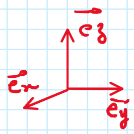
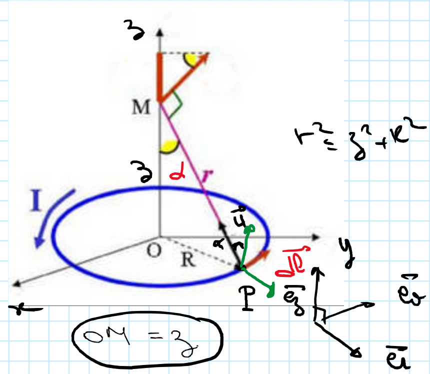
Exercice 5 spire circulaire



On considère une spire circulaire de rayon R, de centre ***O***, d'axe ***(Oz)***, parcourue par un courant d'intensité ***I***. Soit un point ***M*** de son axe ***(Oz)*** (figure ci-contre).



$r^2 = z^2 + R^2$

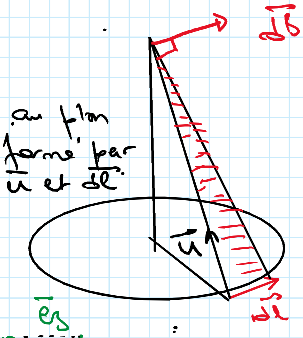
$OM = z$

a) Calculez $\vec{B}(M)$à l'aide de la loi de Biot Savart. Donnez, l'expression du champ en fonction de z (abscisse de M) et du rayon R.

Au voisinage du point P, on considère $\vec{dl}$

$$I\,\vec{dl} \longrightarrow \vec{dB} = \frac{\mu_0 I}{4\pi}\,\frac{\vec{dl} \wedge \vec{u}}{r^2} \qquad \vec{u} = \frac{\vec{PM}}{PM}$$

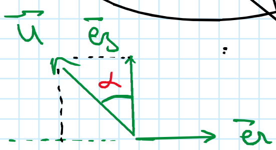
$\vec{dB} \perp$ au plan formé par $\vec{u}$ et $\vec{dl}$.



En coord. cylindriques : (r, θ, z)

$(\vec{e_r}, \vec{e_\theta}, \vec{e_z})$

$$\vec{dl} \wedge \vec{u} \quad ; \quad \begin{cases} \vec{dl} = R\,d\theta\,\vec{e_\theta} \; ; \\ \vec{u} = -\sin\alpha\,\vec{e_r} + \cos\alpha\,\vec{e_z} \end{cases}$$



$$\vec{dl} \wedge \vec{u} = R\,d\theta\,\vec{e_\theta} \wedge (-\sin\alpha\,\vec{e_r} + \cos\alpha\,\vec{e_z})$$

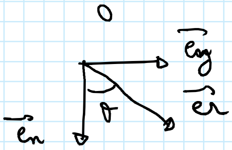
$$= -R\,d\theta\sin\alpha\,\underbrace{(\vec{e_\theta} \wedge \vec{e_r})}_{-\vec{e_z}} + R\,d\theta\cos\alpha\,\underbrace{(\vec{e_\theta} \wedge \vec{e_z})}_{\vec{e_r}}$$

$$= R\,d\theta\sin\alpha\,\vec{e_z} + R\,d\theta\cos\alpha\,\vec{e_r}$$

$$\vec{dB} = \frac{\mu_0 I}{4\pi r^2} R\sin\alpha\,d\theta\,\vec{e_z} + \frac{\mu_0 I}{4\pi r^2}\cos\alpha\,d\theta\,\vec{e_r}$$

$$\vec{B}_{(M)} = \int \vec{dB} \Rightarrow \vec{B}_{(M)} = \frac{\mu_0 I}{4\pi r^2} R\sin\alpha \left(\int_0^{2\pi} d\theta\right)\vec{e_z} + \frac{\mu_0 I}{4\pi r^2} R\cos\alpha\left(\int_0^{2\pi} d\theta\,\vec{e_r}\right)$$

$$\vec{e_r} = \cos\theta\,\vec{e_x} + \sin\theta\,\vec{e_y}$$



$$\int_0^{2\pi} d\theta\,\vec{e_r} = \underbrace{\int_0^{2\pi}\cos\theta\,d\theta\,\vec{e_x}}_{= 0} + \underbrace{\int_0^{2\pi}\sin\theta\,d\theta\,\vec{e_y}}_{= 0}$$

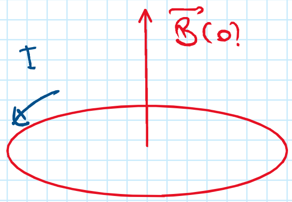
$$\vec{B}_{(M)} = \frac{\mu_0 I}{4\cancel{\pi} r^2} R\sin\alpha\, 2\cancel{\pi}\,\vec{e_z} \quad \text{or} \quad \sin\alpha = \frac{R}{r}$$

• en fct de $\alpha \longrightarrow$ $\vec{B}(M) = \dfrac{\mu_0 I}{2R}\sin^3\alpha\,\vec{e_z}$

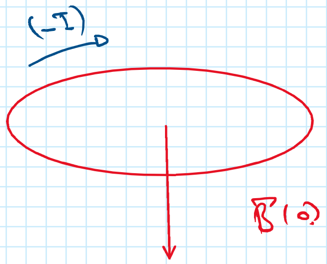
• en fonct de z $\longrightarrow$ $\boxed{\vec{B}(M) = \frac{\mu_0 I}{2} \frac{R^2}{(z^2+R^2)^{3/2}} \vec{e_z}}$ $r = (z^2+R^2)^{1/2}$

b) Déduire le champ crée au centre O de la spire.

Au centre de la spire $(M \equiv O)$ $\longrightarrow$ $\alpha = 0$ ce qui correspond à $z = 0$



$\boxed{\vec{B}(0) = \frac{\mu_0 I}{2R} \vec{e_z}}$



$I \longrightarrow (-I)$

$\vec{B}(0) = -\frac{\mu_0 I}{2R} \vec{e_z}$