Midterm 2, Math 2552

PLEASE PRINT YOUR NAME CLEARLY IN ALL CAPITAL LETTERS

First Name _____ Last Name _____

GTID Number: _____

Student GT Email Address: _____@gatech.edu

Section Number (A1, A2 or A3) _____ TA Name _____

INSTRUCTIONS (PLEASE READ)

Formatting and Timing

- You should only need 75 min to take the exam, but students will have 3 hours to submit the exam, from the time that it is released.
- **Show your work** and justify your answers for all questions unless stated otherwise.
- Please write neatly, and use dark and clear writing so that the scan is easy to read.
- Please write your name or initials at the top of every page.
- Please solve the questions in the exam in the order they are given.
- You do not need to print the exam. As long as you solve problems in the order they are given (just like the written homework sets), you can write your answers on your own paper.But students can print the exam and write their answers on the printed copy if they prefer.

Submission

- Students should scan their work and submit it through Gradescope. There should be an **assignment** in Gradescope for this exam. The process for submitting your work will be similar to what you have used for homework.
- Work must be submitted today by 12:30 PM ET.
- Please upload your work as a single PDF file. If this is not possible you can email your work to your instructor.
- During the upload process in Gradescope, please indicate which page of your work correspondsto each question in the exam.

Questions

- If there are questions during the exam, students can ask them on BlueJeans, email their instructor or message them through Canvas.
- Our course Piazza forum will be temporarily inactive for 24 hrs on the day of the exam.
- If you run into any technical issues or any unanticipated emergencies, please email your instructor as soon as you can.

Integrity

- Students can use any resources while taking these tests including online calculators and Mathematica.
- Students cannot communicate with anyone during these tests including using Reddit or online message boards
- Students cannot use solutions provided from another student or third party.
- In other words: do your own work but you can use technology to solve problems.

1. (9 points) Consider the homogeneous system of equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{x} = \begin{pmatrix} 1 & 0 & -3 \\ -2 & -1 & 6 \\ 1 & 0 & -3 \end{pmatrix} \vec{x}.$$

(a) (6 points) Find the general form of the solutions of the equation.

(b) (3 points) Find the unique solution of the equation such that

$$\lim_{t \to +\infty} \vec{x}(t) = \begin{pmatrix} 9 \\ 0 \\ 3 \end{pmatrix} \quad \text{and} \quad (\vec{x})'(0) = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}.$$

2. (9 points) Suppose a mass weighing 1kg stretches a spring $\frac{9.8}{4}$m at equilibrium. Suppose that a time zero the spring is pulled down an additional 1m and set in motion with an initial downward velocity of 2m/s (Remember that we defined positive values for y and y' to be in the downwards direction). Moreover, there is an external force acting on the spring of the form $F(t) = 2\cos(2t)$N. You may ignore damping, and assume that the gravitational constant g is equal to $9.8\text{m}/s^2$.

   Write down the initial value problem describing this dynamical system and solve it using the method of undetermined coefficients.

3. (9 points) Consider the non-homogeneous first-order system

$$\vec{x}' = \begin{pmatrix} 1 & 3 \\ 0 & 2 \end{pmatrix} \vec{x} + \begin{pmatrix} e^t \\ e^{2t} \end{pmatrix}.$$

(a) (3 points) Find the eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 1 & 3 \\ 0 & 2 \end{pmatrix}$.

(b) (6 points) Find a particular solution to the non-homogeneous system.

4. (9 points) Consider the second order differential equation:

$$y'' - 8y' + 16y = \frac{e^{4t}}{1+t}.$$

(a) (3 points) Find y_1 and y_2, two linearly independent solutions of the homogeneous part of the equation.

(b) (6 points) Find a particular solution to the non-homogeneous equation.
(*Hint: if you are blocked with an integral, you may need to write* $t = 1 + t - 1$.)

5. (9 points) Consider the second-order differential equation

$$t^2y'' - 3ty' + 4y = 0, \qquad t > 0. \tag{1}$$

(a) (3 points) Find an integer n such that $y_1(t) = t^n$ solves the equation.

(b) (3 points) Set $y_2(t) = v(t)y_1(t)$ where v is an unknown function.
Write down the differential equation satisfied by v such that y_2 solves (1).

(c) (3 points) Solve the previous equation and write down a fundamental set of solutions for the equation (1).

6. (3 points) Consider the following IVP:

$$(t-2)y'''(t) + \ln(t)y''(t) - \sqrt{t-4}y' + \frac{1}{t-3}y = \tan t, \qquad y(\alpha) = 2.$$

What is the largest interval for which a solution of the following initial value problem is certain to exist in the three following cases:

- $\alpha = 1$,
- $\alpha = 2.5$,
- $\alpha = 4$.

7. (2 points) A small number of points will be allocated for presentation, neatness, and organization. Please ensure that

- Your work is legible in the scan.
- Your name or initials are at the top of every page.
- Questions are answered in the order in which they were given.
- During the upload process you have indicated which pages correspond to which question, and made sure that none of your pages are upside down or sideways (you can also change the orientation of the pages when you upload in Gradescope). Ensuring that these criteria are met helps ensure that your exam is graded efficiently and accurately.