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Probabilistic sample(or random sample, sample):
 One realizable phenomenon from the probabilistic problem you want to solve
 Or one sampled case

Sample space Ω :
 Set which contains all possible samples

Task of defining sample sapce:
 Define which phenomenon is possible to occur
 and which phenomenon is impossible to occur

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Sample space when you toss the coin
 $\Omega = \{H, T\}$

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Some cases has set of entire real numbers as sample space
 $\Omega = \mathbf{R}$

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Possible events: possible sub sets of sample space Ω

Sample space $\Omega = \{H, T\}$
 Possible events: $\phi, \{H\}, \{T\}, \{H, T\}$

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Probability is function which takes all events
 and which outputs number

$P(A) = 0.1$
 $P()$ is function P
 A is event
 0.1 is probability value

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Kolmogorov's axioms

1. $P(\text{all events}) \geq 0$
2. $P(\Omega) = 1$
3. If $A \cap B = \phi$, then $P(A \cup B) = P(A) + P(B)$

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Interpretion about probability value:

1. Frequentist
2. Baysian

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Information of "event A has high probability value",
"event B has medium probability value",
"event C has low probability value", ...
is called "probability distribution"

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Elementary event (or atomic event):
Elementary event has one number of sample

$P(\text{Diamond}) = 0.1$
 $P(\text{Heart}) = 0.2$
 $P(\text{Spade}) = 0.3$
 $P(\text{Clover}) = 0.4$

Then, you can calculate probability value of all kinds of events,
which has 2 samples, 3 samples, etc
according to 3rd rule of Kolmogorov's axiom

$$P(\text{Heart}, \text{Spade}) = 0.2 + 0.3 = 0.5$$

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Probability mass function:
Probability mass function defines probability values
to each elementary event,
when there are only finite number of events

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$P(\{1\}) = 0.2$
 $P()$ is probability function
 $\{1\}$ is event which has one sample
0.2 is probability value for event $\{1\}$

$p(1) = 0.2$
 $p()$ is probability mass function
1 is elementary event which has number 1
0.2 is probability value for elementary event 1

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$P(\{1, 2\}) = 0.2$
 $P()$ is probability function
 $\{1, 2\}$ is event which has 2 samples
0.2 is probability value for event $\{1, 2\}$

$p(1, 2)$ can be defined

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Simple interval event A: $A = \{a \leq x < b\}$

$$A = \{a \leq x < b\} \rightarrow P(A) = P(\{a \leq x < b\}) = P(a, b)$$

$$P(B) = P(\{-2 \leq x < 1\}) + P(\{2 \leq x < 3\}) = P(-2, 1) + P(2, 3)$$

$$P(B) = P(\{-2 \leq x < 3\}) - P(\{1 \leq x < 2\}) = P(-2, 3) - P(1, 2)$$

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Cumulative distribution function:

From above, you used 2 numbers to define "interval" or "simple interval event A"

To use only one number, you can use negative infinity

$$\begin{aligned} S_{-1} &= \{-\infty \leq X < -1\} \\ S_0 &= \{-\infty \leq X < 0\} \\ S_1 &= \{-\infty \leq X < 1\} \\ S_2 &= \{-\infty \leq X < 2\} \\ &\vdots \\ S_x &= \{-\infty \leq X < x\} \end{aligned}$$

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In interval $\{a \leq x < b\}$

simple interval probability $P(a, b) = P(-\infty, b) - P(-\infty, a)$

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Probability density function:

derivative of Cumulative distribution function

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Probability distribution function:

Probability mass function

Cumulative distribution function

Probability density function

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