Lens Distortion Correction by Analysing Peak Shape in Hough Transform Space

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***Abstract*—Lens distortion is the result of radial variations in the lens magnification. It occurs with most low cost wide angle cameras and affects image quality. As a result, straight lines, particularly those in the periphery, appear curved. One dominant lens distortion correction approach is based on linear features within the image. This kind of correction method usually requires detecting edges or lines, and the Hough transform is a powerful tool to do so. In the Hough transform space, straight and curved lines yield patterns with different shapes. In this paper, a proof of concept is explored whereby lens distortion parameters are derived directly from analysing the shape of peaks within Hough transform space. Subsequently, by using this information, the lens distortion can be corrected. This is demonstrated both through simulation and correction of real world images.**

I. Introduction

In computer vision and digital image processing, the basic pin-hole camera model is commonly used [1]. It assumes that each point in the image is generated as a direct projection of a real world point through the optical centre. However, due to the undesirable effect of lens distortion, real cameras rarely follow the ideal pin-hole model [2]. Real camera images suffer from radial lens distortion to a greater or lesser extent. However, with wide angle or fish-eye lenses the problem becomes more serious [3]. There are generally two components of lens distortion: radial lens distortion and tangential distortion, however Tsai [4] has demonstrated that the effect of tangential distortion is insignificant in many cases, and can therefore be neglected. Radial lens distortion results from a different magnification in the centre of the image to that at the edges, causing points in the image to deviate in a non-linear fashion from their ideal position of a pin-hole camera model. For barrel lens distortion, the image will have a higher magnification in the centre of the image than the periphery while pincushion distortion has the opposite effect. As a result of radial lens distortion, straight lines, especially those located near the edges of the image, will appear curved. Radial lens distortion is symmetric about the centre of distortion, and does not affect the image at the distortion centre.

For many cameras, radial lens distortion can be neglected, however, for image processing tasks or machine vision systems, where measurements are made from the images, lens distortion cannot be accepted. Not only does radial lens distortion influence the visual quality of the image, it is also critical in many applications where shape recognition, localisation, classification and detection is essential [1].

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To correct lens distortion, geometric camera calibration is needed. It aims to correct the lens distortion by mapping the pixels in the image based on the features of real-world points. There are two categories of geometric camera calibration methods based on the features used [5] [6]. The first type of approach uses feature points, consisting of dots, circles or corners which can be easily extracted from an image of a calibration grid [7]. The second approach is linear feature based [8] [9] [10] [11], where the features are one or more straight lines or edges, either from a calibration grid or naturally occurring in the scene. This method assumes that the linear features without distortion should be straight, thus, this principle can be used for lens distortion correction. For this approach some form of edge or line detection method is usually required.

The Hough transform is a reliable way to detect linear features [5]. The Hough transform works by converting the original image space into a parameter space, referred to here as the Hough transform space. Each point in image space will vote in Hough transform space for the parameters of all lines consistent with that point. A line in image space will result in multiple votes at the point in Hough transform space corresponding to the line's parameters. By finding the local maxima (peaks) in parameter space, it is possible to locate the dominant lines in the original image. A straight line image will give a peak in the Hough transform space concentrated at a point. A curved line will yield a 'blurred' peak because the curve cannot be represented by a single pair of line parameters. Fig. 1 illustrates the shape of peaks in Hough transform space corresponding to a straight line and a curved line.

We assume there is a relationship between the shape of the blurred peak and the lens distortion. In this paper, a lens distortion correction method based on analysing the shape of peaks in Hough transform space is proposed. This method is based on estimating lens distortion parameters by locating feature points in Hough transform space and analysing those feature points.

