HERHALINGSOEFENINGEN 5ASO 5u

① $f(x) = 9x^3 - 27x^2 + 20x - 4 = 0$

$$\begin{array}{c|ccc||c} & 9 & -27 & 20 & -4 \\ 2 & & 18 & -18 & 4 \\ \hline & 9 & -9 & 2 & 0 \end{array}$$

$x = 2 \quad \vee \quad 9x^2 - 9x + 2 = 0$

$D = 9 \qquad x_1 = \frac{1}{3} \qquad x_2 = \frac{2}{3}$

nulw van f: $\quad 2, \frac{1}{3}$ en $\frac{2}{3}$

② $9x^2 - 2x^3 < 12x - 4$

$-2x^3 + 9x^2 - 12x + 4 < 0$

$$\begin{array}{c|ccc||c} & -2 & 9 & -12 & 4 \\ 2 & & -4 & 10 & -4 \\ \hline & -2 & 5 & -2 & 0 \end{array}$$

$-2x^2 + 5x - 2 = 0$

$D = 9$

$x_1 = 2 \qquad x_2 = \frac{1}{2}$

multipl. is 2

x		$1/2$		2	
$-2x^3+9x^2-12x+4$	$+$	0	$-$	0	$-$

$V = \left]\frac{1}{2}, 2\right[\cup \left]2, +\infty\right[$

③ $\dfrac{2x}{x-2} - \dfrac{2}{x+2} + \dfrac{x-6}{x^2-4} = 0 \qquad x^2 - 4 = (x-2)(x+2)$

$$\frac{2x(x+2) - 2(x-2) + x - 6}{(x-2)(x+2)} = 0$$

$$\frac{2x^2 + 4x - 2x + 4 + x - 6}{(x-2)(x+2)} = 0$$

$$\frac{2x^2 + 3x - 2}{(x-2)(x+2)} = 0$$

T: $\quad D = 25 \quad x_1 = \cancel{-2} \quad x_2 = \frac{1}{2}$

N: $\quad x = 2 \;, \quad x = -2$

$V = \left\{\frac{1}{2}\right\}$

④ $T: \; x+3=0 \Leftrightarrow x=-3$

$N: \; x^2+x-6=0 \qquad D = 1-4\cdot1\cdot(-6) = 25$

$$x_1 = \frac{-1-5}{2} = -3$$

$$x_2 = \frac{-1+5}{2} = 2$$

a) $\operatorname{dom} f = \mathbb{R}\setminus\{-3, 2\}$

b)

x		-3		2	
$f(x)$	$-$	/	$-$	/	$+$

$m = 2$

c) perforatie voor $x=-3$

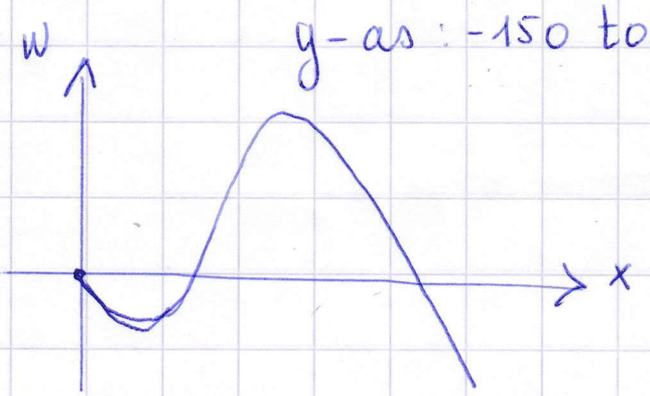
VA: $x = 2$

$x+3 \;\big|\; x^2+x-6$ → 0 $\Rightarrow$ HA: $y=0$

⑤ a) $x < 0$

b) venster x-as: 0 tot 15

y-as: -150 tot 250



c) $-\frac{3}{2}x^3 + 24x^2 - 72x > 0$

$-\frac{3}{2}x^3 + 24x^2 - 72x = 0$

$\Leftrightarrow -3x^3 + 48x^2 - 144x = 0$

$\Leftrightarrow x(-3x^2 + 48x - 144) = 0$

$\Leftrightarrow x = 0 \;\vee\; -3x^2 + 48x - 144 = 0$

$D = 48^2 - 4(-3)(-144) = 576$

$$x_1 = \frac{-48-24}{-6} = 12 \qquad x_2 = \frac{-48+24}{-6} = 4$$

x		0		4		12	
$W(x)$	+	0	−	0	+	0	−

Antw: winst bij productie van 4 tot 12 stuks
verlies bij " van 0 tot 4 "
en meer dan 12 stuks

d) $y_2 = 108$ calc, intersect
$x_1 = 6$ $x_2 = 11,08...$
$\Rightarrow$ bij een productie van 6 tot 11 stuks

e) calc, max $x = 8,861$ $y = 202,81$
$\Rightarrow$ De winst is max bij de productie van 9 stuks

