HOMEWORK 7: MATH 3215-C (PROBABILITY AND STATISTICS)

DUE WEDNESDAY, OCTOBER 14TH, 8 P.M. ATL

- All problems are worth 2.5 points (20 total) and you can get a partial point.
- **If you use any help from anyone or from anywhere, mention it in your work.**
- To get full credit you need to submit full answers.

Problem 1. *Let X and Y be independent random variables. Show that, for any two functions $g, h : \mathbb{R} \to \mathbb{R}$, the random variables $g(X)$ and $h(Y)$ are also independent.*

Problem 2. *Find the formula for the least squares regression line of the Trinomial distribution with parameters (n, p_S, p_I).*

(Hint:

$$\begin{aligned} E[XY] &= \sum_{x=0}^{n} \sum_{y=0}^{n-x} \frac{xyn!}{x!y!(n-x-y)!} p^x q^y (1-p-q)^{n-x-y} \\ &= \sum_{x=0}^{n} x \frac{n! p^x}{x!(n-x)!} \sum_{y=0}^{n-x} \frac{y(n-x)!}{y!(n-x-y)!} q^y (1-p-q)^{n-x-y}. \end{aligned}$$

From the binomial formula

$$g(t) = \sum_{y=0}^{m} e^{ty} \binom{m}{y} q^y r^{m-y} = [r + qe^t]^m.$$

Notice that,

$$g'(0) = \sum_{y=0}^{m} y \binom{m}{y} q^y r^{m-y} = [r + qe^t]^m = mq(r+q)^{m-1}.$$

From here, by taking $m = n - x$ and $r = 1 - p - q$

$$\begin{aligned} E[XY] &= \sum_{x=0}^{n} x \frac{n! p^x}{x!(n-x)!} (n-x) q (1-p-q+q)^{n-x-1} \\ &\text{(notice that term for x=n is zero so)} \\ &= \sum_{x=0}^{n-1} x \frac{n! p^x}{x!(n-x)!} (n-x) q (1-p)^{n-x-1} \\ &\text{(use the formula for the mean of bionoial with parameters } (n-1, p)\text{)} \\ &= nq \sum_{x=0}^{n-1} x \frac{(n-1)!}{x!(n-1-x)!} p^x (1-p)^{n-1-x} \\ &= nq(n-1)p. \end{aligned}$$

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Problem 3. *Do problem 4.1-1 (c,d).*

Problem 4. *Do problem 4.1-4.*

Problem 5. *Do problem 4.1-9.*

Problem 6. *Do problem 4.2-2.*

Problem 7. *Do problem 4.2-5.*

Problem 8. *Do problem 4.2-8.*