PHYS 3201 — Assignment #1

Due: 8/28/20

1. A small ball is fastened to a long rubber band and twirled around in such a way that the ball moves in an elliptical path given by the equation

$$\vec{r}(t) = b\cos\omega t\hat{\mathrm{i}} + 2b\sin\omega t\hat{\mathrm{j}},$$

where b and ω are constants. Find the speed of the ball as a function of t. In particular, find v at $t = 0$ and $t = \pi/2\omega$, at which times the ball is, respectively, at its minimum and maximum distances from the origin.

2. A buzzing fly moves in a helical path given by the equation

$$\vec{r}(t) = b\sin\omega t\hat{\mathrm{i}} + b\cos\omega t\hat{\mathrm{j}} + ct^2\hat{\mathrm{k}}$$

Show that the magnitude of the acceleration of the fly is constant, provided b, ω and c are constant.

3. Repeat Problem #2 using cylindrical coordinates where $R = b$, $\phi = \omega t$ and $z = ct^2$.

4. A bee goes out from its hive in a spiral path given in plane polar coordinates by

$$r = be^{kt} \qquad\qquad \theta = ct$$

where b, c and k are positive constants. Show that the angle between the velocity and acceleration vectors remains constant as the bee moves outward (Hint: Find $\vec{v}\cdot\vec{a}/va$).

5. An ant crawls on the surface of a ball of radius b in such a manner that the ant's motion is given in spherical coordinates by the equations

$$r = b \qquad\qquad \phi = \omega t \qquad\qquad \theta = \frac{\pi}{2}\left[1 + \frac{1}{4}\cos(4\omega t)\right]$$

Find the speed of the ant as a function of time t. What sort of path is represented by the equations?

6. Prove that

$$\frac{d}{dt}[\vec{r}\cdot(\vec{v}\times\vec{a})] = \vec{r}\cdot(\vec{v}\times\dot{\vec{a}})$$

7. A particle moves with v =constant along the curve $r = k(1+\cos\theta)$ (a cardioid). Show that

$$\dot{\theta} = \frac{v}{\sqrt{2kr}}$$

and

$$\ddot{\theta} = \frac{\dot{\theta}^2 \sin\theta}{2(1+\cos\theta)}.$$

From these two relations show that

$$\vec{a} \cdot \hat{r} = -\frac{3}{4}\frac{v^2}{k}$$