## Formulas

• Bayes' Theorem: If  $B_1, \ldots, B_k$  form a partition of the sample space S, and A is any event, then for any  $i = 1, \ldots, k$ ,

$$\mathbb{P}(B_i \mid A) = \frac{\mathbb{P}(A \mid B_i)\mathbb{P}(B_i)}{\sum_{j=1}^k \mathbb{P}(A \mid B_j)\mathbb{P}(B_j)}.$$

- The following are probability mass functions for distributions we discussed.
  - 1. Bernoulli with parameter p:

$$f(x) = \begin{cases} p & x = 1\\ 1 - p & x = 0\\ 0 & \text{otherwise} \end{cases}$$

2. Geometric with parameter p:

$$f(x) = \begin{cases} (1-p)^{x-1}p & x = 1, 2, \dots \\ 0 & \text{otherwise} \end{cases}$$

3. Binomial with parameters n, p:

$$f(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x} & x = 0, 1, 2, \dots, n \\ 0 & \text{otherwise} \end{cases}$$

4. Hypergeometric with parameters  $N_1, N_2, n$ :

$$f(x) = \begin{cases} \frac{\binom{N_1}{x}\binom{N_2}{n-x}}{\binom{N_1+N_2}{n}} & \text{for } \max\{0, n-N_2\} \le x \le \min\{N_1, n\} \\ 0 & \text{otherwise} \end{cases}$$

5. Negative binomial with parameters r, p:

$$f(x) = {x-1 \choose r-1} p^r (1-p)^{x-r}, \ x = r, r+1, \dots$$

6. Poisson with parameter  $\lambda$ :

$$f(x) = e^{-\lambda} \frac{\lambda^x}{x!}, \ x = 0, 1, 2, \dots$$

7. Trinomial with parameters  $n, p_A, p_B, p_C$ :

$$f(x,y) = \frac{n!}{x! u! (n-x-y)!} p_A^x p_B^y p_C^{n-x-y}$$
 for  $x, y \ge 0$  and  $x+y \le n$ .

- The following are probability distribution functions for distributions we discussed.
  - 1. Uniform on [a, b]:

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{if } x \in [a, b] \\ 0 & \text{otherwise.} \end{cases}$$

2. Exponential with parameter  $\theta$ :

$$f(x) = \begin{cases} \frac{1}{\theta} e^{-x/\theta} & \text{if } x \ge 0\\ 0 & \text{otherwise.} \end{cases}$$

3. Gamma with parameters  $\alpha, \theta$ :

$$f(x) = \begin{cases} \frac{1}{\Gamma(\alpha)\theta^{\alpha}} x^{\alpha - 1} e^{-x/\theta} & \text{if } x \ge 0\\ 0 & \text{otherwise.} \end{cases}$$

4. Chi-square with r degrees of freedom:

$$f(x) = \begin{cases} \frac{1}{\Gamma(r/2)2^{r/2}} x^{r/2-1} e^{-x/2} & \text{if } x \ge 0\\ 0 & \text{otherwise.} \end{cases}$$

5. Normal with parameters  $\mu, \sigma^2$ :

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right).$$

• Least squares regression line:

$$y = \mu_Y + \rho \frac{\sigma_Y}{\sigma_X} (x - \mu_X).$$