

## Midterm 3, Math 2552

**PLEASE PRINT YOUR NAME CLEARLY IN ALL CAPITAL LETTERS**

First Name \_\_\_\_\_ Last Name \_\_\_\_\_

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Section Number (A1, A2 or A3) \_\_\_\_\_ TA Name \_\_\_\_\_

## INSTRUCTIONS (PLEASE READ)

### Formatting and Timing

- You should only need 75 min to take the exam, but students will have 3 hours to submit the exam, from the time that it is released.
- **Show your work** and justify your answers for all questions unless stated otherwise.
- Please write neatly, and use dark and clear writing so that the scan is easy to read.
- Please write your name or initials at the top of every page.
- Please solve the questions in the exam in the order they are given.
- You do not need to print the exam. As long as you solve problems in the order they are given (just like the written homework sets), you can write your answers on your own paper. But students can print the exam and write their answers on the printed copy if they prefer.

### Submission

- Students should scan their work and submit it through Gradescope. There should be an **assignment** in Gradescope for this exam. The process for submitting your work will be similar to what you have used for homework.
- Work must be submitted today by 12:30 PM ET.
- Please upload your work as a single PDF file. If this is not possible you can email your work to your instructor.
- During the upload process in Gradescope, please indicate which page of your work corresponds to each question in the exam.

### Questions

- If there are questions during the exam, students can ask them on BlueJeans, email their instructor or message them through Canvas.
- Our course Piazza forum will be temporarily inactive for 24 hrs on the day of the exam.
- If you run into any technical issues or any unanticipated emergencies, please email your instructor as soon as you can.

### Integrity

- Students can use any resources while taking these tests including online calculators and Mathematica.
- Students cannot communicate with anyone during these tests including using Reddit or online message boards
- Students cannot use solutions provided from another student or third party.
- In other words: do your own work but you can use technology to solve problems.

1. (8 points) Compute the Laplace Transform of the following functions:

(a) (4 points)

$$f(t) = e^{2t}t \sin(3t).$$

(b) (4 points)

$$g(t) = \begin{cases} 2 & \text{if } 0 \leq t < 2, \\ t & \text{if } 2 \leq t < 3, \\ t^2 - 5t + 9 & \text{if } t \geq 3. \end{cases}$$

2. (8 points) Compute the Inverse Laplace Transform of the following functions:

(a) (4 points)

$$F(s) = \frac{e^{-2\pi t}}{s^2 + 2s + 10}.$$

(b) (4 points)

$$G(s) = \frac{9s - 16}{(s^2 + 4)(s - 5)}.$$

3. (6 points) Using the Laplace Transform, solve the following IVP:

$$\begin{cases} y''(t) - y'(t) - 6y(t) = -36t, \\ y(0) = -1, \quad y'(0) = 16. \end{cases}$$

4. (9 points) The goal of this exercise is to compute the Laplace Transforms of the function:

$$y(t) = \cos(\alpha t)^2,$$

where  $\alpha$  is a real number. We want to use three different methods.

- (a) (4 points) Compute the first and second derivatives  $y'$  and  $y''$ . Using them, prove that  $y$  is a solution of the IVP:

$$\begin{cases} y'' + 4\alpha^2 y = 2\alpha^2, \\ y(0) = 1, \quad y'(0) = 0. \end{cases}$$

Use this IVP to compute  $Y(s)$ .

- (b) (2 points) Using the trigonometry formula  $\cos(\alpha t)^2 = \frac{1 + \cos(2\alpha t)}{2}$  and the Laplace Transform of the cosine, compute  $Y(s)$ .

- (c) (2 points) Using Euler's Formula  $\cos(\alpha t) = \frac{e^{i\alpha t} + e^{-i\alpha t}}{2}$ , express  $y(t)$  in functions of exponential and constant terms. Use this expression to compute  $Y(s)$ .

- (d) (1 point) Compare your results with the  $s$ -Domain function

$$\frac{s^2}{(s^2 + \alpha^2)^2} = (\mathcal{L}[\cos(\alpha t)](s))^2.$$

Is it an expected comparison?

5. (9 points) For the four followings systems, compute the critical points.

(a) (2 points)

$$\begin{cases} x' = (x - y + 2)(y - x + 3), \\ y' = (x - y)(y - x + 1). \end{cases}$$

(b) (2 points)

$$\begin{cases} x' = x^2 - 2x + y^2 - 4, \\ y' = x^2 + 2x + y^2 - 4. \end{cases}$$



(c) (2 points)

$$\begin{cases} x' = xy, \\ y' = y - x^2 + 1. \end{cases}$$

(d) (3 points)

$$\begin{cases} x' = xy - 1, \\ y' = (x - y)(x + y). \end{cases}$$

6. (8 points) Consider the following system representing two species  $x$  and  $y$  competing for the same food supply:

$$\begin{cases} x' = x(4 - 2x - y), \\ y' = y(3 - x - y). \end{cases}$$

- (a) (2 points) Compute the critical points.

- (b) (4 points) For each critical point, construct the linear system near the critical point and classify the stability of the critical point for the linear system.

(c) (2 points) If you start with a positive population of both species  $x$  and  $y$ , what could you expect for the ratio of both species to be as  $t$  goes to infinity.

7. (2 points) A small number of points will be allocated for presentation, neatness, and organization. Please ensure that

- Your work is legible in the scan.
- Your name or initials are at the top of every page.
- Questions are answered in the order in which they were given.
- During the upload process you have indicated which pages correspond to which question, and made sure that none of your pages are upside down or sideways (you can also change the orientation of the pages when you upload in Gradescope). Ensuring that these criteria are met helps ensure that your exam is graded efficiently and accurately.