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Distributed Robust Time-Varying Formation Control of Multi-Agent Systems under Disturbances

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Abstract: This work considers the problem of time-varying formation tracking control of second-order multi-agent systems under disturbances. A distributed robust time-varying formation control law is proposed including distributed finite-time estimators of the leader's states and sliding mode time-varying formation controllers. Firstly, each agent can quickly estimate the leader's states through the communication network in the distributed finite-time estimators based on the sliding mode estimation. Secondly, utilizing the estimates of the leader's states, a sliding mode time-varying formation controller is designed using the prescribed time modification function. Unlike traditional distributed control law relying on the exchanges of the agents' states, the proposed control design achieves great robustness and stability of the overall system by exchanging the estimates of the leader's states. According to Lyapunov stability analysis, we prove that the proposed approach enables the sliding variables to achieve fast finite-time convergence. Furthermore, simulation examples are provided to illustrate the performance.

Keywords: multi-agent systems; distributed control; time-varying formation control; sliding mode control.

1. INTRODUCTION

In recent years, the distributed control of multi-agent systems has been significantly researched due to its higher efficiency compared to traditional centralized control methods [1]. Its practical implementation has advanced and been utilized in various fields such as robot teams [2], transportation systems [3], drone formations [4], [5], sensor networks [6], and automated vehicles [7]. Formation control, one of the most actively researched topics in multi-agent systems, typically aims to guide multiple agents to satisfy specified constraints on their states [8]. In general, these constraints involve maintaining a specific formation shape among agents. Furthermore, time-varying formations offer greater advantages when dealing with environmental changes and dynamic tracking tasks [9], [10], [11].

In distributed formation control, the main focus of most research is on stabilizing the errors among agent states [8], [12], [13]. However, these traditional distributed control methods couple the cooperation among agents with the individual regulations of each agent. This concept heavily relies on the communication of neighbor agent states to achieve coordination. It led to several issues. Firstly, the information of neighbor agent states is directly used in distributed controllers. This implies that the states of any individual agent can influence the overall system through the communication network. This introduces potential risks when dealing with environmental changes or dynamic tracking tasks. Secondly, as controllers serve dual roles of cooperation and trajectory tracking, the design complexity increases significantly to the complexity of models, thus constraining practicality. Thirdly, redesigning for various tasks entails high additional costs. To face these issues, a design philosophy that separates cooperation and individual regulations is being pursued, while some existing distributed control approaches introduced observers to estimate the common references [14], [15], [16].

We contend that the key to multi-agent systems is the interaction among the agents. It is imperative to ensure that the interaction among multiple agents is efficient and conducive to the tasks, rather than introducing risks to the system stability. To this end, this paper proposes a distributed robust time-varying formation control law.

The contributions of this paper are as follows. The first contribution is the proposals for distributed robust time-varying formation control law including distributed estimators and sliding mode formation controllers. By partitioning cooperation and tracking tasks into distributed estimations and formation controllers, the central feature of this method lies in exchanging the estimates of the leader's states among agents without the neighbor agent's states. It has great flexibility in the overall system and enables rapid convergence. The second contribution is the proposal of sliding mode time-varying formation controllers based on the prescribed time modification function. The prescribed time modification function solves the peaking problem for the initial input. For each agent, the disturbances and the leader's accelerations are unknown. Formation control is achieved using the estimates of the leader's states and the time-varying formation shape function. By the Lyapunov stability theory, the fast finite-time stability of sliding variables is proven, with confirmation through simulation examples.

The remainder of the paper is organized as follows. In Section 2, a concise explanation of mathematical notations, graph theory, and finite-time stability theory is provided. In Section 3, we discuss the problem formulation of time-varying formation control for multi-agent systems under disturbances and traditional distributed con-

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trol design. In Section 4, We present the design of distributed finite-time estimations of leader's states, sliding mode formation control based on the prescribed time modification function, and stability analysis using finite-time stability theory. In Section 5, simulation examples are included to illustrate the effectiveness and performance of the proposed control method. Finally, conclusions are drawn in Section 6.

2. PRELIMINARIES

2.1. Notations

For $x \in R$, the sign function $\mathrm{sgn}(x)$ is defined as follows: $\mathrm{sgn}(x) = 1$, for $x > 0$; $\mathrm{sgn}(x) = 0$, for $x = 0$; $\mathrm{sgn}(x) = -1$, for $x < 0$. Additionally, for $0 < \alpha < 1$, the function $\mathrm{sig}(x)^\alpha$ is defined as $\mathrm{sig}(x)^\alpha = \mathrm{sgn}(x)|x|^\alpha$.

