Georgia Institute of Technology

Physics 3201 – Classical Mechanics I

Quiz #2

October 26, 2020; 9:30am – 10:20am

Instructor:	Prof. D. Ballantyne
Time Allowed:	50 minutes + 10 minutes to submit to the Instructor

To Be Answered On Paper, Scanned and Uploaded to Canvas

This exam consists of 3 multi-part problems. Each question is marked out of 10. The exam is out of 30. Answer all questions. Make sure to include justification for your answers. *Correct answers without appropriate justification may not be awarded full (or any) marks.*

This exam consists of 3 pages, including the cover page (this one). Students should count the pages on their exam prior to beginning and report any omissions to their instructor.

This is an open-notes/open-book exam. Students may consult any class materials posted to Canvas, the assigned textbook, or the other books listed under "Other Resources" in the class syllabus. No other resources, either on-line (i.e., Chegg.com) or physical (i.e., your roommate's notes), can be used while taking this test. Communication between students is also not allowed during the exam window. However, students are encouraged to contact the instructor if they need clarification on any of the test questions.

Students who make use of unauthorized materials, communicate with each other during the exam, or appear to engage in similar dishonest practices may be dismissed from the exam and subject to further academic discipline.

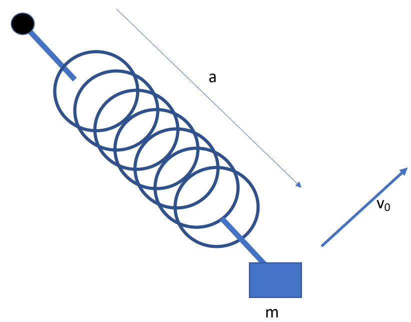

Figure 1: Diagram for Problem #1. The system lies in a horizontal plane.

1. (10 pts) A bead of mass m is attached to the end of a weightless spring of equilibrium length a, whose other end is fixed, so that the spring is free to rotate in a horizontal plane (Fig. 1). The force exerted by the spring is k times its extension.

   At time $t = 0$ the bead is given an impulse that starts it moving at right angles to the spring with velocity v_0. The maximum radial distance attained is $2a$.

   (a) (2 pts) Write down the equations of motion in polar coordinates.

   (b) (3 pts) Use conservation of angular momentum to show that particle's speed when it reaches its maximum radial distance is $v = v_0/2$.

   (c) (3 pts) Use conservation of energy to show that, given this motion, v_0 must have been given by

   $$v_0^2 = \frac{4}{3}\frac{ka^2}{m}$$

   (d) (2 pts) Find the value of $\ddot{r}$ when $r = 2a$ in terms of only k, a, and m.

2. (10 pts) Consider the orbit of a test particle of mass m experiencing the isotropic central force $\vec{f} = f(r)\hat{r}$. If θ is the polar angle and $u = 1/r$, then the particle's orbit can be described by $u = a(1 + b\theta)$, where a and b are constants.

   (a) (5 pts) Find the force function, $f(r)$, responsible for this motion.

   (b) (5 pts) If $\theta = 0$ at time $t = 0$, show that

   $$\theta(t) = \frac{\ell a^2 t}{1 - \ell a^2 bt},$$

   where $\ell = r^2\dot{\theta}$.
   (FYI: $\int \frac{dx}{(a+bx)^2} = -\frac{1}{b(a+bx)}$)

3. (10 pts) Consider a 3D linear isotropic harmonic oscillator with frequency $\omega_0^2 = k/m$ where k and mass m are constants.

   (a) (3 pts) What is the potential energy function $V(r)$ for this oscillator?

   (b) (7 pts) Suppose that at time $t = 0$, the oscillator is at the origin (i.e., $\vec{r} = 0$) and has speed $|\vec{v}| = v_0$. What is the maximum value of $|\vec{r}|$ attained by the particle, r_{max}?