TEXAS A&M UNIVERSITY

MEEN 689-601 Multidisciplinary System Design Optimization (MSADO)
Spring 2018

Assignment 3

You are expected to solve **Part (a)** individually and **Part (b)** in your project team. Each person must submit their own Part (a) but you should submit Part (b) as a group. Please indicate the name(s) of your teammate(s). **Please submit any code you used to answer the questions in Part (a).**

Topics: sensitivity analysis, gradient-based search algorithms, comparison of algorithm performance, single objective optimization of team design projects

Part (a)

Sensitivity Analysis
(a-1)

In Lecture 11, numerous methods of estimating derivatives were discussed. You are going to test their accuracy on a set of given functions. You should estimate the first-derivative using (1) a first-order finite difference, (2) a second-order central difference, and (3) a complex step estimate. In addition, you should estimate the second derivative using (1) a second-order estimate, and (2) a complex step second derivative estimate. Note, for the complex step, the second derivative estimate is:

$$f''(x) = \frac{2}{\Delta x^2}\left[f(x) - \Re\mathrm{e}\left(f(x+i\cdot\Delta x)\right)\right].$$

Plot the error between the analytical first derivatives and your approximations on one plot and the error between the analytical second derivative and your approximations on a second plot, both using a log-scale (the command loglog() in Matlab). Use step sizes from 1×10^{-15} to 10. (The command logspace() in Matlab may help.)

(a) For the function $f(x) = x^2$, at $x = 1$.

(b) For the function $f(x) = x^3$, at $x = 1$.

(c) For the function $f(x) = e^x$, at $x = 1$.

(d) Comment about the change in the accuracy of the central difference and the complex step between (a) and (b).

(e) Does any step size appear to estimate the first derivative well? The second derivative?

Caution: if the error is zero it will not be shown on a log-scale so please take care in plotting your results.

(a-2)

This problem is to use sensitivity analysis on revenue management for a very simplified airline pricing model. We will assume that an airline has one flight per day from Houston to Boston and they use an airplane that seats 150 people. The airline wishes to determine three prices, p_i, where $(i = 1,2,3)$, one for seats in each of the three fare buckets it will use. The fare buckets are designed to maximize revenue by separating travelers into groups, for instance 14 day advance purchase, leisure travelers, and business travelers. The airline models demand for seats in each group using the formula:

$$D_i = a_i \exp\left(-\frac{1}{a_i} p_i\right),$$

where D_i is the people that want to fly given price p_i. The remaining parameters are $a_1 = 100$, $a_2 = 150$, and $a_3 = 300$. Please note, for simplicity you may assume that each D_i is a continuous variable.

(a) Formulate the revenue maximization problem for this flight as an optimization problem.

(b) What are the optimal prices and how many people are expected to buy a ticket in each fare bucket?

(c) Using sensitivity analysis, if the airline were to squeeze three additional seats onto this flight (i) How much do you expect revenue to change? (ii) By how much should the airline change each price?

Comparison of Optimization Algorithms

(a-3)

Consider the following three optimization problems:

The Rosenbrock Function

This function is known as the "banana function" because of its shape; it is described mathematically in Equation 1. In this problem, there are two design variables with lower and upper limits of $[-5,5]$. The Rosenbrock function has a known global minimum at $[1,1]$ with an optimal function value of zero.

$$\text{Minimize } f(\mathbf{x}) = 100\left(x_2 - x_1^2\right)^2 + (1-x_1)^2 \tag{1}$$

The Eggcrate Function

This function is described mathematically in Equation 2. In this problem, there are two design variables with lower and upper bounds of $[-2\pi, 2\pi]$. The Eggcrate function has a known global minimum at $[0,0]$ with an optimal function value of zero.

$$\text{Minimize } f(\mathbf{x}) = x_1^2 + x_2^2 + 25\left(\sin^2 x_1 + \sin^2 x_2\right) \tag{2}$$

Golinski's Speed Reducer

This hypothetical problem represents the design of a simple gearbox such as might be used in a light airplane between the engine and propeller to allow each to rotate at its most efficient speed. The gearbox is depicted in Figure 1 and its seven design variables are labeled. The objective is to minimize the speed reducer's weight while satisfying the 11 constraints imposed by gear and shaft design practices. A full problem description can be found in: Ray, T., "Golinski's Speed Reducer Problem Revisited," *AIAA Journal*, Vol. 41, No. 3, 2003, pp. 556-558. A known feasible solution obtained by a sequential quadratic programming (SQP) approach (Matlab's fmincon) is a 2994.34 kg gearbox with the following values for the seven design variables: $[3.5000, 0.7000, 7.3000, 7.7153, 3.3502, 5.2867]$. This is a feasible solution with four active constraints, but is it an optimal solution?

