Guidelines and Submission: The submission link for this assignment is here: **Google Form**.

Please make sure you are accessing the Submission Link with your BracU email/account. You should cite all sources you used outside of the course material. Make sure to look at the "**We are expecting**" blocks below each problem to see what we will be grading for in each problem! Dont copy from our peers, or anywhere else. Slightest of dishonesty will bring you 00 marks in the whole!

Problems

1. **(10 pt.) [BFS and DFS A]**
   Give one example of a *connected undirected* graph on four vertices, A, B, C, and D, so that both DFS and BFS search *discover* the vertices in the **same** order when starting at vertex A. Then give one example of a *connected undirected* graph on four vertices, A, B, C, and D where BFS and DFS discover the vertices in a **different** order when started at A. Assume that both DFS and BFS iterate over neighbors in *alphabetical order*.

   Above, *discover* means the time that the algorithm first reaches the vertex.

   [**We are expecting:** A drawing of your two graphs **and** an ordered list of vertices discovered by BFS and DFS for each of them.]

2. **(5 pt.) [BFS & DFS Tree]** Run both BFS and DFS on the graph below, starting from node A. While performing each search, visit the outgoing neighbors of a vertex in alphabetical order. For each search, draw the resulting tree and list vertices in the order in which they were first visited.
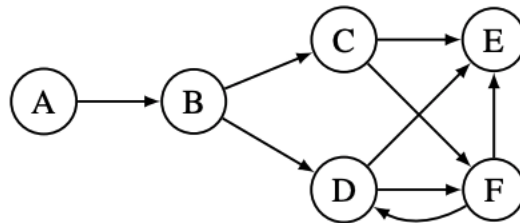
   [**We are expecting:** For each of BFS and DFS: a graph on which the algorithm is run, a tree, and a list containing the order of vertices]

3. **(5 pt.) [Robust Fuel Distribution]** You are given a fuel system to analyze its robustness. Here we will consider the robustness of the fuel distribution system under a few simplifications:

- Each edge in the graph is a pipe.

- Each edge is directed (fuel flow is not bidirectional).

- Each node in the graph is a combination refinery/distribution terminal (nodes both produce and consume fuel products, so they can have both incoming and outgoing edges).

Assume the system has n nodes and m edges. Design an algorithm that verifies whether there is a way to transport fuel from any node in the graph to any other node in the graph. Your algorithm should run in $O(n + m)$.

[**We are expecting:** A description of the algorithm, a brief explanation of why the algorithm works and an informal justification of the algorithm's running time.]

4. **(10 pt.) [DPDC]** The Dhaka Power Distribution Company [DPDC] network contains n power plants and n^3 buildings, pairs of which may be directly connected to each other via bidirectional wires. Every building is powered by either: having a wire directly to a power plant, or having a wire to another building that is recursively powered. Note that the network has the property that no building could be powered by more than one power plant. DPDC has secured funds to install an emergency generator in one of the power plants, which would provide backup power to all buildings powered by it, were the power plant to fail. Given a list L of all the wires in the network, describe an $O(n^6)$-time algorithm to determine the power plant where DPDC should install the generator that would provide backup power to the most buildings upon plant failure.

[**We are expecting:** A description of the algorithm, a brief explanation of why the algorithm works and an informal justification of the algorithm's running time.]

5. **(10 pt.) [Topological Exercise]** Please answer the following questions about the unweighted directed graph G below:
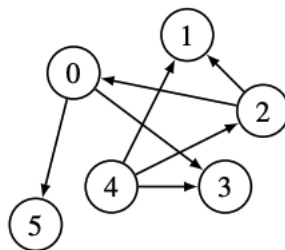
(a) **(5 pt.)** State a topological ordering of G. Then state and justify the number of distinct topological orderings of G.

(b) **(5 pt.)** State a single directed edge that could be added to G to construct a *simple graph* with no topological ordering. Then state and justify the number of distinct single edges that could be added to G to construct a *simple graph* with no topological ordering