Sample Quiz 2, Math 1554, Spring 2020

PLEASE PRINT YOUR NAME CLEARLY IN ALL CAPITAL LETTERS

First Name ____________________ Last Name ____________________

GTID Number: ______________________

Student GT Email Address: _________________________@gatech.edu

Section Number (e.g. A4, M2, QH3, etc.) ________ TA Name _________________________

Circle your instructor:

Student Instructions

- Print your name and GTID darkly and neatly on the cover page.
- You will have 20 minutes to complete this quiz.
- Notes, books, cell phones, and all electronic devices are not allowed.
- Use dark and clear writing: your exam will be scanned into a digital system.
- The quiz is 1 page and double sided. Be sure to complete both sides.
- Leave a 1 inch border around the edges of exams.
- Any work done on scratch paper will be collected and will not be graded.

You do not need to justify your reasoning for questions in this quiz.

1. (6 points) Fill in the blanks.

   (a) (1 point) If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$, then $A^{-1} = \begin{pmatrix} \quad & \quad & \quad \\ & & \\ & & \end{pmatrix}$.

   (b) (2 points) If $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{pmatrix}$, then A has the LU factorization $A = LU$, where

   $U = \begin{pmatrix} \quad & \quad & \quad \\ & & \\ & & \end{pmatrix}$ and $L = \begin{pmatrix} \quad & \quad & \quad \\ & & \\ & & \end{pmatrix}$.

   (c) (1 point) E is an elementary matrix. $EB = C$, and $B = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 1 & 4 \\ 0 & 1 & 0 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$.

   $E = \begin{pmatrix} \quad & \quad & \quad \\ & & \\ & & \end{pmatrix}$

   (d) (1 point) By using homogeneous coordinates, 2D transform $(x_1, x_2) \to (x_1 + 1, x_2 - 3)$ can be represented with the product $A\vec{x}$, where $A = \begin{pmatrix} \quad & \quad & \quad \\ & & \\ & & \end{pmatrix}$.

   (e) (1 point) Suppose A, B, C, and X are invertible $n \times n$ matrices, and $\begin{pmatrix} B & 0 \\ 0 & C \end{pmatrix} \begin{pmatrix} X \\ A \end{pmatrix} = \begin{pmatrix} B + I \\ B \end{pmatrix}$. Express A in terms of B and C. $A = \boxed{\qquad\qquad}$.

2. (2 points) Let A be an $n \times n$ matrix. Fill in the circles next to the statements that guarantee that A is invertible; leave the other circles empty.

   ○ Every vector in $\mathbb{R}^n$ is in the span of the columns of A.

   ○ The homogeneous linear system $A\vec{x} = \vec{0}$ has a non-trivial solution.

3. (2 points) Indicate whether the statements are true or false.

true	false	
○	○	An example of an upper triangular matrix is $A = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$.
○	○	If E_1 and E_2 are $n \times n$ elementary matrices, then $E_1 E_2 = E_2 E_1$.

Answers

No justification needed for any questions in this quiz. Answers were only graded for completion, not accuracy.

1. Fill in the blanks.

   (a) $A^{-1} = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5 \end{pmatrix}$.

   (b) $U = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \end{pmatrix}$, $L = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$

   (c) $E = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

   (d) $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix}$

   (e) $CA = B \Rightarrow A = C^{-1}B$.

2. Statement indicates A is invertible.

   - matrix must be invertible
   - matrix cannot be invertible (not every column would be pivotal)

3. True/false.

   (a) True (elements below main diagonal are zero)

   (b) False