Wind Turbine Technology & Aerodynamics
Assignment 1

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1 Introduction

1.1 Overview

In this report we will use the data from a wind turbine to estimate power, torque, and generator characteristics as well as the optimal pitch and rotation speed settings for turbine operation. "The Tjaereborg machine" is an experimental 2MW HAWT for which the design geometry is available publicly. An algorithm was developed based on the Blade Element Momentum theory to estimate the tangential and normal pressure loading along the blade span. This algorithm was implemented to investigate how the pitch angle and rotation speed would be controlled to optimize the power coefficient and limit the power output to rated power.

1.2 Assumptions

Throughout most of the calculations and assessments made in this assignment, the following assumptions were made. If there are any changes to these assumptions, they will be mentioned in their respective questions.

$\rho_{air} = 1.225kg/m^3$	Number of blades = 3	Blade Radius = 30.56m
Rated Power = 2 MW	Cut-in Wind Speed = 5 m/s	Cut-out Wind Speed = 25m/s

Blade Element Momentum theory also makes key assumptions. We assume the incoming flow is steady, uniform, and non-turbulent. Additionally, we assume that each element analyzed is independent of any other, meaning there are no aerodynamic interactions between the blade elements.

2 Questions

2.1 Question 1

An important part of designing a wind turbine is ensuring that it provides as much energy as can be extracted from the wind. To approximate how efficient a wind turbine can be with the site conditions, the optimum power coefficient, $C_{p,opt}$, had to be calculated. This significant value was determined by applying the blade element momentum theory (BEM) which finds the appropriate axial and angular induction factors for each element of the blade in an iterative process. The algorithm implemented the Prandtl tip loss factor to fix the assumption that we are dealing with an infinite number of blades. The Glauert correction factor was also utilized to adjust the relation between axial induction factor values past 1/3 and the thrust coefficient to fix the problems with the 1-D momentum theory as shown below.

$$C_T = \begin{cases} 4a(1-a)F, & \text{if } a \leq \frac{1}{3} \ . \\ 4a(1-\frac{1}{4}(5-3a)a)F, & \text{if } a > \frac{1}{3}. \end{cases}$$

The a values in the previous equation were found using the Newton-Raphson method. Once the axial induction factor values were known, it was easy to obtain the angle,ϕ between the rotor plane and the relative wind. Using this instrumental value, it was possible to calculate the angle of attack to interpolate for the lift and drag coefficients that corresponded to the airfoil at each element.

We compared various tip speed ratios, $\lambda = \frac{\omega r}{V_o}$, and pitch angles, θ_p as seen in Figure 1 to find the optimal quantities that achieve the desired $C_{p,opt}$. Tip speed ratios were varied by assuming that the angular speed was constant at, $\omega = 2.3rad/s$ and changing the possible wind speeds. The range of pitch angles that was tested was between -2^o and 2^o because the lower pitch angles allow values of angle of attack that maximize lift and minimize drag. Also the wind speed values shown in figure 1 were kept between 7 and 14 m/s after analyzing the entire range of wind speeds from 5 to 25 m/s in order to clearly magnify the maximum C_p range. With these settings, a higher tangential pressure can be achieved and, consequently, a higher moment and increased power generation can be reached as seen in the following equation

$$M = \int_1^N rp_t dr,$$

where r is the radial location of the radial element dr, p_t is the tangential pressure at that location, and N is the number of radial elements used to compute the integral. In our case, this integral was computed using the `trapz()` function in Matlab which applies the trapezoidal rule to integrate over the designated domain. The power was simply calculated by multiplying the angular speed with the moment.In our case, the λ_{opt} and $\theta_{p,opt}$ were calculated to be 7.48 and -0.29^o respectively which resulted in a $C_{p,opt}$ of 0.48.

