國立暨南國際大學 學年度第 學期 期中/期末 考試試卷

記分	89
教師簽章	

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1, a) 1287
b) 3744
c) 123552

2, a) 1024
b) 1024

3, a) $r \to q$
b) $p \to \neg q$
c) $[(s \wedge r) \to q]$

4, $2^5 = 32$

5, a) false
b) true

6, a) false
b) true
c) false

7, a) $\exists x \exists y [(x<y) \wedge \forall z (x>z>y)]$
b) $\exists x \exists y [(|x|=|y|) \wedge (x \neq \pm x)]$

8 a) by the rule of product, $5 \times 10 \times 3 \times 2 = 300$
b) by the rule of product, $5 \times 3 \times 2 = 30$

9, a) (OOOIICCLLGAS)'s arrangement is equal to $\binom{12}{3,2,2,2}$ =
$\frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 2 \times 1 \times 2 \times 1 \times 2 \times 1} = 2^2 \times 3 \times 11 \times 2 \times 5 \times 3^2 \times 2^3 \times 7 \times 5 \times 3$
$= 2^6 \times 3^4 \times 5^2 \times 7 \times 11$

b) (CCLLGS) arrangement $\frac{6!}{2!2!}$, and seven hole select six hole to put vowels in $\binom{7}{6}$, and vowels' arrangement $\frac{6!}{3!2!}$, so there are
$\frac{6!}{2!2!} \times 7 \times \frac{6!}{3!2!} = \frac{6 \times 120}{4} \times 7 \times \frac{6 \times 120}{6 \times 2} = 180 \times 7 \times 60 = 75600$ ways.

10, 分兩邊 $\binom{16}{10}\binom{6}{6}$，環排，$\frac{10!}{10}$，$\frac{6!}{6}$，所以方法數為 $\binom{16}{10}\binom{6}{6}(9!)(5!)$ (by the rule of product)

1. 4
2. 5
3. 4
4. 2
5. 4
6. 6
7. 3
8. 5
9. 5
10. 5
11. 1
12. 5
13. ~~8~~ 10
14. 10
15. 20

11. $\binom{n}{21} = 65780 = 22\times23\times26\times5 = \frac{22\times23\times24\times25\times26}{24\times5}$

$22 \overline{)65780}$ = 2990, $23 \overline{)2990}$ = 130, 13×10, 26×5 $= \binom{26}{21}$, so the n = 26

+1

12. a) 題意等價於 $(x_1+x_2+x_3+x_4)=8$, $x_i>0$, $x_1'+x_2'+x_3'+x_4'=4$ $\binom{4+4-1}{4}=35$

b) $= (x_1+x_2+x_3+(2x_4+1))=8$, $x_1+x_2+x_3+2x_4=7$

當 1) $x_4=0$, $x_1+x_2+x_3=7$, $\binom{3+7-1}{7}=\binom{9}{2}=36$

2) $x_4=1$, $x_1+x_2+x_3=5$ $\binom{3+5-1}{5}=\binom{7}{2}=21$

3) $x_4=2$, $x_1+x_2+x_3=3$ $\binom{3+3-1}{3}=\binom{5}{2}=10$

4) $x_4=3$, $x_1+x_2+x_3=1$ $\binom{3+1-1}{1}=\binom{3}{1}=3$

所以共有 36+21+10+3=70 種方法 (by the rule of sum)

13. a) 題意 $\equiv$ $x_1+x_2+x_3=6$, $x_4+x_5=9$, $x_i\geq 0$, $1\leq i\leq 5$

所以共有 $\binom{3+6-1}{6}\binom{2+9-1}{9}=\binom{8}{2}\binom{10}{1}=28\times10=280$ 種方法 (by the rule of produce)

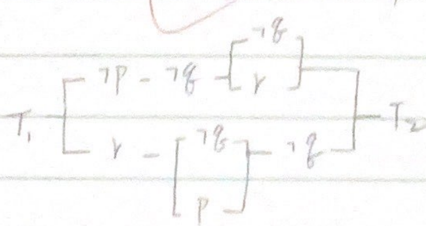
b) 題意 $\equiv$ $x_1+x_2+x_3+u=6$, $x_4+x_5+w=9$, $x_i\geq 0$, $1\leq i\leq 5$, $u\geq 0$, $w\geq 0$.

因此共有 $\binom{4+6-1}{6}\binom{3+9-1}{9}=\binom{9}{3}\binom{11}{2}=\frac{9\times8\times7}{6}\times\frac{11\times10}{2}=4620$ 種方法 (by the rule of produce)

14. $[(\neg p\vee q)\wedge\neg q\wedge(\neg q\vee r)]\vee[(p\wedge r\wedge\neg q)\vee(r\wedge\neg q)]$

$\Leftrightarrow [[(\neg p\wedge\neg q)\vee(q\wedge\neg q)]\wedge(\neg q\vee r)]\vee[((p\wedge r)\wedge\neg q)\vee(r\wedge\neg q)]$ (Distributive Laws and Associative Laws) $\Leftrightarrow [[(\neg p\wedge\neg q\vee F_0)]\wedge(\neg q\vee r)]\vee[((r\wedge\neg q)\vee(p\wedge r))\wedge((r\wedge\neg q)\vee\neg q)]$ (Inverse Laws and Distributive Laws)

$\Leftrightarrow [(\neg p\wedge\neg q)\wedge(\neg q\vee r)]\vee[r\wedge(\neg q\vee p)\wedge\neg q$ (Idenity Laws, Distributive Laws, and Absorption Laws)



15. a) 1) $p\wedge\neg q$ premise

2) p The rule of Conjuctive Simplification and step (1)

3) $\neg q$ The rule of Conjuctive Simplification and step (1)

4)
$\neg p \vee (\neg q \vee r)$ $\quad p \to q \Leftrightarrow \neg p \vee q$

5) $\neg q \vee r$ step (2) and rule of Disjunctive Syllogism.

6) T_0, 所以 r 是未知的

b) 1) $(r \wedge s) \to (p \vee t)$ premise

2) $\neg(r \wedge s) \vee (p \vee t)$ step (1) and $p \to q \Leftrightarrow \neg p \vee q$

3) $\neg t$ premise

4) $\neg(r \wedge s) \vee p$ step (2), (3) and the rule of Disjunctive Syllogism

5) $\neg r \vee \neg s \vee p$ step (4) and Demorgan's Laws

6) $q \to (u \wedge s)$ premise

7) $\neg q \vee (u \wedge s)$ step (6) and $p \to q \Leftrightarrow \neg p \vee q$

8) $u \to r$ premise

9) $\neg u \vee r$ step (8) and $p \to q \Leftrightarrow \neg p \vee q$

10) $\neg(q \to p)$ premise

11) $\neg(\neg q \vee p)$ step 10 and $p \to q \Leftrightarrow \neg p \vee q$

12) $\neg\neg q \wedge \neg p$ step 11 and Demorgan's Laws

13) $q \wedge \neg p$ step 12 and Double Negation

14) q step 13 and the rule of Conjuctive Simplification

15) $(u \wedge s)$ step 7, 14 the rule of Disjunctive Syllogism

16) u step 15, and the rule of Conjuctive Simplification

17) r step 9, 16 and the rule of Disjuctive Syllogism

18) $\neg p$ step 13, and the rule of conjuctive Simplification

19) $\neg s$ step 6, 17, 18 and the rule of Disjunctive Syllogism

20) s step 15 the rule of Conjuctive Simplification

21) $\therefore q \to p$ step 19), 20) it is contradiction, so $q \to p$ is true