



**DATA
61**

Constraint Programming applied to the Multi-Skill Project Scheduling Problem

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Intro: The Problem



The Multi-Skill Project Scheduling Problem (MSPSP) is a variant of the Resource Constrained Project Scheduling Problem (RCPSP)

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Aim: Find the fastest way to complete all the activities

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- Skills

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Constraints

- Activity constraint: Precedence relations between activities
- Skill constraint: Activities require skills
- Worker constraint: Workers each have a variety of skills

Intro: Example



Table: Workers' Skills

	Alice	Bob	Carl	Dora
Programmer	-	✓	✓	✓
DB Designer	✓	-	-	-
Webmaster	✓	✓	-	✓

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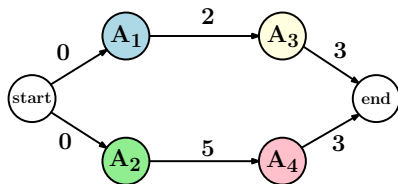


Figure: Precedence Graph

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Table: Skill Requirement

	A_1	A_2	A_3	A_4
Programmer	-	1	2	1
DB Designer	1	-	-	1
Webmaster	1	1	-	-

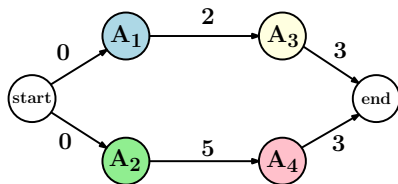


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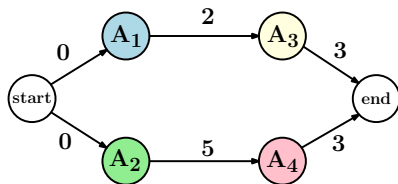


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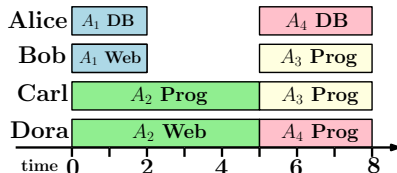


Figure: Schedule

Intro: The Literature



- French research group
 - ▶ Principal researchers: Odile Belleguez-Morineau, Emmanuel Néron, Carlos Montoya
 - ▶ Exact branch and bound methods
 - ▶ Lower bounds
 - ▶ Adapted data from PSPLib

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 - ▶ Randomised search heuristics
 - ▶ Generated their own data

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Model



- Objective
 - ▶ Minimise the total project duration

Model



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- Two main decisions

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 1. Scheduling decisions
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- Two main decisions
 1. Scheduling decisions
 - Activity start times
 2. Assignment decisions
 - Workers to activities
 - Skill contribution of workers

Model: Constraints Outline



- Precedence relations are respected

Model: Constraints Outline



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- Workers perform only one activity at a time

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Model: Constraints Outline



- Precedence relations are respected
- Workers perform only one activity at a time
- Workers cannot multi-task
- Skill requirement is satisfied
 - ▶ A worker for each skill must be present to perform the activity

Model: Decision Variables



Decision Variables

Primary	s_i	Start time of activity $i \in V$
	y_{ir}^s	1 iff resource $r \in R$ contributes with skill $s \in S$ to activity $i \in V$
Auxiliary	o_{ij}	1 iff activities i and j overlap for $(i, j) \in U$
	x_{ir}	1 iff resource $r \in R$ is assigned activity $i \in V$

Model: Basic Constraints



$$s_i + p_i \leq s_j \quad \forall (i, j) \in E$$

$$\sum_{r \in R} y_{ir}^s = sr_i^s \quad \forall i \in V, \forall s \in S$$

$$\sum_{s \in S} \sum_{i \in V: s_i \leq t < s_i + p_i} y_{ir}^s \leq 1 \quad \forall r \in R, \forall t \in \left\{0, 1, \dots, \sum_{i \in V} p_i\right\}$$

$$y_{ir}^s \leq mast_{rs} \quad \forall i \in V, \forall r \in R, \forall s \in S$$

Model: Choice of Constraints



Unary Resource Constraint

- Each worker only performs one activity at a time

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Unary Resource Constraint

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Possible ways of modelling

1. Time-indexed decomposition
2. Global constraints (either disjunctive or cumulative)
3. Order constraints

Model: Unary Resource Constr.



- Time-indexed decomposition

$$\sum_{s \in S} \sum_{i \in V: s_i \leq t < s_i + p_i} y_{ir}^s \leq 1 \quad \forall r \in R, \forall t \in \left\{0, 1, \dots, \sum_{i \in V} p_i\right\}$$

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- Cumulative global constraint

$$\text{cumulative}((s_i)_{i \in V}, (p_i)_{i \in V}, (x_{ir})_{i \in V}, 1) \quad \forall r \in R$$

Model: Unary Resource Constr.



