MIDTERM 2
MATH 3215-C (PROBABILITY AND STATISTICS)

TUESDAY, OCTOBER 20

INSTRUCTOR: ARMENAK PETROSYAN

IMPORTANT: Please read carefully (1pt)

- You have a **12 hour** window to take and submit your exam (**7 am - 7 pm**).
- Be warned: **exam ends at 7 pm** (e.g. if you start at 6 pm, you only have 1 hour).
- After you opened this file you have 100 minutes to finish the work and 20 minutes to submit it **(120 minutes in total)**.
- If you run into difficulties submitting on GradeScope, email the files to the instructor before the 120 minutes expire and before 7 pm. **Late submissions will not be accepted**.
- If you encounter technical problems, email the instructor as soon as possible.
- You **CAN** use the course textbook and the lecture notes/slides for reference.
- You **CAN** use any fact we presented in class without proving them; anything else used must be proved.
- You **CANNOT** get any help from or collaborate with anyone.
- Posting the problems online to get help or to let others know what the problems are will be a violation; it will be reported and result in a penalty.
- **To get full credit you need to write complete answers.**
- Numerical answers must be up to 4 decimal precision.
- The total amount of points for this exam is 75. Different problems have different weights.
- Be wise with your time. You can handwrite your answers on a different paper, and submit a photocopy. Make sure it is readable. No need to print the problem sheet or copy the problems.

Calculator and software use

- You can use a calculator for arithmetic computations.
- To find the values of standard distributions, you **MUST** use the tables in the appendix of the textbook (e.g. if it is asking to find a cdf value for the normal distribution, you need to reduce to the standard normal and find the value from the table in the back of the book).
- You may verify your answers for yourself with a calculator.
- You can plot the graphs by hand or you may use any graphical software.

Problem 1 (15pt). *For the following functions, check if there exists a number c for which $f(x)$ is a pdf.*
(a) $f(x) = c(1 - x^2)$, $x \in [-1, 1]$.
(b) $f(x) = c(2 - x^2)$, $x \in [-2, 2]$.
If such c exists, compute the expected value and the variance of a random variable X with pdf $f(x)$.

Problem 2 (10pt). *Assume X has $N(\mu, \sigma^2)$ distribution (normal distribution with mean μ and variance σ^2). Find the probability that the value of X is at most 1.55σ distance away from the mean.*

Problem 3 (12pt). *A web page gets 150 visits on average per hour. Assuming visits are governed by a Poisson process, what is the probability that the first 50 visits will happen within the first 50 minutes?*

(Hint: let X be the time of the 50th visit. Observe that the distribution of $Z = 2\frac{60}{150}X$ is one of the distributions with a table in the Appendix of the textbook).

Problem 4 (12pt). *Assume the length of a side of a cube is a random variable that has exponential distribution with parameter $\theta = 2$. Compute the expected volume of the cube.*

Problem 5 (20pt). *The following table contains values of the joint pmf of two discrete random variables. The top row and the leftmost column are the corresponding ranges of the random variables.*
(a) Are the random variables independent. Explain your answer.
(b) Compute the least squares line:

$$y = \frac{\text{Cov}(X, Y)}{\text{Var}(X)}(x - \mu_X) + \mu_Y.$$

(c) Compute the conditional mean $m(x) = \mu_{Y|x}$.

Y \ X	-1	0	1	2
-1	0.01	0.04	0.03	0.1
0	0.08	0.22	0.24	0.06
1	0.03	0.12	0.03	0.04

TABLE 1.

Problem 6 (5pt)**.** *Let X and Y be two discrete random variables on the same space of outcomes S. We proved the following two facts:*

(1) If X and Y are independent then $\mathrm{Cov}(X, Y) = 0$ *(in class).*

(2) If X and Y are independent then, for any two functions $g, h : (-\infty, \infty) \to (-\infty, \infty)$, the random variables $g(X)$ and $h(Y)$ are also independent (as a homework problem).

Consequently, if X and Y are two independent random variables then, for any two functions $g, h : (-\infty, \infty) \to (-\infty, \infty)$,

$$\mathrm{Cov}(g(X), h(Y)) = 0.$$

Show that the opposite of the above statement is true as well: if, for any two functions $g, h : (-\infty, \infty) \to (-\infty, \infty)$,

$$\mathrm{Cov}(g(X), h(Y)) = 0$$

then X and Y are independent.

(Hint: fix any $(x, y) \in \mathrm{Range}(X, Y)$ and select appropriate functions f, g such that $\mathrm{Cov}(X, Y) = 0$ becomes $f(x, y) = f_X(x) f_Y(y)$.)