FINAL
MATH 3215-C (PROBABILITY AND STATISTICS)

THURSDAY, DECEMBER 3

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IMPORTANT: Please read carefully (1pt)

- You have a **14 hour** window to take and submit your exam (**7 am - 9 pm**).
- Be warned: **exam ends at 9 pm** (e.g. if you start at 7 pm, you only have 2 hours).
- After you opened this file you have 170 minutes to finish the work and 30 minutes to submit it **(200 minutes in total)**.
- If you run into difficulties submitting on GradeScope, email the files to the instructor before the 200 minutes expire and before 9 pm. **Late submissions will not be accepted**.
- If you encounter technical problems, email the instructor as soon as possible.
- You **CAN** use the course textbook and the lecture notes/slides for reference.
- You **CAN** use any fact we presented in class without proving them; anything else used must be proved.
- You **CANNOT** get any help from or collaborate with anyone.
- Posting the problems online to get help or to let others know what the problems are will be a violation; it will be reported and result in a penalty.
- To get full credit you need to write complete answers.
- Numerical answers must be up to 4 decimal precision.
- The total amount of points for this exam is 150. Different problems have different weights.
- Be wise with your time. You can handwrite your answers on a different paper, and submit a photocopy. Make sure it is readable. No need to print the problem sheet or copy the problems.

Calculator and software use

- You can use a calculator for arithmetic computations.
- To find the values of standard distributions, you **MUST** use the tables in the appendix of the textbook (e.g. if it is asking to find a cdf value for the normal distribution, you need to reduce to the standard normal and find the value from the table in the back of the book).
- You may verify your answers for yourself with a calculator.
- You can plot the graphs by hand or you may use any graphical software.

Problem 1 (19pt). *A shepherd has 100 white and 10 black sheep in his flock. He randomly chooses 5 to sell in the market. What is the probability that at least one of the selected sheep is black?*

Problem 2 (20pt). *According to the U.S. Census Bureau, persons under age* 18 *years form the* 22.3%*, and persons of age* 65 *years and over form the* 16.5% *of the U.S. population. Let* X *be the number of persons aged 18-64 years in a sample of 100 people. What is the approximate probability that*

(a) $X = 50$*?*
(b) $55 < X \leq 65$*?*

Problem 3 (30pt). *Let the joint pdf of* X *and* Y *be*

$$f(x, y) = c \quad \text{for} \quad 0 \leq x \leq 2, \quad x^3 - 0.5 \leq y \leq x^3 + 0.5.$$

(a) Find c*.*
(b) Sketch the region where $f(x, y) > 0$*.*
(c) Calculate and plot the least squares line.
(d) Calculate and plot $E(Y|x)$*.*

Problem 4 (30pt). *Let* $X_1, \ldots, X_n$ *be a random sample from continuous distribution with pdf*

$$f(x; \theta) = \frac{3x^2}{\theta}, \quad 0 \leq x \leq \sqrt[3]{\theta}.$$

(a) Find the mean and variance of the X_i*-s.*
(b) Find the maximum likelihood estimator of θ*.*
(c) Find a constant c *such that*

$$\tilde{\theta} = c \sum_{i=1}^{n} X_i^3$$

is an unbiased estimator of θ*.*

Problem 5 (30pt)**.** *Let*

$$data = \{0.1, \quad 0.5, \quad 1, \quad 1.2, \quad 3, \quad 0.8, \quad 1.6, \quad -3, \quad 2, \quad 10\}.$$

(a) Compute the 5-number summary of the data.
(b) Draw the box plot with Tukey's inner and outer fences showing outliers and suspected outliers.

Problem 6 (20pt)**.** *The soccer balls for FIFA world cup are regulated to have average weight of 430 grams. However the players testing the balls before the tournament complained that the balls seem to be lighter than what they were expecting them to be. FIFA representative randomly collected 12 balls and measured their weights. Here is the data*

$$\begin{aligned} data = \{&423.90, \quad 420.24, \quad 431.00, \quad 418.76, \\ &428.68, \quad 423.64, \quad 430.65, \quad 432.92, \\ &421.93, \quad 433.97, \quad 426.10, \quad 430.20\} \end{aligned}$$

Assuming ball weights are normally distributed

(a) At 5% significance level, check the null hypothesis $H_0 : \mu = 430$ against the alternative hypothesis of $H_0 : \mu < 430$. What is your conclusion?
(b) Compute the approximate p-value of the test.