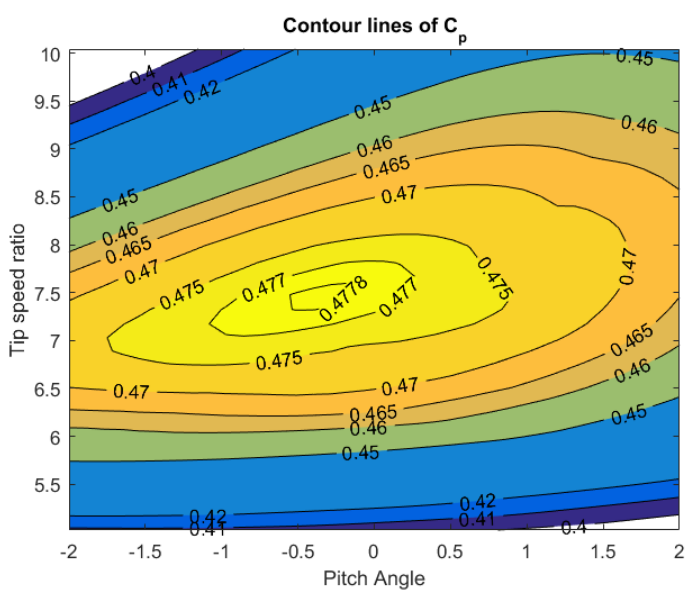

Figure 1: Contour plot of power coefficient versus tip speed ratio and pitch angle

2.2 Question 2

In this question, we considered maximizing the efficiency of a variable speed pitch regulating wind turbine for a different generator and control system than in Question 1. Using the λ_{opt} obtained in the previous question, and by limiting the maximum rotational speed at $\omega_{max} = 2.62\ rad/s$ due to noise concerns, we wanted to determine the wind speeds that would hold the power coefficient at the optimal point. Keeping the blade pitch constant at $\theta_{p,opt}$, the $C_{p,opt}$ can be held for the wind speeds in the given range below as long as tip speed ratio is kept constant as well.

$$5 \leq V_0 \leq 10.70\ m/s$$

The lower bound is the cut-in wind speed, and the upper bound is the highest possible wind speed up to where we can hold λ_{opt}. The wind turbine is extracting as much power as it can during these wind speeds. We can see how the rotational speed should be controlled as the wind speed varies in Figure 2. As shown, for wind speeds below 10.70 m/s, the rotation speed can be controlled to keep the turbine operating at optimum conditions. For wind speeds above 10.70 m/s, ω is held constant at ω_{max}.

