PHYS 3201 — Assignment #10

Due: 11/20/20

1. Use direct integration to show that the gravitational potential is constant at any point inside a thin uniform spherical shell.

2. Assuming Earth to be a uniform solid sphere, show that if a straight hole were drilled from pole to pole, a particle dropped into the hole would execute simple harmonic motion. Show also that the period of this oscillation depends only on the density of the Earth and is independent of the size. What is the period in hours ($R_{\mathrm{Earth}} = 6.4 \times 10^6$ m)?

3. A planet of density ρ_1 (spherical core, radius R_1) is discovered within a thick spherical cloud of dust (density ρ_2, radius R_2). What is the gravitational force on a particle of mass m placed within the dust cloud at radius R_0, where $R_1 < R_0 < R_2$?

4. The isochrone potential is written as

$$\Phi(r) = -\frac{GM}{b + \sqrt{b^2 + r^2}},$$

where b is a constant.

   (a) Show that the circular speed in this potential is

$$v_c^2 = \frac{GMr^2}{(b+a)^2 a},$$

   where $a = \sqrt{b^2 + r^2}$.

   (b) Show that the central density (at $r = 0$) is

$$\rho(0) = \frac{3M}{16\pi b^3}$$

   and at large radii ($r \gg b$)

$$\rho(r) \sim \frac{bM}{2\pi r^4}.$$

5. Consider the following spherical density distribution (Jaffe, W., 1983, MNRAS, 202, 995):

$$\rho(r) = \left(\frac{M}{4\pi r_J^3}\right) \frac{r_J^4}{r^2 (r + r_J)^2},$$

where M and r_J are constants.

   (a) Verify that the total mass of the system is M.

(b) Show that the potential generated by this distribution is

$$\Phi(r) = \frac{GM}{r_J} \ln\left(\frac{r}{r + r_J}\right)$$

(FYI: $\int \frac{dx}{x(a+bx)^2} = \frac{1}{a(a+bx)} - \frac{1}{a^2} \ln(\frac{a+bx}{x})$)

(c) Show that the circular speed, v_c, is approximately constant at $r \ll r_J$ and falls off as $v_c \propto r^{-1/2}$ at $r \gg r_J$.