For $x = [x_1, \ldots, x_n]^T \in R^n$, the sign function $\mathrm{sgn}(x) = [\mathrm{sgn}(x_1), \ldots, \mathrm{sgn}(x_n)]^T$, $\mathrm{sig}(x)^\alpha = [\mathrm{sig}(x_1)^\alpha, \ldots, \mathrm{sig}(x_n)^\alpha]^T$, $|x| = [|x_1|, \ldots, |x_n|]^T$, and $|x|^\alpha = [|x_1|^\alpha, \ldots, |x_n|^\alpha]^T$. Euclidean norms are defined as $\|x\|_2 = \sqrt{x^T x}$. Moreover, the N - dimensional column vector with all elements being 1 is defined as 1_N. The symbol $\otimes$ denotes the Kronecker product. Finally, $\underline{\sigma}(\cdot)$ represents the minimum singular values of a matrix.

2.2. Graph theory

The graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ is considered, where $\mathcal{V} = \{v_1, \cdots, v_N\}$ represents N nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ denotes the edges. The graph $\mathcal{G}$ satisfies properties such as the following (i)–(vii) [17]: (i) An edge $\epsilon_{ik} = (v_i, v_k) \in \mathcal{E}$ indicates that agent k can receive information from agent i. (ii) The adjacency matrix $\mathcal{A} = [a_{ik}] \in R^{N\times N}$ of the graph $\mathcal{G}$ is defined such that $a_{ik} > 0$ if $(v_k, v_i) \in \mathcal{E}$, and $a_{ik} = 0$ otherwise. Additionally, $a_{ii} = 0$. (iii) Not all edges in a directed graph necessarily have bidirectional connections. (iv) If there exists a node i_r such that there is a path to every other node in the graph, the directed graph is said to have a spanning tree. (v) The set of neighbors with incoming to a node v_i is denoted as $\mathcal{N}_i = \{v_k \in \mathcal{V} | (v_k, v_i) \in \mathcal{E}\}$. (vi) Define the in-degree matrix as a diagonal matrix $\mathcal{D} = [d_i] \in R^{N\times N}$, where $d_i = \sum_{k\in\mathcal{N}_i} a_{ik}$. (vii) The Laplacian matrix $\mathcal{L} \in R^{N\times N}$ is defined $\mathcal{L} = \mathcal{D} - \mathcal{A}$.

Define the augmented graph for leader and agents as $\mathcal{G}_0 = (\mathcal{V}_0, \mathcal{E}_0, \mathcal{A}_0)$, where $\mathcal{V}_0 = \{v_0, \cdots, v_N\}$ and $\mathcal{E}_0 \subseteq \mathcal{V}_0 \times \mathcal{V}_0$. Notice that the index 0 refers to the leader and the indices $1, ..., N$ refer to the follower agents. The pinning gain b_i represents the weight of edges from v_0 to v_i. The pinning matrix is defined as $\mathcal{B} = \mathrm{diag}\{b_1, \cdots, b_N\}$. $b_i > 0$ if follower i is connected to the leader, while $b_i = 0$ otherwise.

Assumption 1: The augmented graph $\mathcal{G}_0$ contains a spanning tree with the leader node 0 being the root node. The leader node can also be viewed as a virtual leader and functions as the node responsible for generating the reference signal.

Lemma 1: [18], [19] Let assumption 1 hold, then $\mathcal{H} = \mathcal{L} + \mathcal{B}$ is a non-singular matrix.

2.3. Finite-time stability theory

Lemma 2: [20] For a nonlinear systems $\dot{x}(t) = f(x(t))$, where $f(0) = 0$ and $x(0) = x_0$. Let there exists a continuous function $V(x)$, which is positive definite and satisfy the differential inequality $\dot{V}(x) \le -\alpha V(x)^p$ where $\forall x(t) \ne 0$, $\alpha > 0$ and $p \in (0, 1)$. Then, the system is finite-time stable. $T_1 \le V(x_0)^{1-p}/(\alpha(1-p))$ is the required time. Furthermore, if the differential inequality $\dot{V}(x) \le -\beta V(x) - \alpha V(x)^p$ where $\beta > 0$ is satisfied, the system is fast finite-time stable. $T_2 \le \ln\left(1 + \beta V(x_0)^{1-p}/\alpha\right)/\beta(1-p)$ is the required time.