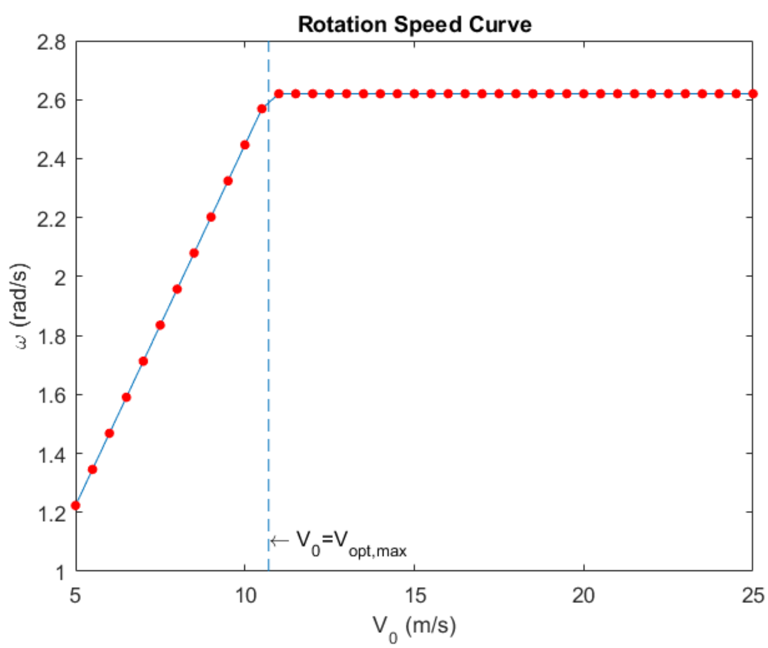

Figure 2: Rotation speed versus wind speed for a speed regulated turbine

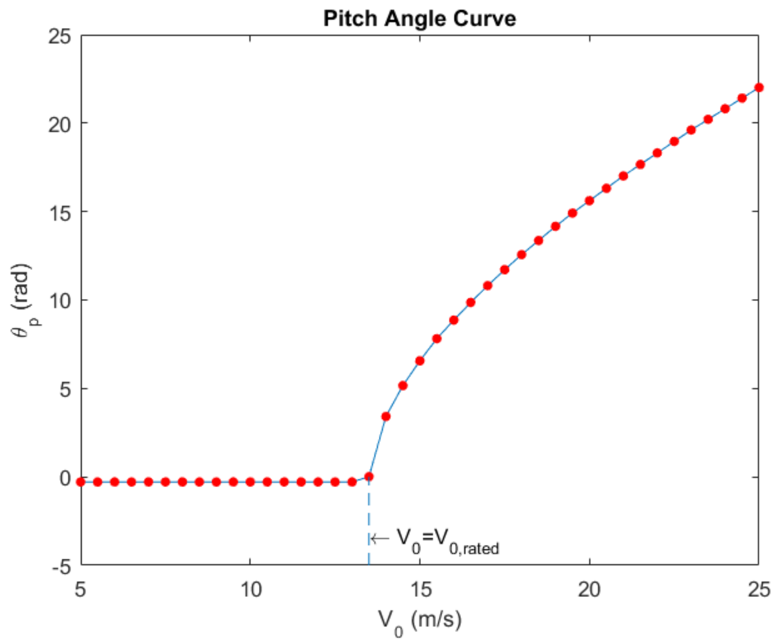

Figure 3: Pitch angles of the blade as wind speed increases

2.3 Question 3

For a pitch regulated wind turbine, the pitch of the blades can be changed by the controller and actuator to restrict the power being generated from exceeding the rated power. This is obviously important for safety and to prevent damage to the generator. Output power can be compared to the rated power and the pitch can be adjusted to reduce this power output and the loads on the turbine. Prior to the power reaching levels near the rated power, the pitch is held at the previously calculated $\theta_{p,opt}$ to ensure that the turbine is extracting the maximum amount of power possible from the wind. Once the wind speed exceeds the rated wind speed, the pitch angle is increased in steady increments to reduce the angle of attack (AOA) on the blades. An example of this pitch angle change can be seen in Figure 3.

At wind speeds above approximately $V_{0,rated} = 13.5\ m/s$, the controller begins to reduce the pitch angle and limit the power to its rated value. It is important to note that in controlling the pitch angle, there are two pitch angles which may be used to maintain power output at its rated value. Notice that once rated power is reached, the pitch angle is increased for our controller. Alternatively, the pitch angle may be decreased at this point to increase angle of attack (AOA) and force the blades to stall (active stall). However these lower angles were not used in this analysis as they will cause high loads to act on the rotor. The higher pitch angles give lower angles of attack as can be seen in the following equation:

$$\alpha = \phi - (\beta + \theta_p)$$

where α is the angle of attack, β is the twist angle, and θ_p is the pitch angle.

