
# Constraint Programming applied to the Multi-Skill Project Scheduling Problem

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# Intro: The Problem


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Constraints

- Activity constraint: Precedence relations between activities
- Skill constraint: Activities require skills
- Worker constraint: Workers each have a variety of skills

# Intro: Example

Table: Workers' Skills

| | Alice | Bob | Carl | Dora |
|---|---|---|---|---|
| **Programmer** | - | ✓ | ✓ | ✓ |
| **DB Designer** | ✓ | - | - | - |
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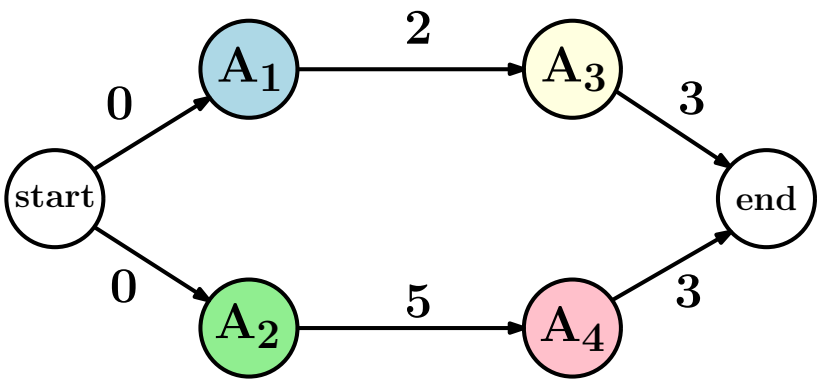


Figure: Precedence Graph

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Table: Skill Requirement

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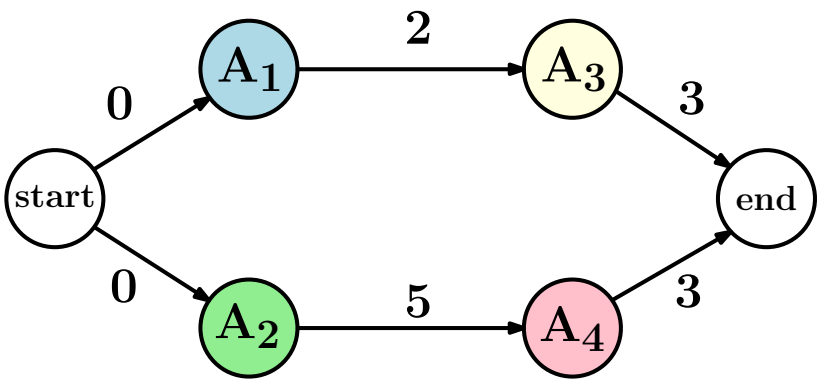


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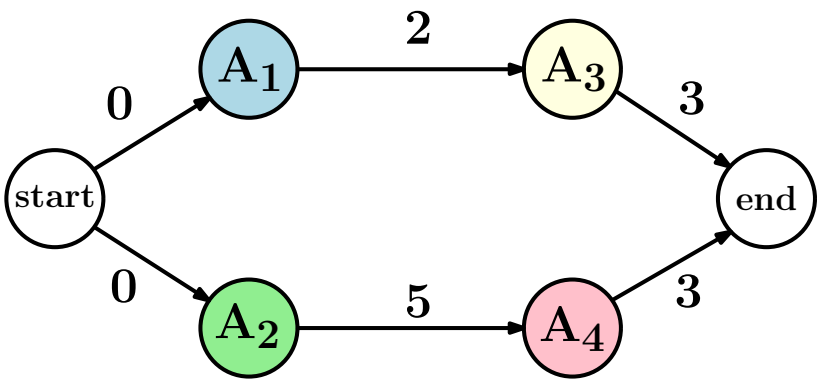


