**COMP3211 Fundamentals of AI** **Name:** _____
**Fall 2019 Final**
**10/12/2018** **Stu ID:** _____
**Time Limit: 1300 Dec 10 – 1259 Dec 11 HKT (GMT+8)**

**Instructions**:

1. This exam contains 7 pages (including this cover page) and 12 questions.
2. This is an open book exam.
3. Observe the honor code. Write the exam on your own. Do not discuss with any other people.
4. Upload your answers as a pdf file (YourStudentID.pdf) to canvas by 1259 on Dec 11 HKT. You can use handwriting or any document editor but must scan or convert your answers to a PDF file.

Grade Table (for teacher use only)

| Question | Points | Score |
|---|---|---|
| State Machines | 6 | |
| Game Theory | 6 | |
| Auction | 6 | |
| CNF | 8 | |
| Representation in PL | 10 | |
| Representation in FOL | 15 | |
| Uncertainty | 12 | |
| MDP | 10 | |
| Admissibility | 4 | |
| Perceptron Learning and GSCA Rule Learning | 12 | |
| Perceptrons/Linearly Separable Functions | 6 | |
| Machine Learning/General Discussion | 5 | |
| Total: | 100 | |

**Question 1: State Machines . . . . . 6 points**

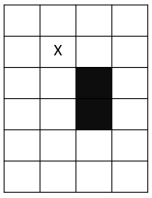
Consider again the boundary-following robot but with only 4 sensors: $s_1, s_2, s_3, s_4$, which correspond to north, east, south, and west, respectively. The robot again has four actions: *south, north, west, east*. We also define the following features:

- $w_i = s_i$, where $i = 1, 2, 3, 4$.
- $w_5 = 1$ iff at the previous step, the robot moved north.
- $w_6 = 1$ iff at the previous step, the robot moved south.

The production system is as follows:

$$
\begin{aligned}
&w_2 w_1 \rightarrow \mathit{south},\\
&w_2 w_3 \rightarrow \mathit{north},\\
&w_2 w_5 \rightarrow \mathit{north},\\
&w_2 w_6 \rightarrow \mathit{south},\\
&w_5 \rightarrow \mathit{south},\\
&w_6 \rightarrow \mathit{north},\\
&w_2 \rightarrow \mathit{north},\\
&1 \rightarrow \mathit{east}.
\end{aligned}
$$

The robot's current position is indicated in the figure below with an "X". Current feature values are: $w_1 = 0, w_2 = 0, w_3 = 0, w_4 = 0, w_5 = 1, w_6 = 0$.




List the next 4 moves of the robot controlled by this production system.

**Question 2: Game Theory . . . . . 6 points**

There are two firms serving the same market. They have constant average costs of \$2 per unit. The firms can choose either a high price (\$10) or a low price (\$5) for their output. When both firms set a high price, total demand is 10,000 units which is split evenly between the two firms. When both set a low price, total demand is 18,000, which is again split evenly. If one firm sets a low price and the other a high price, the low priced firm sells 15,000 units, the high priced firm only 2,000 units. What pricing decisions would the firms make?

Solve the problem by formalizing the strategic situation as a game in normal form between these two firms (constructing the pay-off matrix) and find a solution by computing the pure Nash equilibria.

**Question 3: Auction ..... 6 points**

Consider the single item first-price auction with two bidders $B_1$ and $B_2$, and assume that when there is a tie, $B_2$ will win. Suppose $B_i$ values the item $x_i$, $i = 1, 2$, and the information is common knowledge (thus $B_1$ knows that $B_2$'s value is $x_2$, and vice versa and so on). Suppose further that each can bid with any integers in the interval $[0, 100]$.

- Make this auction into a game in normal form $(\{B_1, B_2\}, R_1, R_2, u_1, u_2)$ by defining $R_i$ (the set of pure strategies for player $B_i$) and $u_i$ (player $B_i$'s utility function). You can assume that both players are risk neutral.
- Suppose $0 \leq x_i \leq 1$ for both $i = 1, 2$. What are the Nash equilibria of your game? You need to consider all possible cases for $x_1$ and $x_2$.

**Question 4: CNF ..... 8 points**

Consider the following KB with two formulas:

$$(p \supset q) \equiv r,$$
$$(p \supset q) \equiv \neg r.$$

Answer the following questions:

1. (4 pts) Convert the KB to a set of clauses.
2. (4 pts) Use resolution to prove that the KB is contradictory.

