Quiz 1

How to turn your work in: This quiz finishes by 11:50 am on 10/23, at which point you must drop your pen and prepare to upload your work to CANVAS. You have until 12:10 am to upload it to CANVAS. If you encounter technical difficulties to upload, send a picture of your work to mourigal@gatech.edu or by text message to 404-747-4969.

Honor code: This quiz is administered under a strict Honor Code. By signing your name on each page of your uploaded quiz, you certify that you took the test with closed book and notes, closed internet, no calculator, and no communications with anyone – you were indeed variable separated from your peers ♡.

Grading Rubric: This quiz contains one exercise [50%] and one problem [50%]. The points you earned for each question will be decided based on the overall performance of the class.

Useful Formulas:

$$(1+\epsilon)^\alpha = 1 + \alpha\epsilon + \frac{\alpha(\alpha-1)}{2!}\epsilon^2$$

$$V = \frac{1}{4\pi\varepsilon_0}\iiint \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\mathrm{d}\tau'$$

$$V = \frac{1}{4\pi\varepsilon_0}\iint \frac{\sigma(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\mathrm{d}a'$$

$$\nabla\cdot\mathbf{E} = \rho/\varepsilon_0 \quad\text{or}\quad \oiint \mathbf{E}\cdot\mathrm{d}\mathbf{a} = \frac{1}{\varepsilon_0}\iiint \rho\,\mathrm{d}\tau$$

$$\Delta V = -\int \mathbf{E}\cdot\mathrm{dl} \quad\text{and}\quad \mathbf{E} = -\nabla V$$

$$\nabla^2 V = -\rho/\varepsilon_0$$

$$\partial V/\partial n|_{\text{above}} - \partial V/\partial n|_{\text{below}} = -\sigma/\varepsilon_0$$

$$W = \frac{1}{2}\sum_i q_i V(\mathbf{r}_i)$$

$$W = \frac{\varepsilon_0}{2}\iiint |\mathbf{E}|^2\mathrm{d}\tau + \frac{1}{2\mu_0}\iiint |\mathbf{B}|^2\mathrm{d}\tau$$

$$V(r,\theta) = \sum_{\ell=0}^{+\infty}\left(A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}}\right)P_\ell(\cos\theta)$$

$$P_0(x) = 1, P_1(x) = x, P_2(x) = 3x^2/2 - 1/2,$$

$$P_3(x) = \frac{1}{2}(5x^3 - 3x)$$

$$\int_{-1}^{+1} P_\ell(x)P_{\ell'}(x) = \frac{2}{2\ell+1}\delta_{\ell,\ell'}$$

$$\int_0^a \sin(n\pi x/a)\sin(n'\pi x/a)\ \mathrm{d}x = \frac{a}{2}\delta_{n,n'}$$

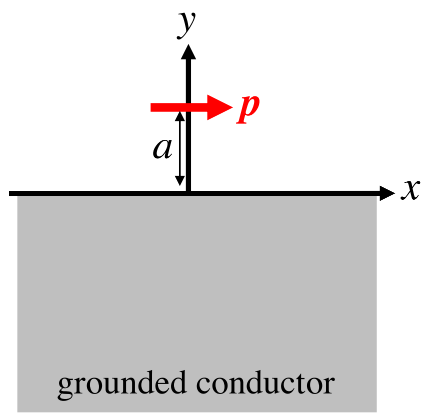
$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{\ell=0}^{+\infty}\frac{1}{r^{\ell+1}}\iiint (r')^\ell P_\ell(\cos\theta')\rho(\mathbf{r}')\mathrm{d}\tau'$$

$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r} + \frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{r^2} + \sum_{\alpha,\beta}\frac{Q_{\alpha\beta}\hat{r}_\alpha\hat{r}_\beta}{r^3}\right)$$

$$\mathbf{p} = \sum_i q_i\mathbf{r}_i \equiv \iiint \rho(\mathbf{r}')\mathbf{r}'\mathrm{d}\tau'$$

$$Q_{\alpha\beta} = \sum_i \frac{q_i}{2}\left(3r_{i\alpha}r_{i\beta} - r_i^2\delta_{\alpha\beta}\right) \equiv \iiint \frac{\rho(\mathbf{r}')}{2}\left(3r'_\alpha r'_\beta - {r'}^2\delta_{\alpha\beta}\right)\mathrm{d}\tau'$$

Exercise: Theory of Image Dipoles [50%]. The goal of this exercise is to *qualitatively* obtain key results about "image electric dipoles". Consider the physical situation below, where a *pure* electric dipole is located at a distance a above an infinitely large grounded conductor. The dipole moment $\mathbf{p}$ is initially parallel to the surface of the conductor.

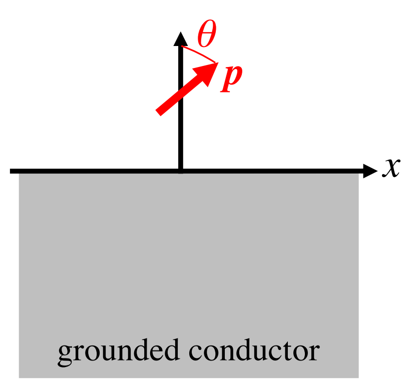


(1) What is the difference between a *pure* dipole and a *physical* dipole? [Be brief]. Draw a possible *physical* dipole corresponding to the above situation.

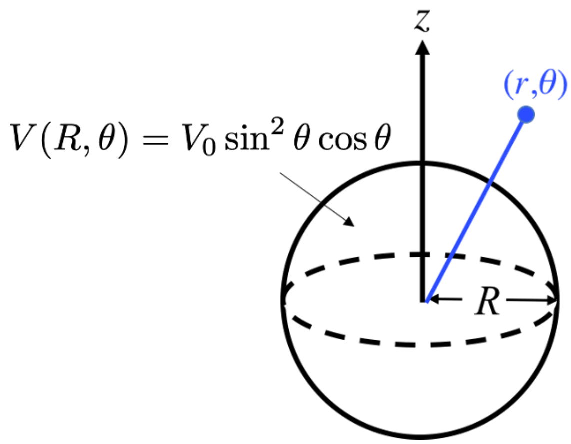
(2) Indicate which image charges you would use to determine the electric potential in the upper-half plane $y > 0$. Give the location and charge value of these image charges and explain your reasoning.

(3) What is the name of the resulting configuration of real + image charges? Based on this, qualitatively draw the electric-field lines in the $y > 0$ plane.

(4) Consider the situation below where the electric dipole has been rotated with respect to the conductor. What image charges would solve for the potential in the upper-plane $y > 0$ now? From this, derive a general rule for image electric dipoles next to a flat conductor.



Problem: Potential on a sphere [50%]. Consider a sphere of radius R on which the surface potential is $V(r=R,\theta) = V_0 \sin^2\theta\cos\theta$.



(1) Calculate the potential for $r \geq R$.

(2) What kind of multipolar moments does the sphere carry ? [Be Brief]