Sample Quiz 4, Math 1554, Fall 2019

PLEASE PRINT YOUR NAME CLEARLY IN ALL CAPITAL LETTERS

First Name ____________________ Last Name ____________________

GTID Number: ______________________

Student GT Email Address: _________________________@gatech.edu

Section Number (e.g. A4, M2, QH3, etc.) _______ TA Name ________________________

Student Instructions

- **Show your work** and justify your answers for all questions unless stated otherwise.
- **Organize your work** in a reasonably neat and coherent way.
- Calculators, notes, cell phones, books are not allowed.
- Use dark and clear writing: your exam will be scanned into a digital system.
- Exam pages are double sided. Be sure to complete both sides.
- Leave a 1 inch border around the edges of exams.
- Any work done on scratch paper will not be collected and will not be graded.

Math 1554, Sample Quiz 4. Your initials: ______________

You do not need to justify your reasoning for questions on this page.

1. (3 points) Indicate whether the statements are true or false.

true	false	
○	○	If $A \in \mathbb{R}^{2\times 2}$, $A = A^T$, and A has distinct eigenvalues λ_1 and λ_2, the corresponding eigenvectors $\vec{v}_1$ and $\vec{v}_2$ are orthogonal.
○	○	The quadratic form $Q = -x^2 - 2xy - y^2$ is negative definite.
○	○	$2x^2 - 2xy + y^2 \geq 0$ for all real values of x and y.

2. (4 points) If possible, give examples of the following.

   (a) A non-zero 2×2 elementary matrix, A, that can be diagonalized as PDP^T.

$$A = \begin{pmatrix} \quad & \quad \\ \quad & \quad \end{pmatrix}$$

   (b) An indefinite quadratic form that has no cross terms, and is expressed in the form $Q = \vec{x}^T A\vec{x}$, where $\vec{x} \in \mathbb{R}^2$.

$$Q(\vec{x}) =$$

3. (3 points) Fill in the blanks.

   (a) A unit vector that that gives the location of the maximum value of $Q(\vec{x}) = x_1^2 - 2x_2^2$ subject to $\vec{x}^T\vec{x} = 1$, $\vec{x} \in \mathbb{R}^2$, is $\begin{pmatrix} \quad \\ \quad \end{pmatrix}$.

   (b) $\vec{p}$ is an eigenvector of A with unit length that corresponds to eigenvalue $\lambda = 12$. The value of $\vec{p}^T A\vec{p}$ is $\boxed{\qquad}$.

   (c) The maximum value of $Q = \vec{x}^T A\vec{x} = \vec{x}^T \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix} \vec{x}$ subject to the constraints $\vec{x}^T\vec{x} = 1$ and $\vec{x} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 0$ is equal to $\boxed{\qquad}$.