TEXAS A&M UNIVERSITY

MEEN 689-601 Multidisciplinary System Design Optimization (MSADO)

Spring 2017

Assignment 4

You are expected to solve **Part (a)** individually and **Part (b)** in your project team. Each person must submit their own Part (a) (**and any code created for the assignment**) but you should submit Part (b) as a group. Please indicate the name(s) of your teammate(s).

Topics: genetic algorithms, mixed-integer optimization, scaling

Part (a)

(a-1)

The following questions refer to a Genetic Algorithm as defined in Lecture 13.

a) Which of the following will most increase population diversity?

(a) Increasing mutation rate

(b) Changing the crossover location

(c) Increasing the amount of elitism

(d) All of the above

(e) None of the above

b) In a binary GA, how many bits are needed to represent numbers between 1 and 10 with a resolution of 0.0001?

(a) 3

(b) 4

(c) 16

(d) 17

(e) 18

c) In a binary GA, what is the minimum number of bits required to represent a discrete design variable that can have values 1, 2, 3, or 4?

(a) 1

(b) 2

(c) 3

(d) 4

d) For a discrete variable that can have the values 1, 2, 3, or 4, in the binary representation of this variable with the minimum number of bits, what would the representation of 3 be?

(a) 1

(b) 01

(c) 10

(d) 111

(e) 010

(f) 0110

e) Given the parents 01001101 and 01100100, which of the following are possible single point crossovers?

A: 11001101 B: 01000100 C: 01001100 D: 01111101

(a) A & B

(b) B & C

(c) A & D

(d) A & C

(e) A, B, C, & D

f) In a population, the fitness function values of each member are: $F_1 = 100$, $F_2 = 800$, $F_3 = 1$, $F_4 = 90$, $F_5 = 9$. Using roulette wheel selection, what is the approximate probability the member with F_1 will be chosen as a parent with one spin of the wheel?

(a) 0.01

(b) 0.10

(c) 0.80

(d) 0.90

(e) 0.99

g) In a binary encoded GA with two design variables, one with 4-bits, and one with 16 bits, what is the total number of possible population members?

(a) 2^{18}

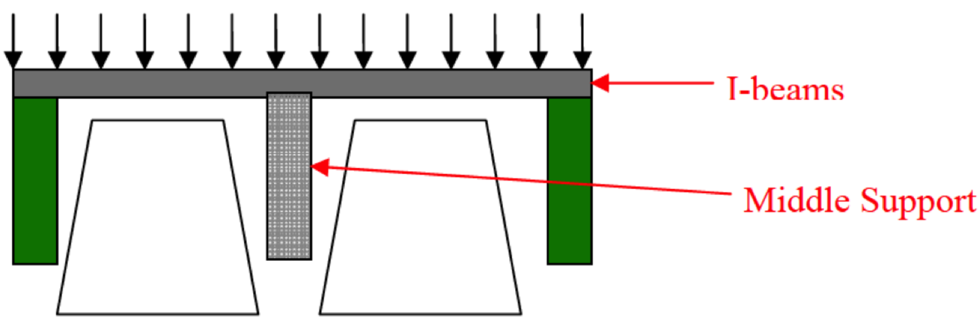
(b) 2^{20}

(c) 2^{24}

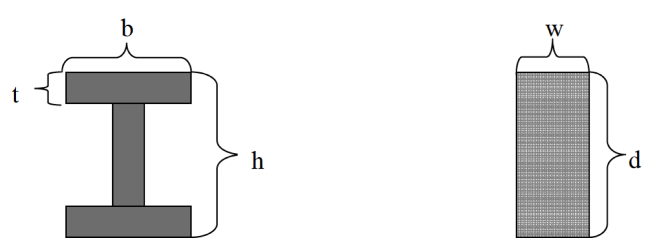
(d) 2^{32}

(a-2)

Your objective is to design the cheapest possible bridge to span a highway. The total span of the bridge (across both halves of the highway) must be $L = 30$ meters, and it must support its own weight and a load $q = 33 \times 10^4$ N/m along its span (a total load of about 1×10^6 N plus its own weight). The bridge span will be supported by between one and four I-beams. In the figure above,



the I-beams would be parallel to each other going into the page, for example it could be one I-beam in the middle of the bridge, or one I-beam on both sides of the bridge, etc. and this will be represented by the design variable, n_{Ibeams}. The shape of the I-beams will be represented by three continuous design variables, the height, h, the flange width, b, and thickness, t. The middle support will be rectangular (when viewed from above), and will have two design variables, the width, w, and depth, d. Let, ρ_{Ibeams} be the density of the material used for the I-beams, then the mass of the



I-beams can be computed using,

$$M_{\text{Ibeams}} = [2bt + (h - 2t)t]\, L \rho_{\text{Ibeams}} n_{\text{Ibeams}}.$$

Let ρ_{Support} be the density of the material used for the support, and $H = 5$m be the height of the bridge above the ground, then the mass of the middle support can be computed using

$$M_{\text{Support}} = wdH\rho_{\text{Support}}.$$

There is a constraint that the stress of the I-beam is less than the material failure stress for the I-beam, $\sigma_{\text{Failure-Ibeams}}$. (Note: g is the gravitational constant (9.81 m/s^2).)