**Question 5: Representation in PL ..... 10 points**

Let's consider a propositional language where

- A="Angelo comes to the party",
- B="Bruno comes to the party",
- C="Carlo comes to the party",
- D="David comes to the party".

Formalize the following sentences:

1. If David comes to the party then Bruno and Carlo come too.
2. David comes to the party if and only if Carlo comes and Angelo doesn't come.
3. Carlo comes to the party only if Angelo and Bruno do not come.
4. Angelo, Bruno and Carlo all come to the party if and only if David doesn't come.
5. If neither Angelo nor Bruno come to the party, then David comes only if Carlo comes.

**Question 6: Representation in FOL ..... 15 points**

Consider a world with balls and boxes, and the following relations:

- $ball(x)$: $x$ is a ball.

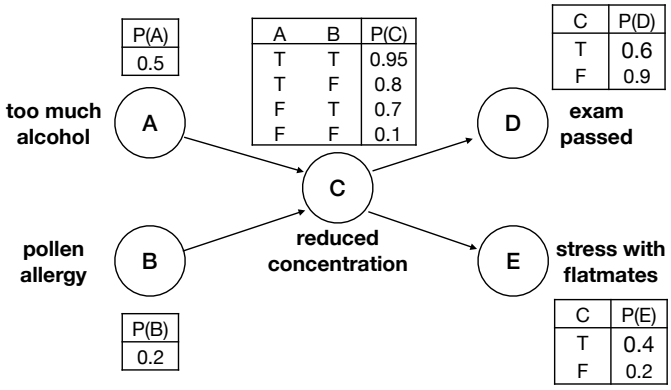
- $box(x)$: $x$ is a box.
- $color(x, y)$: $x$ has color $y$.
- $in(x, y)$: $x$ is in $y$.

Represent the following statements in first-order logic:

1. (2 pts) Some balls are not in any box.
2. (2 pts) Every box has some balls in it.
3. (2 pts) A ball cannot have more than one color.
4. (2 pts) There can be no more than two balls in any box.
5. (2 pts) A box can only contain balls with the same color as itself (boxes also have colors).

**Question 7: Uncertainty ..... 12 points**

Consider the following Bayesian network. Imagine you are at a birthday party of a friend on Sunday and you have an exam on Monday. If you drink too much alcohol at the birthday party, you most likely have problems concentrating the next day, which would reduce the probability that you pass the exam. Another consequence of the reduced concentration might be increased stress with your flatmates, because, e.g., you forget to turn off the radio or the stove. Lack of concentration might also be caused by your pollen allergy, which you suffer from on some summer days.




1. (2 pts) Are A (*too much alcohol*) and B (*pollen allergy*) independent given D (*exam passed*)? Explain your answer using D-separation.
2. (5 pts) Compute the probability of B (*pollen allergy*) given C (*reduced concentration*) is true: $P(B|C)$. There is no need to perform numerical calculations. As long as your formula is right, you will get the full mark.
3. (5 pts) Compute the probability of A (*too much alcohol*) given C (*reduced concentration*) is true and E (*stress with flatmates*) is not true: $P(A|C, \neg E)$. Again, there is no need to perform numerical calculations.

**Question 8: MDP . . . . . 10 points**

Consider a $4 \times 3$ stochastic grid world laid out in the figure below (the crossed-out cell is an obstacle). The agent starts in state (1,1), and has four available actions: *North*, *South*, *West*, *East*. For each action, the agent goes forward with 0.8 probability, goes left and right with 0.1 probability respectively. If there is a wall, the agent stays at current location. For example, if the agent move *East* in cell (1,3), then she'll end up with 0.8 probability in cell (2,3), 0.1 probability in (1,2) (goes right instead), and 0.1 probability in the same cell (1,3) (goes left, which is a wall). At the terminating states (4,2) and (4,3), the only action is *Exit*, after which the agent will be in state *END*. The reward function is defined as follows:

$$R(s,a,s') = R(s') = \begin{cases} -1, & a = Exit \text{ and } s = (4,2) \\ +1, & a = Exit \text{ and } s = (4,1) \\ -0.1, & \text{otherwise} \end{cases}$$

Assume that the discount factor $\gamma = 1.0$.