(6) $180^\circ = \pi$

$1^\circ = \frac{\pi}{180}$

$\underbrace{316^\circ 15' 21''}_{316,255833..} \cdot \frac{\pi}{180} = 5,52 \text{ rad}$

Extra 215°

$1^\circ = \frac{\pi}{180}$

$215^\circ = \frac{215\pi}{180} = \frac{43\pi}{36}$

(7) Geg $\tan\alpha = -3 \quad \alpha \in II$ Gevr $x = 2\sin\alpha - 3\cos\alpha + 1$

Opl $1 + \tan^2\alpha = \frac{1}{\cos^2\alpha}$

$1 + 9 = \frac{1}{\cos^2\alpha} \Leftrightarrow \cos^2\alpha = \frac{1}{10} \Leftrightarrow \cos\alpha = \pm\frac{1}{\sqrt{10}}$

$\overset{\alpha \in II}{\Rightarrow} \cos\alpha = -\frac{\sqrt{10}}{10}$

$\sin^2\alpha = 1 - \cos^2\alpha$

$= 1 - \left(-\frac{\sqrt{10}}{10}\right)^2 = 1 - \frac{10}{100} = 1 - \frac{1}{10} = \frac{9}{10}$

$\Rightarrow \sin\alpha = \pm\frac{3}{\sqrt{10}} = \pm\frac{3\sqrt{10}}{10}$

$\overset{\alpha \in II}{\Rightarrow} \sin\alpha = \frac{3\sqrt{10}}{10}$

$x = 2 \cdot \frac{3\sqrt{10}}{10} - 3 \cdot \frac{-\sqrt{10}}{10} + 1$

$= \frac{6\sqrt{10}}{10} + \frac{3\sqrt{10}}{10} + 1$

$= \frac{9\sqrt{10}}{10} + 1$

(8) a) $LL = \sin^2\alpha\,(1-\sin^2\beta) - (1-\sin^2\alpha)\sin^2\beta$

$= \sin^2\alpha - \cancel{\sin^2\alpha\,\sin^2\beta} - \sin^2\beta + \cancel{\sin^2\alpha\,\sin^2\beta}$

$= \sin^2\alpha - \sin^2\beta = RL$

b) $LL = \left(\cos\left(\frac{\pi}{4}+\alpha\right)\right)^2$

$= \left(\cos\frac{\pi}{4}\cdot\cos\alpha - \sin\frac{\pi}{4}\cdot\sin\alpha\right)^2$

$= \left(\frac{\sqrt{2}}{2}\cos\alpha - \frac{\sqrt{2}}{2}\sin\alpha\right)^2$

$= \left(\frac{\sqrt{2}}{2}\right)^2\left(\cos\alpha - \sin\alpha\right)^2$

$= \frac{2}{4}\cdot\left(\underline{\cos^2\alpha} - 2\sin\alpha\cos\alpha + \underline{\sin^2\alpha}\right)$

$= \frac{1}{2}\cdot\left(1 - 2\sin\alpha\cdot\cos\alpha\right)$

c) $\csc 2x + \cot 2x = \cot x$

$\frac{1}{\sin 2x} + \frac{\cos 2x}{\sin 2x} = \frac{1+\cos 2x}{\sin 2x}$

$= \frac{\cancel{1} + 2\cos^2 x \cancel{-1}}{2\sin x\cos x} = \frac{\cancel{2}\cos^{\cancel{2}} x}{\cancel{2}\sin x\,\cancel{\cos x}} = \cot x$

d) $\frac{\sin x + \sin 3x + 2\sin 2x}{\cos x + \cos 3x + 2\cos 2x}$

Simpson $= \frac{\cancel{2}\sin 2x\cdot\cos(-x) + \cancel{2}\sin 2x}{\cancel{2}\cos 2x\cdot\cos(-x) + \cancel{2}\cos 2x}$

$\overset{T}{=} \frac{\sin 2x\,\cancel{(\cos x + 1)}}{\cos 2x\,\cancel{(\cos x+1)}} = \tan 2x$

