

# Basic Data Structures: Arrays and Linked Lists

Neil Rhodes

Department of Computer Science and Engineering  
University of California, San Diego

Data Structures

<http://bit.ly/algospecialization>

# Outline

```
long arr[] = new long[5];
```

```
long arr[5];
```

```
arr = [None] * 5
```

1	5	17	3	25
---	---	----	---	----

1	5	17	3	25
8	2	36	5	3

# Definition

Array:

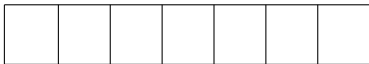
Contiguous area of memory



# Definition

## Array:

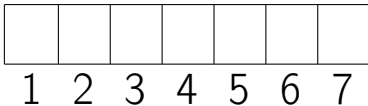
Contiguous area of memory consisting of equal-size elements



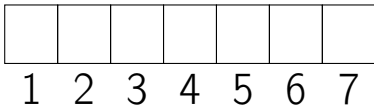
## Definition

### Array:

Contiguous area of memory consisting of equal-size elements indexed by contiguous integers.

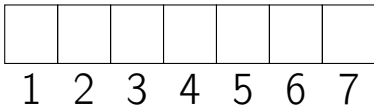


# What's Special About Arrays?



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Constant-time access

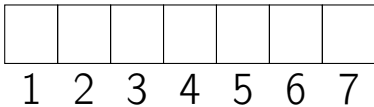




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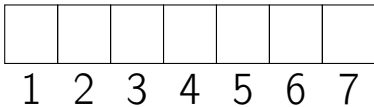
array\_addr



# What's Special About Arrays?

Constant-time access

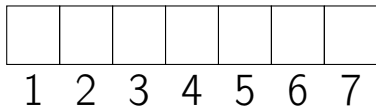
$\text{array\_addr} + \text{elem\_size} \times ( \quad )$



# What's Special About Arrays?

Constant-time access

$\text{array\_addr} + \text{elem\_size} \times (i - \text{first\_index})$



# Multi-Dimensional Arrays


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(1, 1)					

# Multi-Dimensional Arrays

			(3,4)		

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			(3,4)		

$$(3 - 1) \times 6$$

# Multi-Dimensional Arrays

			(3,4)		

$$(3 - 1) \times 6 + (4 - 1)$$



# Multi-Dimensional Arrays

			(3,4)		

$$\text{elem\_size} \times ((3 - 1) \times 6 + (4 - 1))$$

# Multi-Dimensional Arrays

			(3,4)		

$$\text{array\_addr} + \\ \text{elem\_size} \times ((3 - 1) \times 6 + (4 - 1))$$

$(1, 1)$
$(1, 2)$
$(1, 3)$
$(1, 4)$
$(1, 5)$
$(1, 6)$
$(2, 1)$
$\vdots$

Row-major

$(1, 1)$
$(1, 2)$
$(1, 3)$
$(1, 4)$
$(1, 5)$
$(1, 6)$
$(2, 1)$
$\vdots$

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$(1, 3)$
$(1, 4)$
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$(1, 1)$
$(2, 1)$
$(3, 1)$
$(1, 2)$
$(2, 2)$
$(3, 2)$
$(1, 3)$
$\vdots$

Row-major

$(1, 1)$
$(1, 2)$
$(1, 3)$
$(1, 4)$
$(1, 5)$
$(1, 6)$
$(2, 1)$
$\vdots$

Column-major

$(1, 1)$
$(2, 1)$
$(3, 1)$
$(1, 2)$
$(2, 2)$
$(3, 2)$
$(1, 3)$
$\vdots$

# Times for Common Operations

	Add	Remove
Beginning		
End		
Middle		

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	Add	Remove
Beginning		
End		
Middle		

5	8	3	12			
---	---	---	----	--	--	--



# Times for Common Operations

	Add	Remove
Beginning	$O(1)$	
End		
Middle		

5	8	3	12	4		
---	---	---	----	---	--	--

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Beginning		$O(n)$
End	$O(1)$	$O(1)$
Middle		

