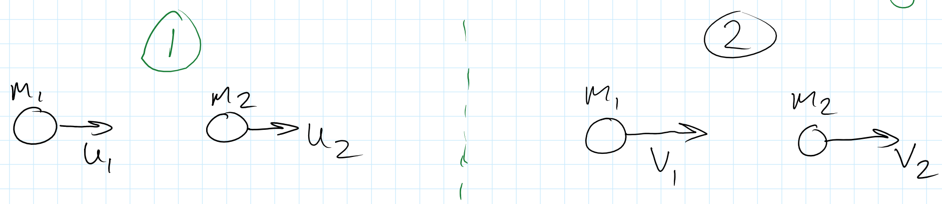
Direct Collisions:

Collisions that occur on a single straight line



Since everything is on one line, momentum conservation eq'n is $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$ (note: u_1, u_2, v_1, v_2 can be negative)

Often $Q \neq 0$, but hard to know a priori; also, Q will likely depend on the strength of the collision (e.g. heat generated).

Therefore, it is convenient to introduce ϵ, the coefficient of restitution, which depends on the relative speeds of the particles

$$\epsilon = \frac{|v_2 - v_1|}{|u_2 - u_1|} \rightarrow \left[(u_2 - u_1)\epsilon = (v_2 - v_1) \right]$$

ϵ depends on the composition of the 2 bodies & is approx. constant for a wide range of velocities

Ex: Show that for an elastic collision $\epsilon = 1$.

In an elastic collision

$$\tfrac{1}{2} m_1 u_1^2 + \tfrac{1}{2} m_2 u_2^2 = \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2$$

or $$m_2 u_2^2 - m_2 v_2^2 = m_1 v_1^2 - m_1 u_1^2$$

Cons. of mom. is $m_2 u_2 - m_2 v_2 = m_1 v_1 - m_1 u_1$

$\div$ the first eqn. by the 2nd & get

$$u_2 + v_2 = v_1 + u_1$$

$$\rightarrow v_2 - v_1 = -u_2 + u_1$$

$$V_2 - V_1 = -(u_2 - u_1)$$

Sub. into $\epsilon = \dfrac{|V_2 - V_1|}{|u_2 - u_1|} = \dfrac{|-(u_2-u_1)|}{|u_2-u_1|} = \dfrac{|u_2-u_1|}{|u_2-u_1|} = 1$ ∎

In the case of an inelastic collision, the 2 bodies stick together so $V_2 = V_1$ & $\epsilon = 0$. So, $0 \le \epsilon \le 1$.

So, in the general case, start from mom. consv.

$$m_1 V_1 = m_2 u_2 + m_1 u_1 - m_2 V_2$$

$$V_1 = \left(\frac{m_2}{m_1}\right) u_2 + u_1 - \left(\frac{m_2}{m_1}\right) V_2$$

Sub. this into $\epsilon = \dfrac{V_2 - V_1}{-(u_2 - u_1)} = \dfrac{V_2 - \left(\frac{m_2}{m_1}\right)u_2 - u_1 + \left(\frac{m_2}{m_1}\right)V_2}{-u_2 + u_1}$

$$\epsilon(-u_2 + u_1) = V_2\left(1 + \frac{m_2}{m_1}\right) - \left(\frac{m_2}{m_1}\right)u_2 - u_1$$

$$\therefore V_2\left(1 + \frac{m_2}{m_1}\right) = \epsilon(-u_2 + u_1) + \left(\frac{m_2}{m_1}\right)u_2 + u_1$$

$$V_2 = \frac{-\epsilon u_2 + \epsilon u_1 + \left(\frac{m_2}{m_1}\right)u_2 + u_1}{\left(1 + \frac{m_2}{m_1}\right)}$$

$$V_2 = \frac{-\epsilon m_1 u_2 + m_1 \epsilon u_1 + m_2 u_2 + m_1 u_1}{m_1 + m_2}$$

$$\Rightarrow \quad V_2 = \frac{(m_1 + \epsilon m_1)u_1 + (m_2 - \epsilon m_1)u_2}{m_1 + m_2}$$

Similarly, $\quad V_1 = \dfrac{(m_1 - \epsilon m_2)u_1 + (m_2 + \epsilon m_2)u_2}{m_1 + m_2}$

Limiting cases:- if collision is perfectly elastic, $\epsilon = 1$ & $m_1 = m_2$, $u_2 = 0$

$V_1 = 0$ & $V_2 = u_1$

First particle is brought to rest ; velocity is completely tranferred.

- If collision is perfectly inelastic, $\epsilon = 0$

$$V_1 = \frac{m_1 u_1}{(m_1+m_2)}, \quad V_2 = \frac{m_1 u_1}{m_1+m_2}$$

Same velocity as expected

- if 2^{nd} body is initially at rest ($u_2 = 0$)

$$V_1 = \frac{(m_1 - \epsilon m_2)u_1}{m_1+m_2}, \quad V_2 = \frac{(m_1 + \epsilon m_1)u_1}{m_1+m_2}$$

Exercise: Show that for a general, non-elastic collision

$$Q = \frac{1}{2}\left(\frac{m_1 m_2}{m_1+m_2}\right)(V_2 - V_1)^2(1-\epsilon^2)$$

Consider a situation where $u_2 = 0$, so the initial KE is $T = \frac{1}{2}mu_1^2$; the final KE is $T' = \frac{1}{2}mV_1^2 + \frac{1}{2}m_2V_2^2$. $\therefore$ The fractional loss of KE in the collision

$$\frac{T-T'}{T} = \frac{\cancel{\frac{1}{2}}m_1u_1^2 - \cancel{\frac{1}{2}}m_1V_1^2 - \cancel{\frac{1}{2}}m_2V_2^2}{\cancel{\frac{1}{2}}m_1u_1^2}$$

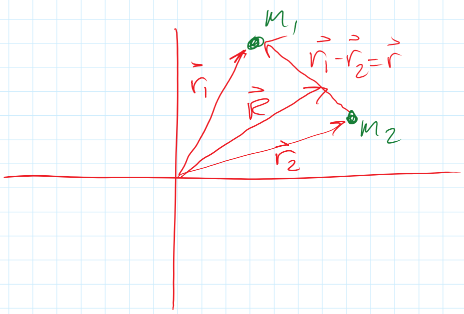
$$= \frac{m_1\cancel{u_1^2} - m_1\frac{(m_1-\epsilon m_2)^2\cancel{u_1^2}}{(m_1+m_2)^2} - \frac{m_2(m_1+\epsilon m_2)^2\cancel{u_1^2}}{(m_1+m_2)^2}}{m_1\cancel{u_1^2}}$$

$= ASA = \ldots$

$$\frac{T-T'}{T} = \frac{(1-\epsilon^2)m_2}{(m_1+m_2)}$$

For Oblique Collisions, its often easier to work in center-of-mass coordinates since $\vec{p}_1 + \vec{p}_2 = \vec{q}_1 + \vec{q}_2$ leads to many complications.

Definitions: Consider 2 particles w/ positions $\vec{r}_1$ & $\vec{r}_2$ & masses m_1 & m_2. If the internal force is $\vec{F}$ & the particles are in a uniform grav. field then the e.o.m are



$$m_1\ddot{\vec{r}}_1 = m_1\vec{g} + \vec{F}$$
$$m_2\ddot{\vec{r}}_2 = m_2\vec{g} - F$$

Define the center of mass position vector $\vec{R} = \dfrac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2}$

and, as normal, the relative position $\vec{r} = \vec{r}_1 - \vec{r}_2$