3. PROBLEM FORMULATION

3.1. System Model

Consider $i = 1, \ldots, N$ follower agents by double-integral dynamics that are written as

$$\dot{x}_{i1}(t) = x_{i2}(t),\ \dot{x}_{i2}(t) = u_i + d_i, \tag{1}$$

where $x_{i1}(t) = [x_{i1,1}, \ldots, x_{i1,n}]^T$, $x_{i2}(t) = [x_{i2,1}, \ldots, x_{i2,n}]^T \in R^n$ is the states of the i-th agent, and $u_i \in R^n$ is the control input of the i-th agent. $d_i \in R^n$ is unknown and bounded disturbances. n is the system dimension. The leader agent is described as

$$\dot{x}_{01}(t) = x_{02}(t),\ \dot{x}_{02}(t) = u_0(t), \tag{2}$$

where $x_{01}(t), x_{02}(t) \in R^n$ is the position and velocity states of the leader agent. The $u_0(t) \in R^n$ is a bounded function as the acceleration of the leader agent and unknown for any follower agent.

3.2. Control task

This article aims to achieve the time-varying formation control based on leader-follower, which is specified as

$$\begin{aligned} &\lim_{t\to\infty} \|x_{i1}(t) - x_{01}(t) - p_i(t)\|_2 = 0, \\ &\lim_{t\to\infty} \|x_{i2}(t) - x_{02}(t) - \dot{p}_i(t)\|_2 = 0, \end{aligned} \tag{3}$$

where the time-varying formation shape function $p_i(t), \dot{p}_i(t) \in R^n$ denotes the relative position and relative velocity between the i-th agent and the leader agent, which are used for the desired time-varying formation shape. However, $\ddot{p}_i(t)$ is a bounded function and unknown for any follower agent.

3.3. Traditional distributed formation control

As shown in Fig.1, the traditional distributed formation control law [8] can be designed as

$$\begin{aligned} u_i = &-\rho_1 \textstyle\sum_{k\in\mathcal{N}_i} a_{ik}(x_{i1} - x_{k1} - p_i + p_k) \\ &-\rho_2 \textstyle\sum_{k\in\mathcal{N}_i} a_{ik}(x_{i2} - x_{k2} - \dot{p}_i + \dot{p}_k) \\ &-\rho_3 b_i(x_{i1} - x_{01} - p_i + x_{i2} - x_{02} - \dot{p}_i), \end{aligned} \tag{4}$$

which relies on the exchanges of states among agents, and $\rho_1, \rho_2, \rho_3 > 0$ are design parameters. The formation task is transformed into stabilization tasks of the errors among the agent's states in (4).

However, for the i-th agent, the states of the neighbor k-th agent x_{k1} and x_{k2} are influenced by the disturbances d_k. The controller of i-th agent must handle the disturbances from other agents, which is already a challenging task, not to mention the additional complexities from time-varying formation shape function $\ddot{p}_i$ and the leader's accelerations u_0. As introduced in Section 1, the reason lies in the reliance on exchanges of the agent's states. Therefore, unlike the above traditional distributed control law, we propose the following control design as shown in Fig.2.