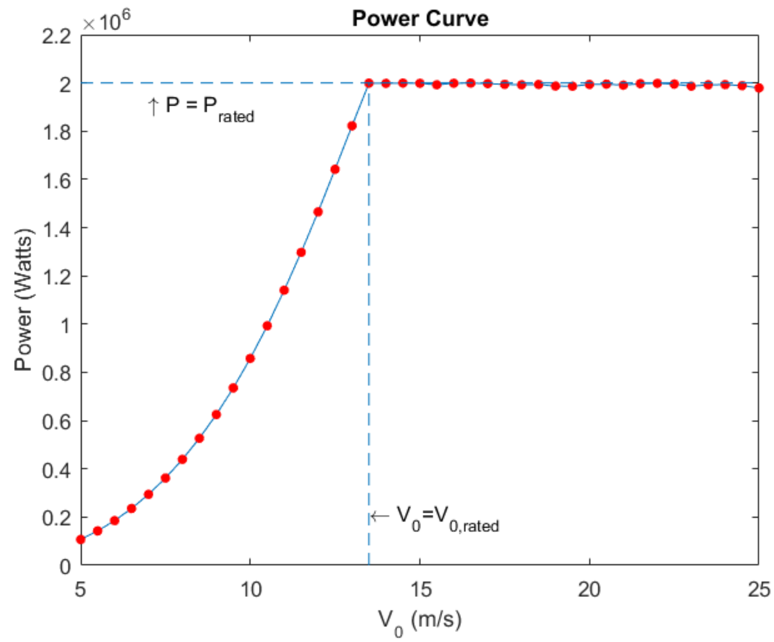

Figure 4: Power curve of the wind turbine from cut-in to cut-out wind speeds

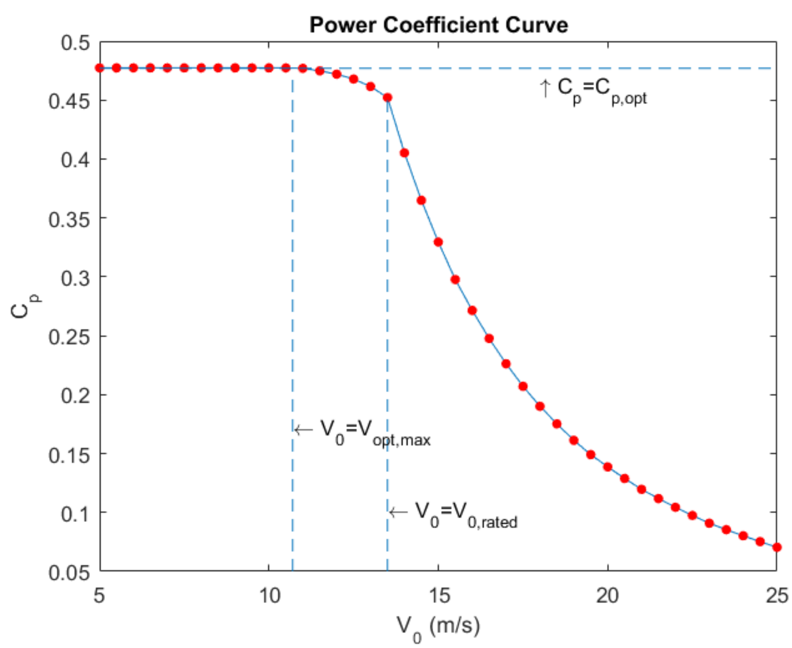

Figure 5: Power coefficient of the wind turbine from cut-in to cut-out wind speeds

Consequently, this effect reduces the lift on the blade, leading to reduced power generation and lower loads. This choice was forced into the code by only allowing increases in pitch angle for every iteration in calculating normal and tangential loads via the BEM algorithm.

Shown in figure 4 is the power generated by the wind turbine through all operational wind speeds. The power is proportional to the cube of the wind speed up until the rated power is reached, where the controller and actuator adjusts the pitch angle to reduce the torque applied to the blades by the wind. Notice there are still fluctuations in the power curve above rated power. These are caused by limited numerical precision in the BEM algorithm and do not include the fluctuations caused by unsteadiness of a real-time controller i.e. this model assumes an instantaneous adjustment of the pitch angle with changes in wind speed, where in reality there is for example lag time, overshoot, and feedback that need to be dealt with separately.

At wind speeds below 10.70 m/s, the turbine is operating at optimum point $C_{p,opt}$ (see Figure 5). As wind speed increases, the rotation speed is limited to ω_{max} and the tip speed ratio is no longer optimum. This results in a losses in efficiency, shown by a decreasing power coefficient. When the wind speed reaches $V_{0,rated}$, the pitch angle is also not at the optimum value and there is a sharp drop in C_p as wind speed increases. Since the available power to be extracted from the wind increases with the cube of the wind speed, and the power of our turbine is held to its rated power, the ratio of the extracted power to the total available power, or C_p, decreases proportionally to V_0^3.

