PHYS 3201 — Assignment #7

Due: 10/16/20

1. The most energy-efficient way to send a spacecraft to the Moon is to boost its speed while it is in circular orbit about the Earth such that its new orbit is an ellipse. The boost point is the perigee of the ellipse, and the point of arrival at the Moon is the apogee. Calculate the percentage increase in speed required to achieve such an orbit. Assume that the spacecraft is initially in a low-flying circular orbit about Earth. The distance between Earth and the Moon is approximately $60R_e$, where R_e is the radius of Earth.

2. A particle moves in an elliptical orbit in an inverse-square force field. Prove that the product of the minimum and maximum speeds is equal to $(2\pi a/\tau)^2$, where a is the semi-major axis and τ is the period.

3. Prove that the time-average of the potential energy of a particle moving in an elliptical orbit in a central inverse-square force field is $-k/a$, where a is the semi-major axis.

4. A comet is moving in a parabolic orbit lying in the plane of Earth's orbit. Regarding Earth's orbit as circular of radius a, show that the points where the comet intersects Earth's orbit are given by

$$\cos\theta_0 = -1 + \frac{2p}{a}$$

   where p is the perihelion distance of the comet defined at $\theta = 0$.

5. Use the results of the previous problem to show that the total time interval the comet remains inside Earth's orbit is

$$T = \int_{\text{entering}}^{\text{leaving}} dt = \frac{2^{1/2}}{3\pi}\left(\frac{2p}{a}+1\right)\left(1-\frac{p}{a}\right)^{1/2} \text{ yrs}$$

   and that the maximum value of this time interval occurs when $p = a/2$ and is $(2/3\pi)$ yrs (77.5 days). (*Hint: Use conservation of angular momentum twice: once to change the integral into one over θ, and once to evaluate J at a useful location. You'll also need to use Kepler's 3rd Law to get rid of a GM factor. This will also convert the expression into units of years.*)