

科目名稱：離散數學

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系/所別：資工系

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記分

教師簽章

89

1,	a) 1281	1. ④
	b) 3744	2. ⑤
	c) 12352	3. ④
2,	a) 1024	4. ②
	b) 1024	5. ④
3,	a) $r \rightarrow g$	6. ⑥
	b) $p \rightarrow \neg q$	7. ③
	c) $[(s \wedge r) \rightarrow g]$	8. ⑤
4,	$2^5 = 32$	9. ⑤
5,	a) false	10. ⑤
	b) true	11. ①
6,	a) false	12. ⑤
	b) true	13. ⑩
	c) false	14. ⑨
7,	a) $\exists x \exists y [(x < y) \wedge \neg (x > z > y)]$	15. ⑩
	b) $\exists x \exists y [(x = y) \wedge (x \neq \pm x)]$	
8	a) by the rule of product, $5 \times 10 \times 3 \times 2 = 300$	
	b) by the rule of product, $5 \times 3 \times 2 = 30$	
9,	a) (00011CCLLGAS)'s arrangement is equal to $\binom{12}{3,2,2,1,1,1,1} = \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3! \times 2! \times 2! \times 1! \times 1! \times 1! \times 1!} = 2^6 \times 3 \times 11 \times 2 \times 5 \times 3^2 \times 7 \times 5 \times 3 = 2^6 \times 3^4 \times 5^2 \times 7 \times 11$	
	b) (CCLLGSS) arrangement $\frac{6!}{2!2!1!1!1!1!}$, and seven hole select six hole to put vowels in $\binom{7}{6}$, and vowels' arrangement $\frac{6!}{2!2!1!1!1!1!}$, so there are $\frac{6!}{2!2!} \times 7 \times \frac{6!}{2!2!} = \frac{6 \times 20 \times 6!}{2 \times 2} = \frac{6 \times 120 \times 6!}{4} = 180 \times 7 \times 60 = 75600$ ways.	
10,	分兩塊 $\binom{16}{10} \binom{6}{6} = \text{環排} \cdot \frac{10!}{10} \cdot \frac{6!}{6}$. 所以方法數為 $\binom{16}{10} \binom{6}{6} (9!)(5!)$ (by the rule of product)	

11, $\binom{n}{21} = 65780 = 22 \times 23 \times 26 \times 5 = \frac{22 \times 23 \times 24 \times 25 \times 26}{24 \times 5} = \binom{26}{21}$, so the $n = 26 + 1$

12, a) 題意等價於 $(x_1 + x_2 + x_3 + x_4) = 8$, $x_i > 0$, $x'_1 + x'_2 + x'_3 + x'_4 = 4$ $\binom{4+4-1}{4} = 35$

b) $(x_1 + x_2 + x_3 + (2x_4 + 1)) = 8$, $x_1 + x_2 + x_3 + 2x_4 = 7$

當 1) $x_4 = 0$, $x_1 + x_2 + x_3 = 7$, $\binom{3+7-1}{2} = \binom{9}{2} = 36$

2) $x_4 = 1$, $x_1 + x_2 + x_3 = 5$, $\binom{3+5-1}{2} = \binom{7}{2} = 21$

3) $x_4 = 2$, $x_1 + x_2 + x_3 = 3$, $\binom{3+3-1}{2} = \binom{5}{2} = 10$

4) $x_4 = 3$, $x_1 + x_2 + x_3 = 1$, $\binom{3+1-1}{1} = \binom{3}{1} = 3$

所以共有 $36 + 21 + 10 + 3 = 70$ 種方法 (by the rule of sum)

13, a) 題意 $\equiv x_1 + x_2 + x_3 = 6$, $x_4 + x_5 = 9$, $x_i \geq 0$, $1 \leq i \leq 5$

所以共有 $\binom{3+6-1}{2} \binom{2+9-1}{1} = \binom{8}{2} \binom{10}{1} = 28 \times 10 = 280$ 種方法 (by the rule of product)

b) 是意 $\equiv x_1 + x_2 + x_3 + u = 6$, $x_4 + x_5 + w = 9$, $x_i \geq 0$, $1 \leq i \leq 5$, $u \geq 0$, $w \geq 0$.

