# Laws of Mechanics

Considering an isolated system of N bodies

$$i = 1, \ldots, N$$

isolated = all other bodies are so remote they do not influence our system (no external forces)

The position of the $i^{th}$ body wrt an inertial frame is $\vec{r}_i(t)$

inertial = non-accelerating frame

The velocity & acc'n of the particle is $\dot{\vec{r}}_i(t)$ & $\ddot{\vec{r}}_i(t)$

Each body is characterized by a scaler constant, called mass $\rightarrow m_i$

Define the body's momentum as $\vec{p}_i = m_i \vec{v}_i = m_i \dot{\vec{r}}_i$

The equation of motion specifies how the body will move

$\dot{\vec{p}}_i = m_i \ddot{\vec{r}}_i = \vec{F}_i$ where $\vec{F}_i$ is the total force acting on the body [Newton's $2^{nd}$ Law]

This force is composed of the sum of forces due to the other bodies in the system. Denoting the force on the $i^{th}$ body due to the $j^{th}$ body by $\vec{F}_{ij}$ then

$$\vec{F}_i = \sum_{j=1}^{N} \vec{F}_{ij} \quad \text{(vector sum)}$$

The 2-body forces $\vec{F}_{ij}$ must satisfy Newton's $3^{rd}$ Law which states that action & reaction are equal & opposite

$$\vec{F}_{ji} = -\vec{F}_{ij}$$

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$\vec{F}_{ij}$ is a function of only the relative positions & velocities of the bodies $\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$ & $\vec{v}_{ij} = \vec{v}_i - \vec{v}_j$

∴ Any inertial frame will give same answer or experimental result.

Newton's 1st Law ($\vec{v}_i = \text{const}$ when $\vec{F}_i = 0$)

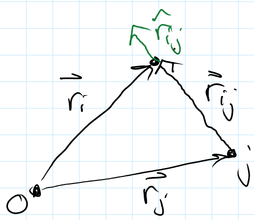
→ An inertial frame is one where this Law holds.

If Newton's Laws are valid in 1 reference frame, then they are valid in any other frame that is in unifor motion wrt it (Newtonian relativity).

If forces are known as functions of position & velocities then e.o.m. will predict future motion of bodies. Given initical conditions, solve thes eqns to find position at any later time.

Goal of Classical Mechanics

Need to identify forces. An important class are central, conservative forces & have the form $\vec{F}_{ij} = f(r_{ij})\,\hat{r}_{ij}$




if $f(r_{ij}) > 0$ the force is repulsive

$f(r_{ij}) < 0$ the force is attractive

e.g. gravity $f(r_{ij}) = -\dfrac{G m_i m_j}{r_{ij}^2}$, electrostatic

The chief feature of conservative forces is the existence of a quantity which is conserved, the energy of the system

(Frictional forces, e.g., have the effect of moving energy from large-scale motion to small-scale motions in the interior of, bodies [heat!] ∴ appear non-conservative on a large scale)

## One-Dimensional Motion

Choose the x-axis to lie in the direction of the particle's motion. Then e.o.m. $m\ddot{x} = F(x, \dot{x}, t)$

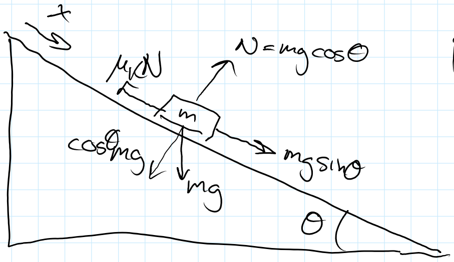
a) $F = \text{const.}$

$$\ddot{x} = \frac{dv}{dt} = \frac{F}{m} = \text{constant} = a$$

$\therefore\ \dot{x} = v = at + v_0$ where $v_0$ = initial velocity

and $x = \frac{1}{2}at^2 + v_0 t + x_0$ where $x_0$ = initial position

Application? A body falling freely near Earth's surface (neglecting air resistance). In that case $a = |\vec{g}| = 9.8\,\mathrm{m\,s^{-1}}$ ∴ $F = mg$ = weight




Block on inclined plane.

if smooth plane $\ddot{x} = \frac{F}{m} = g\sin\theta$

What is rough plane w/ frictional force ∝ normal force

$$f = \mu_K N = \mu_K mg\cos\theta$$

where $\mu_K$ = coefficient of kinetic friction

Then $\ddot{x} = (\sin\theta - \mu_K\cos\theta)g$

∴ Block will accelerate if $\sin\theta > \mu_K\cos\theta$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta > \mu_K$$

$$\theta > \tan^{-1}\mu_K$$

angle of kinetic friction

What happens if $\theta = \tan^{-1}\mu_K \Rightarrow a = 0$