PHYS 3201 — Assignment #8
Due: 10/23/20

1. A comet is first seen at a distance of d astronomical units from the Sun and it is traveling with a speed of q times the Earth's speed. Show that the orbit of the comet is hyperbolic, parabolic, or elliptic, depending on whether the quantity $q^2 d$ is greater than, equal to, or less than 2, respectively.

2. A particle moves in an elliptical orbit in an inverse-square-law central force field. If the ratio of the maximum angular velocity to the minimum angular velocity of the particle in its orbit is n, then show that the eccentricity of the orbit is

$$e = \frac{\sqrt{n} - 1}{\sqrt{n} + 1}$$

3. A particle moves in a potential, $V(r) = -C/(3r^3)$, where C is a constant.

   (a) Given J, find the effective potential and sketch it.

   (b) Determine the maximum value of the effective potential.

   (c) Let the particle come in from infinity with speed v_0 and impact parameter b. In terms of C, m, and v_0, what is the largest value of b (call it b_{max}) for which the particle is captured by the potential? What is the "cross-section" for capture, πb_{max}^2, for this potential?

4. Show that the most efficient way to change the energy of an elliptical orbit for a single short engine thrust is by firing the rocket along the direction of travel at perigee.

5. An α-particle of energy 4 keV (1 eV $= 1.6 \times 10^{-19}$ J) is scattered by an aluminium atom through an angle of $90°$. Calculate the distance of closest approach to the nucleus.