. Boutin, S. Puri, D. Hover, V. Bolkhovsky, W. Oliver, A. Blais, and I. Siddiqi, Physical Review X **6**, 031004 (2016).
[13] R. Tanaś and Z. Ficek, Journal of Optics B: Quantum and Semiclassical Optics **6**, S610 (2004).
[14] F.-L. Li, P. Peng, and Z.-Q. Yin, Journal of Modern Optics **53**, 2055 (2006).
[15] J. You, Z. Liao, S.-W. Li, and M. S. Zubairy, Physical Review A **97**, 023810 (2018).
[16] Z. Ficek and P. Drummond, Physical Review A **43**, 6247 (1991).
[17] Z. Ficek and P. Drummond, EPL (Europhysics Letters) **24**, 455 (1993).
[18] J.-T. Shen and S. Fan, Physical review letters **95**, 213001 (2005).
[19] J.-T. Shen and S. Fan, Physical review letters **98**, 153003 (2007).
[20] V. Yudson and P. Reineker, Physical Review A **78**, 052713 (2008).
[21] Z. Liao, X. Zeng, S.-Y. Zhu, and M. S. Zubairy, Physical Review A **92**, 023806 (2015).
[22] Z. Liao, H. Nha, and M. S. Zubairy, Physical Review A **94**, 053842 (2016).
[23] Q. Turchette, Phys. Rev. A **58**, 4056 (1998).
[24] K. Murch, Nature (London) **499**, 62 (2013).
[25] N. Bergeal, R. Vijay, V. Manucharyan, I. Siddiqi, R. Schoelkopf, S. Girvin, and M. Devoret, Nature Physics **6**, 296 (2010).
[26] Z. Wang, S. Shankar, Z. Minev, P. Campagne-Ibarcq, A. Narla, and M. Devoret, arXiv preprint arXiv:1807.04849 (2018).
[27] W. Qin, A. Miranowicz, P.-B. Li, X.-Y. Lü, J.-Q. You, and F. Nori, Physical review letters **120**, 093601 (2018).
[28] M. O. Scully and M. S. Zubairy, *Quantum optics* (1999).