Chapter #1: BASIC CONCEPTS

Fundamentals of Data Structure in C

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Overview : System life cycle

```
Requirements
- describe informations(input,
  output, initial)
Analysis
- bottom-up, top-down
Design
- data objects and operations
  performed on them
Coding
- choose representations for data
  objects and write algorithms for
  each operation
```

Overview : System life cycle

Verification
- correctness proofs
  select algorithms that have been proven correct
- testing
  working code and sets of test data
- error removal
  If done properly, the correctness proofs and system test indicate erroneous code

Algorithm specification

Definition
- a finite set of instructions
- accomplish a particular task

Criteria
- zero or more inputs
- at least one output
- definiteness(clear, unambiguous)
- finiteness(terminates after a finite number of steps)
  ↑ different from program

Algorithm specification

- Ex 1.1 [Selection sort]

  sort n(≥1) integers

```
From those integers that are currently
  unsorted, find the smallest and place it
  next in the sorted list.
```

```
for (i=0; i<n; i++) {
  Examine list[i] to list[n-1] and suppose
  that the smallest integer is at list[min];

  Interchange list[i] and list[min];
}
```

Algorithm specification

<u>finding the smallest integer</u>

assume that minimum is list[i]

compare current minimum with list[i+1] to list[n-1] and find smaller number and make it the new minimum

<u>interchanging minimum with list[i]</u>

<u>function</u>

swap(&a,&b) easier to read

<u>macro</u>

swap(x,y,t) no type-checking

Algorithm specification

- Ex 1.2 [Binary search]

  assumption

  <u>sorted</u> n(≥1) distinct integers
  stored in the array *list*

  return

  <u>index *i*</u>
  (if ∃*i*, *list*[*i*] = *searchnum)*
  <u>or -1</u>
  (otherwise)

Algorithm specification

denote *left* and *right*

left and right ends of the list to be searched

<u>initially</u>, *left*=0 and *right*=n-1

let *middle*=(*left*+*right*)/2

middle position in the list

compare *list[middle]* with the *searchnum* and adjust *left* or *right*

Algorithm specification

```
compare list[middle] with searchnum
   1)searchnum < list[middle]
      set right to middle-1
   2)searchnum = list[middle]
      return middle
   3)searchnum > list[middle]
      set left to middle+1

if searchnum has not been found
and there are more integers to check
   recalculate middle
   and continue search
```

Algorithm specification

```
while(there are more integers to check) {
  middle=(left+right)/2;
  if(searchnum < list[middle])
   right=middle-1;
  else if(searchnum == list[middle])
   return middle;
  else left=middle+1;
}
```

- **determining if there are any elements left to check**
- **handling the comparison (through a function or a macro)**

Algorithm specification

```
int binsearch(int list[],int searchnum,
                        int left,int right) {
  int middle;
  while(left <= right) {
   middle = (left + right) / 2;
   switch(COMPARE(list[middle],searchnum)) {
   case -1: left = middle + 1;
            break;
   case  0: return middle;
   case  1: right = middle - 1;
   }
  }
  return -1;
}
```

➔ COMPARE() returns -1, 0, or 1

Recursive Algorithms

direct recursion
- call themselves

indirect recursion
- call other function that invoke the calling function again

recursive mechanism
- extremely powerful
- allows us to express a complex process in very clear terms

◆ any function that we can write using assignment, if-else, and while statements can be written recursively

Recursive Algorithms

- Ex 1.3 [Binary search]

  transform iterative version of a binary search into a recursive one

  establish boundary condition that terminate the recursive call

  1)success

  list[*middle*]=*searchnum*

  2)failure

  left & *right* indices cross

  implement the recursive calls so that each call brings us one step closer to a solution

Recursive Algorithms

```
int binsearch(int list[],int searchnum,int left,int
   right) {
   int middle;
   if(left <= right) {
     middle=(left+right)/2;
     switch(COMPARE(list[middle], searchnum)) {
     case -1 : return
          binsearch(list,searchnum,middle+1,right);
     case  0 : return middle
     case  1 : return
          binsearch(list,searchnum,left,middle-1);
     }
   }
   return -1;
}
```

