Project_Study_3

Reconstruction From Two Planar Cross Sections

1. Paper review

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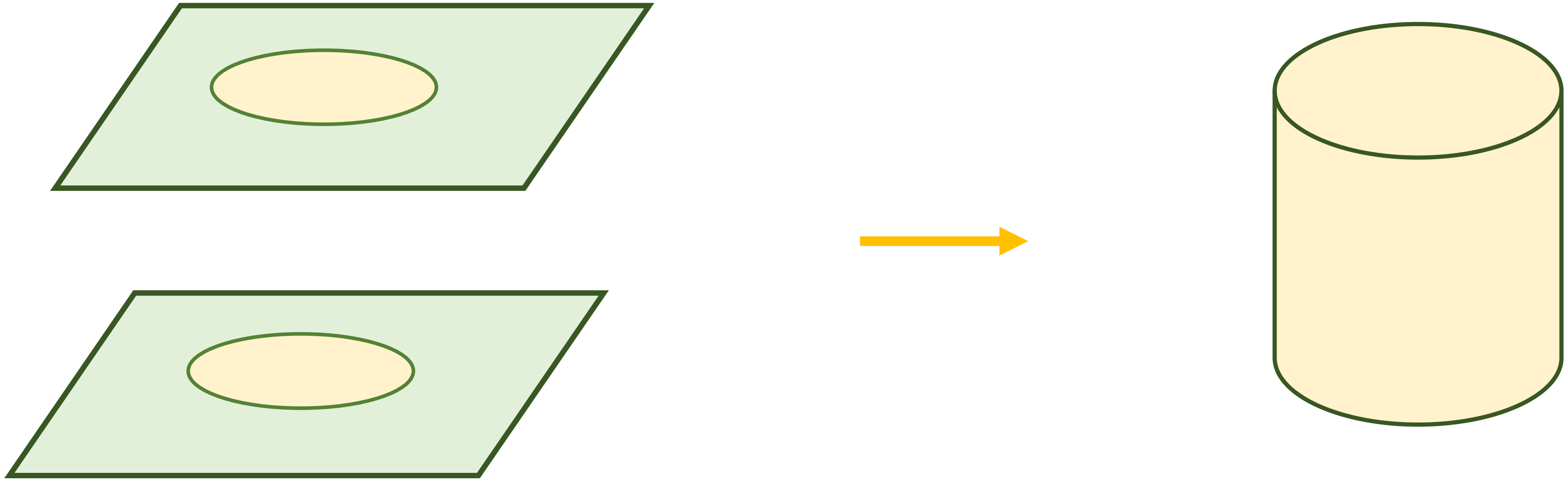
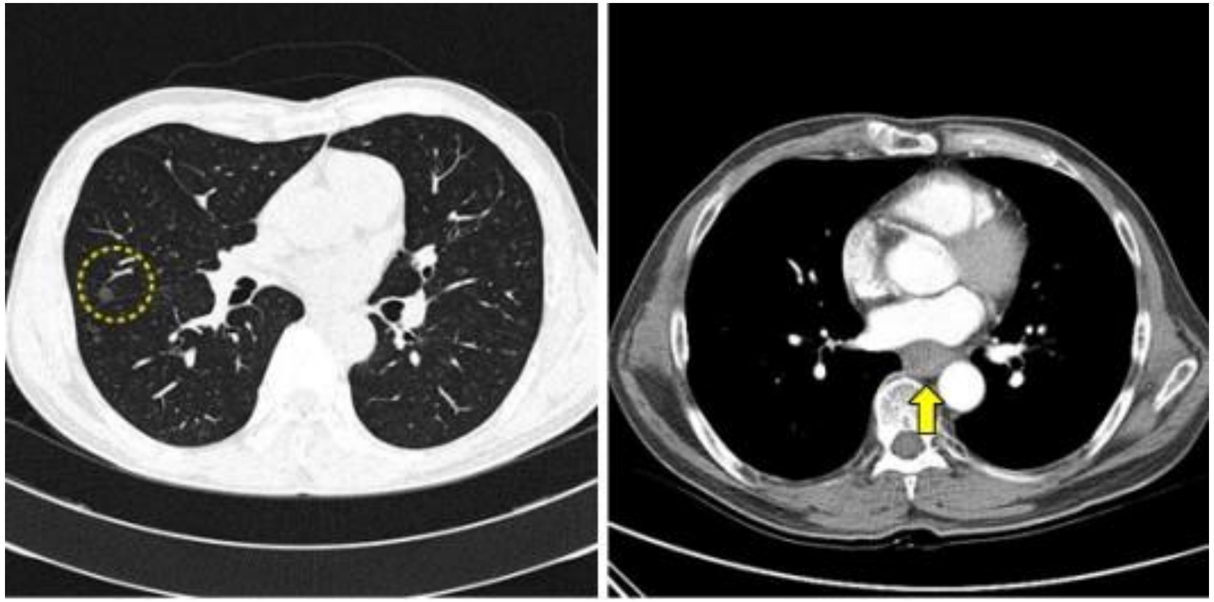
1. Abstract & Introduction



