Georgia Institute of Technology

Physics 3201 – Classical Mechanics I

Quiz #1

September 28, 2020, 9:30am – September 29, 2020, 11:30am

Instructor: Prof. D. Ballantyne

Time Allowed: 26 hours

To Be Answered On Paper, Scanned and Uploaded to Canvas

This exam consists of 3 multi-part problems and 1 single-part problem. All problems are marked out of 10. The exam is out of 40. Answer all questions. Make sure to include justification for your answers. *Correct answers without appropriate justification may not be awarded full (or any) marks.*

This exam consists of 2 pages, including the cover page (this one). Students should count the pages on their exam prior to beginning and report any omissions to their instructor.

This is an open-notes/open-book exam. Students may consult any class materials posted to Canvas, the assigned textbook, or the other books listed under "Other Resources" in the class syllabus. No other resources, either on-line (i.e., Chegg.com) or physical (i.e., your roommate's notes), can be used while taking this test. Communication between students is also not allowed during the exam window. However, students are encouraged to contact the instructor if they need clarification on any of the test questions.

Students who make use of unauthorized materials, communicate with each other during the exam, or appear to engage in similar dishonest practices may be dismissed from the exam and subject to further academic discipline.

1. (10 pts) Suppose an electron of mass m_e moves in the xy plane in such a way that its polar coordinates are

$$R(t) = R_0 e^{-\lambda t},\ \phi(t) = \lambda t,$$

where R_0 and λ are constants. Find the net force on the electron in Cartesian coordinates, $\vec{F} = F_x\hat{i} + F_y\hat{j}$, where the components (F_x, F_y) are given as explicit functions of x and y.

2. (a) (3 pts) For a particle with mass m subject to a conservative force, the total energy is (in one dimension)

$$E = \frac{1}{2}mv^2 + V(x),$$

where $V(x)$ is the potential energy. Differentiate this expression with respect to time t to show that E is a constant provided Newton's Second Law holds.

(b) (7 pts) Consider a particle with mass m in a conservative force field described by a potential energy function $V(x) = A\cos kx$, where A and k are constants. If the particle velocity is $v = 0$ at $x = 0$ find its velocity for arbitrary values of x, $v(x)$.

3. (10 pts) A moving object with mass m can be described with the potential energy function $V(x) = mk|x|$, where $k > 0$ is a constant.

   (a) (3 pts) What is the force when $x > 0$, and when $x < 0$?

   (b) (3 pts) Sketch the function V and describe the motion.

   (c) (4 pts) If the object starts from rest at $x = -a$, calculate the time it takes to reach $x = a$.

4. (10 pts) There exists a damped oscillator subject to a periodic driving force $F(t) = F_1\cos\omega_1 t$. The oscillator has been running for a long enough time that the transient motion has disappeared.

   (a) (5 pts) Show that when the system is in steady-state (i.e., $dE/dt = 0$), the power P, the rate at which the force does work, exactly balances the rate at which energy is dissipated against the resistive force.

   (b) (2 pts) Show that the time average power is $\langle P\rangle = m\gamma\omega_1^2 a_1^2$.

   (c) (2 pts) Show that the average power $\langle P\rangle$ is a maximum, as a function of ω_1, at $\omega_1 = \omega_0$.

   (d) (1 pts) Find the values of ω_1 for which $\langle P\rangle$ has half its maximum value.