SOLUTIONS

Quiz 2, 1:30, Math 1554, Spring 2020

Administer on Thu Feb 13, 2020 at 1:30 am

PLEASE PRINT YOUR NAME CLEARLY IN ALL CAPITAL LETTERS

First Name _____ Last Name _____

GTID Number: _____

Student GT Email Address: _____@gatech.edu

Section Number (e.g. A4, M2, QH3, etc.) _____ TA Name _____

Circle your instructor:

Dr. Barone, Dr. Mayer, Dr. Belegradek

Student Instructions

- Print your name and GTID darkly and neatly on the cover page.
- You will have 20 minutes to complete this quiz.
- Notes, books, cell phones, and all electronic devices are not allowed.
- Use dark and clear writing: your exam will be scanned into a digital system.
- The quiz is 1 page and double sided. Be sure to complete both sides.
- Leave a 1 inch border around the edges of exams.
- Any work done on scratch paper will be collected and will not be graded.

Quiz 2, 1:30. Your initials: _____

You do not need to justify your reasoning for questions on this page.

1. (3 points) Indicate whether the following statements are true or false.

	true	false
a) If A and B are $n \times n$ and invertible, then the inverse of AB is $A^{-1}B^{-1}$.	◯	◯
b) If A is an $n \times n$ matrix, and the equation $A\vec{x} = \vec{b}$ does not have at least one solution for every $\vec{b}$ in $\mathbb{R}^n$, then A is not invertible.	◯	◯
c) All elementary matrices can be reduced to the identity matrix by using only one row operation.	◯	◯

For each part, give 1 point for the correct answer, zero points otherwise. No work needed to be shown, and no partial credit should be awarded. Solutions:
a) False. The inverse of AB is $B^{-1}A^{-1}$.
b) True. A cannot have pivots in every row, and so A cannot be invertible.
c) True.

2. (7 points) Fill in the blanks.

(a) (1 point) If A, B, X, and Y are invertible $n \times n$ matrices and $\begin{pmatrix} 0 & A \\ A & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} B \\ C \end{pmatrix}$, express X in terms of A and C. $X =$ $\boxed{}$

(b) (2 points) $A = \begin{pmatrix} 1 & 1 \\ 2 & 5 \end{pmatrix}$ has an LU factorization with $L = \begin{pmatrix} & \\ & \end{pmatrix}$ and $U = \begin{pmatrix} & \\ & \end{pmatrix}$.

(c) (1 point) The inverse of the matrix $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$ is $A^{-1} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$.

(d) (1 point) Consider the production model $\vec{x} = C\vec{x} + \vec{d}$ for an economy with two sectors, where $C = \begin{pmatrix} .0 & .5 \\ .6 & .2 \end{pmatrix}$, and $\vec{d} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$. The augmented matrix which can be used to solve the system for $\vec{x}$ is $\begin{pmatrix} & \\ & \end{pmatrix}$.

(e) (1 point) A 4×4 matrix which performs a translation by the vector $\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}^T$, using homogeneous coordinates, is $\begin{pmatrix} & & & \\ & & & \\ & & & \\ & & & \end{pmatrix}$.

(f) (1 point) Suppose A has an LU factorization $A = LU$, with $U = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$. How many pivot columns does A have? $\boxed{}$

For each part, give 1 point for the correct answer, zero points otherwise. No work needed to be shown, and no partial credit should be awarded. Solutions:

a) $AX = C \Rightarrow X = A^{-1}C$. And note that $X \neq CA^{-1}$

b) $L = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, U = \begin{pmatrix} 1 & 1 \\ 0 & 3 \end{pmatrix}$. Give 1 point for L, 1 point for U.

c) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$.

d) $\left(\begin{array}{cc|c} 1 & -.5 & 5 \\ -.6 & .8 & 3 \end{array}\right)$. Students could, technically, apply any number of row operations to this augmented matrix, but it is not necessary to do so.

e) $\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$.

f) 2.