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	Add	Remove
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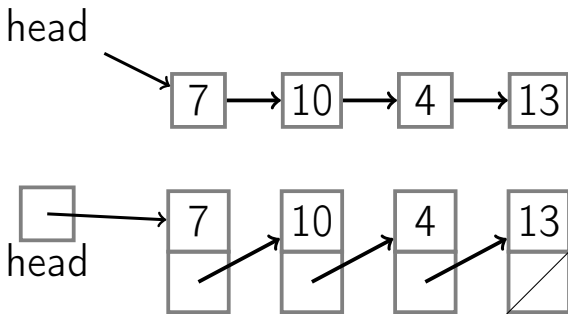
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# Summary

- Array: contiguous area of memory consisting of equal-size elements indexed by contiguous integers.
- Constant-time access to any element.
- Constant time to add/remove at the end.
- Linear time to add/remove at an arbitrary location.

# Outline

# Singly-Linked List



Node contains:

- key
- next pointer



# List API

PushFront(Key)                      add to front

## List API

PushFront(Key)	add to front
Key TopFront()	return front item

## List API

<code>PushFront(Key)</code>	add to front
<code>Key TopFront()</code>	return front item
<code>PopFront()</code>	remove front item

## List API

PushFront(Key)	add to front
Key TopFront()	return front item
PopFront()	remove front item
PushBack(Key)	add to back
	also known as Append

## List API

PushFront(Key)	add to front
Key TopFront()	return front item
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Boolean Find(Key)	is key in list?

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Boolean Find(Key)	is key in list?
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Boolean Empty()	empty list?

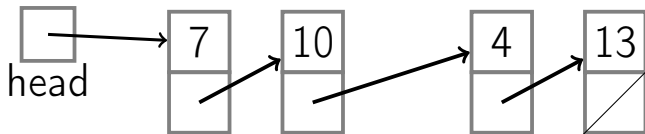
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AddBefore(Node, Key)	adds key before node

## List API

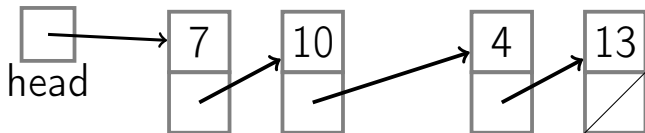
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AddBefore(Node, Key)	adds key before node
AddAfter(Node, Key)	adds key after node

# Times for Some Operations



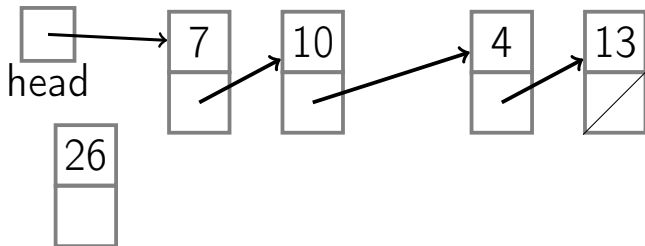
# Times for Some Operations

PushFront



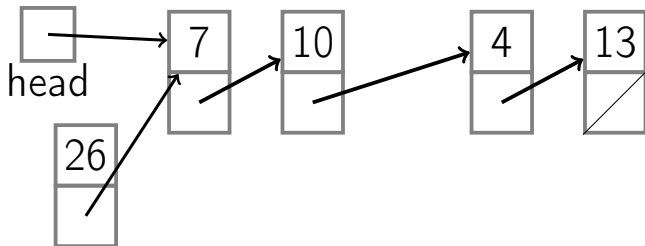
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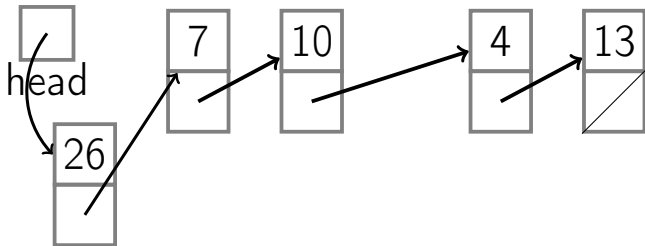
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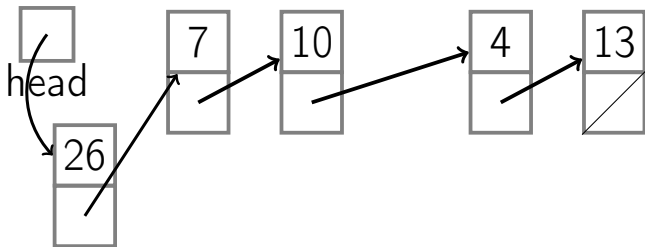
PushFront  $O(1)$





