nr 6 p 100 a, b, d, f

a) $f'(a) = \lim\limits_{x \to a} \frac{x^4 - a^4}{x - a} = \frac{0}{0}$

$= \lim\limits_{x \to a} \frac{(x^2-a^2)(x^2+a^2)}{x-a} = \lim\limits_{x \to a} \frac{\cancel{(x-a)}(x+a)(x^2+a^2)}{\cancel{x-a}}$

$= \lim\limits_{x \to a} (x+a)(x^2+a^2) = (a+a)(a^2+a^2)$

$= 2a \cdot 2a^2 = 4a^3$

b) $f'(a) = \lim\limits_{x \to a} \frac{\frac{1}{x^3} - \frac{1}{a^3}}{x-a} = \lim\limits_{x \to a} \frac{\frac{a^3-x^3}{x^3 \cdot a^3}}{x-a} =$

$= \lim\limits_{x \to a} \frac{a^3-x^3}{x^3 \cdot a^3 (x-a)} = \frac{0}{0} = \lim\limits_{x \to a} \frac{\underset{\cancel{-(x-a)}}{(a-x)}(a^2+ax+x^2)}{x^3 \cdot a^3 \cancel{(x-a)}}$

$= \lim\limits_{x \to a} \frac{-(a^2+ax+x^2)}{x^3 \cdot a^3} = \frac{-(a^2+a^2+a^2)}{a^3 \cdot a^3} = \frac{-3a^2}{a^6}$

$= -\frac{3}{a^4}$

d) $f'(a) = \lim\limits_{x \to a} \frac{\sqrt[4]{x} - \sqrt[4]{a}}{x-a} = \frac{0}{0}$

$= \lim\limits_{x \to a} \frac{(\sqrt{x} - \sqrt{a})(\sqrt{x} + \sqrt{a})}{(x-a)(\sqrt[4]{x} + \sqrt[4]{a})(\sqrt{x} + \sqrt{a})}$

$= \lim\limits_{x \to a} \frac{\cancel{x-a}}{\cancel{(x-a)}(\sqrt[4]{x} + \sqrt[4]{a})(\sqrt{x} + \sqrt{a})}$

$= \frac{1}{2\sqrt[4]{a} \cdot \underset{\sqrt[4]{a^2}}{\underset{\shortparallel}{2\sqrt{a}}}} = \frac{1}{4\sqrt[4]{a^3}}$

d) $f(x) = x^2+2x$

$$f'(a) = \lim_{x \to a} \frac{x^2+2x-a^2-2a}{x-a} = \frac{0}{0}$$

$$= \lim_{x \to a} \frac{x^2-a^2+2(x-a)}{x-a}$$

$$= \lim_{x \to a} \frac{\cancel{(x-a)}(x+a+2)}{\cancel{x-a}} = 2a+2$$

e) $f(x) = -x^3+x$

$$f'(a) = \lim_{x \to a} \frac{-x^3+x+a^3-a}{x-a} = \frac{0}{0}$$

$A^3-B^3 = (A-B)(A^2+AB+B^2)$ of Horner

$$= \lim_{x \to a} \frac{(a-x)(a^2+ax+x^2)-(a-x)}{x-a}$$

$$= \lim_{x \to a} \frac{\overset{-(x-a)}{(a-x)}(a^2+ax+x^2-1)}{\cancel{x-a}}$$

$$= -a^2-a^2-a^2+1 = -3a^2+1$$

f) $f(x) = \dfrac{1}{\sqrt[3]{x^2}}$

$$f'(a) = \lim_{x \to a} \frac{\frac{1}{\sqrt[3]{x^2}} - \frac{1}{\sqrt[3]{a^2}}}{x-a} = \frac{0}{0}$$

$$= \lim_{x \to a} \frac{\left(\sqrt[3]{a^2} - \sqrt[3]{x^2}\right)}{\sqrt[3]{x^2} \cdot \sqrt[3]{a^2}\,(x-a)} \cdot \frac{\left(\sqrt[3]{a^4} + \sqrt[3]{a^2} \cdot \sqrt[3]{x^2} + \sqrt[3]{x^4}\right)}{\left(\sqrt[3]{a^4} + \sqrt[3]{a^2} \cdot \sqrt[3]{x^2} + \sqrt[3]{x^4}\right)}$$

$$= \lim_{x \to a} \frac{\cancel{x-a}}{\sqrt[3]{x^2}\,\sqrt[3]{a^2}\,\cancel{(x-a)}\left(\sqrt[3]{a^4} + \sqrt[3]{a^2} \cdot \sqrt[3]{x^2} + \sqrt[3]{x^4}\right)}$$

$$= \frac{1}{\sqrt[3]{a^2} \cdot \sqrt[3]{a^2}\; 3 \cdot \sqrt[3]{a^4}} = \frac{1}{3 \sqrt[3]{a^8}}$$

$$= \frac{1}{3a^2 \sqrt[3]{a^2}}$$