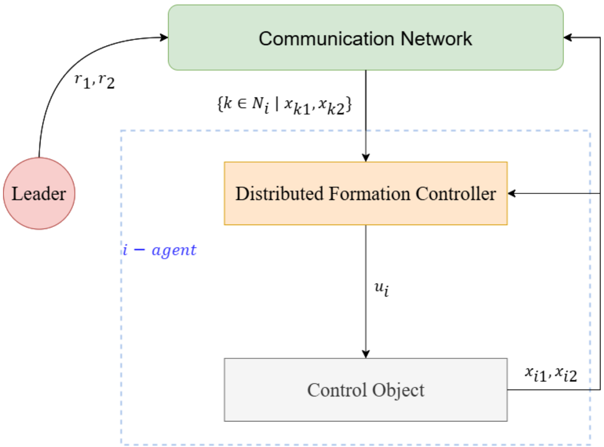

Fig. 1 Block diagrams for the traditional design

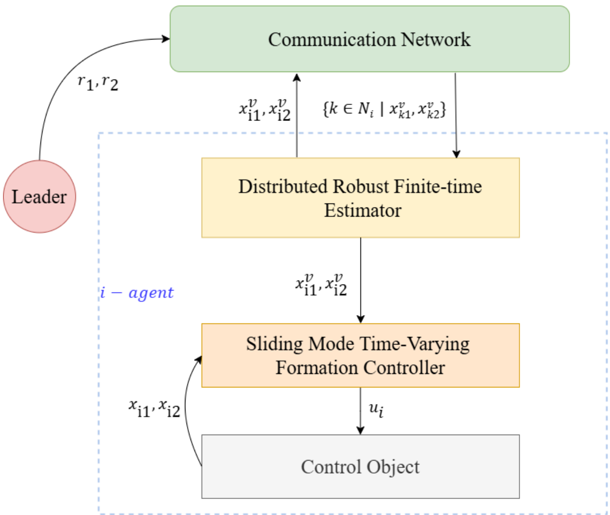

Fig. 2 Block diagrams for the proposed design

4. MAIN RESULT

To achieve the control task, the formation tracking position and velocity error vectors are defined as

$$\begin{aligned} e_{i1}(t) &= x_{i1}(t) - x_{01}(t) - p_i(t), \\ e_{i2}(t) &= x_{i2}(t) - x_{02}(t) - \dot{p}_i(t). \end{aligned} \tag{5}$$

Then, we obtain the dynamics of errors as

$$\begin{aligned} \dot{e}_{i1}(t) &= e_{i2}(t), \\ \dot{e}_{i2}(t) &= u_i + d_i - u_0 - \ddot{p}_i. \end{aligned} \tag{6}$$

The control task results in all agents' tracking errors e_{i1} and e_{i2} converging to zero. Firstly, distributed finite-time estimations [21] for consensus tracking control are introduced to estimate the leader's states in Section 4.1. Secondly, we design sliding mode time-varying formation controllers based on the prescribed time modification function in Section 4.2.

4.1. Distributed estimation of leader's states

Assuming the leader's states are known for some follower agents and can be published to several follower agents. The communication information is denoted as

$$r_1 = x_{01}, \quad r_2 = x_{02}. \tag{7}$$

Then, we introduce a virtual single-integrator for each agent as $\dot{x}^v_{ij} = u^v_{ij}$ $(j = 1, 2)$,where x^v_{i1}, x^v_{i2} represents the estimate of the leader position and velocity information r_1, r_2 for the i-th agent. Therefore, for $j = 1, 2$, $u^v_{ij} \in R^n$ denotes the control input for x^v_{ij} to estimate the leader state r_j.

Define the estimation error as $\epsilon^v_{ij} = x^v_{ij} - r_j$. We have the dynamics of estimation error as $\dot{\epsilon}^v_{ij} = u^v_{ij} - r_{j+1}$ where r_3 denotes as u_0. Therefore, for $i = 1, \ldots, N$, $j = 1, 2$, the global expression of the above equations becomes

$$\begin{aligned} \dot{x}^v_j &= u^v_j, \\ \epsilon^v_j &= x^v_j - 1_N \otimes r_j, \\ \dot{\epsilon}^v_j &= u^v_j - 1_N \otimes r_{j+1}, \end{aligned} \tag{8}$$

where $x^v_j = [{x^v_{1j}}^T, \ldots, {x^v_{Nj}}^T]^T$, $u^v_j = [{u^v_{1j}}^T, \ldots, {u^v_{Nj}}^T]^T$, $\epsilon^v_j = [{\epsilon^v_{1j}}^T, \ldots, {\epsilon^v_{Nj}}^T]^T \in R^{Nn}$ is the global expression of estimate, input and estimation error.

The estimate of the agents is exchanged through the communication graph $\mathcal{G}_0$ defined in assumption 1. Then, we define the local estimation error as

$$\epsilon^v_{s,ij} = \textstyle\sum_{k \in N_i} a_{ik}(x^v_{ij} - x^v_{kj}) + b_i(x^v_{ij} - r_j), \tag{9}$$

where $b_i, a_{ik} \in R$ are the elements of the pinning matrix $\mathcal{B}$ and the adjacency matrix $\mathcal{A}$. Define the global expression of the local estimation error as $\epsilon^v_{s,j} = [{\epsilon^v_{s,1j}}^T, \ldots, {\epsilon^v_{s,Nj}}^T]^T \in R^{Nn}$. According to (8) and (9), we obtain the following equation.

$$\epsilon^v_{s,j} = (\mathcal{H} \otimes I_n)\epsilon^v_j, \tag{10}$$

where $\mathcal{H} = \mathcal{L} + \mathcal{B}$ is defined in Lemma 1. Furthermore, according to Lemma 1, we have the relation $\|\epsilon^v_j\|_2 \leq \|\epsilon^v_{s,j}\|_2 / \underline{\sigma}(\mathcal{H})$. Therefore, the estimation error will converge as the local estimation error converges.