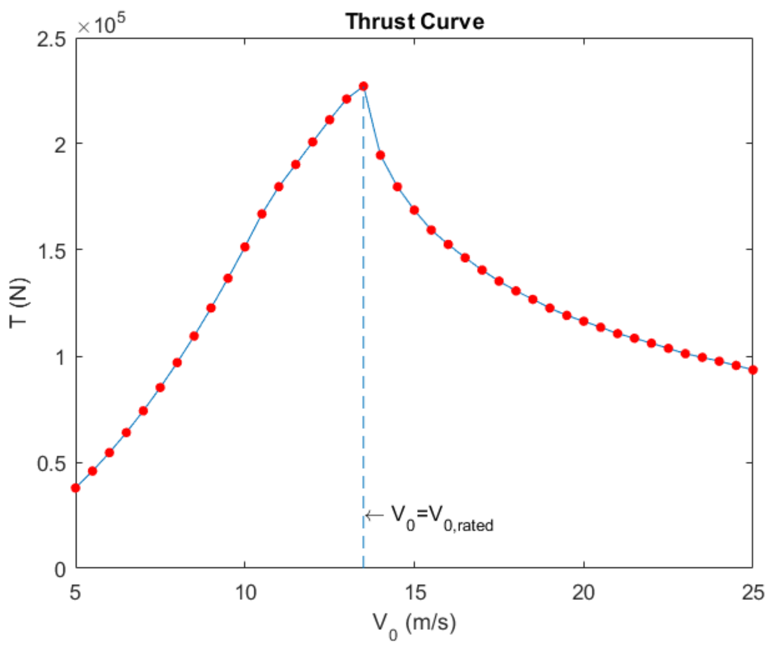

Figure 6: Thrust curve of the wind turbine from cut-in to cut-out wind speeds

Looking at Figure 6, the thrust acting on the blades gradually rises until the $V_{0,rated}$ is reached. This can be explained by the fact that the pitch angle is constant up until that point where after the pitch angle starts to be steadily increased. Again this will lower the angle of attack which in turn lowers the normal loads acting on the blades. A similar pattern can be seen with the thrust coefficient in Figure 7. The power coefficient is held constant at the optimum point through wind speeds up until $V_0 = 10.7 m/s$ where after the tip speed ratio starts to decrease steadily from our optimal value due to the limitation on rotational speed until $V_{0,rated}$. From this point on, the pitch angle is raised from the optimum pitch angle, which as explained before, will drop the angle of attack and lower the normal loads acting on the blades. Therefore the thrust coefficient also diminishes rapidly.

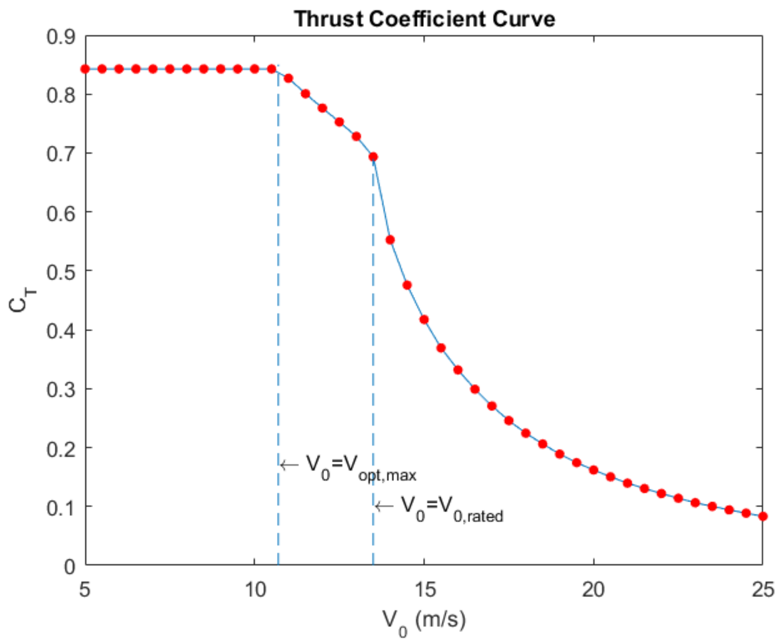

Figure 7: Thrust coefficient of the wind turbine from cut-in to cut-out wind speeds