⑨ $\cos\frac{\pi}{10} + \cos\frac{5\pi}{10} + \cos\frac{9\pi}{10} + \cos\frac{13\pi}{10} + \cos\frac{17\pi}{10}$

$\downarrow S \quad \downarrow A \quad \downarrow G$

$= \cancel{\cos\frac{\pi}{10}} + \cos\frac{\pi}{2} - \cancel{\cos\frac{\pi}{10}} - \cos\frac{3\pi}{10} + \cos\left(-\frac{3\pi}{10}\right)$

$\overset{T}{=} \cos\frac{\pi}{2} - \cancel{\cos\frac{3\pi}{10}} + \cancel{\cos\frac{3\pi}{10}} = 0$

Deel 2: Seminarie wiskunde

a) $z_1 + z_2 - z_3 = 3-2i-1-i-4-5i = -2-8i$

b) $z_1 \cdot z_2 = (3-2i)(-1-i) = -3-3i+2i+2i^2$

$= -3-i-2 = -5-i$

c) $\overline{z_2} = \overline{(-1-i)} = -1+i$

d) $z_3^2 = (4+5i)^2 = 16+40i+25\underbrace{i^2}_{=-1} = -9+40i$

e) $z_1^2 - 2z_2z_3 + i(z_1+3z_2)$

$= (3-2i)^2 - 2(-1-i)(4+5i) + i(3-2i+3(-1-i))$

$= 9-12i+4\underbrace{i^2}_{=-1} + (2+2i)(4+5i) + i(\cancel{3}-2i-\cancel{3}-3i)$

$= 5-12i+8+10i+8i+\underbrace{10i^2}_{-10}\underbrace{-5i^2}_{+5} = 8+6i$

f) $(z_1-\overline{z_2})(\overline{z_3}-iz_1)$

$= (3-2i-(-1+i))(4-5i-i(3-2i))$

$= (3-2i+1-i)(4-5i-3i+2i^2)$

$= (4-3i)(2-8i)$

$= 8-32i-6i+24i^2$

$= -16-38i$

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e) $\frac{3-2i}{3+2i} = \frac{(3-2i)^2}{9-4i^2} = \frac{9-12i+4i^2}{9+4} = \frac{5-12i}{13}$

j) $\frac{2i+5}{3-i} + \frac{4+i}{1+3i} = \frac{(2i+5)(1+3i)+(4+i)(3-i)}{(3-i)(1+3i)}$

$= \frac{2i + 6\overset{-1}{i^2} + 5 + 15i + 12 - 4i + 3i - i^2 = -1}{3 + 9i - i - 3i^2 = -1}$

$= \frac{12+16i}{6+8i} = \frac{2\cancel{(6+8i)}}{\cancel{6+8i}} = 2$

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j) $(x+yi)^2 = 1 + 2\sqrt{6}\,i \qquad x, y \in \mathbb{R}$

$x^2 + 2xyi + y^2i^2 = 1 + 2\sqrt{6}\,i$

$x^2 - y^2 + 2xyi = 1 + 2\sqrt{6}\,i$

$\begin{cases} x^2 - y^2 = 1 \\ xy = \sqrt{6} \end{cases} \Leftrightarrow \begin{cases} x^2 - y^2 = 1 \\ y = \frac{\sqrt{6}}{x} \end{cases} \Leftrightarrow \begin{cases} x^2 - \frac{6}{x^2} = 1 \\ y = \frac{\sqrt{6}}{x} \end{cases} \quad (1)$

(1) $\frac{x^4 - 6 - x^2}{x^2} = 0 \qquad N: x = 0$

$T: \quad x^4 - x^2 - 6 = 0 \qquad t = x^2$

$t^2 - t - 6 = 0 \qquad D = 25$

$t_1 = -2 \quad \vee \quad t_1 = 3$

$\cancel{x^2 = -2} \quad \vee \quad x^2 = 3$

$x \notin \mathbb{R}$

dus $x = \sqrt{3} \Rightarrow y = \frac{\sqrt{6}}{\sqrt{3}} = \frac{\sqrt{2}\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}} = \sqrt{2}$

$x = -\sqrt{3} \Rightarrow y = \frac{\sqrt{6}}{-\sqrt{3}} = \frac{\sqrt{2}\cdot\cancel{\sqrt{3}}}{-\cancel{\sqrt{3}}} = -\sqrt{2}$

De vkw zijn $\sqrt{3} + \sqrt{2}\,i$ en $-\sqrt{3} - \sqrt{2}\,i$