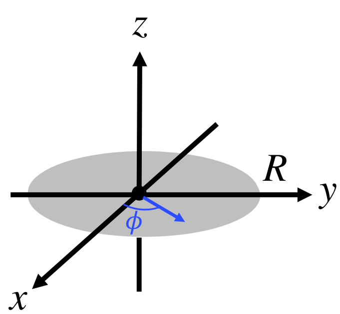
Homework 3

Due Date: All homework submitted by Sunday **09/13** 11:59pm will be graded together. Homework submitted past that time may be graded late. Submit your homework through Canvas as a single pdf file. Do not use solution sets from previous years. You are encouraged to discuss homework assignments with each other, the TAs or myself, but the solutions have to be executed and submitted individually.

Exercise 1 [10%]. Let $\Phi(\mathbf{r}) = \Phi(x, y, z)$ be any function that can be expanded in a Taylor series around point $\mathbf{r}_0 = (x_0, y_0, z_0)$. Write a Taylor series expansion for the value of Φ at each of the six points $\mathbf{r}_1 = (x_0 + \delta, y_0, z_0)$, $\mathbf{r}_2 = (x_0 - \delta, y_0, z_0)$, $\mathbf{r}_3 = (x_0, y_0 + \delta, z_0)$, $\mathbf{r}_4 = (x_0, y_0 - \delta, z_0)$, $\mathbf{r}_5 = (x_0, y_0, z_0 + \delta)$ and $\mathbf{r}_6 = (x_0, y_0, z_0 - \delta)$, which symmetrically surround the point $\mathbf{r}_0$ at a small distance δ. Expand up to third order in δ. Show that if Φ satisfies Laplace's equation then $\Phi(\mathbf{r}_0) = \frac{1}{6}\sum_i \Phi(\mathbf{r}_i)$ through terms of the third order in δ (keep contribution up to δ^3 if present).

Problem A [40%]. A flat insulating disk of radius R is placed in the xy plane. It carries a *non-uniform* charge density $\sigma = \sigma_0 \cos(\phi/4)$ where ϕ is the usual azimuthal angle in the xy plane, as shown, and $\sigma_0 > 0$.



(1) Draw the surface charge density on the disk (using a top view).

(2) Using the direct-formula for $V(\mathbf{r})$, find the potential $V(s = 0, z)$ along the z-axis for any z. Use a cylindrical coordinate system.

(3) From your results for $V(\mathbf{r})$, calculate the electric field $\mathbf{E}$ along the z-axis for $z > 0$ and for $z < 0$.

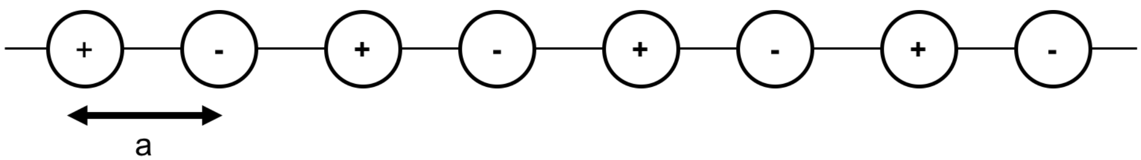
(4) Why is it much harder to calculate $\mathbf{E}$ directly? Would Gauss's law be of any use here?

(5) Plot the projection of the electric field along $\hat{z}$, $\mathbf{E}(z) \cdot \hat{z}$, as a function of z. Using the concept of boundary condition (for instance by calculating the normal derivative of the potential $\partial V/\partial n$ close to the surface) explain the jump that is obversed in the magnitude of the electric field around $z = 0$.

Problem B [25%]. The goal of this problem is to find the total electrostatic energy stored in a uniformly charge sphere of radius R and total charge Q. Note that the charge is uniformly distributed throughout the whole volume – this is not just a shell of charges.

(1) Calculate the total electrostatic energy stored in the sphere. Express your answer in terms of Q, R, and constants of nature. There are many different ways to do this, you might want you to use two different methods so you can check your result. One question you could ask is how much energy it takes to assemble the sphere "shell by shell".

Problem C [25%] Consider an *infinite* chain of alternating point charges q and $-q$ as shown below. The lattice spacing of this one-dimensional salt (Na positive charges, Cl negatice charges) is a.



(1) Find the work *per particle* required to assemble such a configuration.