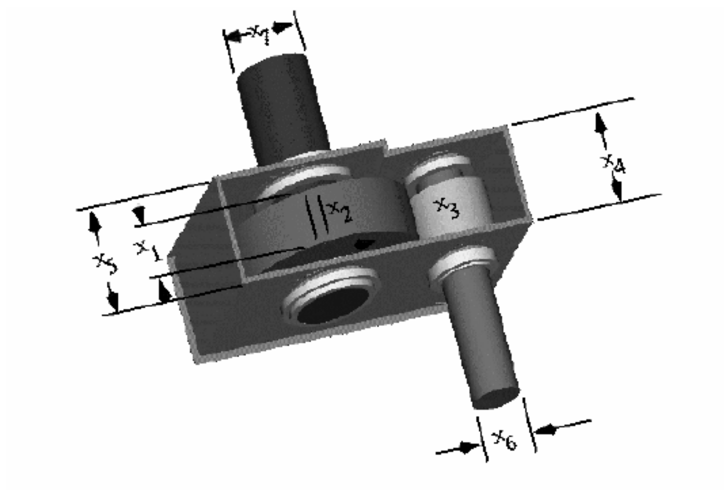

Figure 1: Golinski's speed reducer with 7 design variables

Numerically find the minimum (=optimal) feasible design vector **x** for each of the above three problems using a gradient-based search technique of your choice (e.g., steepest descent, SQP,...). For each run record the starting point you used, the iteration history (objective value on y-axis and iteration number on x-axis), the final point at which the algorithm terminated and whether or not the final solution is feasible. Do at least 10 runs for each problem, but not more than 100.

Discuss the results and insights you get from numerically solving these three nonlinear optimization problems.

(a-4)
Repeat the numerical experiments from part (a-3), but this time using a heuristic technique of your choice (e.g., SA, GA, ...). Explain how you "tuned" the heuristic algorithm.

Compare your two algorithms (the gradient-based approach and the heuristic approach) from above quantitatively and qualitatively for the three problems as follows:

(i) Dependence of answers on initial design vector (start point, initial population)

(ii) Computational effort (CPU time [sec] or FLOPS)

(iii) Convergence history

(iv) Frequency at which the technique gets trapped in a local maximum

In order to answer this question you will need to implement your algorithms in some way (e.g., Matlab). You may use any tool you wish, but be sure to explain what algorithm is being used.

Part (b)

In this assignment you are to take the simulation code that you developed for your project in assignment 2, refine it and couple it with an optimizer. First you should use a gradient-based technique. If you have non-continuous variables keep them at fixed values, or assume they are continuous. (We will use heuristic techniques on your assignment in a later assignment.)

(b-1) Simulation completion
Complete the simulation code you started developing under part (b1) in assignment A2. Replace "placeholder" modules with actual code and rerun the entire analysis. Select interesting design points based on what you learned in (b3) of assignment A2. What are remaining open issues in your project?

(b-2) Gradient-based optimization

b2.1 Algorithm Selection Select a gradient-based algorithm based on the characteristics of your project and the properties of the available algorithms. Rationalize in a few sentences, why your selection seems most appropriate for the problem at hand.

b2.2 Single objective optimization Select a single (scalar) objective function for which to optimize your system. Describe why you selected this objective. Other potential objectives should be turned into equality or inequality constraints or ignored (for now). Using the gradient-based optimization technique identified in (b2.1), try to optimize your system with respect to the one objective function. Can you get the algorithm to converge? Do you obtain an improvement in the design compared to your initial starting point? If not, please give some reasons. You may use any optimization environment of your choice (e.g., Matlab), but please specify in your write-up what you used. What is the optimal solution $\mathbf{x}^*$?

b2.3 Sensitivity analysis Conduct a sensitivity analysis at the optimal point $\mathbf{x}^*$ with respect to $\mathbf{x}$, and a few of your fixed parameters, p. What design variables seem to be the drivers in your problem? Does this match the intuition you had beforehand? What are the active constraints at $\mathbf{x}^*$? How can you tell? Try moving the most important active constraint by some amount. Reoptimize and compare the new optimum with the previous optimum, what do you observe?

b2.4 Global optimum How confident are you that you have found the true global optimum? Explain.

Note: Keep all the results from this assignment handy for A4, where we will extend the work on your project by considering multiple objectives.