# In-Class Midterm 1 Review, Math 1554

1. Consider the matrix $A$ and vectors $\vec{b}_1$ and $\vec{b}_2$.

$$A = \begin{pmatrix} 1 & 4 \\ 2 & 8 \end{pmatrix}, \quad \vec{b}_1 = \begin{pmatrix} -2 \\ -4 \end{pmatrix}, \quad \vec{b}_2 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

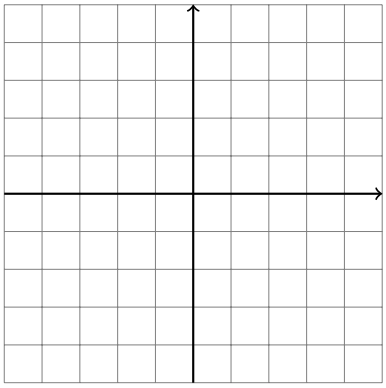
If possible, on the grids below, draw

(i) the two vectors and the span of the columns of $A$,

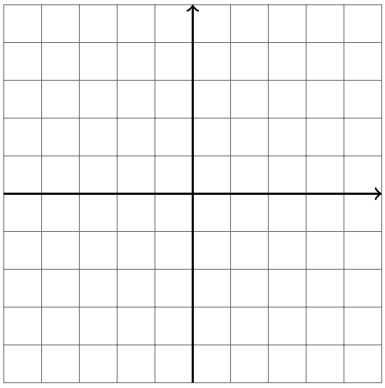
(ii) the solution set of $A\vec{x} = \vec{b}_1$.

(iii) the solution set of $A\vec{x} = \vec{b}_2$.

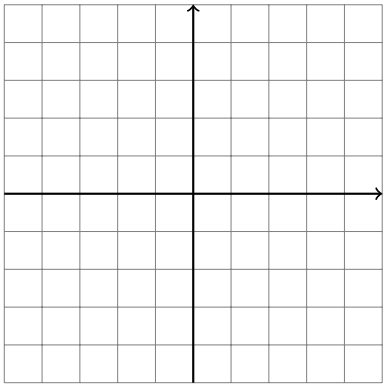
(i) $\vec{b}_1$, $\vec{b}_2$, column span



ii) solution set $Ax = \vec{b}_1$



iii) solution set $Ax = \vec{b}_2$



2. Indicate **true** if the statement is true, otherwise, indicate **false**. For the statements that are false, give a counterexample.

| | true | false | counterexample |
|---|---|---|---|
| a) If $A \in \mathbb{R}^{M \times N}$ has linearly dependent columns, then the columns of $A$ cannot span $\mathbb{R}^M$. | ◯ | ◯ | |
| b) If there are some vectors $\vec{b} \in \mathbb{R}^M$ that are not in the range of $T(\vec{x}) = A\vec{x}$, then there cannot be a pivot in every row of $A$. | ◯ | ◯ | |
| c) If the transform $\vec{x} \to A\vec{x}$ projects points in $\mathbb{R}^2$ onto a line that passes through the origin, then the transform cannot be one-to-one. | ◯ | ◯ | |

3. If possible, write down an example of a matrix with the following properties. If it is not possible to do so, write *not possible*.

   (a) A linear system that is homogeneous and has no solutions.

   (b) A standard matrix $A$ associated to a linear transform, $T$. Matrix $A$ is in RREF, and $T_A : \mathbb{R}^3 \to \mathbb{R}^4$ is one-to-one.

   (c) A $3 \times 7$ matrix $A$, in RREF, with exactly 2 pivot columns, such that $A\vec{x} = \vec{b}$ has exactly 5 free variables.

4. Consider the linear system $A\vec{x} = \vec{b}$, where

$$A = \begin{pmatrix} 1 & 0 & 7 & 0 & -5 \\ 0 & 1 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

   (a) Express the augmented matrix $(\, A \,|\, \vec{b}\,)$ in RREF.

   (b) Write the set of solutions to $A\vec{x} = \vec{b}$ in parametric vector form. Your answer must be expressed as a vector equation.