$$\sigma_{\text{Ibeams}} = \frac{q\left(\frac{L}{2}\right)^2 + M_{\text{Ibeams}}\left(\frac{L}{4}\right)g}{8I_{\text{Ibeam}}n_{\text{Ibeams}}}\left(\frac{h}{2}\right) \leq \sigma_{\text{Failure-Ibeams}},$$

where I_{Ibeam} is the moment of inertia for the I-beam given by

$$I_{\text{Ibeam}} = \frac{(h-2t)^3t}{12} + 2\left[\frac{t^3b}{12} + tb\left(\frac{h}{2} - \frac{t}{2}\right)^2\right].$$

In addition, there is a constraint that the shear stress in the I-beams is less than the material failure stress:

$$\tau_{\text{Ibeams}} = \frac{M_{\text{Ibeams}}g + qL}{4\left[2bt + (h-2t)t\right]n_{\text{Ibeams}}} \leq \sigma_{\text{Failure-Ibeams}}.$$

For the middle support there are two constraints, the column cannot buckle and the stress must be less than the material failure stress. Buckling is based on a requirement that the applied load is less than a critical load:

$$P_{\text{Applied}} = \frac{M_{\text{Ibeams}}g + qL}{2} \leq P_{\text{Crit}},$$

where the critical load is a function of th slowest moment of inertia of the support and the modulus of elasticity of the support material, E_{support},

$$P_{\text{Crit}} = \frac{\pi^2 E_{\text{Support}} \min\left\{\frac{w^3d}{12}, \frac{wd^3}{12}\right\}}{4H^2}.$$

The stress requirement is that the applied stress is less than the support material failure stress,

$$\sigma_{\text{Support}} = \frac{P_{\text{Applied}}}{wd} \leq \sigma_{\text{Failure-Support}}.$$

The bridge span (I-beams) can be made from A1 6061, A36 Steel, A514 Steel, or Titanium; however, the support can be made from A1 6061, A36 Steel, A514 Steel, or Concrete. The reason for the difference is that concrete cannot be loaded in tension. The material properties and prices are listed in the Table:

Material	Density (kg/m^3)	Modulus of Elasticity (GPa = 10^9 N/m^2)	Failure Stress (MPa = 10^6 N/m^2)	Cost($/kg)
A1 6061	2700	70	270	2.05
A36 Steel	7850	210	250	0.62
A514 Steel	7900	210	700	0.90
Titanium	4500	120	760	16.00
Concrete	2400	31	70	0.04

Your objective is to find the dimensions of the I-beams, number of I-beams, and material type for the I-beams, as well as the dimensions of the support and material type for the support to minimize cost of the bridge. Where c_{Ibeams} and c_{Support} are the cost per kilogram of the materials used for the I-beams and Support. The total bridge cost is then:

$$C = c_{\text{Ibeams}} M_{\text{Ibeams}} + c_{\text{Support}} M_{\text{Support}}.$$

(a) Please explain what optimization algorithm you chose to find the cheapest possible bridge and why.

(b) What are the design variables and cost for the cheapest bridge?

(a-3)
Repeat the numerical experiments from part (a-3) of Assignment 3, but this time using a heuristic technique of your choice (e.g., SA, GA, ...). Explain how you "tuned" the heuristic algorithm.

Compare your two algorithms (the gradient-based approach and the heuristic approach) from above quantitatively and qualitatively for the three problems as follows:

(i) Dependence of answers on initial design vector (start point, initial population)

(ii) Computational effort (CPU time [sec] or FLOPS)

(iii) Convergence history

(iv) Frequency at which the technique gets trapped in a local maximum

In order to answer this question you will need to implement your algorithms in some way (e.g., Matlab). You may use any tool you wish, but be sure to explain what algorithm is being used.

Part (b)

(b-1) Scaling

Use a gradient-based algorithm as in Assignment 3 for the question below (pick one algorithm). The current optimal solution $\mathbf{x}^*$ refers to the "optimal" solution in your project found using this algorithm in Assignment 3.

(b-1.1) Compute the n diagonal entries of the Hessian at your current optimal solution, $H(\mathbf{x}^*)$. Use finite differencing to evaluate these entries. (Note: if you have a large number of design variables, you may do these steps for any 10 design variables.)

(b-1.2) If any of the entries computed in **(b-1.1)** are greater than 10^2 or less than 10^-2, then the corresponding design variable should be scaled. Note that $H(i,i) \sim 1/x_i^2$. Thus, if $H(i,i) \sim 10^{-4}$, then the appropriate scaling of the design variable x_i is $10^{-2}x_i$. Compute the scaling required for each design variable to make $H(i,i) \sim \mathscr{O}(1)$. Consider only scalings of the form $10^{-2}, 10^{-1}, 10^1, 10^2$, etc, (i.e., worry about magnitude only).

(b-1.3) Redefine your design variable using the scalings computed in **(b-1.2)** and re-run the optimizer, starting from the previous "optimal" solution $\mathbf{x}^*$. do you see any change in the optimal solution?

Please note: If you do not have any continuous design variables in your problem, please instead analyze the scaling of your constraints. Study whether scaling certain constraints in your problem affects convergence or convergence rates.

(b-2) Heuristic Optimization

In Assignment 3 you used a gradient search technique to optimize the system for your design project. In this assignment you are to apply a heuristic technique (SA, GA, PSO, or Tabu search) to your design project.

(b-2.1) Describe which technique you chose and why.

(b-2.2) Attempt to optimize the system and compare the answers with the answers you received with the gradient search technique.

(b-2.3) Tune the parameters of the algorithm and observe the differences in behavior. What appears to be the best algorithm tuning parameter settings for your project?

(b-2.4) How confident are you that you have found the true global optimum?