b) F as a function of position

e.o.m. $F(x) = m\ddot{x}$

Use chain rule on RHS: $\ddot{x} = \frac{d\dot{x}}{dt} = \frac{dx}{dt}\frac{d\dot{x}}{dx} = v\frac{dv}{dx}$

$$\therefore F(x) = mv\frac{dv}{dx} = \frac{m}{2}\frac{d(v^2)}{dx}$$

Define $T = \frac{1}{2}mv^2$ as the kinetic energy

$$\text{then } F(x) = \frac{dT}{dx} \rightarrow T - T_0 = \int_{x_0}^{x} F(x)\,dx$$

Work done on particle by $F(x)$ from $x_0 \rightarrow x$ is the change in KE of the particle

Define a function V such that $-\frac{dV}{dx} = F(x)$. $V(x)$ is the potential energy.

$$\therefore \int_{x_0}^{x} F(x)\,dx = -\int_{x_0}^{x} dV = -V(x) + V(x_0) = T - T_0$$

Re-writing, $T + V(x) = T_0 + V(x_0)$ = constants = E, the total energy

$\therefore$ Forces that are functions of position (in 1D) are conservative

$\therefore$ Conservative forces can be written in terms of a potential energy

Solve the energy equation to find motion $(T = \frac{1}{2}mv^2)$

$$\therefore v = \frac{dx}{dt} = \pm\sqrt{\frac{2}{m}\left[E - V(x)\right]} \leftarrow$$

$$\rightarrow \quad t - t_0 = \int_{x_0}^{x} \frac{dx}{\pm\sqrt{\frac{2}{m}\left[E - V(x)\right]}}$$

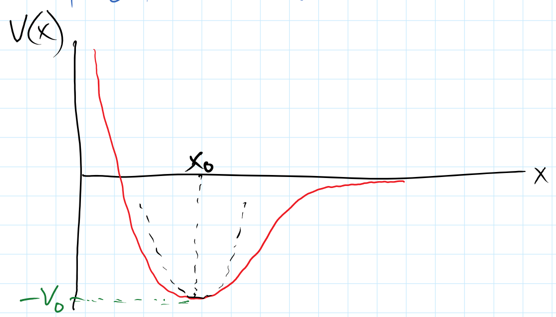
Note, that v is only real for values of x such that $E \geq V(x)$

Physically, the particle is confined to a region or regions where $V(x) \le E$ is satisfied. When $V(x) = E$, $v = 0$ so particle comes to rest & reverses its motion (turning points)

Example: The Morse Function $V(x)$ approximates the potential energy of a vibrating diatomic molecule as a function of x, the distance or seperation b/w the 2 atoms

$$V(x) = V_0\left(1 - e^{-(x-x_0)/\delta}\right)^2 - V_0$$

where V_0, x_0 & δ are parameters used to describe the observed behavior of a pair of atoms. Show that x_0 is the seperation of the 2 atoms when $V(x)$ is a minimum & that $V(x_0) = -V_0$



Find $\frac{dV}{dx} = 0$ & solve for x

$$\frac{2V_0}{\delta}\left(1 - e^{-(x-x_0)/\delta}\right)\left(e^{-(x-x_0)/\delta}\right) = 0$$

$$\rightarrow \left(1 - e^{-(x-x_0)/\delta}\right) = 0$$

$$\ln(1) = -\frac{(x-x_0)}{\delta}$$

$$0 = \frac{x_0 - x}{\delta} \implies x = x_0$$

Sub. into $V(x)$, $V(x_0) = -V_0$

Since $F = -\frac{dV}{dx}$ if $\frac{dV}{dx} = 0$, then $F = 0$ & atoms are in equilibrium

Show that for separation distances x close to x_0, $V(x)$ is parabolic & that the resulting force is linear and directed to the equil. pos'n.

Taylor's Series about $x = x_0$

$$V(x) \cong V(x_0) + \frac{dV}{dx}(x_0)(x - x_0) + \frac{1}{2}\frac{d^2V}{dx^2}(x_0)(x - x_0)^2 + \cdots$$

$$\frac{dV}{dx} = \frac{2V_0}{\delta}\left(e^{-(x-x_0)/\delta} - e^{-2(x-x_0)/\delta}\right)$$

$$\frac{d^2V}{dx^2} = \frac{2V_0}{\delta}\left(e^{-(x-x_0)/\delta}\left(-\frac{1}{\delta}\right) - e^{-2(x-x_0)/\delta}\left(-\frac{2}{\delta}\right)\right)$$

$$\frac{d^2V}{dx^2}(x_0) = \frac{2V_0}{\delta}\left(-\frac{1}{\delta} + \frac{2}{\delta}\right) = \frac{2V_0}{\delta^2}$$

$$\therefore\ V(x) \simeq -V_0 + \frac{2V_0}{\delta^2}(x - x_0)^2 = \frac{V_0}{\delta^2}(x - x_0)^2 - V_0 \quad \text{parabola}$$

$$\therefore\ F(x) = -\frac{dV}{dx} = -\frac{2V_0}{\delta^2}(x - x_0)$$ Force is linear & restorative.