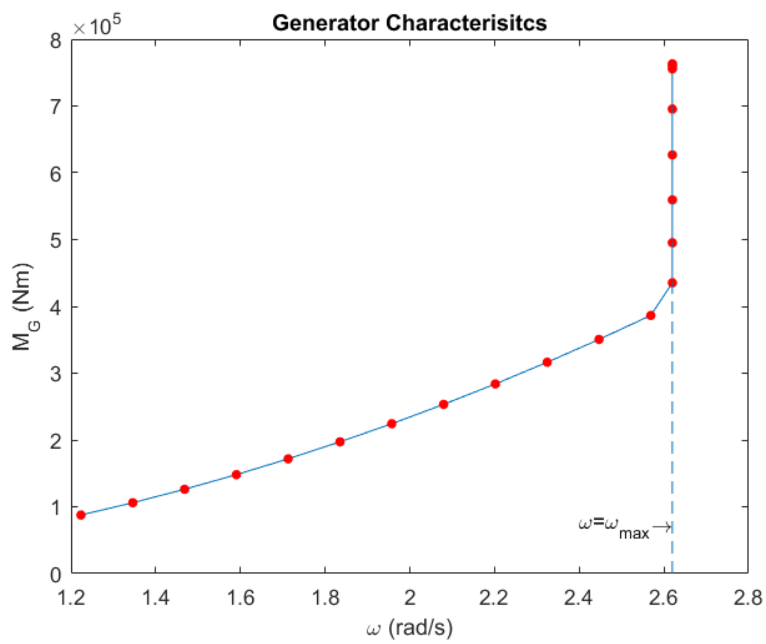

Figure 8: Desired generator characteristics

Figure 8 shows the generator characteristics. Below ω_{max}, the rotor is operating at optimum tip speed ratio. Once ω_{max} is reached, the torque becomes solely a function of power which can be controlled through the pitch angle. Once the wind speeds reach $V_{0,rated}$, power and thus torque becomes constant.

2.4 Question 4

One of the most important aspects of justifying the installation of a wind turbine in a certain is calculating the possible annual energy production (AEP) that a wind turbine can generate. Knowing the approximate power that can be produced from the wind turbine is crucial in outlining the economic and environmental benefits of installing the turbine. For convenience, a Weibull distribution with given scale parameter, $A = 7m/s$ and shape parameter $k = 1.9$ was used as a model of the probability density of the wind speeds. Weibull distributions can often be helpful in accurately modelling a wind speed distribution. In a more realistic situation, actual wind speeds would have to be measured at the site in ten minute intervals for at least a year to have a more accurate estimation and idea of AEP. By taking the integral of the Weibull distribution from one bin to the next bin of wind speeds, we calculated the probability of that certain wind speed of occurring. Through repeating this method for each bin of wind speed, it was possible to calculate the total annual energy production. It was calculated by summing the products of the average power generated at each wind speed and the probabilities at each wind speed as shown:

$$AEP = \sum_{i=1}^{N-1} \frac{1}{2}(P(V_{i+1} + P(V_i) \times f(V_i < V_o < V_{i+1}) \times 8760$$

N was the number of wind speed bins and was equal to 41 in our case to reduce the computational efforts of having too many bins. A more accurate and precise calculation can be made if more bins had been utilized. The total annual energy production in our case was computed to be about 3.16 million kWh.

2.5 Question 5

For a stall regulated turbine, it is not possible to adjust the pitch angle as was with the pitch regulated wind turbine due to the fact that the blades are bolted to the hub of the turbine. Therefore, it was imperative that a pitch angle was found that would limit the power for all wind speeds below the rated power, which in this case was 2.5MW. A range of low values for pitch angles were analyzed as shown in Figure 9 to find the limiting pitch angle as the low angles would help actively stall the blades when approaching the rated power. The limiting pitch angle was calculated to be -2.6^o and the figure showing the power at this pitch angle is shown in Figure 10. As can be seen in this figure, the power peaks at 2.5MW, but does not plateau at this rated power like the pitch regulated turbine because the pitch is always constant. In fact the produced power steadily drops off after the peak power production.

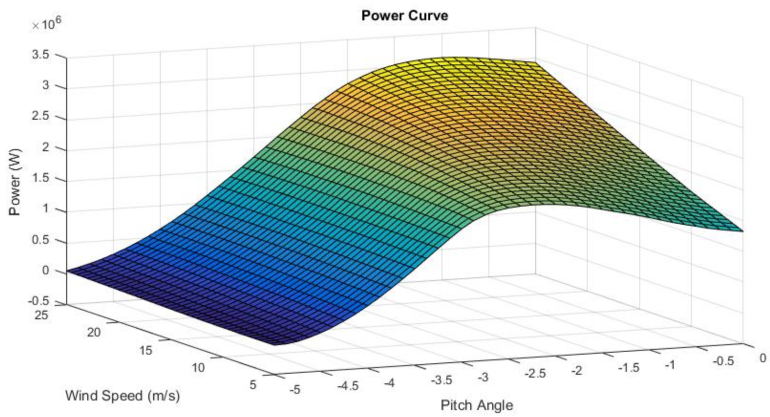

Figure 9: Surface plot of generated power with changing wind speeds and pitch angles

