CSIE Probability Exam I

Tue, Nov 9, 2021

Attempt all the questions. Justify your answers unless otherwise specified. Give your answer in terms of fractions, exp(·), etc., if needed.

Name: 吳侑珉 . Student ID No.: [illegible]

75

good!

1. At NCNU, the number of the undergrads(U) is five times the number of the graduate students(G), $\mathrm{P}[U] = 5\mathrm{P}[G]$. In addition, the students are equally likely to have no credit card(C_0), one credit card(C_1), or two or more credit cards(C_2), $\mathrm{P}[C_0] = \mathrm{P}[C_1] = \mathrm{P}[C_2] = 1/3$. Among the students with no credit card(C_0), only 20% are graduate students, and the same is true with the students with one credit card(C_1), $\mathrm{P}[G|C_0] = \mathrm{P}[G|C_1] = 0.2$. When you meet a graduate student, the chance that she or he has two or more credit cards is 0.2, $\mathrm{P}[C_2|G] = 0.2$. Finish the following table, and find the probability when you meet a student with one or more credit cards, she or he is an undergrad $\mathrm{P}[U|C_1 \cup C_2]$. (20pts)

		C_0	C_1	C_2
5	U	$4/15$	$4/15$	$3/10$
1	G	$1/15$	$1/15$	$1/30$

$\frac{\cancel{160}}{\cancel{600}} \; 30 \; 15 \qquad \frac{1}{3} \times \frac{4}{5} = \frac{4}{15}$

$1 \quad \frac{\cancel{180}}{\cancel{600}} \; \cancel{30} \; 93 \qquad \frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$

$\frac{20}{600}$

500

100

160 160 180

40 40 20

$15 \times 6 = 90$

$\mathrm{P}[U|C_1 \cup C_2] = \frac{340}{400} = \frac{17}{20}$

20/20

2. Roll a fair ten-sided die five times. The outcome is independent from roll to roll. Let X_k = the k-th outcome, $k = 1, 2, ..., 5$. Find $\mathrm{P}[\max\{X_1, X_2, ..., X_5\} = 7]$. (15pts)

$\mathrm{P}[\max\{X_1, X_2, \ldots X_5\} = 7] = \mathrm{P}[\max\{X_1, X_2, \ldots X_5\} \le 7] - \mathrm{P}[\max\{X_1, X_2, \ldots X_5\} \le 6]$

$= (7/10)^5 - (6/10)^5$

$= \frac{9031}{100000}$

$$\begin{array}{r} 49 \\ 49 \\ \hline 441 \\ 356 \\ \hline 4001 \\ 7 \\ \hline 28007 \end{array} \qquad \begin{array}{r} 28007 \\ 7776 \\ \hline 20231 \end{array} \qquad \begin{array}{r} 16807 \\ 7776 \\ \hline 9031 \end{array}$$

15/15

3. You roll two fair dice until you get a double. N is the number of the rolls. Find E[N], and the probability that the double shows up in the first, second, or third roll, P[$N \le 3$]. (20pts)

$P = 6/36 = 1/6$

$E[N] = \frac{1}{p} = 6$

$P[1] = P[2] = P[3] = 1/6$

$P[N \le 3] = 1 - (\frac{5}{6})^3$ → 都沒中的機率

$= 1 - \frac{125}{216} = \frac{91}{216}$ ※

$\begin{array}{r} 216 \\ -125 \\ \hline 91 \end{array}$

20/20

4. At a base station, the number X of the messages it receives during 6:00-6:30am is a Poisson random variable with E[X] = 3. I is an indicator random variable such that $I = 1$ if at least one message shows up; otherwise $I = 0$. Find P[$I = 0$], P[$I = 1$], and E[I]. (20pts)

$E[X] = 3 = \alpha$

$P[x] = \frac{\alpha^x \cdot e^{-\alpha}}{x!} = \frac{3^x \cdot e^{-3}}{x!}$

$P[I=0] = P[X=0] = \frac{3^0 \cdot e^{-3}}{0!} = e^{-3}$

$P[I=1] = 1 - P[X=0]$

$= 1 - \frac{3^0 \cdot e^{-3}}{0!}$

$= 1 - e^{-3}$

$E[I] = P = P[I=1] = 1 - e^{-3}$

20/20