Few works have investigated the use of the Hough transform for camera lens distortion correction. Cucchiara et al. [1] introduced a Hough transform based radial lens distortion correction method. It adjusts the lens distortion parameter to

Fig. 1. The left image is the straight line and the line curved by lens distortion, the right image is the peak of the straight line and the blurred peak of the curved line in the Hough transform space.

maximise the votes in Hough transform space which correspond to a single straight line in image space. Lee et al. [8], proposed a Hough transform based method to correct distortion in wide-angle cameras. They used the entropy of the Hough transform after distortion correction to measure straightness of all distorted lines in an image. They then adjusted the lens distortion parameters to maximise the entropy. Aleman-Flores et al. [12], proposed a line detection and lens distortion correction method, by introducing a third parameter to the Hough transform which represents the lens distortion. It effectively tries all values of the lens distortion parameters and selects the best. However, none of these methods focus on the shape of peaks in the Hough transform space when the image is distorted.

Furukawa and Shinagawa [5] analysed the shape of the 'butterfly' in the Hough transform space for accurate and robust line segment extraction. They used the shape of the 'butterfly' patterns around the peak to identify the endpoints of the line segment in the image. It is their work which inspired us for using the shape of peaks to directly estimate the lens distortion.

This paper is organised as follows. Section II reviews lens distortion models and the Hough transform. Section III analyses the shape of peaks in Hough transform space and investigates the relationship between the distortion in image space and the shape of the Hough transform pattern. The basic steps of our method are presented in this section. Simulation results and examples of correcting real world image are covered in Section IV. Section V provides the conclusions and future research goals.

II. Camera Model and Hough Transform

A. Lens distortion model

A radial shift of coordinates modifies the distance of every pixel from the image centre only. With this notion, the lens distortion function can be described as:

$$r_d = f(r_u) \tag{1}$$

where r_d is the radius of a point in the distorted image, r_u is the radius of the corresponding point in the original undistorted image. Any distortion $f()$ can be approximated by a Taylor series expansion. Generally, an odd model can represent lens distortion while maintaining radial symmetry:

$$r_d = r_u + \kappa_1 r_u^3 + \kappa_2 r_u^5 + \kappa_3 r_u^7 + ... \tag{2}$$

where κ_i are the radial distortion coefficients.

It has been demonstrated, that for typical camera lenses, a low order approximation is sufficient and more terms only cause numerical instability [1]. According to Li and Lavest [13] the first order component corrects most of the radial distortion of typical lenses.

We used a reverse mapping model to suit the features of the Hough transform. In this model, the undistorted radius is represented as a function of the distorted radius:

$$r_u = f(r_d). \tag{3}$$

Here, only the first order model was considered. We assume that this is a sufficiently accurate approximation for our target application. The first order Taylor series approximation of (3) is given by:

$$r_u \approx r_d + \kappa {r_d}^3 = r_d(1 + \kappa {r_d}^2). \tag{4}$$

Note the value of lens distortion parameter in (4) is different from that in (2). We invert (4) to generate the distorted image for our simulations:

$$r_d = \omega - \frac{1}{3\kappa\omega} \tag{5}$$

where

$$\omega = -\sqrt[3]{\sqrt{\left(\frac{r_u}{2\kappa}\right)^2 + \frac{1}{27\kappa^3}} - \frac{r_u}{2\kappa}}. \tag{6}$$