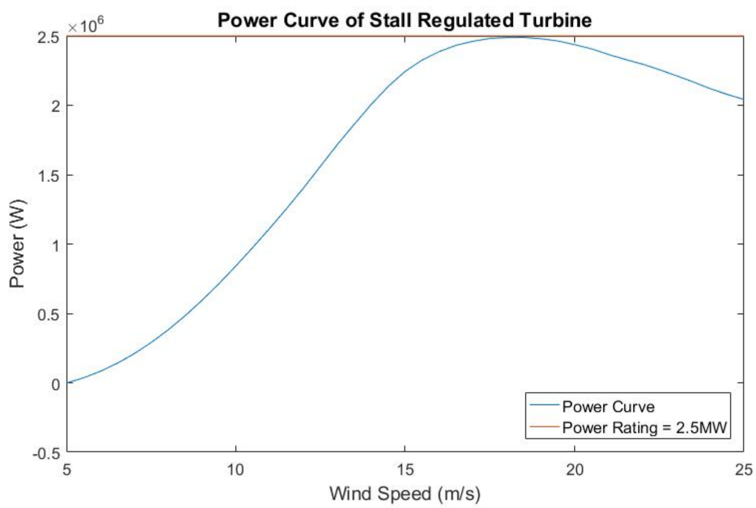

Figure 10: Power curve of stall regulated turbine at -2.6^o

2.6 Question 6

Optimizing the yield of energy production for a stall regulated turbine can be done by analyzing the optimal angle $\theta = (\beta + \theta_p)$ with the rotor plane for each element along the blade. For this question, the eighth element was used as an example at the location, 27.46m, which was the closest element to the desired location of 28m. The annual energy production was calculated for this section by finding the optimal angle through slight changes from the original θ, and by using the same procedure as in Question 4. The new angle θ was computed to be about 2.62^o compared to the old angle of -1.63^o. For the optimal angle, the total annual energy production was calculated to be about 1.94 $\times 10^4$ kWh/m; whereas, the original angle for stall regulation has a total AEP of about 1.36 $\times 10^4$ kWh/m. It is logical that the calculated optimal angle produces more energy per meter than the old angle because we don't care about the power rating, which may be exceeded in the case of the optimal angle.

3 Conclusion

The BEM theory provides a way to estimate the steady loads on a wind turbine when the blade geometry and drag and lift data for blade sections are known. Inputting a range of pitch angles and tip speed ratios, we saw how the power behaves as a function of wind speed which could be used in designing and operating a Variable speed pitch regulated wind turbine. With a VSPR turbine, the power output can be controlled to keep output from exceeding the rated power, which is of concern for integrating the turbine with the power grid and for the maintaining the safety of the generator.

With the power curve known and with data on the probability density function of wind speeds at a particular site, the annual energy production for the turbine may be estimated, a key metric in the planning and development of a wind turbine.

A stall-regulated turbine was investigated to find that although the design is mechanically more simple due to a fixed pitch angle and constant rotational speed, stall-regulated turbines result in a lower power output than VSPR turbines.

Since in BEM theory, the blade elements are considered independent of each other, they can be optimized one by one. As an example, this was performed for one element to choose a local pitch angle, θ, that maximizes annual energy production. When performed for a range several elements along the span, the optimal values could be used as design inputs for a turbine blade.

Wind Turbine Technology & Aerodynamics
Assignment 1 Progress Report

The following table shows the percentage of work completed by each team member. The assignment credit was divided into 3 equal parts.

	Young Kwon	**Erik Haugen**
Matlab Code	75%	25%
Report Writing	50%	50%
Editing/Revision	25%	75%
% of Assignment Completed	**50%**	**50%**

By signing this document, each person agrees that the credit given to each team member is accurate and fair.

s163075	Young	05/10/2016
Student Number	**Signature**	**Date**

s163135	Erik Haugen	05/10/2016
Student Number	**Signature**	**Date**