國立暨南國際大學　　學年度第　　學期期中期末考試試卷

記分	90
教師簽章	

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計分	
	1) 6　2) 0　3) 4　4) 5
	5) 10　6) 8　7) 6　8) 8
	9) 7　10) 10　11) 6　12) 10　13) 10

1. a) $1-0 = 0+1$ true

b) $y-0 = y+0^2$ true

c) false

2. b、d、~~e~~ are true

3. a) $(14+1)(9+1)(8+1)(10+1)(3+1)(5+1)(10+1) = 3\times5\times2\times5\times3^2\times11\times2^2\times2\times3\times11$

$= 2^4\times3^4\times5^2\times11^2$

b) let $x^2 \geq 2^2\cdot9^2\cdot5^2\cdot11^2$

2	3	5	11
2, 4, 6, 8, 10, 12, 14	3^4, 3^6, 3^8	5^2, 5^4, 5^6, 5^8	11^2

$7\times3\times\cancel{4}\times6\times1\times3\times6 = 2^4\times3^4\times7$ ~~4~~

(ii) ?

4. 假設 n 是偶數 且 $n+11$ 也是偶數.

令 $n=2a$, $a\in\mathbb{Z}$, 則 $n+11=2a+11$

因為 $2\cdot a$ 是偶數, 11 是奇數 ⇒ $n+11$ 是奇數 (contradiction)

因此 if n is odd, then $n+11$ is even.

5. $(A\cap B)\cup(A\cap B\cap\bar{C}\cap D)\cup(\bar{A}\cap B)$

$\Leftrightarrow (A\cap B)\cup((A\cap B)\cap(\bar{C}\cap D))\cup(\bar{A}\cap B)$ Associative Law

$\Leftrightarrow (A\cap B)\cup(\bar{A}\cap B)$ Absorption Law

$\Leftrightarrow B\cap(A\cup\bar{A})$ Distributive Law

$\Leftrightarrow B\cap U$ Inverse Law

$\Leftrightarrow B$ Identity Law

6. $\underline{1}\ \underline{1}\ \underline{1}\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_$

依據排容原理，總方法數為 111在首加 0000在尾扣 111在首且 0000在尾

$2^7 + 2^6 - 2^3 = 128 + 64 - 8 = 184$

7. a)可能性有 $6^3 = 216$ 种

首次為 1 的情況 $5 \times 5 = 25$ 种

" 2 " $4 \times 4 = 16$ 种

" 3 " $3 \times 3 = 9$ 种

" 4 " $2 \times 2 = 4$ 种

" 5 " $1 \times 1 = 1$ 种　總合為 $\frac{5 \times 6 \times 11}{6} = 55$

So, the probability of (a) is $\frac{55}{216}$

b) $a < b < c \Rightarrow \binom{6}{3} = 20$, the probability of (b) is $\frac{20}{216} = \frac{5}{54}$　$\overline{(A \cap B)} = \bar{A} \cup \bar{B}$

8. Assume $A \cap B = \{1, 2, 3\}$　$U = \{1, 2, 3, 4, 5, 6, 7, 8, \underbrace{9, 10, 11, 12}_{(A \cup B)'}\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

a) $C : \{\boxed{1, 2, 3}, 4, 5, 6, 7, 8\}$ C's subsect : $2^5 = 32$, even number $= \binom{5}{1} + \binom{5}{3} + \binom{5}{5} = 2^4 = 16$

b) $D : \{4, 5, 6, 7, 8, \boxed{9, 10, 11, 12}\}$ D's subsect : $2^5 = 32$, even number $= \binom{5}{0} + \binom{5}{2} + \binom{5}{4} = 2^4 = 16$

9. let $S(n) = n^2 < 2^n < n!$, for $n \in Z^+$, $n > 4$

$S(5) = 5^2 = 25 < 2^5 = 32 < 5! = 120$

Assume $S(k) = k^2 < 2^k < k!$, for $k \geq 5$, $k \in Z^+$

then $S(k+1) = (k+1)^2 < 2^{k+1} < (k+1)!$

$\Rightarrow k^2 + (2k+1) < 2^k + 2^k < (k+1)(k!)$　-3

$\because 2k+1 < 2^k$, for $k \geq 5$, $k+1 \geq 6 \Rightarrow \underbrace{(k!) + (k!) + \dots + (k!)}_{k+1 個} \Rightarrow 2^k + 2^k < (k+1)!$

$\therefore S(n)$ is true.

