Homework 9

Due Date: All homework submitted by Sunday **11/08** 11:59pm will be graded together. Homework submitted past that time may be graded late. Submit your homework through Canvas as a single pdf file. Do not use solution sets from previous years. You are encouraged to discuss homework assignments with each other, the TAs or myself, but the solutions have to be executed and submitted individually.

Problem A [50%] Two circular coils of radius a made of N turns of a wire (negligible width), each carrying current I in the same sense, are parallel with the xy plane with their centers at $(0, 0, \pm s/2)$. On the z axis the magnetic field is $\mathbf{B} = B(z)\hat{\mathbf{z}}$, in the z direction; and at $z = 0$, i.e., halfway between the coils, $\partial B/\partial z$ is 0.

(1) Draw the physical situation with just one coil. Consider first the case of a single coil at $z = +s/2$. Derive the magnetic field created by the coil on the on the z-axis.

(2) Draw the physical situation with the two coils. Calculate the magnetic field on the z-axis and determine s such that $\partial^2 B/\partial^2 z$ is 0 at $z = 0$ on the z axis. This configuration is called **Helmhotz coils**, and it produces a very uniform magnetic field in a neighborhood of the origin. Show that for this configuration $\partial^3 B/\partial^3 z$ is also 0 at $z = 0$.

(3) (For fun) Use a computer program to plot the magnetic field $\mathbf{B}(z)$ as a function of position z along the axis of the Helmhotz coils.

Note: Helmhotz coils are often used in laboratories to cancel out the Earth's magnetic filed in a small region of space. However, if a large magnetic-field-free region is needed, say room sized, the necessary coils would be impractically large. In practice, to produce a field-free volume one surrounds it with a material with high magnetic susceptibility, e.g., Mumetal, which concentrates the magnetic field in the walls.

Problem B [50%] Consider a thin (infinite) conducting sheet parallel to the xy plane at $z = 0$. It carries a uniform surface current density $\mathbf{K} = K_0\hat{\mathbf{x}}$.

(1) Draw the physical situation. Use the Biot-Savart law to find the $\mathbf{B}$ field both above and below the sheet, by integration. The integral is slightly nasty. Before you start asking Mathematica for help, simplify as much as possible! Set up the integral, be explicit about terms, what your integration limits are, etc. Then, make clear mathematical and/or physical arguments based on symmetry to convince yourself of the direction of the $\mathbf{B}$ field above and below the sheet, and to argue how $\mathbf{B}(x, y, z)$ depends (or doesn't) on x and y.

(2) Now solve the above problem using Ampere's law. Please be explicit about what Amperian loop(s) you are drawing and why. What assumptions are you making/using?

(3) Now let's add a second parallel sheet at $z = +a$ with a surface current running the other way (formally, this means $\mathbf{J} = -K_0\delta(z-a)\hat{\mathbf{x}} + K_0\delta(z)\hat{\mathbf{x}}$). Use the superposition principle (do not start from scratch again) to find $\mathbf{B}$ between the two sheets, and also above and below.

(4) Now the sheet of current has become a thick slab of current. The slab is infinite in (both) x and y, but finite in z, inviting to use the volume current density $\mathbf{J}$, rather than $\mathbf{K}$. The slab has thickness $2h$ (It runs from $z = -h$ to $z = +h$). $\mathbf{J}$ depends on height linearly, i.e. $J = J_0|z|\hat{\mathbf{x}}$ inside the slab and 0 above or below the slab. Find the $\mathbf{B}$ field (magnitude and direction) everywhere in space (above, below, and also, most interesting, inside the slab!).