

To get a rough estimate of the effects of the tidal field consider a perfectly smooth, spherical Earth completely covered w/ an ocean that responds immediately to the field. So, the height of the ocean $h(\theta)$ is an equipotential surface completely in balance b/w the Earth's & Moon's grav. pull

$$g_E h(\theta) = \frac{GM r^2}{a^3} \left(\frac{3}{2} \cos^2 \theta - \frac{1}{2} \right)$$

$$\text{where } g_E = \frac{GM}{r^2}$$

$$\therefore h(\theta) = h_0 \left(\frac{3}{2} \cos^2 \theta - \frac{1}{2} \right) \quad \text{where } h_0 = \frac{m r^4}{M a^3}$$

for the Moon this is $h_0 \approx 0.36 \text{ m}$; for the Sun $h_0 \approx 0.16 \text{ m}$

These are approx. the right magnitudes for the tidal bulges in the deep oceans. Coastal tides depend strongly on local topography ; resonance may also enhance tides.

Two-Body Problems

Collisions

We are interested in 2 bodies interacting through very rapid internal forces that are only large when the bodies are close together (External forces like gravity are assumed to affect each particle equally ; can be ignored).

As these are internal forces they are equal & opposite so the total linear momentum of the system is conserved:

$$\vec{p}_1 + \vec{p}_2 = \text{constant} \quad \text{where } 1, 2 \text{ are the 2 bodies}$$

$$\text{or } m_1 \vec{u}_1 + m_2 \vec{u}_2 = m_1 \vec{v}_1 + m_2 \vec{v}_2 \quad \begin{array}{l} (\vec{u}_1, \vec{u}_2) \text{ before collisions} \\ (\vec{v}_1, \vec{v}_2) \text{ after collision} \end{array}$$

Most collision forces are conservative (eg. electrostatic repulsion) so can write energy conservation:

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + Q$$

when Q is the net gain or loss in KE that occurs in the collision.

If $Q = 0 \rightarrow$ elastic collision

$Q > 0$, energy loss

$Q < 0$, energy gain (say, an explosion during collision)

Direct Collisions:

Collisions that occur on a single straight line