B. Hough Transform

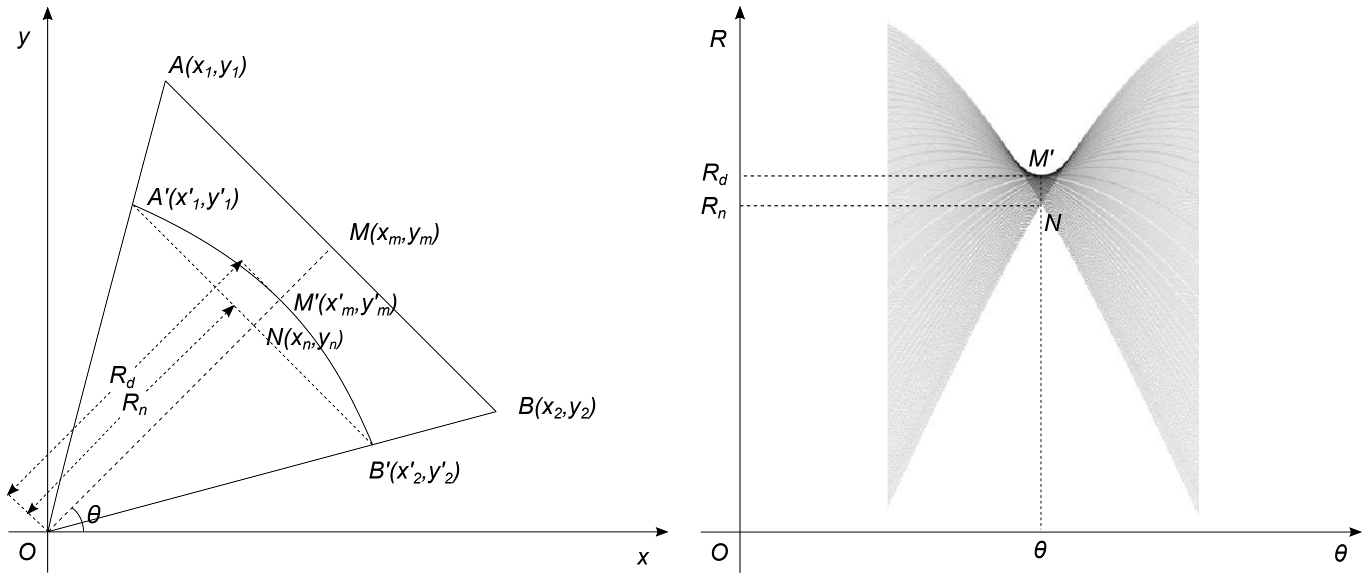
The Hough transform is a feature extraction technique used in image analysis, computer vision, and digital image processing. It has long been used to detect lines and other parameterised shapes within an image. The original Hough transform was invented for machine analysis of bubble chamber photographs by Hough in 1959 [14]. Now, the most commonly used parametrization, known as the *standard* Hough transform, is that introduced by Duda and Hart [15]:

$$\rho = x\cos\theta + y\sin\theta \tag{7}$$

where ρ is the distance from the origin to the closest point on the straight line and θ is the angle between the x axis and the line connecting the origin with that closest point.

III. Peak Shape in Hough Transform Space

In this section, the shape of peaks in the Hough space is analysed as a function of the distortion of lines. Based on features of the Hough transform peak, we propose a lens distortion correction method.



(a) (b)

Fig. 2. Image (a) is the straight line and the line curved by lens distortion, Image (b) is the blurred peak of the curved line in the Hough transform space.

A. Analysis of the Shape of Peaks in Hough Transform Space

We start our analysis with a simplified case, where the line in image space is symmetrical about the origin. Fig. 2 shows a straight line and the curved line caused by lens distortion, and the corresponding blurred peak in Hough transform space. Points in image space are named as shown in Fig. 2(a). $\overline{AB}$ is the original undistorted straight segment with point M being the closest to the origin O. $\overset{\frown}{A'B'}$ is the curved line caused by lens distortion. The line $\overline{A'B'}$ connects the end points of the curved line. In this case, by symmetry $\overline{OM}$ is also perpendicular to $\overline{A'B'}$, and N is the closest point on line $\overline{A'B'}$ to the origin. We also define R_d as the length of $\overline{OM'}$, and R_n as the length of $\overline{ON}$. ΔR is length of $\overline{M'N}$.

