Project_Study_3

Group Screening

1. Article review

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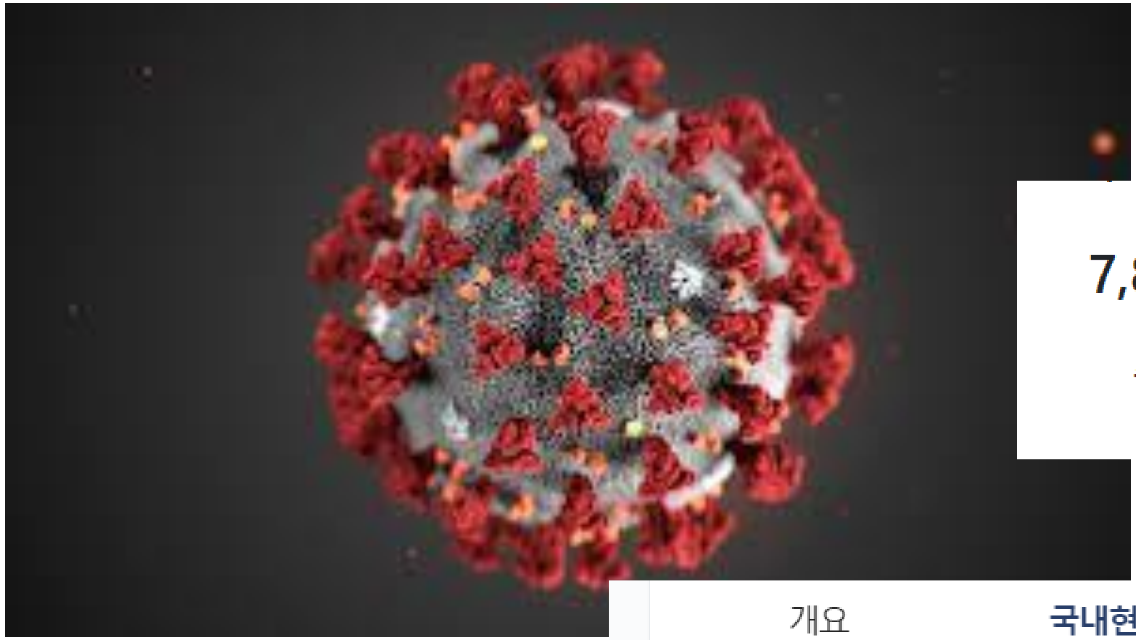
1.Introduction