The dynamics of (10) is expressed as $\dot{\epsilon}^v_{s,j} = (\mathcal{H} \otimes I_n)(u^v_j - 1_N \otimes r_{j+1})$ for $j = 1, 2$. Then, we can design the virtual inputs for estimation [21] as follows.

$$\begin{aligned} u^v_1 &= -l^v_{a,1}\epsilon^v_{s,1} - l^v_{b,1}\mathrm{sig}(\epsilon^v_{s,1})^\alpha + x^v_2, \\ u^v_2 &= -l^v_{a,2}\epsilon^v_{s,2} - l^v_{b,2}\mathrm{sgn}(\epsilon^v_{s,2}), \end{aligned} \tag{11}$$

where the parameters $l^v_{a,1}$, $l^v_{b,1}$, $l^v_{a,2}$, $l^v_{b,2} > 0$ are scalar gains, and $0 < \alpha < 1$.

Lemma 3: For distributed finite-time estimation (11), there exists a finite-time T^v_f at which all the estimation errors converge to zero [21].

Remark 1: By selecting a sufficiently large $l^v_{a,1}, l^v_{a,2}$, and a relatively large $l^v_{b,1}, l^v_{b,2}$, a finite time T^v_f exists. For each follower agent, the convergence time of the distributed finite-time estimation T^v_f is significantly smaller compared to the formation tracking control mentioned below.

4.2. Sliding mode time-varying formation control

To solve the peaking problem of the initial inputs, which is due to the large initial tracking errors, we introduce the prescribed time modification function to formation tracking errors as

$$\begin{aligned} e_{mi1}(t) &= e_{i1}(t) - \xi_i(t), \\ e_{mi2}(t) &= e_{i2}(t) - \dot{\xi}_i(t), \end{aligned} \tag{12}$$

where $\xi_i(t) = \Big(e_{i1}(T_f^v) + \dot{e}_{i1}(T_f^v)t\Big)\exp(-\frac{\lambda_i t^2}{T_F - t})$ if $t \in [T_f^v, T_F)$, while $\xi_i(t) = 0$ otherwise. $\lambda_i > 0$ and $T_F > 0$ are design parameters. Furthermore, for $t \geq T_F$, we have $\xi_i(t) = \dot{\xi}_i(t) = \ddot{\xi}_i(t) = 0$.

Remark 2: We set $e_{mi1} = e_{mi2} = 0$ for $t < T_f^v$. As a result of the prescribed time modification function, the tracking errors are modified to zero for $t = T_f^v$. The tracking errors are modified for $T_f^v \leq t \leq T_F$ and $e_{mi1} = e_{i1}$, $e_{mi2} = e_{i2}$ for $t > T_F$.

Remark 3: According to Lemma 3, for $t < T_f^v$, we lack the correct estimates. Therefore, we set the control inputs to zero to maintain the original agent's states. As T_f^v is sufficiently small, the agents do not escape far away.

According to (6), we obtain the dynamics of modified tracking errors as follows:

$$\begin{aligned} \dot{e}_{mi1} &= e_{i2} - \dot{\xi}_i(t), \\ \dot{e}_{mi2} &= u_i + d_i - u_0 - \ddot{p}_i - \ddot{\xi}_i(t). \end{aligned} \tag{13}$$

Define the sliding variable for the i-th agent as

$$s_i = c_i e_{mi1} + e_{mi2}, \tag{14}$$

where $s_i \in R^n$, and $c_i > 0$ is a design parameter to satisfy the Hurwitz equation. According to (13) and (14),we obtain the dynamics of sliding variable as

$$\begin{aligned} \dot{s}_i &= c_i(e_{i2} - \dot{\xi}_i) + u_i + d_i - u_0 - \ddot{p}_i - \ddot{\xi}_i, \\ &= c_i e_{i2} + u_i + D_i - c_i\dot{\xi}_i - \ddot{\xi}_i, \end{aligned} \tag{15}$$

where $D_i = d_i - u_0 - \ddot{p}_i$ is the lumped disturbance term. As d_i, u_0, and $\ddot{p}_i$ are bounded functions, D_i is also a bounded function. Therefor, there exist a constant $\bar{D}_i$ that satisfy $\|D_i\|_2 \leq \bar{D}_i (i = 1, \ldots, N)$.

