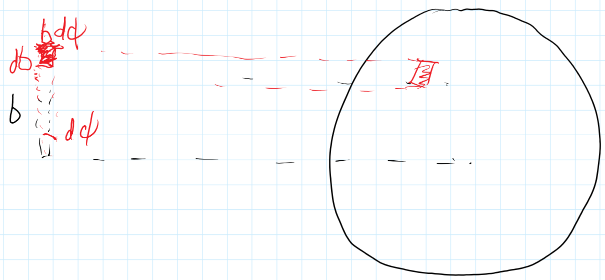
A small change in impact parameter db will lead to a change in the scattering angle $d\Theta$:

$$db = -\frac{1}{2} R \sin\left(\frac{1}{2}\Theta\right) d\Theta$$

Now, consider a small amount of target area $d\sigma = b|db|d\phi$




$$\therefore d\sigma = \frac{R^2}{2} \cos\left(\frac{\Theta}{2}\right) \sin\left(\frac{\Theta}{2}\right) d\phi \, d\Theta$$

$$= \frac{R^2}{4} \sin\Theta \, d\Theta \, d\phi$$

or, $d\sigma = \frac{R^2}{4} d\Omega$ where $d\Omega = \sin\Theta d\Theta d\phi$ is the solid angle subtended by the area. Units of steradians. $\int d\Omega$ over all angles you get $4\pi$.

$\rightarrow \frac{d\sigma}{d\Omega}$ = differential cross-section $\bar{c}$ tells us how particles are scattered into diff. angles (has units of area)

For scattering by hard spheres $\frac{d\sigma}{d\Omega} = \frac{R^2}{4}$ (isotropic). Other force laws will give different $\frac{d\sigma}{d\Omega}$.

Rutherford Scattering. Firing a +vely charged particle $q$ at a positively charged nucleus w/ charge $Q$. In this case $K = \frac{qQ}{4\pi\epsilon_0}$. The orbit is hyperbolic, so $b = \frac{|K|}{mv^2} \cot\frac{\Theta}{2}$

$$\therefore b = \frac{qQ}{4\pi\epsilon_0 mv^2} \cot\frac{\Theta}{2}$$

$$\therefore\ db = -\frac{qQ}{8\pi\epsilon_0 mv^2}\csc^2\frac{\Theta}{2}\,d\Theta = \frac{-qQ}{8\pi\epsilon_0 mv^2\sin^2\left(\frac{\Theta}{2}\right)}\,d\Theta$$

Know $d\sigma = b|db|d\phi = \dfrac{q^2Q^2\cos\left(\frac{\Theta}{2}\right)d\Theta\,d\phi}{32\pi^2\epsilon_0^2 m^2 v^4\sin^3\left(\frac{\Theta}{2}\right)}$

$\div$ both sides by $d\Omega = \sin\Theta\,d\Theta\,d\phi$

$$\frac{d\sigma}{d\Omega} = \frac{q^2Q^2}{32\pi^2\epsilon_0^2 m^2 v^4}\,\frac{\cos\left(\frac{\Theta}{2}\right)}{\sin(\Theta)\sin^3\left(\frac{\Theta}{2}\right)}$$

$$\sin(\Theta) = \sin\left(2\left(\frac{\Theta}{2}\right)\right) = 2\sin\left(\frac{\Theta}{2}\right)\cos\left(\frac{\Theta}{2}\right)$$

$$\frac{d\sigma}{d\Omega} = \frac{q^2Q^2}{64\pi^2\epsilon_0^2 m^2 v^4}\,\frac{1}{\sin^4\left(\frac{\Theta}{2}\right)}$$

if $E = \frac{1}{2}mv^2 \rightarrow E^2 = \dfrac{m^2v^4}{4}$

$$\boxed{\therefore\ \frac{d\sigma}{d\Omega} = \frac{q^2Q^2}{256\pi^2\epsilon_0^2 E^2}\,\frac{1}{\sin^4\left(\frac{\Theta}{2}\right)}}$$

Rutherford scattering cross-section
- → strong angular dependence
- → strong energy dependence
- → charge dependence

Ex: An $\alpha$-particle emitted by radium w/ $E = 5\text{ MeV}$ suffers a deflection of 90° upon passing a gold nucleus. What is the value of the impact parameter?

$\alpha$ particle has charge $2e$

gold nucleus has charge $79e$

so $b = \dfrac{qQ}{4\pi\epsilon_0 mv^2}\cot\left(\dfrac{\Theta}{2}\right) = \dfrac{158e^2}{4\pi\epsilon_0 2E}\cot(45°)$

$\dfrac{1}{4\pi\epsilon_0} = 9\times10^9\,\dfrac{Nm^2}{C^2}$, $e = 1.60\times10^{-19}\,C$

$$\frac{1}{4\pi\epsilon_0} = 9\times10^9 \frac{N m^2}{C^2}, \quad e = 1.60\times10^{-19}\,C$$

$$E = 5\times10^6\,eV \left(\frac{1.6\times10^{-19}\,J}{1\,eV}\right) = 8\times10^{-13}\,J$$

$$\therefore\ b = \frac{158\left(1.6\times10^{-19}\,C\right)^2\left(9\times10^9\,\frac{N m^2}{C^2}\right)}{2\left(8\times10^{-13}\,J\right)\tan 45^\circ} = 2.3\times10^{-14}\,m$$

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## Rotating or Non-Inertial frames

Up to now we have always considered motion in an inertial frame, but some problems are more convienent to solve in a rotating frame. Also, when considering motion close the Earth's surface, non-inertial forces are crucial to understanding motion (e.g. weather systems). Here, we'll show that converting eq'ns of motion from inertial to rotating frames lead to new 'forces' that appear to influence motion in the rotating frame.