nr 2 p 70

a) $\lim\limits_{x \to 3} \dfrac{2x^2-3x-2}{x^2-2x-3} = \dfrac{2.9-3.3-2}{9-6-3} = \dfrac{7}{0}$

teken teller is positief dicht bij $x=3$ want voor $x=3 \Rightarrow T=7$

teken N afh. van tekentabel

N: $x^2-2x-3=0$ $\quad D=16 \quad x=-1 \quad x=3$

x		-1		3	
x^2-2x-3	$+$	0	$-$	0	$+$

$\Rightarrow \lim\limits_{\substack{x \to 3 \\ <}} f(x) = -\infty$

$\lim\limits_{\substack{x \to 3 \\ >}} f(x) = +\infty$

b) $\lim\limits_{x \to 2} \dfrac{2x^2-3x-2}{x^2-x-2} = \dfrac{0}{0}$

T:

	2	−3	−2
2		4	2
	2	1	0

N:

	1	−1	−2
2		2	2
	1	1	0

$\lim\limits_{x \to 2} \dfrac{\cancel{(x-2)}(2x+1)}{\cancel{(x-2)}(x+1)} = \dfrac{5}{3}$

c) $\lim\limits_{x \to 1} \dfrac{x^2-2x+1}{2x^2+x-3} = \dfrac{0}{0}$

N

	2	1	−3
1		2	3
	2	3	0

$= \lim\limits_{x \to 1} \dfrac{(x-1)^{\cancel{2}}}{\cancel{(x-1)}(2x+3)} = \dfrac{0}{5} = 0$

d) $\lim\limits_{x \to 1} \dfrac{2x^2+x-3}{x^2-2x+1} = \dfrac{0}{0} \overset{zie\ c}{=} \lim\limits_{x \to 1} \dfrac{\cancel{(x-1)}(2x+3)}{(x-1)^{\cancel{2}}} = \dfrac{5}{0}$

x		1	
$x-1$	$-$	0	$+$

$\Rightarrow \lim\limits_{\substack{x \to 1 \\ <}} \dfrac{2x+3^{\,+}}{x-1_{\,-}} = -\infty$

$\lim\limits_{\substack{x \to 1 \\ >}} \dfrac{2x+3^{\,+}}{x-1_{\,+}} = +\infty$

e) $\lim\limits_{x \to 0} \dfrac{x^4+x^3+2x^2}{x^5+x^4+x^3} = \dfrac{0}{0}$

$= \lim\limits_{x \to 0} \dfrac{\cancel{x^2}(x^2+x+2)}{x^{\cancel{3}}(x^2+x+1)} = \dfrac{2}{0}$

$N: \ x(x^2+x+1)=0 \iff x=0 \ \vee \ x^2+x+1=0$

$x^2+x+1=0 \quad D=-3 \quad | \iff x=0$

x		0	
$x(x^2+x+1)$	$-$	0	$+$

$$\lim_{\substack{x\to 0\\ <}} \frac{x^4+x^3+2x^2}{x^5+x^4+x^3} = \lim_{\substack{x\to 0\\ <}} \frac{x^2+x+2}{x(x^2+x+1)} = -\infty$$

$$\lim_{\substack{x\to 0\\ >}} \frac{x^4+x^3+2x^2}{x^5+x^4+x^3} = \lim_{\substack{x\to 0\\ >}} \frac{x^2+x+2}{x(x^2+x+1)} = +\infty$$

f) $$\lim_{x\to -3} \frac{3x^2-8x-3}{3x^2+8x-3} = \frac{27+24-3}{27-24-3} = \frac{48}{0}$$

$N: \ 3x^2+8x-3=0 \quad D=100 \quad x=-3 \quad x=\frac{1}{3}$

x		-3		$1/3$	
N	$+$	0	$-$	0	$+$

$\Rightarrow \lim_{\substack{x\to -3\\ <}} f(x) = +\infty$

$\lim_{\substack{x\to -3\\ >}} f(x) = -\infty$

g) $$\lim_{x\to -1} \frac{-x^2-2x}{x^2+2x+1} = \frac{-1+2}{1-2+1} = \frac{1}{0}$$

$N: \ (x+1)^2=0 \iff x=-1$

x		-1	
N	$+$	0	$+$

$\Rightarrow \lim_{x\to -1} \frac{-x^2-2x}{x^2+2x+1} = +\infty$

h) $$\lim_{x\to -1} \frac{x^3+4x^2+5x+2}{2x^2+3x+1} = \frac{-1+4-5+2}{2-3+1} = \frac{0}{0}$$

T:

	1	4	5	2
-1		-1	-3	-2
	1	3	2	0

N:

