Homework 11

Due Date: All homework submitted by Sunday **11/22** 11:59pm will be graded together. Homework submitted past that time may be graded late. Submit your homework through Canvas as a single pdf file. Do not use solution sets from previous years. You are encouraged to discuss homework assignments with each other, the TAs or myself, but the solutions have to be executed and submitted individually.

Problem A [60%] . **Magnetic coaxial cable**. Consider a very-long straight coaxial cable consisting of an inner cylindrical conductor of radius a and a very thin cylindrical conducting shell of radius $b>a$. The center conductor is made of copper (negligible magnetic susceptibility $\chi_m = 0$) and carries a total current I in the $+\hat{z}$ direction (distributed uniformly). The outer shell is also made of copper, and carries a total return current I (distributed uniformly over the shell) in the $-\hat{z}$ direction. The space between the conductors ($a < s < b$) is filled with *Mourigalium*, a novel ceramic substance of formula $Mo_3U_2Re_3Ga_8Lu_2$, which we will model as a linear paramagnetic material of magnetic susceptibility $\chi_m > 0$.

(1) Calculate the $\mathbf{H}$-field everywhere, namely in regions $s \leq a$, $a < s < b$ and $s \geq b$. Clearly lay out your reasoning about symmetries, invariance, etc. Specify magnitude and direction.

(2) Calculate the $\mathbf{B}$-field everywhere, specifying magnitude and direction. Graph the spatial dependence of the non-zero component(s) of $\mathbf{B}$.

(3) Solve for *any* bound currents (surface, volume). Indicate clearly the direction of any bound currents, keeping in mind that the *Mourigalium* is paramagnetic. Comment on the physical meaning.

(4) The *Mourigalium* becomes a superconductor (a perfect linear diamagnet $\chi_m = -1$) below T_c=260 K. Calculate the $\mathbf{B}$-field everywhere and any bound currents in the above cable if measured in Siberia (average temperature in January $T < T_c$).

Problem B [40%] **Charged sphere spinning**. A uniformly charged solid sphere of radius R carries a total charge Q and is set spinning with angular velocity ω about the z-axis.

(1) What is the magnetic dipole moment of the sphere? Provide magnitude and direction.