To ensure convergence, the input is designed as

$$\begin{aligned} u_i &= u_n^i + u_r^i, \\ u_n^i &= -c_i e_{i2} - k_i s_i + c_i\dot{\xi}_i + \ddot{\xi}_i, \\ u_r^i &= -\eta_i \mathrm{sgn}(s_i), \end{aligned} \tag{16}$$

where u_n^i is the nominal control term for the i-th agent with k_i as feedback gain. u_r^i is the robust control term, where $\eta_i > \bar{D}_i$ is a design parameter.

Theorem 1: By the distributed estimators (11) and the controllers (16), the following results hold.
(i) All the sliding variables $s_i (i = 1, \ldots, N)$ converge to zero within a finite time T_f.
(ii) All the formation tracking errors $e_{i1} (i = 1, \ldots, N)$ and $e_{i2} (i = 1, \ldots, N)$ converge to zero exponentially.

Proof: For $t \geq T_f^v$, considering the Lyapunov function candidate as

$$V_1 = \frac{1}{2}\sum_{i=1}^{N} s_i^T s_i = \frac{1}{2}\sum_{i=1}^{N} \|s_i\|_2^2,$$

we have the time derivative of it as

$$\dot{V}_1 = \sum_{i=1}^{N} s_i^T \dot{s}_i = \sum_{i=1}^{N} s_i^T \big(-k_i s_i + D_i - \eta_i \mathrm{sgn}(s_i)\big).$$

Using the properties, $a^T b \leq \|a\|_2 \|b\|_2$, $a^T \mathrm{sgn}(a) = \|a\|_1$ and $\|a\|_1 \leq \|a\|_2$ for $a, b \in R^n$, the above equation can be rewritten as

$$\begin{aligned} \dot{V}_1 &\leq \sum_{i=1}^{N} \Big(-k_i \|s_i\|_2{}^2 - \big(\eta_i - \|D_i\|_2\big)\|s_i\|_2 \Big) \\ &\leq \sum_{i=1}^{N} \Big(-k_a \|s_i\|_2{}^2 - \eta_a \|s_i\|_2 \Big) \\ &\leq -2k_a V_1 - 2\eta_a \sqrt{V_1}. \end{aligned} \tag{17}$$

where $k_a = \min\{k_1, \ldots, k_N\}$ and $\eta_a = \min\{\eta_1 - \bar{D}_1, \ldots, \eta_N - \bar{D}_N\}$. The sliding variables in sliding mode formation control (16) achieve fast finite-time stability according to Lemma 2. We obtain the convergence time of the overall formation as

$$T_f \leq \frac{\ln\Big(1 + k_a\sqrt{V_1(T_f^v)/\eta_a}\Big)}{k_a} + T_f^v.$$

Then, we have the result (i). According to (14), the result (ii) holds. ■

Remark 4: All agents' estimation errors converge to zero within a finite time T_f^v. Formation tracking error is modified within the prescribed time T_F. The formation tracking error converges to zero within a finite time T_f, where $T_f > T_F \gg T_f^v$.

5. SIMULATION EXAMPLES

A leader-follower second-order multi-agent system as (1) with a leader agent and $N = 10$ follower agents is given. The system dimension is $n = 3$. The communication topology is shown in Fig. 3 to set $\mathcal{A}$ and $\mathcal{B}$.

The leader's states are set as $x_{01} = [R_r(1 - \cos\omega t), R_r \sin\omega t, v_{max}(t + e^{-t} - 1)]^T$, where $\omega = 0.1\pi(1 - e^{-t})$ rad/s, $R_r = 5$ m, $v_{max} = 0.5$ m/s. The initial states of follower agents are set as $x_{i1,j}(0) = x_{01,j}(0) + (-5 \sim 5)$ m for $j = 1, 2$ and $x_{i1,3}(0) = 0$ m, $x_{i2}(0) = [0, 0, 0]^T$ m/s. The initial estimation states are set as $x_{i1}^v(0) = x_{i1}(0)$ and $x_{i2}^v(0) = [0.1, 0.1, 0.1]^T$ m/s. The simulation lasts 30 s and the step length is 0.001 s.

