Sample Quiz 3, Math 1554

PLEASE PRINT YOUR NAME CLEARLY IN ALL CAPITAL LETTERS

First Name ____________________ Last Name ____________________

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Section Number (e.g. A4, QH3, etc.) ________ TA Name _________________________

Student Instructions

- **Show your work** and justify your answers for all questions unless stated otherwise.
- **Organize your work** in a reasonably neat and coherent way.
- Calculators, notes, cell phones, books are not allowed.
- Use dark and clear writing: your exam will be scanned into a digital system.
- Exam pages are double sided. Be sure to complete both sides.
- Leave a 1 inch border around the edges of exams.
- Any work done on scratch paper will not be collected and will not be graded.

You do not need to justify your reasoning for questions in this quiz.

1. (6 points) Fill in the blanks.

   (a) An eigenvector of $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$ is $\vec{v}_1 = \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}$.

   What is the eigenvalue associated with eigenvector $\vec{v}_1$? $\boxed{}$

   (b) For what values of k (if any) does $A = \begin{pmatrix} -2 & k \\ -1 & 0 \end{pmatrix}$ have exactly two distinct real eigenvalues? $\boxed{}$

   (c) For what values of k (if any) is $A = \begin{pmatrix} 2 & 0 \\ k & 2 \end{pmatrix}$ diagonalizable? $\boxed{}$

   (d) The characteristic polynomial of A is $(\lambda - 1)^2(\lambda - 3)\lambda^6$.

   - What is the algebraic multiplicity of the eigenvalue $\lambda = 1$? $\boxed{}$
   - What are the dimensions of matrix A? $\boxed{}$
   - What is the value of $\det(A)$? $\boxed{}$

2. (4 points) Suppose A is an $n \times n$ matrix. Fill in the circles next to the **true** statements; leave the others empty.

   ○ If 2 is an eigenvalue of A, and A is invertible, $\frac{1}{2}$ is an eigenvalue of A^{-1}.

   ○ An example of a regular stochastic matrix is $P = \frac{1}{10}\begin{pmatrix} 9 & 2 & 1 \\ 0 & 7 & 8 \\ 1 & 1 & 1 \end{pmatrix}$.

   ○ If v_1 and v_2 are linearly independent eigenvectors, they must correspond to distinct eigenvalues.

   ○ If a stochastic matrix is not regular then it cannot have a steady state.

Answers

No justification needed for any questions in this quiz. Answers were only graded for completion, not accuracy.

1. Fill in the blanks.

   (a) By multiplying $A\vec{v}_1$ we can determine the eigenvalue for $\vec{v}_1$.

   $$A\vec{v}_1 = \begin{pmatrix} 0 \\ -2 \\ 2 \\ 0 \end{pmatrix} = -2\vec{v}_1$$

   The eigenvalue for this eigenvector is -2.

   (b) The characteristic equation is

   $$0 = (-2-\lambda)(0-\lambda) - (k)(-1) = \lambda^2 + 2\lambda + k$$

   The roots are given by

   $$\lambda = -1 \pm \frac{1}{2}\sqrt{2^2 - 4k}$$

   For there to be two distinct roots we need $2^2 - 4k > 0$, or $k < 1$.

   (c) If $k \neq 0$, we can only construct one linearly independent eigenvector. But if k is zero, there are exactly two linearly independent eigenvectors. Thus, we need $k = 0$ for the matrix to be diagonalizable.

   (d) Characteristic polynomial problem.

      i. 2

      ii. $2 + 1 + 6 = 9$

      iii. Zero is an eigenvalue of A, which implies that A is singular, and singular matrices have a determinant equal to zero.

2. True/False

   (a) True

   (b) True: if we were to calculate P^2 we would see that every element is positive

   (c) False. The $n \times n$ identity matrix has n linearly independent eigenvectors, all of which correspond to eigenvalue $\lambda = 1$.

   (d) False. The identity matrix is stochastic because the sum of the entries of each column is 1. But the matrix is not regular, because I^k will have zero entries for any k. But $I\vec{q} = \vec{q}$ for any $\vec{q}$.