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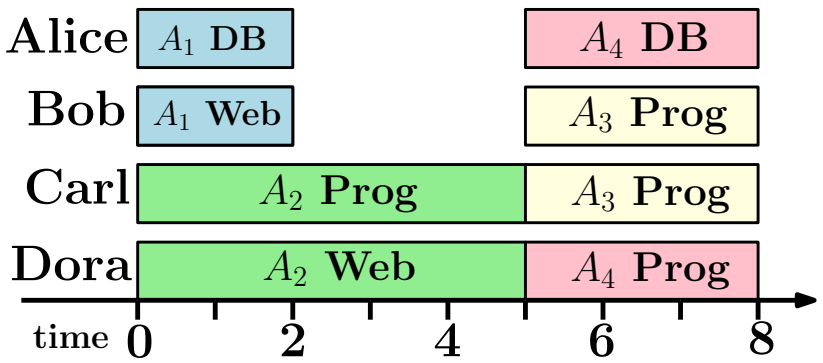


Figure: Schedule

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Domain propagation

- Variables have domains of possible values
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Nogood learning

- Learn from failures
- Record these failures as constraints
- Use these constraints to make inferences

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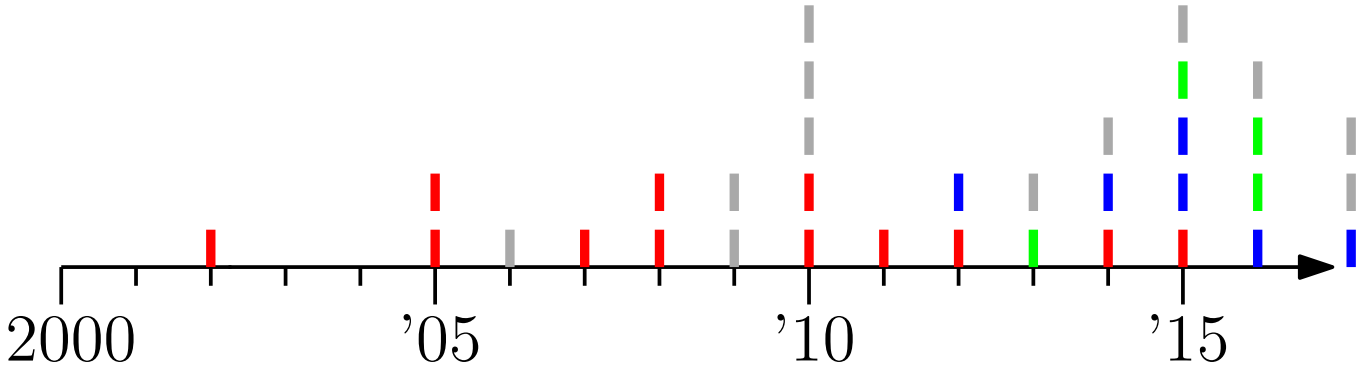
# Intro: Timeline of the Literature

- French
- Portuguese
- Polish
- Other



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  2. Assignment decisions
     - Workers to activities
     - Skill contribution of workers

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- Redundant constraints

# Model: Choice of Constraints

Unary Resource Constraint

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3. Cumulative global constraint

# Model: Decision Variables

| Decision Variables | |
|---|---|
| $s_i$ | Start time of activity $i \in V$ |
| $o_{ij}$ | 1 iff activities $i$ and $j$ overlap for $(i, j) \in U$ |
| $x_{ir}$ | 1 iff resource $r \in \mathscr{R}$ is assigned activity $i \in V$ |
| $y_{irk}$ | 1 iff resource $r \in \mathscr{R}$ contributes with skill $k \in \mathscr{L}$ to activity $i \in V$ |