The initial formation shape is defined as $p_i(0) = i_p(i)R_p\left[\cos(\theta(i)), \sin(\theta(i)), 0\right]^T$ where $i_p(i) = 0.5i$ and $\theta(i) = -\pi/4$ if $i = 1, \ldots, 5$. $i_p(i) = 0.5(i - 5)$ and $\theta(i) = -3\pi/4$ if $i = 6, \ldots, 10$. $R_p = 2\sqrt{2}$ m. For $t \in (t_c, t_c + t_t]$, the formation shape function is defined as $\ddot{p}_i(t) = i_p(i)\left[0, 0.2\sin(2\pi(t - t_c)/t_t), 0\right]^T$, while $\ddot{p}_i = [0, 0, 0]^T$ otherwise, where $t_c = 10$ s and $t_t = 10$ s. The $t \in (t_c, t_c + t_t]$ is the time for the formation changes.

The nonlinear disturbances are chosen as $d_{i,j}(t) = a_i^d \left(\cos\left(b_i^d x_{i1,1} + x_{i2,1}\right) + \sin\left(b_i^d x_{i1,2} + x_{i2,2}\right)\right)$, for $j =$

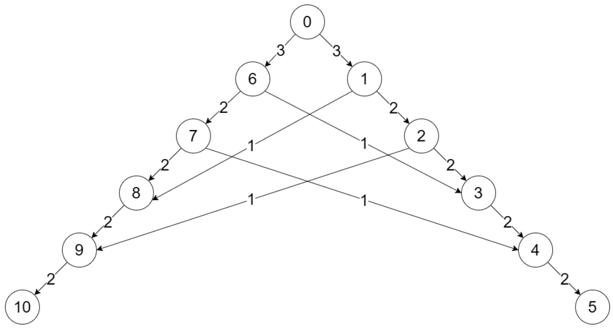

Fig. 3 Communication topology

$1, 2, 3$, where $[a_1^d, \ldots, a_{10}^d] = [-1.8,\ 1.6,\ 1.9,\ -1.7,\ 1.8,\ -1.6,\ 1.9,\ -1.7,\ 1.5, -1.4]$ and $[b_1^d, \ldots, b_{10}^d] = [-1.3, 0.8, 1.5, -0.6, 0.7, -1.2, 0.9, 0.4, 1.1, -0.5]$.

For the distributed estimators, we set $\alpha = 0.2$, $l_{a,1}^v = l_{a,2}^v = 80$, $l_{b,1}^v = l_{b,2}^v = 5$. Based on experience, we have $T_f^v \leq 0.05$ s and set $\lambda_i = 1$, $T_F = 5$ s for the prescribed time function. For the sliding mode time-varying formation controllers, we set $c_i = 1$, $k_i = 2$, $\eta_i = 7$. We have the following inequalities: $\max_{i \in 1,\ldots,N}\{\|d_i\|_2\} < 5$, $\|u_0\|_2 < 1$, $\max_{i \in 1,\ldots,N}\{|\ddot{p}_i\|_2\} \leq 1$. The robust control gain η_i satisfies the inequality $\max_{i \in 1,\ldots,N}\{\|D_i\|_2\} \leq \max_{i \in 1,\ldots,N}\{\|d_i\|_2\} + \|u_0\|_2 + \max_{i \in 1,\ldots,N}\{\|\ddot{p}_i\|_2\} < 7 \leq \min_{i \in 1,\ldots,N}\{\eta_i\}$.

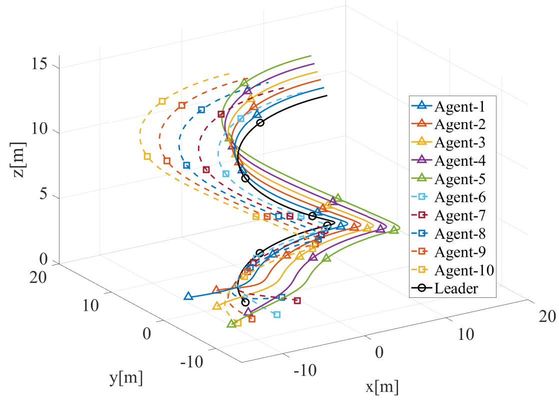

Fig. 4 Time-varying formation control

The time-varying formation tracking control is shown in Fig.4, where the formation shape has changed. The estimates of the leader's states $\epsilon_{i1}^v, \epsilon_{i2}^v$ is shown in Fig. 5. It can be confirmed that the distributed estimators can quickly estimate the leader's states $T_f^v \leq 0.05$ s. The sliding mode time-varying formation tracking performance is shown in Fig. 6, Fig. 7 and Fig. 8 where the agents' states $x_{i1}, x_{i2}, x_{01}, x_{02}$, the formation tracking errors e_{i1}, e_{i2} and the control inputs u_i are indicated. The proposed control approach achieves time-varying formation control with both great robustness and rapid responsiveness. Additionally, thanks to the prescribed time modification function, the proposed formation controllers deliver an excellent tracking performance and suppress the peak of initial control input.