Recursive Algorithms

- Ex 1.4 [Permutations]

```
given a set of n(≥1) elements
print out all possible permutations
  of this set

eg) if set {a,b,c} is given,
    then set of permutations is
    {(a,b,c), (a,c,b), (b,a,c),
     (b,c,a), (c,a,b), (c,b,a)}
```

Recursive Algorithms

```
if look at the set {a,b,c,d},
the set of permutations are
   1)a followed by all permutations
     of (b,c,d)
   2)b followed by all permutations
     of (a,c,d)
   3)c followed by all permutations
     of (a,b,d)
   4)d followed by all permutations
     of (a,b,c)
```

➔ **"followed by all permutations" :**
clue to the recursive solution

Recursive Algorithms

```
void perm(char *list,int i,int n) {
   int j,temp;
   if(i==n) {
     for(j=0;j<=n;j++)
       printf("%c", list[j]);
     printf("     ");
   }
   else {
     for(j=i;j<=n;j++) {
       SWAP(list[i],list[j],temp);
       perm(list,i+1,n);
       SWAP(list[i],list[j],temp);
     }
   }
}
```

Recursive Algorithms

initial function call is
<u>perm(*list*,0,*n*-1);</u>

recursively generates permutations
<u>until *i*=*n*</u>

Data Abstraction

- Data type

```
definition
   - a collection of objects and
   - a set of operations that act on
     those objects
```

basic data type
  char, int, float, double
composite data type
  array, structure
user-defined data type
pointer data type

Data Abstraction

- Abstract data type (ADT)

definition
- **data type** that is organized in such a way that
- **the specification** of the objects and **the specification** of the operations on the objects is separated from
- **the representation** of the objects and **the implementation** of the operations

Data Abstraction

```
specification
- names of every function
- type of its arguments
- type of its result
- description of what the function
  does

classify the function of data type
- creator/constructor
- transformers
- observers/reporters
```

Data Abstraction

- Ex 1.5 [Abstract data type]

structure Natural_Number(Nat_No) is
objects: an ordered subrange of the integers starting at zero and ending at the max. integer on the computer
functions: for all x, y ∈ Natural_Number; TRUE, FALSE ∈ Boolean and where +, -, <, and == are the usual integer operations,

```
Nat_No Zero() ::= 0
Nat_No Add(x,y) ::= if ((x+y)<=INT_MAX) return x+y
                    else return INT_MAX
Nat_No Subtract(x,y) ::= if (x<y) return 0
                         else return x-y
Boolean Equal(x,y) ::= if (x==y) return TRUE
                       else return FALSE
Nat_No Successor(x) ::= if (x==INT_MAX) return x
                        else return x+1
Boolean Is_Zero(x) ::= if (x) return FALSE
                       else return TRUE
```

end Natural_Number

Data Abstraction

objects and **functions** are two main sections in the definition

function Zero is a **constructor**

function Add, Substractor, Successor are **transformers**

function Is_Zero and Equal are **reporters**

Performance Analysis

Performance evaluation

- performance analysis
  - machine independent
  - complexity theory
- performance measurement
  - machine dependent

space complexity

the amount of memory that it needs to run to completion

time complexity

the amount of computer time that it needs to run to completion

Space complexity

fixed space requirements

- don't depend on the number and size of the program's inputs and outputs
- eg) instruction space

variable space requirement

- the space needed by <u>structured variable</u> whose size depends on the particular instance, I, of the problem being solved

Space complexity

total space requirement S(P)

$$\boxed{S(P) = c + S_P(I)}$$

- c : constant representing the <u>fixed space requirements</u>

- $S_p(I)$: function of some *characteristics* of the instance I

Space complexity

- Ex 1.6

```
float abc(float a, float b, float c) {
  return a+b+b*c+(a+b-c)/(a+b)+4.00;
}
```

```
input - three simple variables
ouput - a simple variable

fixed space requirements only
   Sabc(I) = 0
```

