Sample Quiz 3, Math 1554

PLEASE PRINT YOUR NAME CLEARLY IN ALL CAPITAL LETTERS

First Name _____ Last Name _____

GTID Number: _____

Student GT Email Address: _____@gatech.edu

Section Number (e.g. A4, QH3, etc.) _____ TA Name _____

Student Instructions

- **Show your work** and justify your answers for all questions unless stated otherwise.
- **Organize your work** in a reasonably neat and coherent way.
- Calculators, notes, cell phones, books are not allowed.
- Use dark and clear writing: your exam will be scanned into a digital system.
- Exam pages are double sided. Be sure to complete both sides.
- Leave a 1 inch border around the edges of exams.
- Any work done on scratch paper will not be collected and will not be graded.

You do not need to justify your reasoning for questions in this quiz.

1. (6 points) Fill in the blanks.

   (a) An eigenvector of $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$ is $\vec{v}_1 = \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}$.

   What is the eigenvalue associated with eigenvector $\vec{v}_1$? $\boxed{}$

   (b) For what values of k (if any) does $A = \begin{pmatrix} -2 & k \\ -1 & 0 \end{pmatrix}$ have exactly two distinct real eigenvalues? $\boxed{}$

   (c) For what values of k (if any) is $A = \begin{pmatrix} 2 & 0 \\ k & 2 \end{pmatrix}$ diagonalizable? $\boxed{}$

   (d) The characteristic polynomial of A is $(\lambda - 1)^2(\lambda - 3)\lambda^6$.

   - What is the algebraic multiplicity of the eigenvalue $\lambda = 1$? $\boxed{}$
   - What are the dimensions of matrix A? $\boxed{}$
   - What is the value of $\det(A)$? $\boxed{}$

2. (4 points) Suppose A is an $n \times n$ matrix. Fill in the circles next to the **true** statements; leave the others empty.

   ◯ If 2 is an eigenvalue of A, and A is invertible, $\frac{1}{2}$ is an eigenvalue of A^{-1}.

   ◯ An example of a regular stochastic matrix is $P = \frac{1}{10}\begin{pmatrix} 9 & 2 & 1 \\ 0 & 7 & 8 \\ 1 & 1 & 1 \end{pmatrix}$.

   ◯ If v_1 and v_2 are linearly independent eigenvectors, they must correspond to distinct eigenvalues.

   ◯ If a stochastic matrix is not regular then it cannot have a steady state.