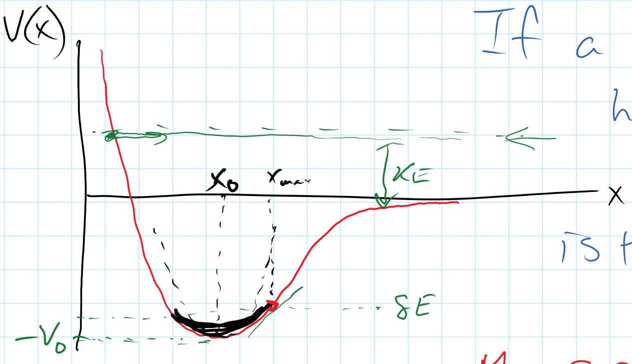


If a diatomic molecule has energy δE higher than its minimum possible value (its binding energy), what is the max. separation of the 2 atoms?

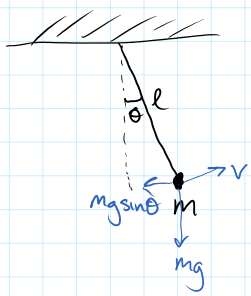
Max separation will occur when all the energy is potential. At these turning points

$$V(x) = -V_0 + \delta E \simeq \frac{V_0}{\delta^2}(x-x_0)^2 - V_0$$

$$\therefore\ x_{max/min} = x_0 \pm \delta\sqrt{\frac{\delta E}{V_0}}$$

This oscillation is symmetric due to parabolic shape of V(x) near equil. As molecule heats up, on avg. it will spend more time at large distances. This is why most substances expand when heated.

Ex: The Simple Pendulum



A simple pendulum w/ mass m supported by a light rod of length ℓ (ignore mass of rod). The mass m starts w/ velocity v from the equil. pos'n. What kinds of motion are possible for diff. values of v?

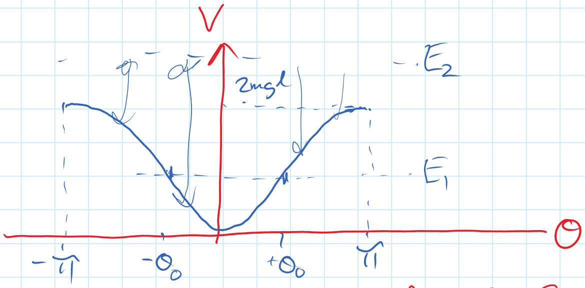
Initially, for small angles, the arc through which m moved is $\theta = \frac{x}{\ell}$, so restoring force $F = -mg\sin\theta$

$$F = -\frac{dV}{dx} = -mg\sin\theta \quad \text{but } dx = \ell\, d\theta$$

$$\therefore\ V = \int_0^\theta mg\ell\sin\theta\, d\theta = -mg\ell\cos\theta\Big]_0^\theta = -mg\ell(\cos\theta - 1)$$

$$\therefore\ V = \int_0 mgl\sin\theta\, d\theta = -mgl\cos\theta\Big]_0 = -mgl(\cos\theta - 1)$$

$$= mgl(1-\cos\theta)$$



Energy equation at start

$V = 0$, $E = \frac{1}{2}mv^2$

If $E < 2mgl$, E_1, the motion will be confined b/w the 2 angles $\pm\theta_0$ where $V(\theta_0) = E_1$

$$\frac{1}{2}mv^2 = mgl(1-\cos\theta_0)$$

$$1-\cos\theta_0 = \frac{v^2}{2gl}$$

These are the turning points where KE vanishes & all energy is PE.

If the mass is pushed hard enough that $E > 2mgl$ (E_2) then the KE will never vanish, and when the mass is inverted $T = \frac{1}{2}mv^2 - 2mgl \neq 0$, mass will rotate continuously

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Velocity Dependent Forces (non-conservative)

Since $F(\dot{x})$ we don't have a PE, & cannot write down an energy eq'n.

Resistive force.

Consider $F = -av$ where $a > 0$ is a constant

If $x = x_0$ & $v = v_0$ at $t = 0$

$$F = -av = m\frac{dv}{dt}$$

$$-\frac{a}{m}\int_0^t dt = \int_{v_0}^{v(t)} \frac{dv}{v}$$

$$-\frac{a}{m}t = \ln\left(\frac{v}{v_0}\right) \Rightarrow v(t) = v_0 e^{-\frac{a}{m}t}$$

$$-\frac{a}{m}t = \ln\left(\frac{v}{v_0}\right) \Rightarrow v(t) = v_0 e^{-\frac{a}{m}t}$$

Pos'n : $\frac{dx}{dt} = v_0 e^{-\frac{a}{m}t}$

$$\int_{x_0}^{x} dx = v_0 \int_0^t e^{-a/m\, t}\, dt = -\frac{v_0 m}{a} e^{-\frac{a}{m}t}\Big]_0^t = -\frac{v_0 m}{a}\left(e^{-\frac{a}{m}t} - 1\right)$$

$$\therefore\ x = x_0 + \frac{v_0 m}{a}\left(1 - e^{-\frac{a}{m}t}\right)$$

So, as $t \to \infty$, $x - x_0$ = distance moved in $0 < t < \infty$

$$= \frac{v_0 m}{a}$$