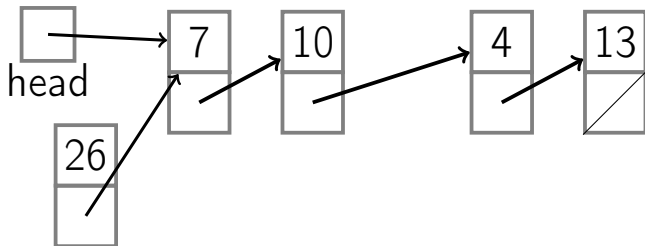
# Times for Some Operations

PopFront



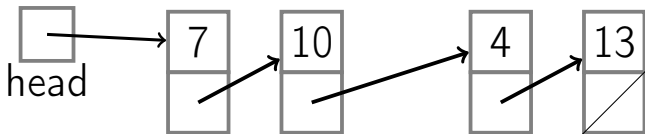
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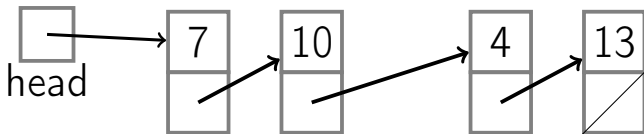
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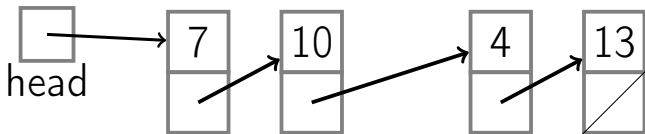
# Times for Some Operations

PushBack  
(no tail)



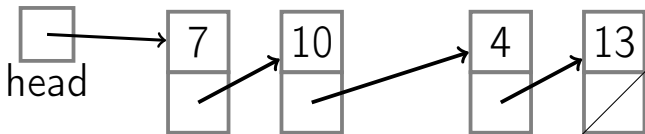
# Times for Some Operations

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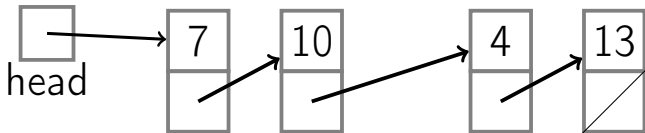
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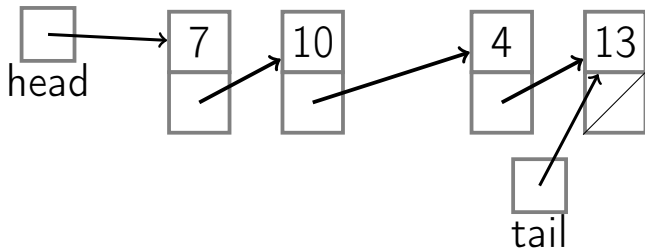