In Hough transform space, one significant point is N, which as its coordinates are the parameters to the parameters of the dotted line $\overline{A'B'}$. Another significant point in Hough transform space is M' which corresponds to the tangent to arc $\overset{\frown}{A'B'}$ through M' in image space. The distance from M' to N in Hough transform space should also be equal to ΔR. Let R_1 be the length of $\overline{OA}$, R_1' be the length of $\overline{OA'}$ and R_u be the length of $\overline{OM}$. Because $\triangle AMO$ is similar to $\triangle A'NO$.

$$\frac{R_u}{R_n} = \frac{R_1}{R_1'} \Rightarrow R_u = R_n\left(\frac{R_1}{R_1'}\right). \tag{8}$$

Based on the reverse lens distortion model (4):

$$R_1 = R_1'(1+\kappa {R_1'}^2) \Rightarrow \frac{R_1}{R_1'} = 1+\kappa {R_1'}^2. \tag{9}$$

Substituting (9) into (8) gives:

$$R_u = R_n(1+\kappa {R_1'}^2) = R_n + \kappa R_n {R_1'}^2. \tag{10}$$

Also from the model:

$$R_u = R_d(1+\kappa {R_d}^2) = R_d + \kappa {R_d}^3. \tag{11}$$

Subtracting (10) from (11) gives ΔR as:

$$\Delta R = R_d - R_n = \kappa(R_n {R_1'}^2 - {R_d}^3). \tag{12}$$

From (12) the lens distortion coefficient κ can be represented by:

$$\kappa = \frac{\Delta R}{R_n {R_1'}^2 - {R_d}^3}. \tag{13}$$

The value of R_1' can be estimated in Hough transform space from the slope of curve at N, which corresponds to the trace from the A' (or B' when it is symmetric). We define point $A' = (x_1', y_1')$ corresponding to the end point of the arc $\overset{\frown}{A'B'}$. For N we have:

$$R_n = x_1' cos\theta + y_1' sin\theta \tag{14}$$

and the slope of the curve at N is:

$$\frac{\mathrm{d}R}{\mathrm{d}\theta} = -x_1' sin\theta + y_1' cos\theta. \tag{15}$$

Solving (14) and (15) gives the coordinates of A' as

$$\begin{cases} x_1' = R_n cos\theta - \dfrac{\mathrm{d}R}{\mathrm{d}\theta} sin\theta \\ y_1' = R_n sin\theta + \dfrac{\mathrm{d}R}{\mathrm{d}\theta} cos\theta \end{cases} \tag{16}$$

$${R_1'}^2 = {x_1'}^2 + {y_1'}^2 = {R_n}^2 + \left(\frac{\mathrm{d}R}{\mathrm{d}\theta}\right)^2. \tag{17}$$

Substituting (17) into (13) gives:

$$\kappa = \frac{R_d - R_n}{{R_n}^3 - {R_d}^3 + R_n\left(\frac{\mathrm{d}R}{\mathrm{d}\theta}\right)^2}. \tag{18}$$

This is only true when the original straight line is symmetrical about the centre of distortion.

B. Straight Line Based Method

In generalising the above analysis to arbitrary lines, we make 3 observations:

1) M is no longer the midpoint of $\overline{AB}$.
2) A, B have different distance to M, so $\overline{A'B'}$ is no longer parallel to line $\overline{AB}$.
3) N occurs at a different θ to M.

From the distortion model in equation (4), if the coordinates of the distorted point are known, those of the original point on the straight line can be found. We define the coordinates of points as shown in Fig. 2(a), and R_2 be the length of $\overline{OB}$ and R_2' be the length of $\overline{OB'}$, R_m' be the length of $\overline{OM'}$.

Since the points A, B, M all belong to the original straight line, the slope of segments $\overline{AM}$ and $\overline{BM}$ should be the same:

$$\frac{y_m - y_1}{x_m - x_1} = \frac{y_2 - y_m}{x_2 - x_m}. \tag{19}$$

Substituting in the lens distortion model gives:

$$\frac{y_m'(1+\kappa {R_m'}^2) - y_1'(1+\kappa {R_1'}^2)}{x_m'(1+\kappa {R_m'}^2) - x_1'(1+\kappa {R_1'}^2)} = \frac{y_2'(1+\kappa {R_2'}^2) - y_m'(1+\kappa {R_m'}^2)}{x_2'(1+\kappa {R_2'}^2) - x_m'(1+\kappa {R_m'}^2)}. \tag{20}$$

In (20) the lens distortion coefficient κ can be estimated if the location of points A', B' and M can be determined.

