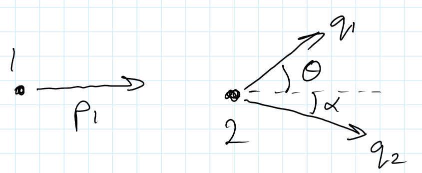
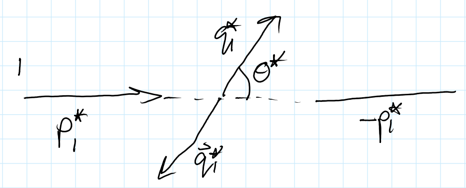
Oblique, Elastic Scattering

Ready to consider the general case of oblique scattering of 2 particles. Crucial in nuclear physics. The calculations are easier c.o.m frame, but want to relate them to the experimental/lab frame where the target particle is at rest.

Lab frame:



Center of mass frame



Since collision is elastic

$$T^* = \frac{p^{*2}}{2\mu} = \frac{q^{*2}}{2\mu} \rightarrow p^* = q^*$$

Relate the lab & rest frame coords:

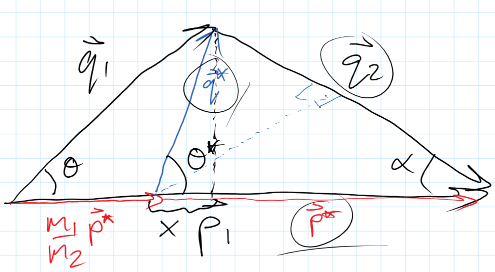
In lab frame $\vec{p}_2 = 0$, so $\dot{\vec{R}} = \frac{p^*}{m_2}$ & $\underline{\vec{p}_1 = \frac{m_1}{m_2}\vec{p}^* + \vec{p}^* = \frac{M}{m_2}p^*}$

after collision $\vec{q}_1 = m_1\dot{\vec{R}} + \vec{q}^* = \underline{\frac{m_1}{m_2}\vec{p}^* + \vec{q}^*}$

$$\vec{q}_2 = \vec{p}^* - \vec{q}^*$$

We also have $\vec{p}_1 = \vec{q}_1 + \vec{q}_2$

Based on these relations we can draw this diagram



Can use these triangles to evaluate the angles as functions of energies & momenta.

The $\vec{p}^*$, $\vec{q}^*$ & $\vec{q}_2$ vectors form an iscoloes triangle (since $p^* = q^*$)

m_2

triangle (since $p^* = q^*$)

$\therefore 2\alpha + \theta^* = \pi$ or $\alpha = \frac{1}{2}(\pi - \theta^*)$

$$\cos\alpha = \frac{\frac{1}{2}q_2}{p^*} \rightarrow q_2 = 2p^*\cos\alpha = 2p^*\cos\left(\tfrac{1}{2}(\pi-\theta^*)\right)$$
$$= 2p^*\left(\cancel{\cos\tfrac{\pi}{2}\cos\tfrac{\theta^*}{2}} + \sin\tfrac{\pi}{2}\sin\tfrac{\theta^*}{2}\right)$$
$$= 2p^*\sin\frac{\theta^*}{2}$$

The KE of the target particle in the lab frame after the collision is $T_2 = \frac{q_2^2}{2m_2} = \frac{2p^{*2}}{m_2}\sin^2\frac{\theta^*}{2}$

The KE of the incoming particle in the lab frame is

$$T = \frac{p_1^2}{2m_1} = \frac{M^2 p^{*2}}{2m_1 m_2}$$

$\therefore$ The fraction of the incoming KE that is transferred to the target particle is $\frac{T_2}{T} = \frac{4m_1m_2}{M^2}\sin^2\left(\frac{\theta^*}{2}\right)$

Max. transfer occurs when $\theta^* = \pi$ (ie a head-on collision);

then the max is $\frac{4m_1m_2}{M^2}$ which can only be close to unity is $m_1 \simeq m_2$.

For a proton/α-particle collision $\left(\frac{m_1}{m_2} = 4 \text{ or } \frac{1}{4}\right)$ then transfer is 64%

" " e^-/p^+ " $\left(\frac{m_1}{m_2} = \frac{1}{1836}\right)$ " " 0.2%

(neglecting rel. effects)

From the triangles we see

$$\cos\theta = \frac{\left(\frac{m_1}{m_2}\right)p^* + x}{q_1} \quad ; \quad \cos\theta^* = \frac{x}{q^*} \rightarrow x = q^*\cos\theta^* = p^*\cos\theta^*$$

$\rightarrow q^*\sin\theta^* = p^*\sin\theta^*$

$$\therefore \cos\theta = \frac{\left(\frac{m_1}{m_2}\right)p^* + p^*\cos\theta^*}{q_1}$$

$$\therefore \cos\theta = \frac{\left(\frac{m_1}{m_2}\right)p^* + p^*\cos\theta^*}{q_1}$$

$$\rightarrow \tan\theta = \frac{p^* \sin\theta^*}{\left(\frac{m_1}{m_2}\right)p^* + p^*\cos\theta^*} = \frac{\sin\theta^*}{\left(\frac{m_1}{m_2}\right) + \cos\theta^*}$$

If the target is much more massive than the incoming particle $m_2 >> m \rightarrow \tan\theta \simeq \tan\theta^*$ or $\theta \simeq \theta^*$; the 2 scattering angles will be equal.

Another interesting case is when $m_1 = m_2$ then

$$\tan\theta = \frac{\sin\theta^*}{1+\cos\theta^*} = \tan\left(\frac{\theta^*}{2}\right)$$

$\rightarrow \theta = \frac{\theta^*}{2}$, the deflection angle is $1/2$ of the value in the c.o.m frame

in this case, $\alpha = \frac{1}{2}(\pi - \theta^*) = \frac{1}{2}(\pi - 2\theta) = \frac{\pi}{2} - \theta$

$\hookrightarrow$ the 2 particles leave at $\perp$ angles as seen in the lab. frame