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PopBack  $O(n)$   
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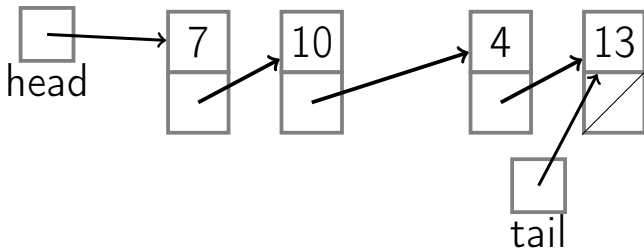
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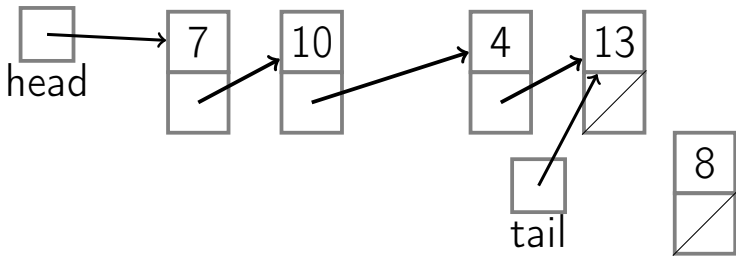
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PushBack  
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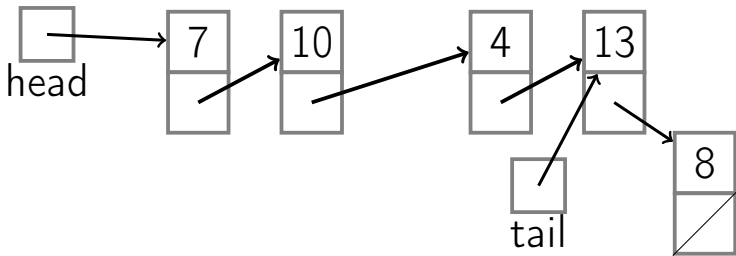
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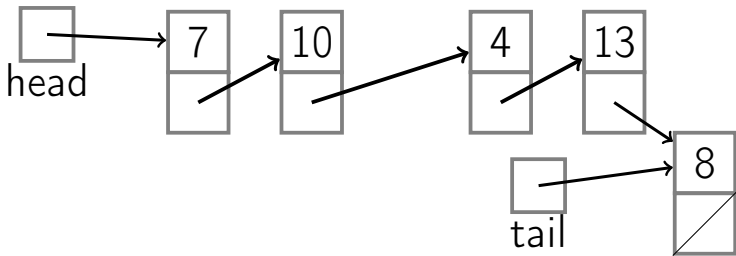
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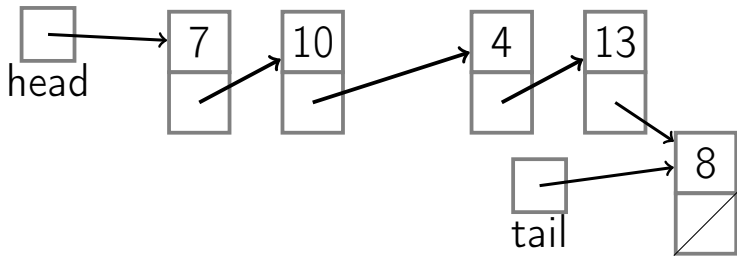
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PushBack  $O(1)$   
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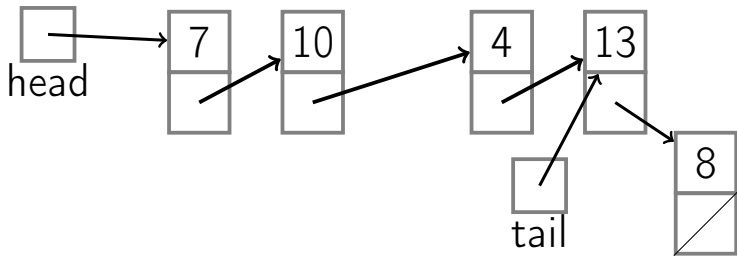
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PopBack  
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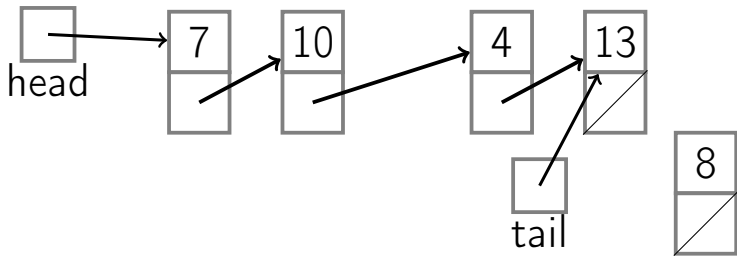
# Times for Some Operations

PopBack  
(with tail)



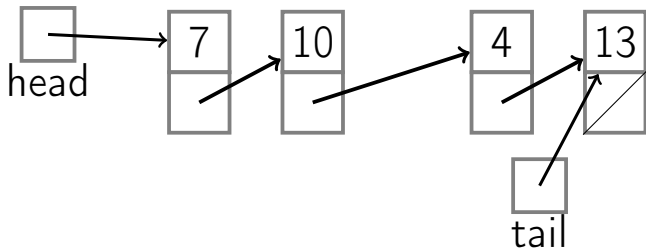
# Times for Some Operations

PopBack  
(with tail)



