Homework 1

Due Date: Week of Friday **08/28**. All homework submitted by Sunday **08/30** 11:59pm will be graded together. Homework submitted past that time may be graded late. Submit your homework through Canvas as a single pdf file. Do not use solution sets from previous years. You are encouraged to discuss homework assignments with each other, the TAs or myself, but the solutions have to be executed and submitted individually.

Problem A [20%]. **Some classic results on triple products.**

(a) Prove the Grassmann identity for arbitrary vectors $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C} \in \mathbb{R}^3$:

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}\,(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}\,(\mathbf{A} \cdot \mathbf{B})$$

(b) Use the Grassmann identity to derive the Jacobi identity:

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) + \mathbf{B} \times (\mathbf{C} \times \mathbf{A}) + \mathbf{C} \times (\mathbf{A} \times \mathbf{B}) = \mathbf{0}$$

Problem B [20%]. **Working with non-uniform charge densities.**

A hollow sphere (outer radius R, thickness $t < R$, inner radius $R - t$) of an unknown material carries a volume charge density $\rho(r, \theta, \phi) = \alpha e^{-\beta r^3} |\sin(\theta)|$ where α and β are constants.

(a) What is the total charge Q on the **sphere** in terms of α and β ?

(b) What are the dimensions (physical units) of α and β ?

(c) Imagine that the thickness t is much smaller than R, *i.e.* $t \ll R$, such that the charge carried by the sphere can be approximated by a surface charge density σ. What is the expression for $\sigma(\theta, \phi)$?

TURN PAGE

Problem C [60%]. **Charged cylindric can.**

An aluminum can of La Croix (Pamplemousse flavor) has been charged electrically. Consider that the walls, top and bottom of the can are infinitely thin, and that the can is well modeled by a closed cylinder of radius R and height h carrying a uniform <u>surface</u> charge density σ. Take the reference of cylindrical coordinates, point O, at the center of the can.

(a) Using Coulomb's law, calculate the magnitude of the electric field at a point P (coordinate z) located on the axis of the cylinder ($s = 0$) but outside the can ($|z| > h$). Would it be possible to solve this problem using Gauss' law?

(b) By taking the limit $z \to \infty$, give the first two terms in the "far-field" expansion of the can's electric field.