From Hough transform space, in Fig. 2(b), N corresponds to the parameters of the $\overline{A'B'}$ in image space. However this is insufficient to calculate the coordinates of A' and B'. So we use (16) to estimate the coordinates of A' and B'. The upper edge of the Hough transform peak in Fig. 2(b) corresponds to the arc $\overset{\frown}{A'B'}$. The coordinates of M' can be located in image space from the minimum of the curve in Hough transform space:

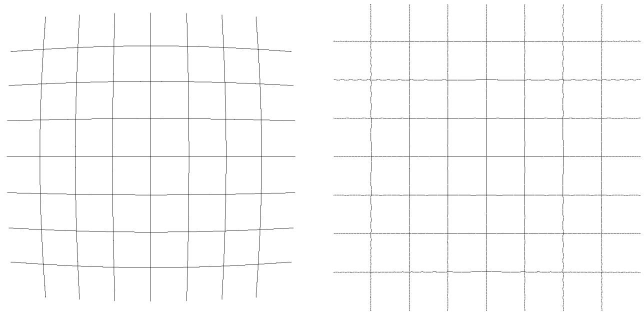
$$\begin{cases} x_m' = R_d \cos\theta \\ y_m' = R_d \sin\theta. \end{cases} \tag{21}$$

These three points can be substituted into equation (20) which is then solved for the lens distortion coefficient κ.

IV. Lens Distortion Correction

First, a Canny edge detection filter is applied to the distorted images in order to extract the most representative segments, lines and arcs from the scene. Then the detected edges are transformed by the Hough transform. If the original image contains many curves or straight lines there will be several peaks in Hough transform space. Therefore, it is necessary to select the best peak for analysis. The best peak should represent the most distorted line, so we select the peak corresponding to the largest radius, because the level of distortion increases with the distance from the centre of the image. Based on the last section we separated our shape analysis into the following steps:

1) Locate the blurred peak.
2) Locate point N.
3) Measure the slope of curves at both sides of point N, and calculate A' and B' from (16).
4) Locate point M', and calculate the third point from (21).
5) Estimate the lens distortion parameter from (20).



(a) Distorted image (b) Corrected image

Fig. 3. (a) is the distorted image with $\kappa = 6.4564 \times 10^{-7}$, (b) is the correction result from our method with a estimated $\kappa = 6.6254 \times 10^{-7}$.

V. Result and Discussion

A. Simulation Result

For simulation, an 800×800 image of a grid with lines every 100 pixels was distorted with a known κ, with the centre of distortion in the centre of the image. Fig. 3 shows the distorted image and the result of our correction method. The bin resolution in Hough transform space was 1131×628, corresponding to a radial resolution of 1 pixel and an angular resolution of 0.05 rad.

Since the grid is symmetric about the centre of the image, we tested both the method based on (18) and (20), with the results shown in Table I. Since the solution depends on accurately finding points N and M' in Hough transform space, the accuracy of estimating κ depends strongly on the resolution of Hough space. When mapping points into Hough transform space, the accuracy of the results can be affected by quantisation (rounding). A higher resolution Hough transform space will limit the influence of rounding. To test this a high-resolution Hough transform space was used, which doubled the angular resolution to 1256 bins, and doubled the radial resolution to 0.5 pixels per bin. The result in Table I shows that with significant lens distortion, both low and high-resolution methods showed an acceptable accuracy. However, when the lens distortion parameter becomes small, the accuracy of the low resolution method deteriorated. Meanwhile, the accuracy of the high-resolution method remains acceptable. So it is possible to increase the accuracy by mapping the original image into a higher resolution Hough transform space when the lens distortion is slight.

B. Real World Image

The proposed method requires at least one straight line visible in the image. In contrast with the simulation, other features within the image may interfere with the curved lines in the real world image. Therefore, a pre-processing method is necessary to segment the curved line from the background.