	2	3	1
-1		-2	-1
	2	1	0

$$= \lim_{x\to -1} \frac{(x^2+3x+2)\cancel{(x+1)}}{(2x+1)\cancel{(x+1)}} = \lim_{x\to -1} \frac{x^2+3x+2}{2x+1}$$

$$= \frac{1-3+2}{-2+1} = \frac{0}{-1} = 0$$

i) $\lim\limits_{x \to -2} \dfrac{-7}{2x+4} = \dfrac{-7}{0}$

$x = -2$

$$\begin{array}{c|ccc} x & & -2 & \\ \hline N & - & 0 & + \end{array} \Rightarrow \begin{cases} \lim\limits_{\substack{x \to -2 \\ <}} \dfrac{-7}{2x+4} = +\infty \\ \lim\limits_{\substack{x \to -2 \\ >}} \dfrac{-7}{2x+4} = -\infty \end{cases}$$

j) $\lim\limits_{x \to 0} \dfrac{2}{x^3 - x^2} = \dfrac{2}{0}$

$N: \; x^3 - x^2 = 0 \Leftrightarrow x^2(x-1) = 0 \Leftrightarrow \underbrace{x = 0}_{m=2} \vee x = 1$

$$\begin{array}{c|ccccc} x & & 0 & & 1 & \\ \hline x^3 - x^2 & - & 0 & - & 0 & + \end{array} \Rightarrow \lim\limits_{x \to 0} \frac{2}{x^3 - x^2} = -\infty$$

k) $\lim\limits_{x \to 1} \dfrac{x^3 - x^2 - x + 1}{x^3 + x^2 - x - 1} = \dfrac{0}{0}$

$$T: \begin{array}{c|ccc|c} & 1 & -1 & -1 & 1 \\ 1 & & 1 & 0 & -1 \\ \hline & 1 & 0 & -1 & 0 \end{array} \qquad N: \begin{array}{c|ccc|c} & 1 & 1 & -1 & -1 \\ 1 & & 1 & 2 & 1 \\ \hline & 1 & 2 & 1 & 0 \end{array}$$

$= \lim\limits_{x \to 1} \dfrac{(x^2-1)\cancel{(x-1)}}{(x^2+2x+1)\cancel{(x-1)}} = \lim\limits_{x \to 1} \dfrac{x^2-1}{x^2+2x+1} = \dfrac{1-1}{1+2+1} = \dfrac{0}{4} = 0$

l) niet

m) $\lim\limits_{x \to 4} \dfrac{8 - 2x}{x^2 - 16} = \dfrac{0}{0}$

$= \lim\limits_{x \to 4} \dfrac{2(4-x)}{(x-4)(x+4)} = \lim\limits_{x \to 4} \dfrac{-2\cancel{(x-4)}}{\cancel{(x-4)}(x+4)} = \lim\limits_{x \to 4} \dfrac{-2}{x+4} = \dfrac{-1}{4}$

s) $\lim\limits_{x \to \underline{+}\infty} \dfrac{3x^2+5x-1}{x-3} = \lim\limits_{x \to \underline{+}\infty} \dfrac{3x^2}{x} = \lim\limits_{x \to \underline{+}\infty} 3x = \underline{+}\infty$

t) $\lim\limits_{x \to \underline{+}\infty} \dfrac{2x^2-x+16}{x^3+6x} = \lim\limits_{x \to \underline{+}\infty} \dfrac{2x^2}{x^3} = \lim\limits_{x \to \underline{+}\infty} \dfrac{2}{x} = 0$

u) $\lim\limits_{x \to \underline{+}\infty} \dfrac{7x^2-8x+5}{x^2-x^3} = \lim\limits_{x \to \underline{+}\infty} \dfrac{7x^2}{-x^3} = \lim\limits_{x \to \underline{+}\infty} \dfrac{-7}{x} = 0$

w) $\lim\limits_{x \to \underline{+}\infty} \dfrac{(2x-5)^5}{(3x+1)^6} = \lim\limits_{x \to \underline{+}\infty} \dfrac{32\cancel{x^5}}{729\cancel{x^6}} = \lim\limits_{x \to \underline{+}\infty} \dfrac{32}{729x}$

$= \dfrac{32}{729(\underline{+}\infty)} = 0$

y) $\lim\limits_{x \to \underline{+}\infty} \left(2x-1+\dfrac{3}{x+1}\right) = \lim\limits_{x \to \underline{+}\infty} \dfrac{2x^2+2x-x-1+3}{x+1}$

$= \lim\limits_{x \to \underline{+}\infty} \dfrac{2x^{\cancel{2}}}{\cancel{x}} = \lim\limits_{x \to \underline{+}\infty} 2x = 2(\underline{+}\infty) = \underline{+}\infty$