Space complexity

- Ex 1.7 [Iterative version]

```
float sum(float list[], int n) {
  float tempsum = 0;
  int i;
  for(i = 0; i < n; i++)
    tempsum += list[i];
  return tempsum;
}
```

```
output - a simple variable
input - an array variable
```

Space complexity

Pascal pass arrays **by value**

entire array is copied into temporary storage before the function is executed

$S_{sum}(I) = S_{sum}(n) = n$

C pass arrays **by pointer**

passing <u>the address of the first element</u> of the array

$S_{sum}(n) = 0$

Space complexity

- Ex 1.8 [Recursive version]

```
float rsum(float list[],int n) {
  if(n) return rsum(list,n-1) + list[n-1];
  return 0;
}
```

handled recursively

compiler must save
- the parameters
- the local variables
- the return address

for each recursive call

Space complexity

space needed for one recursive call

number of bytes required for the two parameters and the return address

6 bytes needed on 80386

2 bytes for pointer list[]

2 bytes for integer n

2 bytes for return address

assume array has n=MAX_SIZE numbers,

total variable space S_{rsum}(MAX_SIZE)

S_{rsum}(MAX_SIZE) = 6 * MAX_SIZE

Time complexity

The time T(P),taken by a program P, is the sum of its compile time and its run(or execution) time

- We really concerned only with the program’s execution time, Tp

count the number of operations the program performs

- give a machine-independent estimation

Time complexity

- Ex 1.9 [Iterative summing of a list of numbers]

```
float sum(float list[], int n) {
  float tempsum=0;
  count++; /* for assignment */
  int i;
  for(i = 0; i < n; i++) {
    count++; /* for the for loop */
    tempsum += list[i];
    count++; /*for assignment*/
  }
  count++; /* last execution of for */
  count++; /* for return */
  return tempsum;
}
```

Time complexity

eliminate most of the program statements from Program 1.12 to obtain a simpler program Program 1.13 that **computes the same value for count**

```
float sum(float list[], int n) {
  float tempsum=0;
  int i;
  for(i = 0; i < n; i++)
   count+=2;
  count += 3;
  return tempsum;
}
```

Time complexity

- Ex 1.10 [Recursive summing of a list of numbers]

```
float rsum(float list[], int n) {
  count++;
  if(n) {
    count++;
    return rsum(list,n-1)+list[n-1];
  }
  count++;
  return 0;
}
```

Time complexity

```
when n=0 only the if conditional and
  the second return statement are
  executed (termination condition)

step count for n = 0 : 2
each step count for n > 0 : 2

total step count for function :
 2n + 2
- less step count than iterative
  version, but
- take more time than those of the
  iterative version
```

Time complexity

- Ex 1.11 [Matrix addition]

  determine the step count for a function that adds two-dimensional arrays(rows ´ cols)

```
void add(int a[][M_SIZE],int b[][M_SIZE],int
           c[][M_SIZE],int rows,int cols) {
  int i, j;
  for(i = 0; i < rows; i++)
    for(j = 0; j < cols; j++)
      c[i][j] = a[i][j] + b[i][j];
}
```

Matrix addition

Time complexity

apply step counts to add function

```
void add(int a[][M_SIZE],int b[][M_SIZE],
         int c[][M_SIZE],int rows,int cols) {
  int i,j;
  for(i = 0; i < rows; i++) {
    coutn++;
    for(j = 0; j < cols; j++) {
      count++;
      c[i][j] = a[i][j] + b[i][j];
      count++;
    }
    count++;
  }
  count++;
}
```

Matrix addition with count statements

Time complexity

combine counts

```
void add(int a[][M_SIZE],int b[][M_SIZE],int
           c[][M_SIZE],int rows,int cols) {
  int i, j;
  for(i = 0; i < rows; i++) {
    for(j = 0; j < cols; j++)
      count += 2;
    count += 2;
  }
  count++;
}
```