1. Abstract & Introduction



CT

(computed tomography)

1. Abstract & Introduction

Cahn-Hilliard Equation

-Equation of mathematical physics which describes the process of phase separation.

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Cahn-Hilliard Equation

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Implicit Method

Cahn-Hilliard Equation

$$\frac{\partial \phi(\mathbb{x}, t)}{\partial t} = \Delta \mu(\mathbb{x}, t), \qquad \mathbb{x} \in \Omega, 0 < t \leq T$$

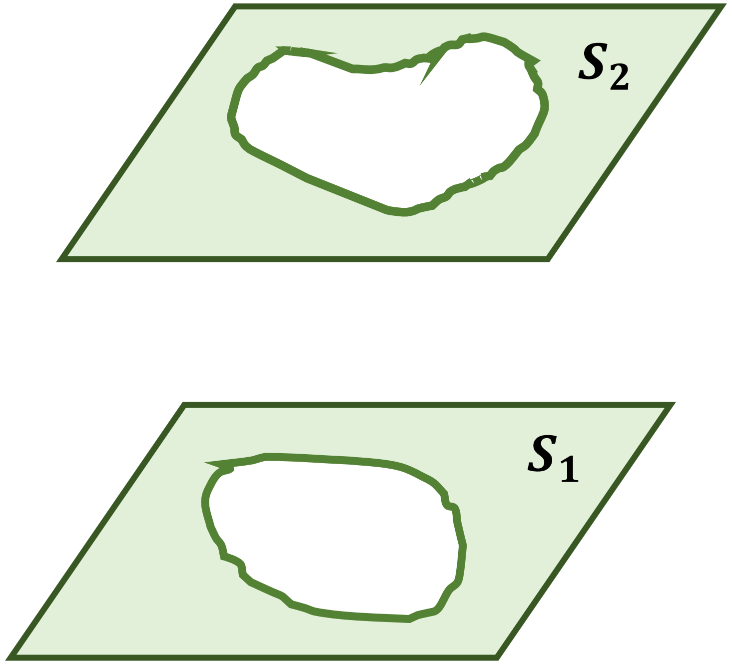
$$\mu(\mathbb{x}, t) = F'\big(\phi(\mathbb{x}, t)\big) - \varepsilon^2 \Delta \phi(\mathbb{x}, \mathrm{t})$$