7,868,820 (-)	80,497 (-)	7,682,571 (-)
총검사자	검사중	결과음성

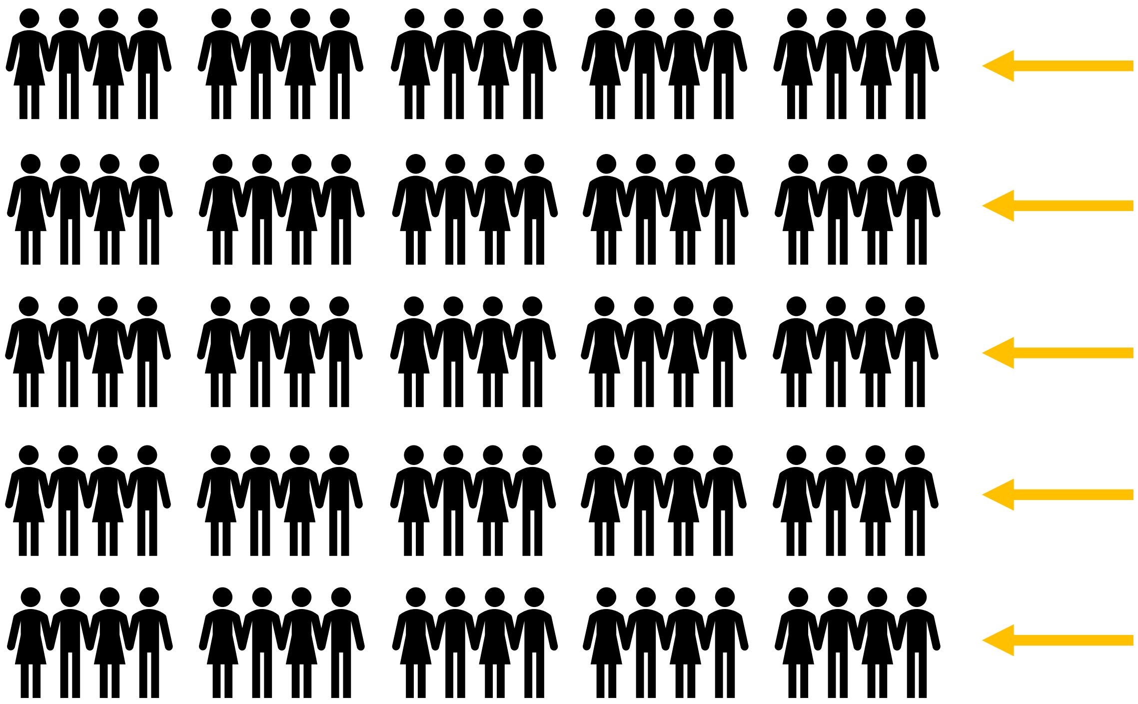
개요 | 국내현황 | 세계현황 | 예방접종 | 거리두기단계

확진환자	검사중	격리해제	사망자
105,752	80,497	96,900	1,748
473▲	698▲	311▲	4▲

일일확진자	국내발생 449	해외유입 24

2. Group Testing

100 People



Nobody : 5

One person : 5+20=25

3. Optimizing the Algorithm

p: prevalence of disease

$p \in [0, 1]$ $\longrightarrow$ Optimal Group Size($n \geq 2$)

X: Number of infections in the group

$X \sim B(n, p)$

3. Optimizing the Algorithm

$\Pr(X = 0) = (1 - p)^n$

$\Pr(x > 0) = 1 - \Pr(x = 0) = 1 - (1 - p)^n$

If $X = 0$, then one only needs to perform $N = 1$ test.

However if $X > 0$, then $N = n + 1$ tests will be needed.

3. Optimizing the Algorithm

$E\ (N)$: Expected number of tests to be performed.

$$E(N) = 1 \times \Pr(X = 0) + (n + 1) \times \Pr(X > 0)$$
$$= n + 1 - n(1 - p)^n$$

3. Optimizing the Algorithm

If $p = 0$ we have $E(N) = 1$.

If $p = 1$ we have $E(N) = n + 1$.

3. Optimizing the Algorithm

$$\frac{E(N)}{n} = 1 + \frac{1}{n} - (1-p)^n.$$

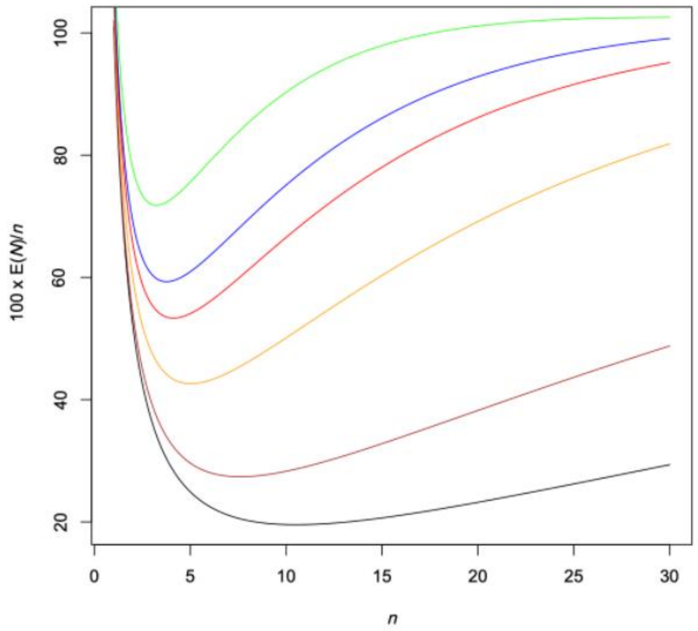
- When $p = 0$ we find $E(N)/n = 1/n$, so it pays to take n as large as possible.
- When p = 1 , we always have

$$\frac{E(N)}{n} = 1 + \frac{1}{n} > 1$$

because the group test is always positive and therefore only adds to the burden.