TABLE I

ACCURACY OF ESTIMATING KNOWN LENS DISTORTION ON THE SIMULATION GRID IMAGE. COMPARED ARE THE LOW RESOLUTION HOUGH TRANSFORM SPACE, BOTH WITH THE SYMMETRIC (18) AND GENERAL (20) ESTIMATES, AND HIGH RESOLUTION HOUGH TRANSFORM SPACE WITH THE GENERAL ESTIMATE (20). THE DISTANCE ERROR IS THE RMS ERROR IN PIXELS OF THE GRID POINT INTERSECTIONS IN FIG.3.

	Low resolution (18)			Low resolution (20)			High resolution (20)		
Distortion parameter κ	Estimated parameter κ	Percentage error	Distance error	Estimated parameter κ	Percentage error	Distance error	Estimated parameter κ	Percentage error	Distance error
1.50×10^{-7}	1.57×10^{-7}	4.73 %	0.18	1.79×10^{-7}	19.43 %	0.75	1.53×10^{-7}	2.43 %	0.09
2.00×10^{-7}	1.97×10^{-7}	1.46 %	0.07	2.18×10^{-7}	9.04 %	0.46	2.09×10^{-7}	4.85 %	0.25
4.00×10^{-7}	3.93×10^{-7}	1.65 %	0.17	3.84×10^{-7}	3.88 %	0.40	4.09×10^{-7}	2.43 %	0.25
6.00×10^{-7}	6.35×10^{-7}	5.92 %	0.92	6.03×10^{-7}	0.61 %	0.09	5.91×10^{-7}	1.44 %	0.22
8.00×10^{-7}	8.62×10^{-7}	7.84 %	1.62	7.91×10^{-7}	1.09 %	0.22	7.92×10^{-7}	1.00 %	0.20
10.00×10^{-7}	10.04×10^{-7}	4.66 %	1.20	10.13×10^{-7}	1.37 %	0.28	10.27×10^{-7}	2.77 %	0.57

(a) (b)

Fig. 4. (a) is a real world image with radial lens distortion, edges of the curved window can be used for lens distortion correction, (b) is the correction result.

(a) (b)

Fig. 5. (a) is a real world image with radial lens distortion, edges of the curved garage can be used for lens distortion correction, (b) is the correction result.

In general, the curved lines are located at the edges within an image, therefore, we use a Canny filter to extract edge information. We use the Hough transform to locate the line, and then remove the noise near the detected line. After pre-processing, the curved line can be used for calculating κ, in the same way as was used in the simulation. The last step is to use the calculated lens distortion parameter to correct the distorted image, Fig. 4 and Fig. 5 show both the distorted and corrected image.

C. Discussion

From the previous results, our method shows encouraging ability in lens distortion correction. In contrast with popular calibration methods, our method does not need a particular calibration pattern or target. Many line based calibration methods usually require accurate line detection, but in significantly distorted images common straight line based detection methods cannot always provide a reliable result [12]. Ours measures the distortion directly in Hough transform space instead of using the line detection method to detect the line first and then analyse the line in image space. However, like other line based methods our method requires linear features to be visible in image space. As it stands our method is based on analysing a single line, making it is hard to estimate the centre of distortion. It is also quite sensitive to accurately measuring the features of the distorted peak in Hough transform space.

VI. CONCLUSION AND FUTURE WORK

This paper demonstrates that it is possible to correct lens distortion based on analysing the shape of peaks within the Hough transform space. From both simulation and correcting real world images, our method provides encouraging results. It is able to directly estimate the lens distortion parameter without an iterative search or optimisation.

Currently, our method works by extracting feature points of a single line in Hough transform space. The next step is to use the data extracted from multiple lines to estimate the location of the centre of distortion (lines through the centre remain straight); and combine the multiple estimates of κ (one from each detected line) to give a more accurate estimate of the lens distortion coefficient.

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