# Times for Some Operations

PopBack  $O(n)$   
(with tail)





# Singly-linked List

## PushFront(*key*)

*node*  $\leftarrow$  new node

*node.key*  $\leftarrow$  *key*

*node.next*  $\leftarrow$  *head*

*head*  $\leftarrow$  *node*

if *tail* = nil:

*tail*  $\leftarrow$  *head*

# Singly-linked List

## PopFront()

```
if head = nil:  
    ERROR: empty list  
head  $\leftarrow$  head.next  
if head = nil:  
    tail  $\leftarrow$  nil
```

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if *tail* = nil:

*head*  $\leftarrow$  *tail*  $\leftarrow$  *node*

# Singly-linked List

## PushBack(*key*)

*node*  $\leftarrow$  new node

*node.key*  $\leftarrow$  *key*

*node.next* = nil

if *tail* = nil:

*head*  $\leftarrow$  *tail*  $\leftarrow$  *node*

else:

*tail.next*  $\leftarrow$  *node*

*tail*  $\leftarrow$  *node*

# Singly-linked List

PopBack()

# Singly-linked List

PopBack()

```
if head = nil:  ERROR: empty list
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# Singly-linked List

## PopBack()

```
if head = nil:  ERROR: empty list
if head = tail:
    head  $\leftarrow$  tail  $\leftarrow$  nil
```



# Singly-linked List

## PopBack()

```
if head = nil:  ERROR: empty list
if head = tail:
    head  $\leftarrow$  tail  $\leftarrow$  nil
else:
    p  $\leftarrow$  head
    while p.next.next  $\neq$  nil:
        p  $\leftarrow$  p.next
```

# Singly-linked List

## PopBack()

