MIDTERM 1
MATH 3215-C (PROBABILITY AND STATISTICS)

TUESDAY, SEPTEMBER 22

INSTRUCTOR: ARMENAK PETROSYAN

IMPORTANT: Please read carefully (1pt)

- You have a **12 hour** window to take and submit your exam (**7 am - 7 pm**).

- Be warned: **exam ends at 7 pm** (e.g. if you start at 6 pm, you only have 1 hour).

- After you opened this file you have 100 minutes to finish the work and 20 minutes to submit it **(120 minutes in total)**.

- If you run into difficulties submitting on GradeScope, email the files to the instructor before the 120 minutes expire and before 7 pm. **Late submissions will not be accepted**.

- If you encounter technical problems, email the instructor as soon as possible.

- You **CAN** use the course textbook and the lecture notes/slides for reference.

- You **CAN** use any fact we presented in class without proving them; anything else used must be proved.

- You **CAN** use any calculator you want.

- You **CANNOT** get any help from anyone.

- Posting the problems online to get help or to let others know what the problems are will be a violation; it will be reported and result in a penalty.

- **To get full credit you need to write complete answers.**

- The total amount of points for this exam is 75. Different problems have different weights.

- Be wise with your time. You can handwrite your answers on a different paper, and submit a photocopy. Make sure it is readable. No need to print the problem sheet or copy the problems.

Problem 1. *(10pt) Post office employs three postmen and they need to deliver post to 120 different households. In how many different ways can they split the job equally?*

Problem 2 (10pt). *Four people are asked to guess a number between 0 and 9. What is the probability that at least two of them will guess the same number (assuming equiprobability space)?*

Problem 3 (10pt). *a) Let A, B be any two events. Show that*
$$P(A \cap B) \geq P(A) + P(B) - 1.$$
b) For any events $A_1, \ldots, A_n$, use induction to show
$$P(A_1 \cap \cdots \cap A_n) \geq P(A_1) + \cdots P(A_n) - n + 1.$$

Problem 4 (10pt). *Are you more likely to win exactly 3 games out of 4 or exactly 5 games out of 8 against an equally strong opponent?*

Problem 5 (12pt). *Urn 1 contains one red ball and three blue balls. Urn 2 contains one red ball. A random ball is drawn from urn 1 and placed into urn 2. Then one ball is drawn at random from urn 2 and it turns out red. What is the conditional probability that the ball remaining in the urn 2 is also red?*

Problem 6 (12pt). *Suppose the cdf of the random variable X is given by*
$$F(x) = \begin{cases} 0 & x < -1 \\ \frac{1}{2} & -1 \leq x < 2 \\ \frac{3}{4} & 2 \leq x < 3 \\ \frac{5}{6} & 3 \leq x < 4 \\ 1 & x \geq 4 \end{cases} .$$
(a) Draw the histogram of X.
(b) Find the mean and variance of X.

Problem 7 (10pt). *A fair coin is flipped 5 times and the number of heads is recorded as X_1. The same coin is then flipped 6 times and the number of heads is recorder as X_2. Find the conditional probability*
$$P(X_1 = 2 | X_1 + X_2 = 4).$$