PHYS 3201 — Assignment #5

Due: 9/25/20

1. A mass m moves along the x-axis subject to an attractive force given by $17\beta^2 mx/2$ and a retarding force given by $3\beta m\dot{x}$, where x is its distance from the origin and β is a constant. A driving force given by $mA\cos\omega t$, where A is a constant, is applied to the particle along the x-axis.

   (a) What value of ω results in steady-state oscillations about the origin with maximum amplitude?

   (b) What is the maximum amplitude?

2. The exponential damping factor γ of a spring suspension system is one-half the critical value. If the undamped frequency is ω_0, find (a) the resonant frequency, (b) the Q factor, (c) the phase angle θ_1 when the system is driven at a frequency $\omega_1 = 2\omega_0$, and (d) the steady-state amplitude at this frequency.

3. For a lightly damped harmonic oscillator $\gamma \ll \omega_0$, show that the driving frequency for which the steady-state amplitude is one-half the steady-state amplitude at the resonant frequency is given by $\omega_1 \approx \omega_0 \pm \gamma\sqrt{3}$.

4. Solve the differential equation of motion of the damped harmonic oscillator driven by a damped harmonic force:
$$F_{\text{ext}}(t) = F_0 e^{-\alpha t}\cos\omega_1 t$$
(*Hint:* $e^{-\alpha t}\cos\omega_1 t = \mathrm{Re}(e^{-\alpha t + i\omega_1 t}) = \mathrm{Re}(e^{\beta t})$, *where* $\beta = -\alpha + i\omega_1$. *Assume a solution of the form* $a_1 e^{\beta t - i\theta_1}$.)

5. An undamped driven harmonic oscillator satisfies the equation of motion $m(\ddot{x} + \omega_0^2 x) = F(t)$. The driving force $F(t) = F_0\sin(\omega_1 t)$ is switched on at $t = 0$. Find $x(t)$ for $t > 0$ for the initial conditions $x = 0$ and $v = 0$ at $t = 0$.

   (*Hint: Look for a particular solution of the differential equation of the form* $x = a_1\sin(\omega_1 t)$ *and determine* a_1. *Add the solution of the homogeneous equation to this to obtain the general solution of the inhomogeneous equation.*)