$$F(x,t) = -2\pi i g_{k_0} e^{ik_0(x-ct)} e^{-(\Gamma/2)(ct-x)} \theta(ct-x)$$
$$= g_{k_0} \int_{-\infty}^{\infty} dk [\frac{e^{ikx-i\nu_k t}}{\nu_k - (\omega - i\Gamma/2)} - \frac{e^{ikx-i\omega t-\Gamma t/2}}{\nu_k - (\omega - i\Gamma/2)}] \quad (1)$$