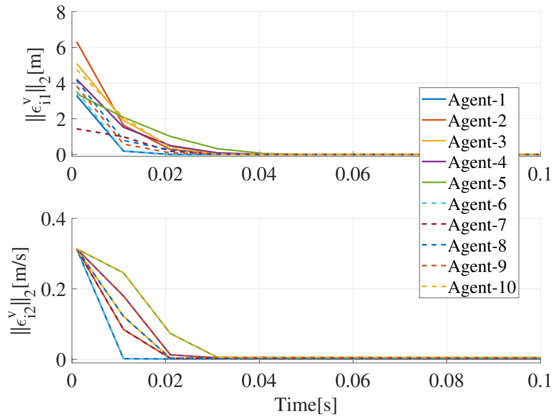

Fig. 5 Estimates errors

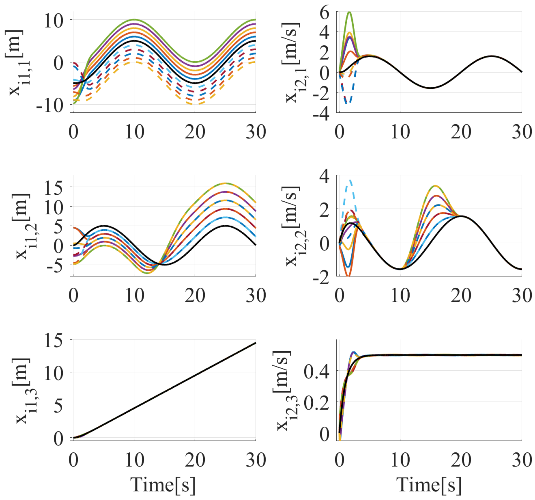

Fig. 6 States of agents

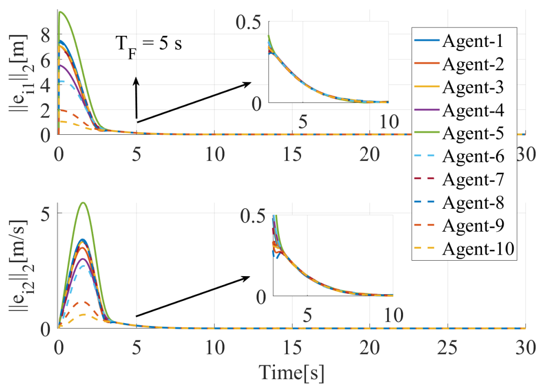

Fig. 7 Formation tracking errors

6. CONCLUSION

In this paper, a control law of distributed robust time-varying formation control has been constructed for multi-agent systems under disturbances. Within this approach, the cooperation and individual regulations are partitioned into distributed estimators and sliding mode formation controllers. This approach simplifies the design of dis-

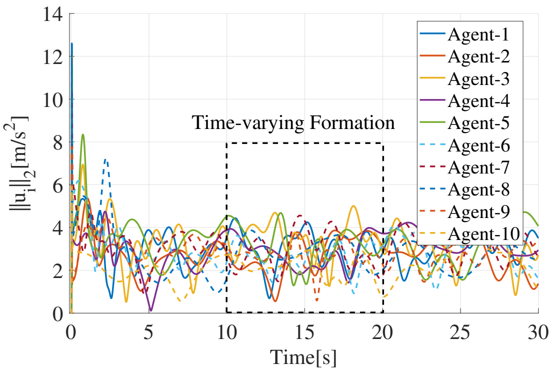

Fig. 8 Control input

tributed control and has flexibility, practicality, and scalability. Using a second-order multi-agent system considering disturbances as an example, we have demonstrated finite-time convergence of sliding variables in time-varying formation controllers. Future research may explore cases involving additional factors such as collision avoidance for multi-agent systems, range-based topology, and optimization.

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