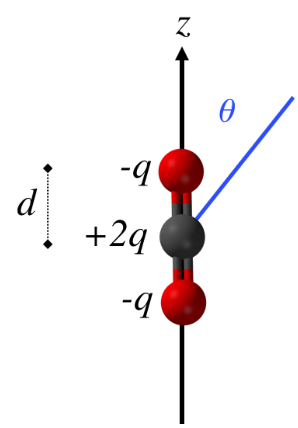
Homework 7

Due Date: All homework submitted by Wednesday **10/21** 11:59pm will be graded together. Homework submitted past that time may be graded late. Submit your homework through Canvas as a single pdf file. Do not use solution sets from previous years. You are encouraged to discuss homework assignments with each other, the TAs or myself, but the solutions have to be executed and submitted individually.

Problem A [60%]. We consider a molecule of carbon dioxide CO_2 where carbon carries a charge $+2q$ and each of the oxygens a charge $-q$. Due to the geometry of the atomic orbitals that participate in chemical bonding, the molecule is *linear*. The distance between a carbon and an oxygen is d. We want to calculate the electric field far from the molecule. Use the carbon atom as the origin of the coordinate system and consider the molecule aligned along the z-axis (see below).



(1) Prove (or discuss why) the first nonzero term in the multipole expansion is the quadrupole ($\ell = 2$).

(2) Calculate the quadrupole moment of the molecule [*Hint: Use the formulas for a discrete distribution*] and the leading term for the electric potential far from the molecule.

(3) Calculate the electric field far from the molecule using the above approximation for the potential (note that both cartesian and spherical coordinates systems can be used). Sketch the electric field along with some equipotential lines.

(4) Write an exact expression for the electric potential. Using a Taylor expansion to second order recover your result from (2). Using Mathematica or your favorite software, can you estimate at which distance R (in units of d) from the center of the molecule is the quadrupolar approximation accurate to 1%?

Problem B [40%]. A sphere of radius R, centered at the origin, carries charge density

$$\rho(r,\theta) = k\frac{R}{r^2}(R-2r)\sin\theta$$

where k is a constant and r and θ are the usual spherical coordinates.

(1) Find the approximate potential for points on the z-axis (corresponding to $\theta = 0$), far from the sphere [*Hint: Use the formulas for a continuous charge distribution*].