① $z = -\sqrt{6} - i\sqrt{2}$

a) $r = \sqrt{6+2} = \sqrt{8} = 2\sqrt{2}$

$\tan\alpha = \frac{\sqrt{2}}{\sqrt{6}} = \frac{\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}\sqrt{3}} = \frac{\sqrt{3}}{3} \Rightarrow \alpha = 30^\circ + k180^\circ$

$\alpha \in \text{III} \Rightarrow \alpha = -150^\circ$

$z = 2\sqrt{2}\left(\cos(-150^\circ) + i\sin(-150^\circ)\right)$

b) $z^7 = (2\sqrt{2})^7 \cdot (\cos(7\cdot(-150^\circ)) + i\sin(7\cdot(-150^\circ)))$

$= 2^7 \cdot 2^3\sqrt{2}\,(\cos(-1050^\circ) + i\sin(-1050^\circ))$

$\overset{G\,(+1080^\circ)}{=} 2^{10}\sqrt{2}\,(\cos 30^\circ + i\sin 30^\circ)$

$= 1024\sqrt{2}\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right)$

$= 512\sqrt{6} + 512\sqrt{2}\,i$

② $10\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right) \overset{S}{=} 10\left(-\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$= 10\cdot\left(-\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = -5\sqrt{3} + 5i$

③ de Moivre: $(\cos\alpha + i\sin\alpha)^n = \cos n\alpha + i\sin n\alpha$

$n=2$: $(\cos\alpha + i\sin\alpha)^2 = \cos 2\alpha + i\sin 2\alpha$

$\cos^2\alpha + 2\cos\alpha\cdot i\sin\alpha + \overset{-1}{i^2}\sin^2\alpha = \cos 2\alpha + i\sin 2\alpha$

$\cos^2\alpha - \sin^2\alpha + i\cdot 2\sin\alpha\cos\alpha = \cos 2\alpha + i\sin 2\alpha$

$\Rightarrow \begin{cases} \cos 2\alpha = \cos^2\alpha - \sin^2\alpha \\ \sin 2\alpha = 2\sin\alpha\cos\alpha \end{cases}$

④ a) $z_1 \cdot z_2 = 12\left(\cos(55^\circ + (-15^\circ)) + i\sin(55^\circ + (-15^\circ))\right)$

$= 12(\cos 40^\circ + i\sin 40^\circ)$

$= 9{,}19 + 7{,}71\,i$

b) $\frac{z_1}{z_2} = 3\left(\cos(55^\circ - (-15^\circ)) + i\sin(55^\circ - (-15^\circ))\right)$

$= 3(\cos 70^\circ + i\sin 70^\circ)$

$= 1{,}03 + 2{,}82\,i$

(5) $$\frac{(\cos 5^\circ + i \sin 5^\circ)^9}{(i\sqrt{2})^5} = \frac{\cos 45^\circ + i \sin 45^\circ}{(i^2)^2 \cdot i \; (\sqrt{2})^2(\sqrt{2})^2\sqrt{2}}$$

$$= \frac{\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) i}{4\sqrt{2}\, i \cdot i}$$

$$= \frac{\frac{\sqrt{2}}{2} i - \frac{\sqrt{2}}{2}}{-4\sqrt{2}}$$

$$= -\frac{\cancel{\sqrt{2}}}{2} \cdot \frac{1}{4\cancel{\sqrt{2}}}\, i + \frac{\cancel{\sqrt{2}}}{2} \cdot \frac{1}{4\cancel{\sqrt{2}}}$$

$$= \frac{1}{8} - \frac{1}{8} i$$