```
if head = nil:  ERROR: empty list
if head = tail:
    head  $\leftarrow$  tail  $\leftarrow$  nil
else:
    p  $\leftarrow$  head
    while p.next.next  $\neq$  nil:
        p  $\leftarrow$  p.next
    p.next  $\leftarrow$  nil; tail  $\leftarrow$  p
```

# Singly-linked List

AddAfter(*node*, *key*)

*node2*  $\leftarrow$  new node

*node2.key*  $\leftarrow$  *key*

*node2.next* = *node.next*

*node.next* = *node2*

if *tail* = *node*:

*tail*  $\leftarrow$  *node2*

Singly-Linked List	no tail	with tail
--------------------	---------	-----------

PushFront(Key)	$O(1)$
----------------	--------

Singly-Linked List	no tail	with tail
PushFront(Key)	$O(1)$	
TopFront()	$O(1)$	

Singly-Linked List	no tail	with tail
PushFront(Key)	$O(1)$	
TopFront()	$O(1)$	
PopFront()	$O(1)$	

Singly-Linked List	no tail	with tail
PushFront(Key)	$O(1)$	
TopFront()	$O(1)$	
PopFront()	$O(1)$	
PushBack(Key)	$O(n)$	$O(1)$

Singly-Linked List	no tail	with tail
PushFront(Key)	$O(1)$	
TopFront()	$O(1)$	
PopFront()	$O(1)$	
PushBack(Key)	$O(n)$	$O(1)$
TopBack()	$O(n)$	$O(1)$



Singly-Linked List	no tail	with tail
PushFront(Key)	$O(1)$	
TopFront()	$O(1)$	
PopFront()	$O(1)$	
PushBack(Key)	$O(n)$	$O(1)$
TopBack()	$O(n)$	$O(1)$
PopBack()	$O(n)$	

Singly-Linked List	no tail	with tail
PushFront(Key)	$O(1)$	
TopFront()	$O(1)$	
PopFront()	$O(1)$	
PushBack(Key)	$O(n)$	$O(1)$
TopBack()	$O(n)$	$O(1)$
PopBack()	$O(n)$	
Find(Key)	$O(n)$	

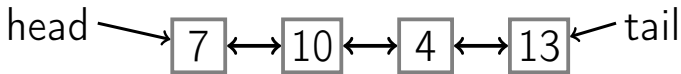
Singly-Linked List	no tail	with tail
PushFront(Key)	$O(1)$	
TopFront()	$O(1)$	
PopFront()	$O(1)$	
PushBack(Key)	$O(n)$	$O(1)$
TopBack()	$O(n)$	$O(1)$
PopBack()	$O(n)$	
Find(Key)	$O(n)$	
Erase(Key)	$O(n)$	

Singly-Linked List	no tail	with tail
PushFront(Key)	$O(1)$	
TopFront()	$O(1)$	
PopFront()	$O(1)$	
PushBack(Key)	$O(n)$	$O(1)$
TopBack()	$O(n)$	$O(1)$
PopBack()	$O(n)$	
Find(Key)	$O(n)$	
Erase(Key)	$O(n)$	
Empty()	$O(1)$	

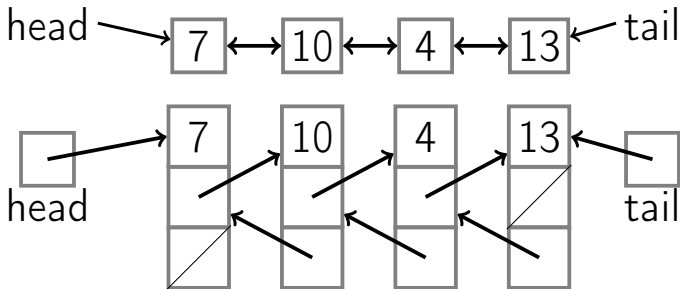
Singly-Linked List	no tail	with tail
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Find(Key)	$O(n)$	
Erase(Key)	$O(n)$	
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AddBefore(Node, Key)	$O(n)$	

Singly-Linked List	no tail	with tail
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# Doubly-Linked List

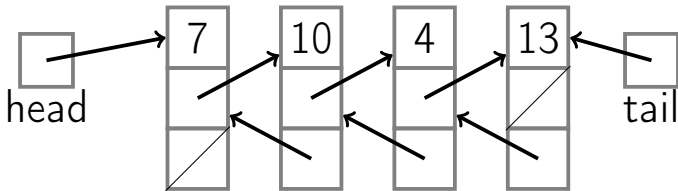


# Doubly-Linked List





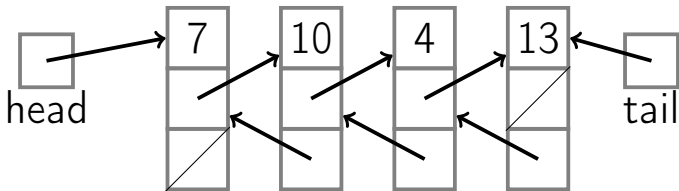
# Doubly-Linked List



Node contains:

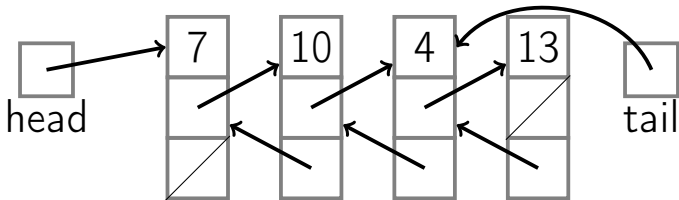
- key
- next pointer
- prev pointer

# Doubly-Linked List



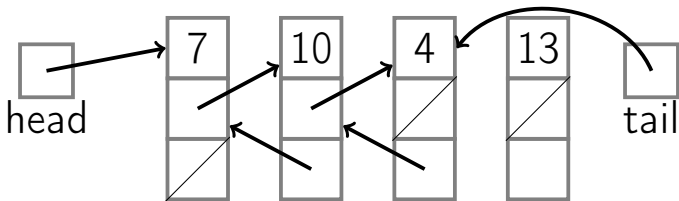
PopBack

# Doubly-Linked List



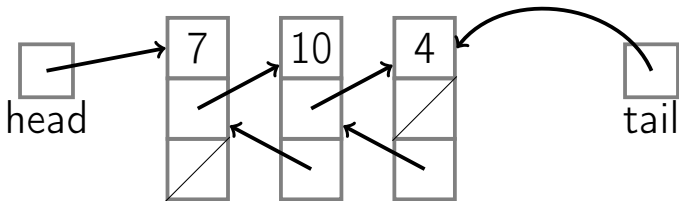
PopBack

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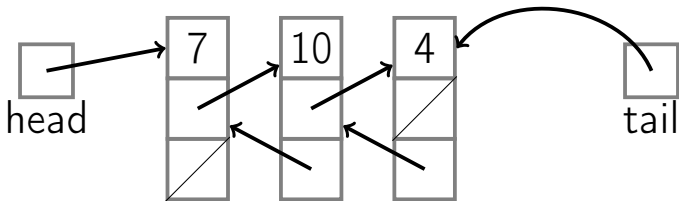
PopBack

# Doubly-Linked List



PopBack

# Doubly-Linked List



PopBack  $O(1)$

# Doubly-linked List

PushBack(*key*)

*node*  $\leftarrow$  new node

*node.key*  $\leftarrow$  *key*; *node.next* = nil

# Doubly-linked List

## PushBack(*key*)

*node*  $\leftarrow$  new node

*node.key*  $\leftarrow$  *key*; *node.next* = nil

if *tail* = nil:

*head*  $\leftarrow$  *tail*  $\leftarrow$  *node*

*node.prev*  $\leftarrow$  nil



# Doubly-linked List

## PushBack(*key*)