3. Optimizing the Algorithm

$100 \times \frac{E(N)}{n}$: represents the average percentage of tests performed as a function of the size, n , of the group.



p (%)	n	상대원가(%)
1	11	20
2	8	27
5	5	43
8	4	53
10	4	59
15	3	72

4. Generalization

- The testing protocol described above is an example of a two-round adaptive algorithm.

Adaptive

4. Generalization

- We can improve the performance of algorithm

Number of round

4. Generalization

- binary division algorithm

a) Take an integer n of the form $n = 2^s$ and perform k rounds of tests, where $k \leq s + 1$.

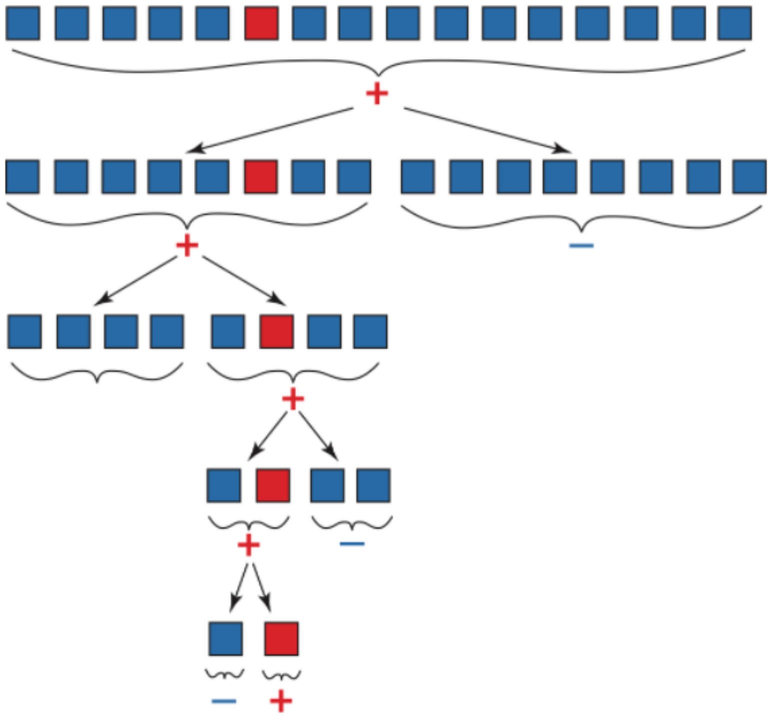
b) In the first round, test a mixture of samples from the whole group.

c) If the test is positive, then the group is divided into two subgroups of 2^{s-1} samples and a mixture of each is tested.

d) This is continued until the k^{th} round, where all members of a subgroup that was positive in the previous round are tested individually.

