

Heap-Sort Algorithm

(a) Algorithms:

The Heap-Sort algorithm involves two main steps:

1. **Building the Max-Heap:**

- Rearrange the elements of the array to satisfy the Max-Heap property.
- Use the **heapify** function to ensure each **subtree** is a valid Max-Heap.

2. **Sorting:**

- Repeatedly extract the largest element (root of the heap) by swapping it with the last element in the heap.
- Reduce the size of the heap and reapply **heapify** to maintain the Max-Heap property.

(b) Analysis of Heap-Sort:

1. **Time Complexity:**

- **Building the Heap:** $O(n)$
- **Heapify during sorting:** For n elements, each heapify operation takes $O(\log n)$. Thus, sorting $O(n \log n)$.
- Total Time Complexity: **$O(n \log n)$**

2. **Space Complexity:**

- Heap-Sort is an in-place algorithm, so it requires only $O(1)$ extra space.

3. **Stability:**

- Heap-Sort is not stable since element positions are swapped in a Max-Heap structure.

(c) Implementation Example:

```
arr = [4, 10, 3, 5, 1]
heap_sort(arr)
print("Sorted array is:", arr)
```

```
Sorted array is: [1, 3, 4, 5, 10]
```