```
node ← new node  
node.key ← key; node.next = nil  
if tail = nil:  
    head ← tail ← node  
    node.prev ← nil  
else:  
    tail.next ← node  
    node.prev ← tail  
    tail ← node
```

# Doubly-linked List

PopBack()

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```
if head = nil:  ERROR: empty list
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## PopBack()

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if head = nil:  ERROR: empty list
if head = tail:
    head  $\leftarrow$  tail  $\leftarrow$  nil
```

# Doubly-linked List

## PopBack()

```
if head = nil:  ERROR: empty list
if head = tail:
    head  $\leftarrow$  tail  $\leftarrow$  nil
else:
    tail  $\leftarrow$  tail.prev
    tail.next  $\leftarrow$  nil
```

# Doubly-linked List

## AddAfter(*node*, *key*)

```
node2 ← new node  
node2.key ← key  
node2.next ← node.next  
node2.prev ← node  
node.next ← node2  
if node2.next ≠ nil:  
    node2.next.prev ← node2  
if tail = node:  
    tail ← node2
```

# Doubly-linked List

AddBefore(*node*, *key*)

```
node2 ← new node  
node2.key ← key  
node2.next ← node  
node2.prev ← node.prev  
node.prev ← node2  
if node2.prev ≠ nil:  
    node2.prev.next ← node2  
if head = node:  
    head ← node2
```

Singly-Linked List	no tail	with tail
PushFront(Key)	$O(1)$	
TopFront()	$O(1)$	
PopFront()	$O(1)$	
PushBack(Key)	$O(n)$	$O(1)$
TopBack()	$O(n)$	$O(1)$
PopBack()	$O(n)$	
Find(Key)	$O(n)$	
Erase(Key)	$O(n)$	
Empty()	$O(1)$	
AddBefore(Node, Key)	$O(n)$	
AddAfter(Node, Key)	$O(1)$	



Doubly-Linked List	no tail	with tail
PushFront(Key)	$O(1)$	
TopFront()	$O(1)$	
PopFront()	$O(1)$	
PushBack(Key)	$O(n)$	$O(1)$
TopBack()	$O(n)$	$O(1)$
PopBack()	<del><math>O(n)</math></del> $O(1)$	
Find(Key)	$O(n)$	
Erase(Key)	$O(n)$	
Empty()	$O(1)$	
AddBefore(Node, Key)	<del><math>O(n)</math></del> $O(1)$	
AddAfter(Node, Key)	$O(1)$	

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- Constant time to insert at or remove from the front.
- With tail and doubly-linked, constant time to insert at or remove from the back.
- $O(n)$  time to find arbitrary element.
- List elements need not be contiguous.
- With doubly-linked list, constant time to insert between nodes or remove a node.