PHYS 3201 — Assignment #4

Due: 9/18/20

1. A particle of mass m moves (in the region $x > 0$) under a force $F = -kx + c/x$, where k and c are positive constants. Find the corresponding potential energy function. Determine the position of equilibrium, and the angular frequency of small oscillations about it.

2. A particle undergoing simple harmonic motion has a velocity $\dot{x}_1$ when the displacement is x_1 and a velocity $\dot{x}_2$ when the displacement is x_2. Find the angular frequency, ω_0, and the amplitude of the motion, a, in terms of the given quantities.

3. In class we calculated the time averages of the kinetic and potential energies of a simple harmonic oscillator over one cycle. Next, calculate the *space* averages of the kinetic and potential energies of a simple harmonic oscillator over one cycle, and comment on the results.

4. Show that the ratio of two successive maxima in the displacement of a damped harmonic oscillator is constant and is equal to $e^{-\gamma(2\pi/\omega_d)}$. (*Note:* The maxima do not occur at the points of contact of the displacement curve with the curve $Ae^{-\gamma t}$.)

5. Given that the amplitude of a damped harmonic oscillator drops to $1/e$ after n complete cycles, show that the ratio of the period of oscillation to the period of the same oscillator with no damping is given by

$$\frac{\tau_d}{\tau_0} = \left(1 + \frac{1}{4\pi^2 n^2}\right)^{1/2} \approx 1 + \frac{1}{8\pi^2 n^2},$$

where the binomial theorem is used in the last expression when n is large. (*Hint:* Start with the result of the previous problem.)