Quiz 1

Student Name: ______________________________ (Please Write Clearly)

Problems: There are 3 problems: A[40%], B[50%], C[30%+10% bonus]. Spend your time accordingly.

Formulas: You may use the Navy Research Laboratory plasma formula booklet provided in class. Calculators are allowed but I suspect will not be useful. In addition, find below some possibly irrelevant information:

Grading: Problems B and C will be graded based on the quality of your explanations. Please be brief but clear! A correct answer with no explanation or indication of how you got the answer will not be worth any points. You may get partial extra credit for showing clearly which steps you would use even if the final answer is not correct.

Maths

$$\int \frac{dx}{x} = \ln x$$

$$\int \frac{dx}{x+a} = \ln(a+x)$$

$$\int \frac{xdx}{x+a} = x - a\ln(a+x)$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln(x+\sqrt{a^2+x^2})$$

$$\int \frac{xdx}{\sqrt{x^2 \pm a^2}} = \sqrt{a^2+x^2}$$

$$\int \frac{dx}{x^2 \pm a^2} = \frac{1}{a}\tan^{-1}(x/a)$$

$$\int \frac{dx}{(x^2 \pm a^2)^{3/2}} = \frac{\pm x}{a^2\sqrt{x^2 \pm a^2}}$$

$$\int \frac{xdx}{(x^2 \pm a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 \pm a^2}}$$

$$\int \cos(ax)dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax)dx = -\frac{1}{a}\cos(ax)$$

Physics

Direct evaluation:

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\iiint \frac{\rho(\mathbf{r}')\boldsymbol{\mathscr{r}}}{\mathscr{r}^3}\mathrm{d}\tau' \quad \text{with} \quad \boldsymbol{\mathscr{r}} = \mathbf{r} - \mathbf{r}'$$

$$\mathbf{V}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\iiint \frac{\rho(\mathbf{r}')}{\mathscr{r}}\mathrm{d}\tau' \quad \text{with} \quad \boldsymbol{\mathscr{r}} = \mathbf{r} - \mathbf{r}'$$

Divergence theorem and Gauss:

$$\nabla \cdot \mathbf{E} = \rho(\mathbf{r})/\varepsilon_0 \quad \text{or} \quad \oint\!\!\oint \mathbf{E}\cdot \mathrm{d}\mathbf{a} = \frac{1}{\varepsilon_0}\iiint \rho(\mathbf{r}')\,\mathrm{d}\tau'$$

Potential:

$$\Delta V = -\int \mathbf{E}\cdot \mathrm{d}\mathbf{l} \quad \text{or} \quad \mathbf{E} = -\nabla V$$

Circulation:

$$\nabla \times \mathbf{E} = \mathbf{0} \quad \text{or} \quad \oint \mathbf{E}\cdot \mathrm{d}\mathbf{l} = 0$$

Neumann Boundary Conditions:

$$\partial V/\partial n = -\sigma/\varepsilon_0$$

Poisson Equation:

$$\nabla^2 V = -\rho/\varepsilon_0$$

Energy:

$$W = \frac{1}{2}\sum_i q_i V(\mathbf{r}_i) \quad \text{or} \quad W = \frac{\varepsilon_0}{2}\iiint |\mathbf{E}|^2 \mathrm{d}\tau$$

Cartesian. $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}; \quad d\tau = dx\,dy\,dz$

Gradient: $$\nabla t = \frac{\partial t}{\partial x}\hat{\mathbf{x}} + \frac{\partial t}{\partial y}\hat{\mathbf{y}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence: $$\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Curl: $$\nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$$

Laplacian: $$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

Spherical. $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}; \quad d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

Gradient: $$\nabla t = \frac{\partial t}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}$$

Divergence: $$\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

Curl: $$\nabla \times \mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial \phi}\right]\hat{\mathbf{r}}$$
$$+ \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial \theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian: $$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial t}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial \phi^2}$$

Cylindrical. $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}; \quad d\tau = s\,ds\,d\phi\,dz$

Gradient: $$\nabla t = \frac{\partial t}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

Divergence: $$\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

Curl: $$\nabla \times \mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial \phi}\right]\hat{\mathbf{z}}$$

Laplacian: $$\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$$