Homework 8

Due Date: All homework submitted by Sunday **11/01** 11:59pm will be graded together. Homework submitted past that time may be graded late. Submit your homework through Canvas as a single pdf file. Do not use solution sets from previous years. You are encouraged to discuss homework assignments with each other, the TAs or myself, but the solutions have to be executed and submitted individually.

Problem A [33%]. A point charge $+q$ is embedded at the center of a linear isotropic dielectric sphere (susceptibility χ and radius R).

(1) Find the electric field everywhere.

(2) Find the polarization everywhere.

(3) Find the bound charge density at the surface and in the interior of the dielectric sphere.

(4) Show that the surface and interior bound charges exactly compensate so that the entire system has a total charge $+q$.

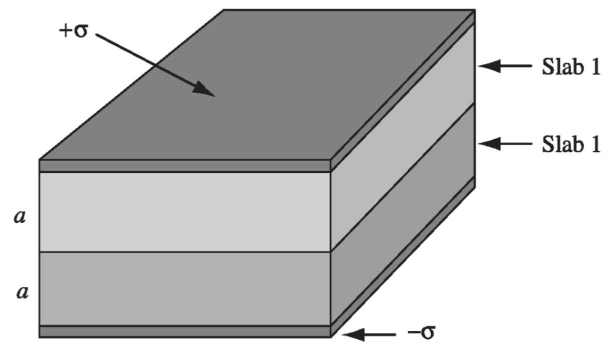
Problem B [33%]. Consider a very long rod (cylinder) of radius a. This rod is made of a very special dielectric material, $\mathrm{BaTiO_3}$, that carries a permanent polarization (for temperatures below 120°C, no need to apply an electric field to polarize the material, it is *spontaneously* polarized). The polarization is spatially non-uniform inside the cylinder with $\mathbf{P}(s, \theta, z) = C\mathbf{s} = Cs\hat{\mathbf{s}}$, where s is the usual cylindrical radial vector from the z-axis, and C is a positive constant. Neglect end effects: the cylinder is very long.

(1) Calculate the bound charges σ_b and ρ_b (on the surface, and in the interior of the rod respectively). What are the units of C? Sketch the charge distribution of the rod.

(2) Next, use the bound charges, along with Gauss' law to find the electric field inside and outside of the cylinder. Provide the direction and magnitude of the electric field.

(3) Find the electric displacement field $\mathbf{D}$ inside and outside the cylinder, and verify that Gauss's law works for $\mathbf{D}$.

Problem C [33%]. The space between the plates of a parallel-plate capacitor is filled with two slabs of linear dielectric material. Each slab has thickness a, so the total distance between the plates is $2a$. Slab 1 has a dielectric constant of 2, and slab 2 has a dielectric constant of 1.5. The free charge density on the top plate is σ and on the bottom plate $-\sigma$



(1) Find the electric displacement $\mathbf{D}$ in each slab.

(2) Find the electric field $\mathbf{E}$ in each slab.

(3) Find the polarization $\mathbf{P}$ in each slab.

(4) Find the potential difference between the plates.

(5) Find the location and amount of all bound charge.