Time complexity

initially count = 0;

total step count on termination :

$2 \cdot rows \cdot cols + 2 \cdot rows + 1$;

Time complexity

- Tabular method

```
construct a step count table
 1) first determine the step count
  for each statement
 - steps/execution(s/e)
 2) next figure out the number of
  times that each statement is
  executed
 - frequency
 3) total steps for each statement
 - (total steps)=(s/e)*(frequency)
```

Time complexity

- Ex 1.12 [Iterative function to sum a list of numbers]:

Statement	s/e	Frequency	Total steps
float sum(float list[],int n) {	0	0	0
float tempsum=0;	1	1	1
int i;	0	0	0
for(i=0;i<n;i++)	1	n+1	n+1
tempsum+=list[i];	1	n	n
return tempsum;	1	1	1
}	0	0	0
total			2n+3

Step count table

Time complexity

- Ex 1.13 [Recursive function to sum a list of numbers]

Statement	s/e	Frequency	Total steps
float rsum(float list[],int n) {	0	0	0
if(n)	1	n+1	n+1
return rsum(list,n-1)+list[n-1];	1	n	n
return 0;	1	1	1
}	0	0	0
total			2n+2

Step count table for recursive summing function

Time complexity

- Ex 1.14 [Matrix addition]

Statement	s/e	Frequency	Total steps
void add(int a[][M_SIZE] ···) {	0	0	0
int i,j;	0	0	0
for(i=0;i<rows;i++)	1	rows+1	rows+1
for(j=0;j<cols;j++)	1	rows· (cols+1)	rows·cols+rows
c[i][j] = a[i][j] + b[i][j];	1	rows·cols	rows·cols
}	0	0	0
total			2·rows·cols+2·rows+1

Step count table for matrix addition

Time complexity

factors: time complexity

1)input size

- depends on size of input(n):

  T(n) = ?

2)input form

- depends on different possible input formats

  average case: A(n) = ?

  worst case: W(n) = ?

- concerns mostly for “worst case”
- worst case gives “upper bound”

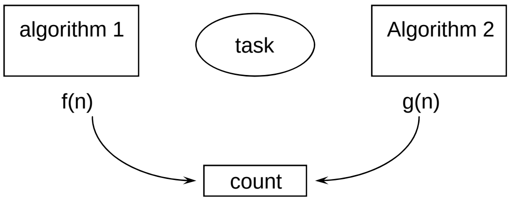
  exist different algorithm for the same task

  which one is faster ?

Asymptotic Notation

comparing time complexities

- exist different algorithms for the same task
- which one is faster ?



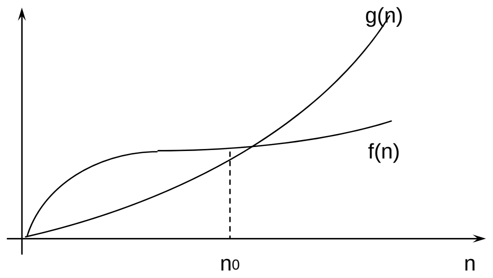
Asymptotic Notation

- Big "OH"

def) $f(n) = O(g(n))$

iff there exist positive constants c and n_0 such that

$f(n) \leq c \cdot g(n)$ for all n, $n \geq n_0$



Asymptotic Notation

- Ex) [$f(n) = 25 \cdot n$, $g(n) = 1/3 \cdot n^2$]

$25 \cdot n = O(n^2/3)$ if let $c = 1$

n	$f(n) = 25 \cdot n$	$g(n) = n^2 / 3$
1	25	1/3
2	50	4/3
.	.	.
.	.	.
.	.	.
75	1875	1875