4. Generalization

- binary division algorithm

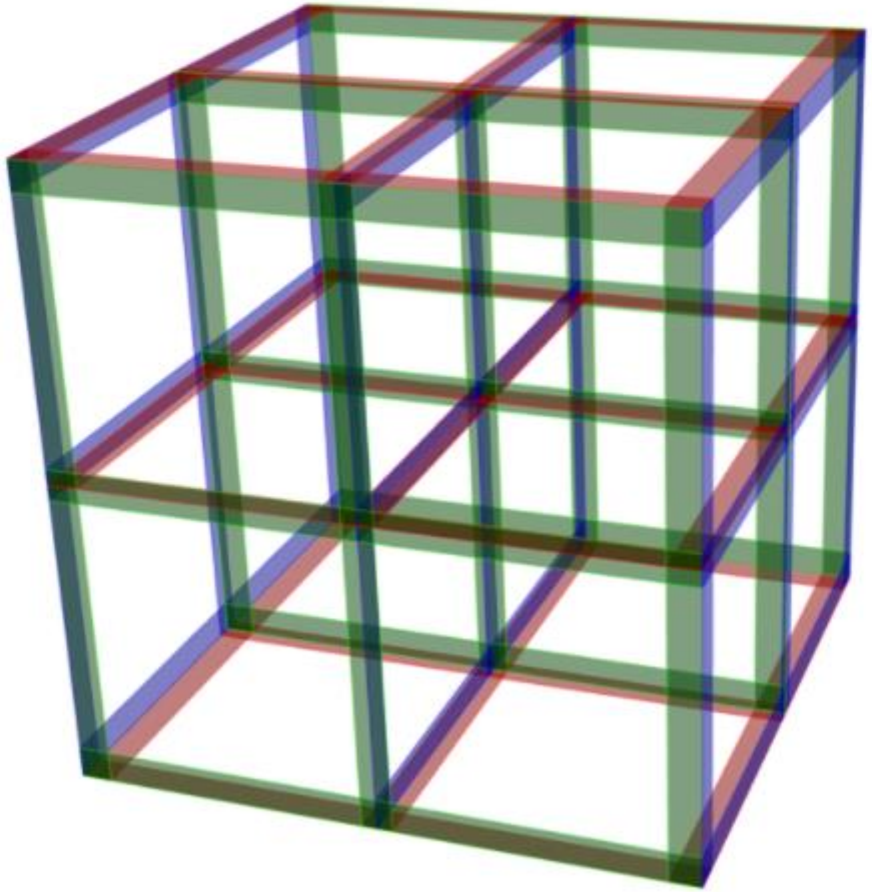


Time

5. A non-adaptive algorithm

- To better control response time, non-adaptive group screening methods may also be considered.
- These protocols involve only one round, allowing all tests to be performed simultaneously.
- They are very effective for case detection if a reliable estimate of the prevalence of the disease is available.