- First order constraint formulation

$$\neg o_{ij} \Leftrightarrow (s_i + p_i \leq s_j) \vee (s_j + p_j \leq s_i) \quad \forall (i, j) \in U$$
$$(x_{ir} \wedge x_{jr}) \Rightarrow \neg o_{ij} \quad \forall (i, j) \in U, r \in R$$

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- Second order constraint formulation

$$(o_{ij} \Rightarrow s_i + p_i \leq s_j) \wedge (\neg o_{ij} \Rightarrow s_j + p_j \leq s_i)$$
$$\forall (i, j) \in U, \exists s \in S : sr_{is} + sr_{js} > \sum_{r \in R} mast_{rs}$$

Data: Overview



- Tested on data from the literature and generated our own data

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set	#instances	n	l	m	Best known results		
					source	%optimal	#unsolved
1a	216	22	4	10-30	Correia et al. 2012	93.98	13
1b	216	42	4	20-60	Almeida et al. 2016	2.31	211

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2a	110	20-51	2-8	5-14	Montoya et al. 2014	43.64	62
2b	77	32-62	9-15	5-19	Montoya et al. 2014	66.20	24
2c	91	22-32	3-12	4-15	Montoya et al. 2014	51.11	44

Data: Complexity Measures



1. Skill Factor

- ▶ $SF \in \{1, 0.75, 0.5, \text{variable}\}$

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- ▶ $SF \in \{1, 0.75, 0.5, \text{variable}\}$

2. Network Complexity

- ▶ $NC \in \{1.5, 1.8, 2.1\}$

3. Modified Resource Strength

- ▶ varied over 3 values
- ▶

$$MRS = \frac{m}{\sum_{i \in V} \sum_{s \in S} sr_{is}}$$

Experiments: Search Strategies



- Basic Search
 - ▶ Start times (s_i)

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 - ▶ Start times (s_i), then worker assignment (x_{ir})
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Experiments: Search Strategies



- Basic Search
 - ▶ Start times (s_i)
- Sequential Searches
 - ▶ Start times (s_i), then worker assignment (x_{ir})
 - ▶ Start times (s_i), then contribution of each worker (y_{ir}^s)
- Priority searches \Rightarrow Group scheduling and assignment decisions of each activity together
 - ▶ priority-ff: choose activity group by smallest start time domain
 - ▶ priority-sml: choose activity group by smallest largest possible start time
 - ▶ priority-sm: choose activity group by smallest possible start time

Experiments: Set 1'a



- Tested on 216 generated instances with 22 activities
- Time limit of 600 seconds

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unary cons.	search	#nodes	%optimal	runtime
cumulative	default	370,174	100.00	10.23s
order-1	default	97,085	100.00	2.73s
order-2	default	54,282	100.00	1.30s

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order-2	priority-ff	41,762	100.00	1.25s
order-2	priority-sml	20,786	100.00	0.68s
order-2	priority-sm	13,241	100.00	0.51s

Experiments: Set 1'b and Set 2



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set	#nodes	%gap	#opt	%opt	mean runtime	#closed
1'b	7,584k	49.3	27/216	12.5	534.6s	—

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set	#nodes	%gap	#opt	%opt	mean runtime	#closed
1'b	7,584k	49.3	27/216	12.5	534.6s	—
2a	2,223k	185.2	81/110	73.6	195.2s	≥33

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1'b	7,584k	49.3	27/216	12.5	534.6s	—
2a	2,223k	185.2	81/110	73.6	195.2s	≥33
2b	816k	22.4	63/77	81.8	122.9s	≥10

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1'b	7,584k	49.3	27/216	12.5	534.6s	—
2a	2,223k	185.2	81/110	73.6	195.2s	≥ 33
2b	816k	22.4	63/77	81.8	122.9s	≥ 10
2c	14k	0.0	91/91	100.0	1.2s	44

Experiments: Complexity Measures



- Results of set 1' b
- Time limit of 600 seconds

Experiments: Complexity Measures



- Results of set 1'*b*
- Time limit of 600 seconds

measure	value	#nodes	#props	%gap	#opt	%opt	runtime(s)
SF	1	5.9m	706k	48.1	6/54	11.1	535.5
	0.75	8.9m	765k	51.8	4/54	7.4	559.9
	0.5	6.7m	914k	44.1	12/54	22.2	489.7
	variable	8.6m	754k	52.4	5/54	9.26	553.5
NC	1.5	7.7m	872k	58.3	6/72	8.3	559.9
	1.8	7.9m	812k	47.7	8/72	11.1	541.3
	2.1	7.0m	671k	41.03	13/72	18.1	502.8
MRS	#1	7.4m	1,106k	79.2	11/72	15.3	525.4
	#2	7.9m	734k	45.6	6/72	8.3	558.4
	#3	7.3m	515k	24.0	10/72	13.9	520.1
Overall		7.5m	785k	49.3	27/216	12.5	534.6

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- Generated a set of benchmark instances

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- Applied the constraint programming solver chuffed to the MSPSP
- Generated a set of benchmark instances
- Created an effective constraint programming model
 - ▶ Together with an application tailored search strategy

Acknowledgements



- Dr. Andreas Schutt
- Dr. Thibaut Feydy
- Adrian Goldwaser

Thanks for listening!

Questions?