離散數學 第一次期中考

2020/10/14

注意事項:

1. 禁止使用計算機、翻譯機、禁止攜帶計算紙；**手機請關機**。
2. **計算與證明題需要計算過程方予計分。**
3. 當然不可以作弊。
4. 請於答案卷左上角填上題號以方便閱卷。
5. 使用兩張答案卷的同學記得兩張都要寫名字，並將之合併在一起交回。
6. 請努力作答。

一、簡答題 (35%)

1. (1-3 no.8) How many ways can a gambler draw five cards from a standard deck and get
   (a) A flush (five cards of the same suit)? $\binom{4}{1}\binom{13}{5}$ (2%)
   (b) A full house (three of a kind and a pair)? $\binom{13}{2}\binom{4}{3}\binom{4}{2} \times 2$ (2%)
   (c) Two pairs? $\binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{11}{1}\binom{4}{1}$ (2%)

2. (1-3 no.27) Determine the sum of all the coefficients in the expansions of
   (a) $(x + y)^{10}$ $= 2^{10}$ (2%)
   (b) $(2s - 3t + 5u + 3v - 10w + 3x + 2y)^{10}$ $= 2^{10} = 1024$ (3%)

3. (2-1 no.4) Let p, q, r, s denote the following statements:
   p: I finish writing my computer program before lunch.
   q: I shall play tennis in the afternoon.
   r: The sun is shining.
   s: The humidity is low.
   Write the following in symbolic form.
   (a) If the sun is shining, I shall play tennis this afternoon. $r \to q$ (2%)
   (b) Finishing the writing of my computer program before lunch is necessary for my playing tennis this afternoon. $p \vee q$ $p \vee (p \to q)$ (2%)
   (c) Low humidity and sunshine are sufficient for me to play tennis this afternoon. (2%)
   $[(s \wedge r) \to q]$

4. (2-1 no.11a) How many rows are needed for the truth table of the compound statement $(p \vee \neg q) \leftrightarrow [(\neg r \wedge s) \to t]$, where p, q, r, s, and t are primitive statements? 2^5 (2%)

5. (2-4 no.3) Let $p(x)$ be the open statement "$x^2 = 3x$," where the universe comprises all integers. Determine whether each of the following statements is true or false.
   (a) $p(-3)$ false (2%)
   (b) $\exists x\, p(x)$ true (2%)

6. (2-4 no.6) let $p(x, y)$, $q(x, y)$ denote the following open statements.

$p(x,y): x^2 \geq y \qquad q(x, y): x + 2 < y$

If the universe for each of x, y consists of all real numbers, determine the truth value for each of the following statements.

(a) $p(-3, 8) \wedge q(1, 3)$ false (2%)

(b) $p(1/2, 1/3) \vee \neg q(1, 2)$ true (2%)

(c) $p(2, 2) \rightarrow q(1, 1)$ false (2%)

7. (2-4 no.25) Let the universe for the variables in the following statements consist of all real numbers. In each case negate and simplify the given statement.

(a) $\forall x\ \forall y\ [(x < y) \rightarrow \exists z\ (x < z < y)]$ $\exists x \exists y [(x<y) \wedge \forall z \neg(x<z<y)]$ (3%)

(b) $\forall x\ \forall y\ [(|x| = |y|) \rightarrow (y = \pm x)]$ (3%)

$\exists x \exists y [(|x|=|y|) \wedge \neg(y=\pm x)] = F.$

二、計算與證明，須有計算過程或說明方予計分 (65%)

8. (1-1 no.3) Buick automobiles come in five models, 10 colors, three engine size, and two transmission types. (a) how many distinct Buicks can be manufactured? $5 \times 10 \times 3 \times 2$ (3%)
(b) If one of the available colors is blue, how many different blue Buicks can be manufactured? (2%)

$5 \times 3 \times 2$

9. (1-1 no.21) (a) How many arrangements are there of all the letters in SOCIOLOGICAL? (2%)
(b) In how many arrangements in part (a) are all the vowels adjacent? CC LL GAS (3%)

I、O $\quad \frac{7!}{2!2!} \times \binom{8}{5} \times \frac{5!}{3!2!}$

10. (1-1 no.37) Sixteen people are to be seated at two circular tables, one of which seats 10 while the other seats six. How many different seating arrangements are possible? (5%)

$\binom{16}{10}\binom{6}{6} \times \frac{10!}{10} \times \frac{6!}{6}$

11. (1-4 no.9) Columba has two dozen each of n different colored beads. If she can select 21 beads (with repetitions of colors allowed) in 65,780 ways, what is the value of n? (5%)

$\binom{n}{21} = 65780$

12. (1-4 no.13) In how many ways can we distribute eight identical white balls into four distinct containers so that

(a) No container is left empty? $x+y+z+u=8$ (2%)

(b) The fourth container has an odd number of balls in it? (3%)

$x+y+z+(2u+1)=8 \Rightarrow x+y+z+2u=7$

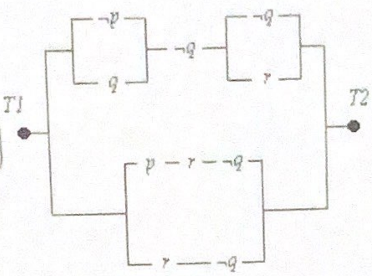
13. (Chap 1 no. 18) (a) Determine the number of nonnegative integer solutions to the pair of equations $x_1 + x_2 + x_3 = 6$, $x_1 + x_2 + x_3 + x_4 + x_5 = 15$, $x_i \geq 0$, $1 \leq i \leq 5$? , $x_4 + x_5 = 9$ (5%)
(b) Answer part (a) with the pair of equations replaced by the pair of inequalities

$x_1 + x_2 + x_3 \leq 6$, $x_1 + x_2 + x_3 + x_4 + x_5 \leq 15$, $x_i \geq 0$, $1 \leq i \leq 5$? (5%)

14. (2-2 EX.2.18) Simplify the network shown below: (10%)

$(\neg p \vee q) \wedge (\neg q) \wedge (\neg q \vee r)] \vee$

$[(p \wedge r \wedge \neg q) \vee (r \wedge \neg q)]$



15. (2-3 EX.2.33, no.11) The following argument is validity or invalidity? Prove it or provide a counterexample.

(a) (10%)

$$\begin{array}{l} p \wedge \neg q \\ \underline{p \rightarrow (q \rightarrow r)} \\ \therefore \neg r \end{array}$$

F0

$[[(\neg q \wedge \neg p) \vee (\neg q \wedge q)] \wedge \neg q \vee r]$

$[\neg q \wedge \neg p] \wedge [\neg q \vee r]$

(b) (10%)

$$\begin{array}{l} u \rightarrow r \\ (r \wedge s) \rightarrow (p \vee t) \\ q \rightarrow (u \wedge s) \\ \underline{\neg t} \\ \therefore q \rightarrow p \end{array}$$

$\neg(q \rightarrow p)$

$\neg(\neg q \vee p)$

$\neg\neg q \wedge \neg p$

$q \wedge \neg p$