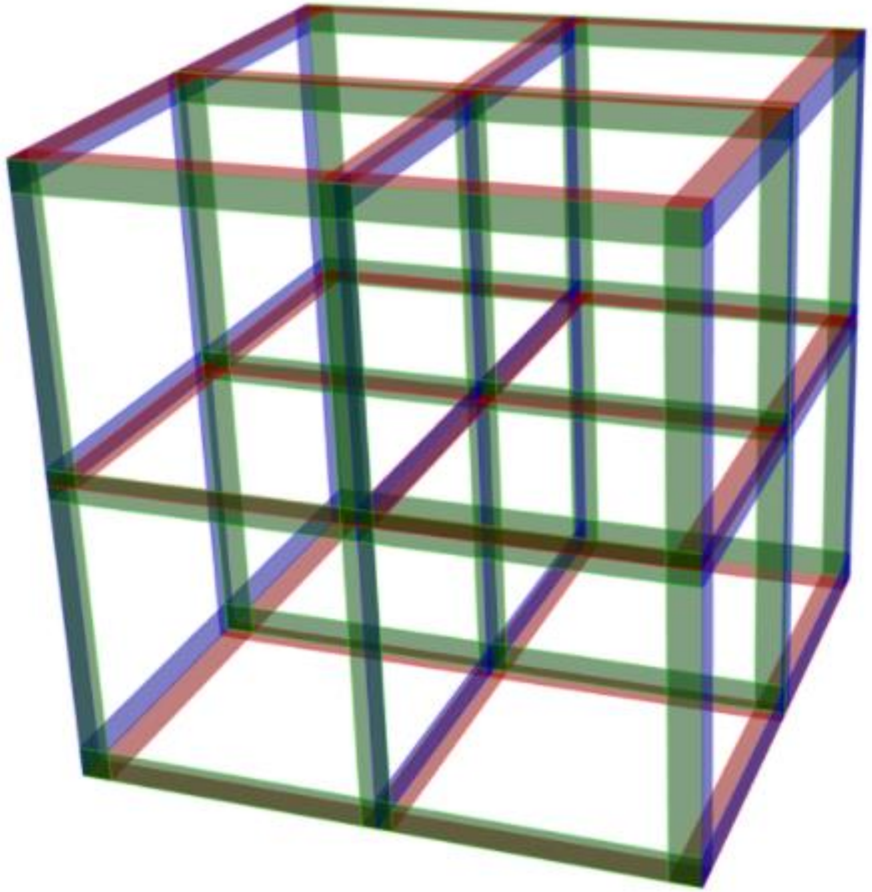
5. A non-adaptive algorithm



Take $n = 3^m$

- Each point of the lattice corresponds to an individual in a random sample of size n.
- The $3^2 = 9$ individuals forming each of the 9 mixtures are located along a red, blue or green slice.

5. A non-adaptive algorithm



- Conduct test number $3m$ simultaneously on the mixture of samples containing 3^{m-1} individuals.
- When $m = 3$, as shown on the left, nine tests are performed on 9 groups.
- Number of Groups: 3^{m-1}
- Number of test runs: $3 \times m$

5. A non-adaptive algorithm

If $n = 4$,

$$X \sim B(3^4, p)$$

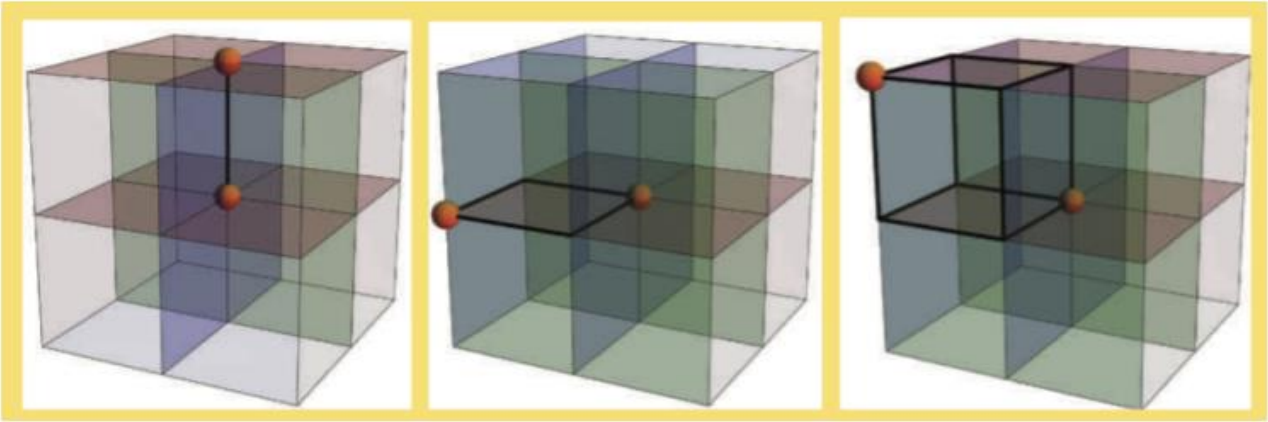
$$\Rightarrow \Pr(X \leq 1) = (1-p)^{81} + 81p(1-p)^{80}$$

- This strategy is interesting if the probability that X ≤ 1 is large.
- But to ensure that $\Pr(X \leq 1) \geq 0.95$, for example, one must have $p \leq 0.44\,\%$
- In other words, the prevalence of the disease must be low.

5. A non-adaptive algorithm

- Costs can easily be controlled if we are clever in the way we conduct the second round when there is more than one infected individual.

(P) For any value of $i \in \{1, \dots, m\}$, there are at most two slices of the form $x_i = s$ and $x_i = t$ leading to a positive test, and there is at least one value of i for which there are exactly two such slices leading to positive tests.



5. A non-adaptive algorithm

k	Number of additional tests
1	0
2	4
3	8
4	16

- Additional Number of trials according to the number of infections

Thank you!