$|25 \cdot n| \leq 1 \cdot |n^2/3|$ for all $n \geq 75$

Asymptotic Notation

$f(n) = O(g(n))$

- $g(n)$ is an upper bound on the value of $f(n)$ for all n, $n \geq n_0$
- but, doesn't say anything about how good this bound is

  $n = O(n^2)$, $n = O(n^{2.5})$

  $n = O(n^3)$, $n = O(2^n)$
- $g(n)$ should be as small a function of n as one can come up with for which $f(n) = O(g(n))$

$f(n) = O(g(n)) \not\Leftrightarrow O(g(n)) = f(n)$

Asymptotic Notation

theorem) if $f(n) = a_m n^m + \ldots + a_1 n + a_0$, then $f(n) = O(n^m)$

proof)

$f(n) \leq |a_k| \cdot n^k + |a_{k-1}| \cdot n^{k-1} + \ldots + |a_1| \cdot n + |a_0|$

$= \{|a_k| + |a_{k-1}|/n + \ldots + |a_1|/n^{k-1} + |a_0|/n^k\} \cdot n^k$

$\leq \{|a_k| + |a_{k-1}| + \ldots + |a_1| + |a_0|\} \cdot n^k$

$= c \cdot n^k$ ($c = |a_k| + |a_{k-1}| + \ldots + |a_1| + |a_0|$)

$= O(n^k)$

Asymptotic Notation

- Omega

def) $f(n) = \Omega(g(n))$

iff there exist positive constants c and n_0 such that

$f(n) \geq c \cdot g(n)$ for all n, $n \geq n_0$

- g(n) is a lower bound on the value of f(n) for all n, $n \geq n_0$
- should be as large a function of n as possible

theorem) if $f(n) = a_m n^m + ... + a_1 n + a_0$ and am > 0, then $f(n) = \Omega(n^m)$

Asymptotic Notation

- Theta

def) $f(n) = \Theta(g(n))$

iff there exist positive constants c_1, c_2, and n_0 such that

$c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$ for all n, $n \geq n_0$

- more precise than both the “big oh” and omega notations
- g(n) is both an upper and lower bound on f(n)

Asymptotic Notation

- Ex 1.18 complexity of matrix addition]

Statement	Asymptotic complexity
void add(int a[][M_SIZE] ···) {	0
int i, j;	0
for(i = 0; i < rows; i++)	Θ(rows)
for(j = 0; j < cols; j++)	Θ(rows·cols)
c[i][j] = a[i][j] + b[i][j];	Θ(rows·cols)
}	0
Total	Θ(rows·cols)

`time complexity of matrix addition`

Practical Complexities

```
class of time complexities
   O(1): constant
   O(log2n): logarithmic
   O(n): linear
   O(n·log2n): log-linear
   O(n^2): quadratic
   O(n^3): cubic
<------------------------------------->
   O(2^n): exponential
   O(n!): factorial
```

polynomial time

exponential time

Practical Complexities

polynomial time
- tractable problem

exponential time
- intractable (hard) problem

eg)
- sequential search
- binary search
- insertion sort
- heap sort
- satisfiablity problem
- testing serializable scheduling

Practical Complexities

		instance characteristic n					
time	name	1	2	4	8	16	32
1	constant	1	1	1	1	1	1
log n	logarithmic	0	1	2	3	4	5
n	linear	1	2	4	8	16	32
n log n	log linear	0	2	8	24	64	160
n^2	quadratic	1	4	16	64	256	1024
n^3	cubic	1	8	64	512	4096	32768
2n	exponential	2	4	16	256	655536	4294967296
n!	factorial	1	2	24	40326	20922789888000	26313×10^{33}

function value

Practical Complexities

If a program needs 2^n steps for execution

n=40 --- number of steps = $1.1*10^{12}$

in computer systems 1 billion steps/sec --- 18.3 min

n=50 --- 13 days

n=60 --- 310.56 years

n=100 --- $4*10^{13}$ years

If a program needs n^{10} steps for execution

n=10 --- 10 sec

n=100 --- 3171 years