PHYS 3201 — Assignment #9
Due: 11/9/20

1. The position of a particle in a fixed inertial frame of reference is given by the vector

$$\vec{r} = (x_0 + R\cos\Omega t)\hat{\imath} + R\sin\Omega t\hat{\jmath},$$

   where x_0, R and Ω are constants.

   (a) Show that the particle moves in a circle with constant speed.

   (b) Find two coupled, first order differential equations of motion that relate the components of position, x' and y', the components of velocity, $\dot{x}'$ and $\dot{y}'$, of the particle relative to a frame rotating with an angular velocity $\vec{\omega} = \omega\hat{k}$.

2. A cockroach crawls with constant speed in a circular path of radius b on a disk rotating with constant angular speed ω. The circular path is concentric with the center of the disk. If the mass of the insect is m and the coefficient of static friction with the surface of the disk is μ_s, how fast, relative to the disk, can the cockroach crawl before it starts to slip if it goes (a) in the direction of rotation and (b) opposite to the direction of rotation? (*Hint: work in polar coordinates in the rotating frame.*)

3. If the bead on the rotating rod we discussed in class (Lecture 28) is initially released from rest (relative to the rod) at its midpoint calculate

   (a) the displacement of the bead along the rod as a function of time

   (b) the time and the velocity (relative to the rod) when the bead leaves the end of the rod.

4. (a) A particle moves in a horizontal plane on the surface of the Earth. Show that the magnitude of the horizontal component of the Coriolis force is independent of the direction of the motion of the particle.

   (b) The horizontal wind speed at co-latitude θ is v. By considering the forces on a small volume of air, show that the pressure gradient required to balance the horizontal component of the Coriolis force, and thus to maintain constant wind direction, is

$$\frac{dP}{dx} = 2\omega\rho v\cos\theta$$

   where ρ is the density of the air.

5. A bullet is fired straight up with initial speed v_0. Assuming g^* is constant, and ignoring air resistance, show that the bullet will hit the ground west of the initial point of upward motion by an amount $4\omega v_0^3\cos\lambda/3(g^*)^2$, where λ is the latitude and ω is Earth's angular velocity.

6. Find the equations of motion for a particle in a frame rotating with *variable* angular velocity $\vec{\omega}$, and show that there is another apparent force of the form $-m\dot{\vec{\omega}} \times \vec{r}$. Discuss the physical origin of this force.