nr 4 p 99

a) $f_1(x) = -3x+5 \qquad P(1,2)$

$f_1'(a) = Df_1(a) = \lim\limits_{x \to a} \dfrac{f_1(x) - f_1(a)}{x-a}$

$P(1,2) \Rightarrow a = 1$ en $f_1(a) = 2$

$f_1'(1) = Df_1(1) = \lim\limits_{x \to 1} \dfrac{-3x+5-2}{x-1}$

$= \lim\limits_{x \to 1} \dfrac{-3x+3}{x-1} = \dfrac{-3+3}{1-1} = \dfrac{0}{0}$

$= \lim\limits_{x \to 1} \dfrac{-3\cancel{(x-1)}}{\cancel{x-1}} = -3$

Antw: $f_1'(1) = -3$

b) $f_2(x) = x^2 - 5x + 6 \qquad P(-2,20) \Rightarrow a = -2$ en $f_2(-2) = 20$

$f_2'(a) = Df_2(a) = \lim\limits_{x \to a} \dfrac{f_2(x) - f_2(a)}{x-a}$

$f_2'(-2) = Df_2(-2) = \lim\limits_{x \to -2} \dfrac{x^2-5x+6-20}{x+2}$

$= \lim\limits_{x \to -2} \dfrac{x^2-5x-14}{x+2} = \dfrac{0}{0}$

$$\begin{array}{c|cc||c} & 1 & -5 & -14 \\ -2 & & -2 & 14 \\ \hline & 1 & -7 & 0 \end{array}$$

$= \lim\limits_{x \to -2} \dfrac{\cancel{(x+2)}(x-7)}{\cancel{x+2}} = -9$

Antw $f_2'(-2) = -9$

c) $f_3(x) = -x^3 + x^2 - x \quad \text{in } P(-1, \dots)$

$a = -1 \Rightarrow f_3(-1) = -(-1)^3 + (-1)^2 - (-1)$

$= 1 + 1 + 1 = 3$

$$f_3'(a) = Df_3(a) = \lim_{x \to a} \frac{f_3(x) - f_3(a)}{x - a}$$

$$f_3'(-1) = \lim_{x \to -1} \frac{-x^3 + x^2 - x - 3}{x + 1} = \frac{1+1+1-3}{-1+1} = \frac{0}{0}$$

$$= \lim_{x \to -1} \frac{\cancel{(x+1)}(-x^2 + 2x - 3)}{\cancel{(x+1)}}$$

$$\begin{array}{r|rrr||r} & -1 & 1 & -1 & -3 \\ -1 & & 1 & -2 & 3 \\ \hline & -1 & 2 & -3 & 0 \end{array}$$

$$= \lim_{x \to -1} (-x^2 + 2x - 3) = -1 - 2 - 3 = -6$$

Antw: $f_3'(-1) = -6$

d) $f_4(x) = -x^2 + 2x \quad \text{in } P(\dots, 0)$

$-x^2 + 2x = 0 \Leftrightarrow -x(x-2) = 0 \Leftrightarrow x = 0 \lor x = 2$

in $P(0,0)$

$$f_4'(0) = \lim_{x \to 0} \frac{-x^2 + 2x - 0}{x - 0} = \lim_{x \to 0} \frac{-x^2 + 2x}{x} = \frac{0}{0}$$

$$= \lim_{x \to 0} \frac{-\cancel{x}(x-2)}{\cancel{x}} = 2$$

in $P(2,0)$

$$f_4'(2) = \lim_{x \to 2} \frac{-x^2 + 2x - 0}{x - 2} = \frac{-4+4}{2-2} = \frac{0}{0}$$

$$= \lim_{x \to 2} \frac{-x\cancel{(x-2)}}{\cancel{x-2}} = -2$$