$\mathbb{x} = (x, y, z)$: Coordinate, $\Omega \subset \mathbb{R}^3$: *Domain*,

$\phi(\mathbb{x}, t)$: $Phase - Field(inside = 1, outside = -1)$

$F(\phi) = 0.25(1 - \phi^2)(1 + \phi^2)$

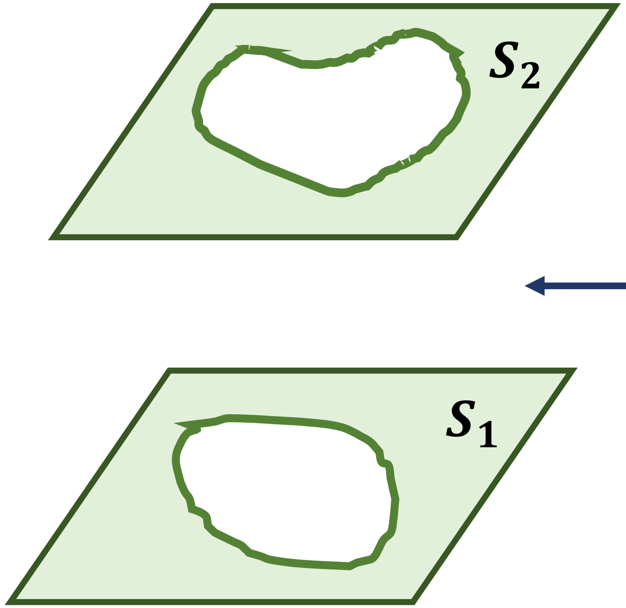
2. Reconstruction Process



$\phi_{top}(x, y)$

$\phi_{bottom}(x, y)$

2. Reconstruction Process



$\phi_{top}(x, y)$

$\phi_{bottom}(x, y)$

$$\phi(x, y, \theta z_2 + (1-\theta) z_1) = \theta \phi_{top}(x, y) + (1-\theta) \phi_{bottom}(x, y)$$

2. Reconstruction Process

Boundary condition

ϕ : $Dirichlet\ boundary$ condition

$Dirichlet\ boundary$ condition : A boundary condition that gives a value to a boundary.

Time

2. Reconstruction Process

Boundary condition

μ: $Neumann\ boundary$ condition

$Neumann\ boundary$ condition : A boundary condition that gives differential values to a boundary.

Space

3. Numerical Solution

Cahn-Hilliard Equation

$$\frac{\partial \phi(\mathbb{x}, t)}{\partial t} = \Delta \mu(\mathbb{x}, t), \qquad \mathbb{x} \in \Omega, 0 < t \leq T$$

$$\mu(\mathbb{x}, t) = F'\big(\phi(\mathbb{x}, t)\big) - \varepsilon^2 \Delta \phi(\mathbb{x}, \mathrm{t})$$

$\Omega_d = (0, L_x) \times (0, L_y) \times (0, L_z)$

$x_i = (i - 0.5h), y_j = (j - 0.5)h, z_k = (k - 0.5h) \quad (1 \leq i \leq N_x, 1 \leq j \leq N_y, 1 \leq k \leq N_z, h = \frac{1}{N_x} = \frac{1}{N_y} = \frac{1}{N_z})$

3. Numerical Solution

Approximation

$$\phi_{ijk}^{n} \approx \phi(x_i, y_j, z_k, n\Delta t) \qquad \Delta t = \frac{T}{N_t}$$

3. Numerical Solution

Discretization

$$\frac{\partial \phi(\mathbb{x},t)}{\partial t} = \Delta \mu(\mathbb{x},t) \qquad \mathbb{x} \in \Omega, 0 < t \leq T$$

$$\mu(\mathbb{x},t) = F'\big(\phi(\mathbb{x},t)\big) - \varepsilon^2 \Delta \phi(\mathbb{x},\mathrm{t})$$

$$\downarrow$$

$$\phi_{ijk}^{n+1} = \phi_{ijk}^{n} + \frac{\Delta t}{h^2}(\mu_{i+1jk}^{n+1} + \mu_{i-1jk}^{n+1} + \mu_{ij+1k}^{n+1} + \mu_{ij-1k}^{n+1} + \mu_{ijk+1}^{n+1} + \mu_{ijk-1}^{n+1})$$

$$\mu_{ijk}^{n+1} = \left(\phi_{ijk}^{n+1}\right)^3 - \phi_{ijk}^{n+1} + \frac{\varepsilon^2}{h^2}(\phi_{i+1jk}^{n+1} + \phi_{i-1jk}^{n+1} + \phi_{ij+1k}^{n+1} + \phi_{ij-1k}^{n+1} + \phi_{ijk+1}^{n+1} + \phi_{ijk-1}^{n+1})$$

3. Numerical Solution

Discretization

Implicit method

$$\frac{\partial u}{\partial t}(x_i, t_j) \approx \frac{u(x_i, t_j) - u(x_i, t_j - k)}{k}$$

central difference method

$$\frac{\partial^2 u}{\partial x^2}(x_i, t_j) \approx \frac{u(x_i - h, t_j) - 2u(x_i, t_j) + u(x_i + h, t_j)}{h^2}$$

3. Numerical Solution

Gauss-Seidel iteration

- To solve the equation, we use A, which is used in Linear system of equations.

Thank you!