# Model

$$\text{Min } s_{n+1} \tag{1}$$

$$\text{s.t.} \quad s_i + p_i \leq s_j \qquad \forall (i,j) \in E \tag{2}$$

$$\neg o_{ij} \Leftrightarrow (s_i + p_i \leq s_j) \vee (s_j + p_j \leq s_i) \qquad \forall (i,j) \in U \tag{3}$$

$$(x_{ir} \wedge x_{jr}) \Rightarrow \neg o_{ij} \qquad \forall (i,j) \in U,\ r \in \mathscr{R}V_i \cap \mathscr{R}V_j \tag{4}$$

$$\sum_{r \in \mathscr{R}V_i} y_{irk} = Req_{ik} \qquad \forall i \in V,\ k \in \mathscr{L}_i \tag{5}$$

$$\sum_{k \in \mathscr{L}_i | Mast_{rk}=1} y_{irk} \leq 1 \qquad \forall i \in V,\ r \in \mathscr{R}V_i \tag{6}$$

$$y_{irk} \leq Mast_{rk} \qquad \forall i \in V,\ r \in \mathscr{R}V_i,\ k \in \mathscr{L} \tag{7}$$

$$y_{irk} \leq x_{ir} \qquad \forall i \in V,\ r \in \mathscr{R}V_i,\ k \in \mathscr{L}_i \tag{8}$$

$$s_i \geq 0 \qquad \forall i \in V \tag{9}$$

# Model: Redundent Constraints

$$\texttt{cumulative}(s,\ p,\ [\ Req_{ik} : i \in V],\ |\mathscr{R}_k|) \qquad \forall k \in \mathscr{L} \tag{10}$$

$$\texttt{cumulative}\left(s,\ p,\ \left[\ \sum_{k \in \mathscr{L}} Req_{ik} : i \in V\right],\ m\right) \tag{11}$$

$$x_{ir} = 0 \quad \forall i \in V,\ r \in \mathscr{R} \backslash \mathscr{R} V_i \tag{12}$$

$$y_{irk} = 0 \quad \forall i \in V,\ r \in \mathscr{R} \backslash \mathscr{R} V_i,\ k \in \mathscr{L} \tag{13}$$

$$y_{irk} = 0 \quad \forall i \in V,\ r \in \mathscr{R},\ k \in \mathscr{L} \backslash \mathscr{L}_i \tag{14}$$

$$y_{irk} = 0 \quad \forall r \in \mathscr{R},\ k \in \mathscr{L} \backslash \mathscr{L}_r,\ i \in V \tag{15}$$

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1. Skill Factor
   - $SF \in \{1,\ 0.75,\ 0.5,\ \mathit{variable}\}$
2. Network Complexity
   - $NC \in \{1.5,\ 1.8,\ 2.1\}$
3. Modified Resource Strength
   - varied over 3 values
   - $$MRS = \frac{m}{\sum_{i \in V} \sum_{k \in \mathscr{L}} Req_{ik}}$$

# Experiments: Constraint Choice

Sample of 72 small instances

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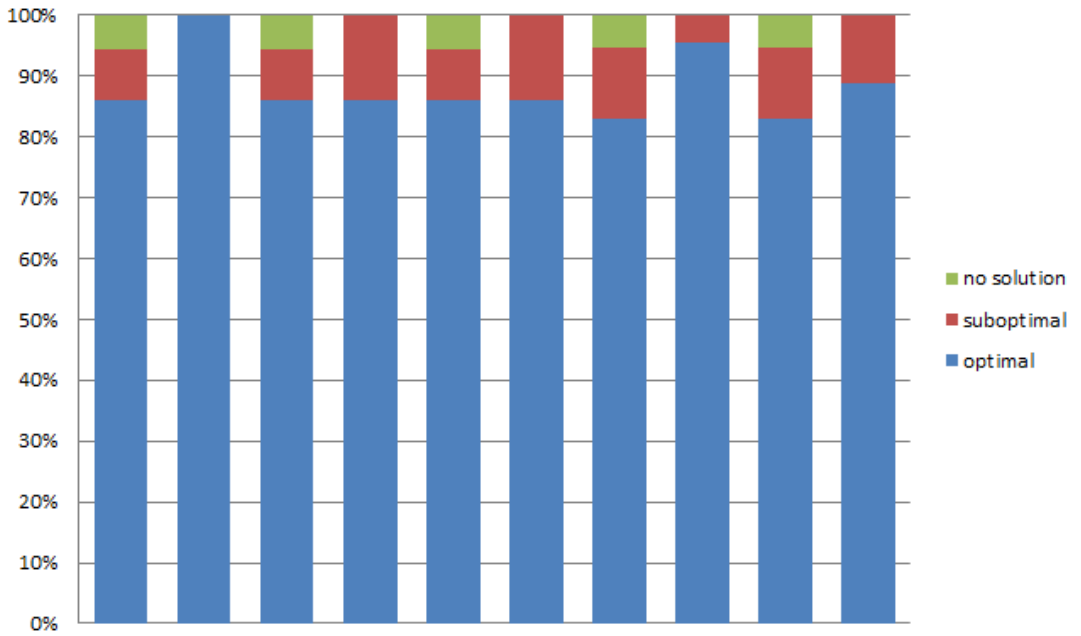


| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| free search | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| edge finding | - | - | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| | bool | | disjunctive | | | | cumulative | | | |

# Experiments: Search Strategies

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- Naïve activity-based

# Experiments: Small Data Set

- Tested on all 216 small instances
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| search strategy | #no soln | %gap | #optimal | %optimal | avg. runtime |
|---|---|---|---|---|---|
| default | 0 | 0.00 | 216 | 100.00 | 3.25s |
| start | 0 | 2.50 | 215 | 99.54 | 1.26s |
| start then worker | 0 | 0.00 | 216 | 100.00 | 2.89s |
| **start then skill** | **0** | **0.00** | **216** | **100.00** | **1.63s** |
| naïve activity-based | 0 | 2.50 | 215 | 99.54 | 0.82s |

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| activity smallest | 0 | 0.00 | 216 | 100.00 | 0.45s |

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| variable selection | #nodes | #props | #no soln | %gap | #opt | %opt | opt rt. | total rt. |
|---|---|---|---|---|---|---|---|---|
| naive | 6,568k | 375m | 0 | 60.36 | 13 | 6.02 | 98.15 | 287.85 |
| **smallest** | **734k** | **71m** | **0** | **53.29** | **15** | **6.94** | **103.51** | **286.35** |
| smallest_largest | 814k | 73m | 0 | 55.50 | 7 | 3.24 | 70.71 | 292.57 |
| first_fail | 1,037k | 80m | 0 | 67.48 | 8 | 3.70 | 113.33 | 293.09 |

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| smallest (fixed) | 3,962k | 409m | 0 | 50.18 | 24 | 11.11 | 55.45 | 272.83 |

# Experiments: Large Data Set

- Time limit of 3600 seconds

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| measure | value | #nodes | #props | %gap | #opt | %opt | opt rt. | total rt. |
|---|---|---|---|---|---|---|---|---|
| SF | 1 | 36m | 4,254m | 48.35 | 7 | 12.96 | 257.55 | 3166.72 |
| | 0.75 | 47m | 4,296m | 49.63 | 4 | 7.41 | 545.44 | 3373.74 |
| | 0.5 | 32m | 4,468m | 42.06 | 20 | 37.04 | 782.20 | 2556.37 |
| | var | 45m | 4,302m | 49.82 | 5 | 9.26 | 130.97 | 3278.79 |
| NC | 1.5 | 41m | 5,007m | 57.33 | 8 | 11.11 | 819.48 | 3291.05 |
| | 1.8 | 42m | 4,381m | 46.09 | 11 | 15.28 | 558.06 | 3135.26 |
| | 2.1 | 36m | 3,602m | 38.99 | 17 | 23.61 | 446.41 | 2855.40 |
| MRS | #1 | 37m | 6,083m | 76.93 | 14 | 19.44 | 327.52 | 2963.68 |
| | #2 | 42m | 4,154m | 43.36 | 8 | 11.11 | 913.89 | 3301.54 |
| | #3 | 39m | 2,753m | 23.94 | 14 | 19.44 | 599.07 | 3016.49 |
| **Overall** | | **40m** | **4,330m** | **47.92** | **36** | **16.67** | **563.43** | **3093.90** |

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# Acknowledgements


- Dr. Andreas Schutt
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- Adrian Goldwaser

Thanks for listening!

Questions?