We perform value iteration on the MDP. The value of each state after 2 iterations are given in the right grid in the following figure.

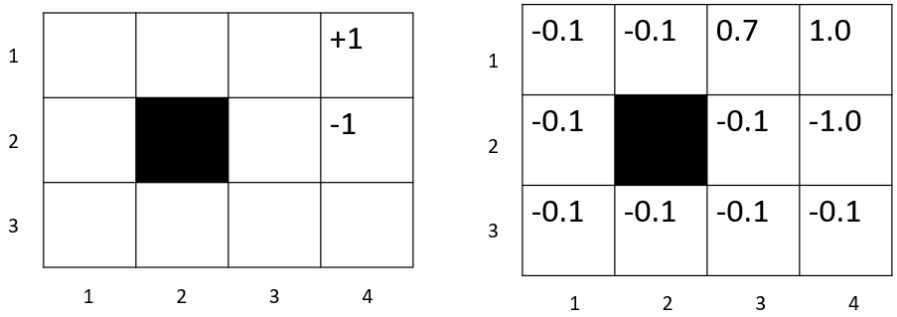



Figure 1: Left: the 4x3 grid world and a policy; right the value of each state after 2 iterations of value iteration.

Now calculate the values of the states (2,1), (3,1), (3,2) after one more iteration.

**Question 9: Admissibility . . . . . 4 points**

Assume you are given three admissible heuristic functions for a given search problem, for simplicity, call them $h_1$, $h_2$, and $h_3$. For each of the following combinations, explain whether or not the resulting $h$ function is admissible.

1. (2 pts) $A : h(n) = h_1(n) + h_2(n) + h_3(n)$
2. (2 pts) $B : h(n) = (h_1(n) + h_2(n) + h_3(n))/3$

**Question 10: Perceptron Learning and GSCA Rule Learning . . . . . 12 points**

Consider the following data set:

| ID | $x_1$ | $x_2$ | $x_3$ | $x_4$ | OK |
|---|---|---|---|---|---|
| 1 | 1 | 0 | 1 | 0 | No |
| 2 | 0 | 0 | 1 | 0 | No |
| 3 | 1 | 0 | 0 | 0 | Yes |
| 4 | 1 | 1 | 0 | 1 | Yes |
| 5 | 0 | 1 | 1 | 1 | No |

where $x_1$, $x_2$, $x_3$ and $x_4$ are some features that should not concern us here.

1. (5 pts) Use these four instances to train a single perceptron using the error-correction procedure. Use the learning rate = 1, and the initial weights all equal to 0. Recall that the threshold is considered to be a new input that always have value "1". Please give your answer by filling in the following table, where weight vector $(w_1, w_2, w_3, w_4, t)$ means that $w_i$ is the weight of input $x_i$, and $t$ is the weight for the new input corresponding to the threshold. Stop when the weight vector converges. If it doesn't converge, explain why not.

| ID | Weight vector $(w_1, w_2, w_3, w_4, t)$ |
|---|---|
| Initial | (0, 0, 0, 0, 0) |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |

2. (2 pts) What is the Boolean function corresponding to your perceptron?
3. (5 pts) From the same training set, apply the GSCA algorithm to try to learn a set of rules. Give the set of rules if it succeeds. If it fails to learn a set of rules, explain why it failed.

**Question 11: Perceptrons/Linearly Separable Functions . . . . . 6 points**

Assume three boolean variables $x$, $y$ and $z$. If $f(x, y, z)$ is a boolean function of $x, y, z$, then the projection $f(0, y, z)$ is the boolean function of $y$ and $z$ with $x$ set to 0 (false).

Answer the following questions

1. (3 pts) Is there a linearly separable function $f(x, y, z)$ such that $f(0, y, z)$ is non-linearly separable? If yes, give an example. If no, explain why.
2. (3 pts) Is there a non-linearly separable function $f(x, y, z)$ such that $f(0, y, z)$ is linearly separable? If yes, give an example. If no, explain why.

**Question 12: Machine Learning/General Discussion . . . . . 5 points**

Consider the statement: "The current machine learning models are brittle and their results not explainable". Answer the following questions:

1. (3 pts) Give an example to illustrate this statement.
2. (2 pts) Do you think this impedes the usefulness of the current machine learning models? Explain your answer.