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$\frac{\begin{array}{r}32\\ \times 32\end{array}}{\begin{array}{r}64\\ 96\\ \hline 1024\end{array}}$ $\frac{\begin{array}{r}240\\ \times 32\end{array}}{\begin{array}{r}480\\ 720\end{array}}$ $\begin{array}{r}7680\\ -6400\\ \hline 1280\end{array}$

22

CSIE Probability Exam II

Tue, Dec 14, 2021

Attempt all the questions. Justify your answers unless otherwise specified. Give your answer in terms of Standard Normal CDF $\Phi(\cdot)$, Standard Normal Complementary CDF $Q(\cdot)$, $\exp(\cdot)$, $\log(\cdot)$, fractions, etc., if needed. Please write down your name and student ID number on your answer sheet.

$Var[X] = E[X^2] - \mu_X^2$

1. Discrete random variable X has the PMF

$$P_X(x) = C^5_x(\frac{1}{2})^5.$$

C^5_1 $C^5_x \cdot \frac{1}{32}$ $120+90+30$ $150+90$ 240

E$[X] = \mu_X$. Var$[X] = \sigma^2_X$. (a) Find P$[|X - \mu_X| \geq \sigma_X]$. (15pts)

8/15

$$P_X(x) = \begin{cases} \frac{5}{32}, & x=1.4 \\ \frac{10}{32} & x=2.3 \\ \frac{1}{32} & x=0.5 \\ 0 & \text{otherwise} \end{cases}$$

$\mu_X = \frac{5}{32}+\frac{20}{32}+\frac{20}{32}+\frac{30}{32}+\frac{5}{32}$

$= \frac{80}{32} = 2.5$ ✓

$Var[X] \Rightarrow E[X^2] = \frac{5}{32}+\frac{80}{32}+\frac{40}{32}+\frac{90}{32}+\frac{25}{32} = \frac{240}{32}$

$Var[X] = \frac{240}{32} - (\frac{80}{32})^2$

$= \frac{240}{32} - \frac{6400}{1024}$

$= \frac{1280}{1024} = \frac{5}{4}$

$\sigma = \frac{\sqrt{1280}}{32}$ 11…

$\sigma < 1$?

$P[|X - \mu_X| \geq \sigma_X]$

$= 1$ #

(b) $Y = \frac{X-\mu_X}{\sigma_X}$. Find E$[Y]$ and Var$[Y]$. (15pts)

$E[Y] = 0$ $\Rightarrow E[X-\mu_X] = 0$

$Var[Y] = 1$

此題為 standardized random variable

5/15

0 ✓
1 ✓
2 ✓
3 ✓
4 ✓
5 ✓

2. A radio station gives out a concert ticket to the sixth caller who knows the birthday of the performer. For each person who calls, the probability is 0.75 of knowing the performer's birthday. All calls are independent. L is the number of calls necessary to find the winner. Find E$[L^2]$. (20pts)

Pascal!

$p = 0.75$, $n = 6$

$E[L] = np = 6 \times 0.75 = 4.5$

$Var[L] = 6 \times 0.75 \times 0.25 = 1.125$

$E[L^2] = Var[L] + E[L]^2$

$= 1.125 + 20.25 = 21.375$

$E[L] = np$

$Var[L] = np(1-p)$

$\begin{array}{r}4.5\\ \times 4.5\\ \hline 225\\ 180\\ \hline 20.25\end{array}$ $\begin{array}{r}4.5\\ \times 2.5\\ \hline 225\\ 90\\ \hline 11.25\end{array}$ $\begin{array}{r}20.250\\ +1.125\\ \hline 21.375\end{array}$

4/20

$\frac{(b-a)^2}{12} = 12$

$(b-a) = \pm 12$

3. X is a uniform continuous random variable with $P[X \leq 6] = 0.5$ and $\text{Var}[X] = 12$. Find the PDF $f_X(x)$ of X. (15pts)

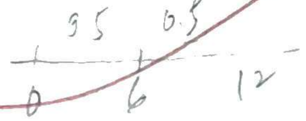
$f_X(x) = \begin{cases} \frac{1}{(b-a)} & a \leq x < b \\ 0 & \text{otherwise} \end{cases}$

$(b-a)^2 = 144$

$b-a = \pm 12$

$f_X(x) = \begin{cases} \frac{1}{12}, & 0 \leq x < 12 \\ 0 & \text{otherwise} \end{cases}$

$\text{Var}[X] = 12 = \frac{(b-a)}{12}$



15/15

4. X is a Gaussian random variable with $P[X > 0] = 0.5$ and $E[X^2] = 16$. Find $P[|X| > 6]$. (15pts)

$-\mu_0^2$: Var

Var 16

6=4

$\begin{cases} \mu_X = 0 \\ \sigma_X = 4 \end{cases}$ ✓

$P[|X|>6] = P[\cancel{-6<X<6}]$

$= \Phi(\frac{6-0}{4}) - \Phi(\frac{-6-0}{4})$

$= \Phi(\frac{6}{4}) - \Phi(-\frac{6}{4})$

$= \Phi(\frac{6}{4}) - (1-\Phi(\frac{6}{4}))$

$= -1+2\Phi(\frac{6}{4})$

$= 2\Phi(\cancel{1.5})-1$

10/15

5. X is a continuous random variables with the PDF

$$f_X(x) = \begin{cases} x/2 & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

$>4\ 8\ 16$

Find $E[X]$ and $\text{Var}[X]$. (20pts)

$E[X] = \int_{-\infty}^{\infty} x f_X(x)\,dx$

$= \int_0^2 x \cdot \frac{x}{2}\,dx$

$= \int_0^2 \frac{x^2}{2}\,dx$

$= \frac{1}{6}x^3 \Big|_0^2$

$= \frac{8}{6} = \frac{4}{3}$ ✓

$E[X^2] = \int_0^2 x^2 \frac{x}{2}\,dx$

$= \int_0^2 \frac{1}{2}x^3\,dx$

$= \frac{1}{8}x^4 \Big|_0^2$

$= 2$

$\text{Var}[X] = E[X^2] - (E[X])^2$

$= 2 - \frac{16}{9}$

$= \frac{2}{9}$ ✓

$\frac{1}{2}x^2$

$\frac{1}{6}x^3$

$\frac{1}{2}x^2$

$\frac{1}{6}x^3$

$\frac{1}{2}x^2$

20/20