Midterm 1, Math 2552

PLEASE PRINT YOUR NAME CLEARLY IN ALL CAPITAL LETTERS

First Name _____ Last Name _____

GTID Number: _____

Student GT Email Address: _____@gatech.edu

Section Number (A1, A2 or A3) _____ TA Name _____

INSTRUCTIONS (PLEASE READ)

Formatting and Timing

- You should only need 75 min to take the exam, but students will have 3 hours to submit the exam, from the time that it is released.
- **Show your work** and justify your answers for all questions unless stated otherwise.
- Please write neatly, and use dark and clear writing so that the scan is easy to read.
- Please write your name or initials at the top of every page.
- Please solve the questions in the exam in the order they are given.
- You do not need to print the exam. As long as you solve problems in the order they are given (just like the written homework sets), you can write your answers on your own paper.But students can print the exam and write their answers on the printed copy if they prefer.

Submission

- Students should scan their work and submit it through Gradescope. There should be an **assignment** in Gradescope for this exam. The process for submitting your work will be similar to what you have used for homework.
- Work must be submitted today by 12:30 PM ET.
- Please upload your work as a single PDF file. If this is not possible you can email your work to your instructor.
- During the upload process in Gradescope, please indicate which page of your work correspondsto each question in the exam.

Questions

- If there are questions during the exam, students can ask them on BlueJeans, email their instructor or message them through Canvas.
- Our course Piazza forum will be temporarily inactive for 24 hrs on the day of the exam.
- If you run into any technical issues or any unanticipated emergencies, please email your instructor as soon as you can.

Integrity

- Students can use any resources while taking these tests including online calculators and Mathematica.
- Students cannot communicate with anyone during these tests including using Reddit or online message boards
- Students cannot use solutions provided from another student or third party.
- In other words: do your own work but you can use technology to solve problems.

1. (8 points) Solve the following initial value problems:

   (a) (4 points)

$$y' = y\sin(t), \qquad y(\frac{\pi}{2}) = -\pi.$$

   (b) (4 points)

$$ty' - y = t^2 e^{-t}, \quad t > 0, \qquad y(1) = 2 - e^{-1}.$$

2. (10 points) Consider the differential equation

$$y' = y^2(y - \sqrt{2})(y + \sqrt{2}).$$

(a) (2 points) Find the equilibria.

(b) (2 points) Draw the phase line and classify the equilibria.

(c) (2 points) Using the phase line, what is $\lim_{t \to +\infty} y(t)$ if $y(0) = 1$?

(d) (2 points) Determine where y is concave up and where it is concave down.

(e) (2 points) Using questions a, b and d, sketch a few solution curves.

3. (8 points) (a) (4 points) A pancake of 390°F is placed in a room of 90°F. After 3 minutes, the temperature of the pancake is 190°F. Write down the differential equation and the initial condition for $T(t)$, the pancake temperature after t minutes, and compute the cooling rate k of the pancake. (You can give an answer with logarithm terms).

(b) (4 points) A tank containing 20g of salt dissolved in 1000 liters of water has two salt solutions pumped in. The first solution of 4g of salt per liter is pumped in at a rate of 15 L/min and the second solution with a concentration of salt of 14g/L is pumped in at a rate of 10 L/min. The tank drains at 25 L/min. Assume the tank is well mixed.

Write down the Initial Value Problem for the quantity of salt $Q(t)$ at time t. What could you expect to be the limit of $Q(t)$ as t goes to infinity?

4. (8 points) Find the general solutions of the following systems of differential equations in terms of real valued functions:

   (a) (4 points) $\dfrac{\mathrm{d}}{\mathrm{d}t}\vec{x} = \begin{pmatrix} 1 & 0 \\ 2 & -3 \end{pmatrix}\vec{x}$.

(b) (4 points) $\dfrac{\mathrm{d}}{\mathrm{d}t}\vec{x} = \begin{pmatrix} -3 & -2 \\ 2 & -3 \end{pmatrix} \vec{x}.$

5. (8 points) Sketch the phase portraits for the following systems:

   (a) (4 points) $\dfrac{\mathrm{d}}{\mathrm{d}t}\vec{x} = \begin{pmatrix} -8 & -5 \\ 5 & 2 \end{pmatrix}\vec{x}$.

(b) (4 points) $\dfrac{\mathrm{d}}{\mathrm{d}t}\vec{x} = A\vec{x}$, knowing that $A\begin{pmatrix} 1-i \\ 1 \end{pmatrix} = (-1+2i)\begin{pmatrix} 1-i \\ 1 \end{pmatrix}$.

6. (3 points) What is the largest interval for which a solution of the following initial value problem is certain to exist:

$$(t-2)y' = \frac{y}{t^2-16} + \cos(t), \quad y(\pi) = -5.$$

7. (2 points) Transform the following differential equation into a system of first order differential equations:

$$my'' + \gamma y' + ky = 3e^t, \qquad \text{where } m > 0,\ \gamma > 0,\ k > 0.$$

8. (2 points) A small number of points will be allocated for presentation, neatness, and organization. Please ensure that

   - Your work is legible in the scan.
   - Your name or initials are at the top of every page.
   - Questions are answered in the order in which they were given.
   - During the upload process you have indicated which pages correspond to which question, and made sure that none of your pages are upside down or sideways (you can also change the orientation of the pages when you upload in Gradescope). Ensuring that these criteria are met helps ensure that your exam is graded efficiently and accurately.

9. (1 point) Please **sign and date** the following GT Honor Code statement.

Georgia Tech Honor Code

Having read the Georgia Institute of Technology Academic Honor Code, I understand and accept my responsibility as a member of the Georgia Tech community to uphold the Honor Code at all times.