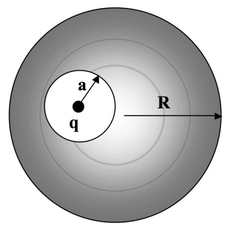
Homework 4

Due Date: All homework submitted by Sunday **09/27** 11:59pm will be graded together. Homework submitted past that time may be graded late. Submit your homework through Canvas as a single pdf file. Do not use solution sets from previous years. You are encouraged to discuss homework assignments with each other, the TAs or myself, but the solutions have to be executed and submitted individually.

Problem A. Polarized Conductor [20%]. A spherical cavity (radius a) is dug out of a larger solid (and overall neutral) conducting sphere of radius R. Note that the cavity is off-centered by a vector $\mathbf{c}$ with respect to the center of the large sphere, which is at the origin. At the center of the small cavity (so at vector $\mathbf{r} = \mathbf{c}$), we put a point charge q. **This problen can be answered qualitatively** with drawings and little to no calculations.

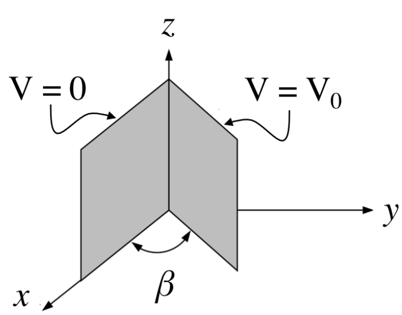


(1) Give the surface charge density σ_a on the cavity wall and σ_R (at $r = R^+$), and sketch the distribution of charges.

(2) Give and then sketch the $\mathbf{E}$ field everywhere: inside the cavity, in the bulk of the sphere, and outside.

(3) Imagine that q is moved a little off to one side, so it is no longer exactly at the center of the cavity, what changes? Please explain qualitatively and/or draw the resulting situation for the surface charge densities.

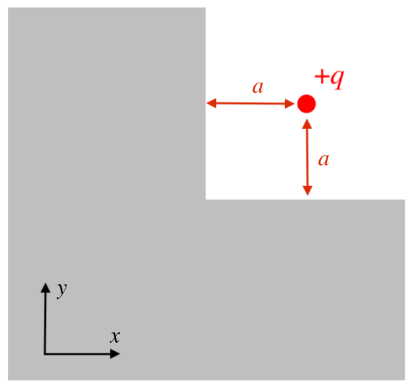
Problem B. The Wedge [20%]. Two very large plates form a wedge of angle β. One plate is held at $V = 0$ and the other one at $V = V_0$ as show below.



(1) Write Laplace's equation for the region between the plates. Use cylindrical coordinates and assume that the electric potential does not depend on z or s. *Hint: You should obtain a 1D differential equation in ϕ – pretty easy to solve no?*

(2) By solving the above Laplace's equation calculate the electric potential and the electric field (magnitude and direction) in the region between the plates assuming the plates have fixed potential as shown.

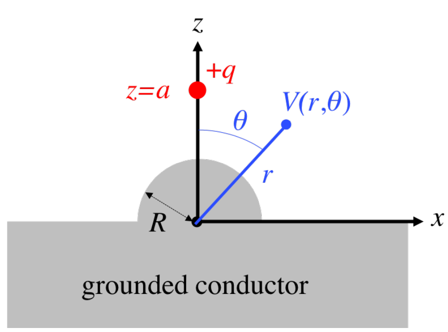
Problem C. The Corner [30%]. A point-charge $+q$ is located next to a very large grounded conductor (distance a). The conductor is bent at a right-angle as shown below. Our goal is to calculate the electric potential generated in the upper-right corner. We assume that the problem is two-dimensional, namely that there is no dependence on the coordinate z. **The use of Mathematica/Wolfram Alpha is allowed.**



(1) Set up a configuration of image charges (what charges do you need, where should they be located, etc) that allows you to solve for the potential in the upper right corner. Calculate the potential in this region.

(2) Find the surface charge density induced on the plane at $x = 0$.

Problem D. The Bump [30%]. Imagine a *grounded* infinite conducting plane in the xy plane, that has a conducting hemispherical bump (radius R) in it, centered at the origin, as shown. A charge q sits a distance a above the plane, i.e. at the point $(x = 0, y = 0, z = a)$.



(1) I claim that you can find the potential V anywhere in the plane above the conductor using the method of images, with **three image charges**. Where should they be? Explain your reasoning; you need to ensure the boundary condition $V = 0$ on the entire conductor.

(2) What is $V(r, \theta)$?

(3) What fraction of the total induced charge is induced on the hemispherical bump?