沒有用歸納法

10. let $S(n) = p \vee (q_1 \wedge q_2 \wedge \dots \wedge q_n) \Leftrightarrow (p \vee q_1) \wedge (p \vee q_2) \wedge \dots \wedge (p \vee q_n)$, for $n \in Z^+$, $n \geq 2$

$S(2) = p \vee (q_1 \wedge q_2) \Leftrightarrow (p \vee q_1) \wedge (p \vee q_2)$ by distributive law

Assume $S(k)$ $p \vee (q_1 \wedge q_2 \wedge \dots \wedge q_k) \Leftrightarrow (p \vee q_1) \wedge (p \vee q_2) \wedge \dots \wedge (p \vee q_k)$ for $k \in Z^+$, $k \geq 2$

then $S(k+1) = p \vee (q_1 \wedge q_2 \wedge \dots q_k \wedge q_{k+1}) \Leftrightarrow (p \vee q_1) \wedge (p \vee q_2) \wedge \dots (p \vee q_k) \wedge (p \vee q_{k+1})$

$\Rightarrow p \vee ((q_1 \wedge q_2 \wedge \dots q_k) \wedge q_{k+1}) \Leftrightarrow ((p \vee q_1) \wedge (p \vee q_2) \wedge \dots \wedge (p \vee q_k)) \wedge (p \vee q_{k+1})$ by Associative Law

$\Rightarrow p \vee (q_1 \wedge q_2 \wedge \dots q_k) \wedge (p \vee q_{k+1}) \Leftrightarrow ((p \vee q_1) \wedge (p \vee q_2) \wedge \dots \wedge (p \vee q_k)) \wedge (p \vee q_{k+1})$ by distributive Law

$\because p \vee (q_1 \wedge q_2 \wedge \dots q_k) \Leftrightarrow (p \vee q_1) \wedge (p \vee q_2) \wedge \dots (p \vee q_k)$, $\therefore S(k+1)$ is true

Hence, $S(n)$ is true.

11. let $n = 2t+1$, for $t \in \mathbb{Z}$

$\Rightarrow 8 \mid ((2t+1)^2 - 1) \Rightarrow 8 \mid 4t^2 + 4t$

let $S(t) = 8 \mid 4t(1+t)$

$S(0) = 8 \mid 4 \cdot 0 \cdot 1 = 8 \mid 0$

Assume $S(k) = 8 \mid (4k \cdot (1+k))$ is true for $k \in \mathbb{Z}$ $\quad 4k^2 + 4k + 8k + 8$

then $S(k+1) = 8 \mid 4(k+1)(1+(k+1)) = 8 \mid 4(k+1)(k+2)$

$= 8 \mid (\underbrace{(4k^2+4k)}_{8的倍数} + \underbrace{8(k+1)}_{8的倍数})$, Hence $S(k+1)$ is true.

So, $S(n)$ is true.

12.

3	427	125	2
	375	104	
2	52	21	2
10	42	20	
	10	1	
	0	1	

$\rightarrow \gcd(427, 125) = 1$

$$\begin{array}{r} 427 \\ -375 \\ \hline 52 \end{array}$$

$7 \overline{)427}$ $\quad 61$

$5 \overline{)125}$, $5 \overline{)25}$, 5

divisors

427	61	7	1
125	25	5	1

$\rightarrow$ so $\gcd(427, 125) = 1$

13. let $S(n) = F_n \le (\frac{5}{3})^n$, for $n \in \mathbb{N}$

$S(1) = 1 \le \frac{5}{3}$ (true)

Assume $S(k) = F_k \le (\frac{5}{3})^{k}$ is true for $k \in \mathbb{N}$

then $S(k+1) = F_{k+1} = F_k + F_{k-1} \le (\frac{5}{3})^{k+1} = \frac{5}{3} \cdot (\frac{5}{3})^k$

$\Rightarrow F_k + F_{k-1} \le (\frac{5}{3})^k + \frac{2}{3}(\frac{5}{3})^k$, since $F_{k-1} < (\frac{5}{3})^{k-1} = \frac{3}{5} \cdot (\frac{5}{3})^k < \frac{2}{3} \cdot (\frac{5}{3})^k$, $S(k+1)$ is true.

Hence, $S(n)$ is true.