因此共有 $\binom{4+6-1}{2} \binom{3+9-1}{1} = \binom{9}{2} \binom{11}{1} = \frac{9 \times 8}{2} \times \frac{11 \times 10}{2} = 4620$ 種方法 (by the rule of product)

14, $[(\neg p \vee q) \wedge \neg q \wedge (\neg q \vee r)] \vee [(p \wedge r \wedge \neg q) \vee (r \wedge \neg q)]$

$\Leftrightarrow [(\neg p \wedge \neg q) \vee (q \wedge \neg q)] \wedge (\neg q \vee r) \vee [(p \wedge r) \wedge \neg q] \vee (r \wedge \neg q)$ (Distributive Laws and Associative Laws) $\Leftrightarrow [(\neg p \wedge \neg q \vee F_0)] \wedge (\neg q \vee r) \vee [(p \wedge r) \wedge (\neg q \vee r)]$ (Inverse Laws and Distributive Laws)

$\Leftrightarrow [(\neg p \wedge \neg q) \wedge (\neg q \vee r)] \vee [r \wedge (\neg q \vee p) \wedge \neg q]$ (Identity Laws, Distributive Laws, and Absorption Laws)

$$T_1 \left[\begin{array}{c} \neg q \\ \neg p - \neg q \left[r \right] \\ r - \left[\neg q \right] \neg q \\ p \end{array} \right] T_2$$

15, a) 1) $p \wedge \neg q$ premise

2) p The rule of conjunctive simplification and step (1)

3) $\neg q$ The rule of conjunctive simplification and step (1)

1) $\neg P \vee (\neg Q \vee R)$ $P \rightarrow Q \Leftrightarrow \neg P \vee Q$

5) $\neg Q \vee R$ step (2) and rule of Disjunctive Syllogism.

6) T , 所以 R 是 T 或 F

b) 1) $(R \wedge S) \rightarrow (P \vee t)$ premise

2) $\neg(R \wedge S) \vee (P \vee t)$ step 1) and $p \rightarrow q \Leftrightarrow \neg p \vee q$

3) $\neg t$ premise

4) $\neg(R \wedge S) \vee P$ step (2), (3) and the rule of Disjunctive Syllogism

5) $\neg R \vee \neg S \vee P$ step (4) and Demorgan's Laws

6) $Q \rightarrow (u \wedge s)$ premise

7) $\neg Q \vee (u \wedge s)$ step (6) and $p \rightarrow q \Leftrightarrow \neg p \vee q$

8) $u \rightarrow r$ premise

9) $\neg u \vee r$ step (8) and $p \rightarrow q \Leftrightarrow \neg p \vee q$

10) $\neg(Q \rightarrow P)$ premise

11) $\neg(\neg Q \vee P)$ step 10 and $p \rightarrow q \Leftrightarrow \neg p \vee q$

12) $\neg \neg Q \wedge \neg P$ step 11 and Demorgan's Laws

13) $Q \wedge \neg P$ step 12 and Double Negation

14) Q step 13 and the rule of Conjunctive Simplification

15) $(u \wedge s)$ step 7, 14 the rule of Disjunctive Syllogism

16) u step 15, and the rule of Conjunctive Simplification

17) r step 9, 16 and the rule of Disjunctive Syllogism

18) $\neg P$ step 13, and the rule of conjunctive simplification

19) $\neg S$ step 6, 17, 18 and the rule of Disjunctive Syllogism

20) S step 15 the rule of Conjunctive Simplification

21) $\therefore Q \rightarrow P$ step 19), 20) it is contradiction, so $Q \rightarrow P$ is true