Formulas for final

- Bayes' Theorem: If $B_1, \ldots, B_k$ form a partition of the sample space S, and A is any event, then for any $i = 1, \ldots, k$,

$$\mathbb{P}(B_i \mid A) = \frac{\mathbb{P}(A \mid B_i)\mathbb{P}(B_i)}{\sum_{j=1}^{k} \mathbb{P}(A \mid B_j)\mathbb{P}(B_j)}.$$

- The following are probability mass functions for distributions we discussed.

  1. Bernoulli with parameter p:

$$f(x) = \begin{cases} p & x = 1 \\ 1-p & x = 0 \\ 0 & \text{otherwise} \end{cases}$$

  2. Geometric with parameter p:

$$f(x) = \begin{cases} (1-p)^{x-1}p & x = 1, 2, \ldots \\ 0 & \text{otherwise} \end{cases}$$

  3. Binomial with parameters n, p:

$$f(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x} & x = 0, 1, 2, \ldots, n \\ 0 & \text{otherwise} \end{cases}$$

  4. Hypergeometric with parameters N_1, N_2, n:

$$f(x) = \begin{cases} \frac{\binom{N_1}{x}\binom{N_2}{n-x}}{\binom{N_1+N_2}{n}} & \text{for } \max\{0, n-N_2\} \leq x \leq \min\{N_1, n\} \\ 0 & \text{otherwise} \end{cases}.$$

  5. Negative binomial with parameters r, p:

$$f(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}, \; x = r, r+1, \ldots$$

  6. Poisson with parameter λ:

$$f(x) = e^{-\lambda} \frac{\lambda^x}{x!}, \; x = 0, 1, 2, \ldots$$

- The following are probability distribution functions for distributions we discussed.

1. Uniform on $[a, b]$:

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{if } x \in [a, b] \\ 0 & \text{otherwise.} \end{cases}$$

2. Exponential with parameter θ:

$$f(x) = \begin{cases} \frac{1}{\theta} e^{-x/\theta} & \text{if } x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

3. Gamma with parameters α, θ:

$$f(x) = \begin{cases} \frac{1}{\Gamma(\alpha)\theta^\alpha} x^{\alpha-1} e^{-x/\theta} & \text{if } x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

4. Chi-square with r degrees of freedom:

$$f(x) = \begin{cases} \frac{1}{\Gamma(r/2)2^{r/2}} x^{r/2-1} e^{-x/2} & \text{if } x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

5. Normal with parameters μ, σ^2:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right).$$

- Least squares regression line:

$$y = \mu_Y + \rho \frac{\sigma_Y}{\sigma_X}(x - \mu_X).$$

- Confidence intervals for the mean when n is large and σ^2 is known (if σ^2 is unknown then replace σ by s and if data is normal z_α or $z_{\alpha/2}$ by $t_\alpha(n-1)$ or $t_{\alpha/2}(n-1)$):
  - two-sided: $\bar{x} \pm z_{\alpha/2}(\sigma/\sqrt{n})$.
  - one sided: $[\bar{x} - z_\alpha(\sigma/\sqrt{n}), \infty)$ or $(-\infty, \bar{x} + z_\alpha(\sigma/\sqrt{n})]$.
- Critical regions for hypothesis testing for normal distribution, σ^2 known (for unknown σ^2, replace σ by s and z_α or $z_{\alpha/2}$ by $t_\alpha(n-1)$ or $t_{\alpha/2}(n-1)$):
  - $H_0 : \mu = \mu_0$, $H_a : \mu > \mu_0$, $\{\bar{x} \geq \mu_0 + z_\alpha(\sigma/\sqrt{n})\}$.
  - $H_0 : \mu = \mu_0$, $H_a : \mu < \mu_0$, $\{\bar{x} \leq \mu_0 - z_\alpha(\sigma/\sqrt{n})\}$.
  - $H_0 : \mu = \mu_0$, $H_a = \mu \neq \mu_0$, $\{|\bar{x} - \mu_0| \geq z_{\alpha/2}(\sigma/\sqrt{n})\}$.