

Information of "event A has high probability value",  
 "event B has medium probability value",  
 "event C has low probability value", ...  
 is called "probability distribution"

=====

Elementary event (or atomic event):  
 Elementary event has one number of sample

$P(\text{Diamond}) = 0.1$   
 $P(\text{Heart}) = 0.2$   
 $P(\text{Spade}) = 0.3$   
 $P(\text{Clover}) = 0.4$

Then, you can calculate probability value of all kinds of events,  
 which has 2 samples, 3 samples, etc  
 according to 3rd rule of Kolmogorov's axiom

$$P(\text{Heart}, \text{Spade}) = 0.2 + 0.3 = 0.5$$

=====

Probability mass function:  
 Probability mass function defines probability values  
 to each elementary event,  
 when there are only finite number of events

=====

$P(\{1\}) = 0.2$   
 $P()$  is probability function  
 $\{1\}$  is event which has one sample  
 0.2 is probability value for event  $\{1\}$

$p(1) = 0.2$   
 $p()$  is probability mass function  
 1 is elementary event which has number 1  
 0.2 is probability value for elementary event 1

=====

$P(\{1, 2\}) = 0.2$   
 $P()$  is probability function  
 $\{1, 2\}$  is event which has 2 samples  
 0.2 is probability value for event  $\{1, 2\}$

$p(1, 2)$  can be defined

=====

Simple interval event A:  $A = \{a \leq x < b\}$

$$A = \{a \leq x < b\} \rightarrow P(A) = P(\{a \leq x < b\}) = P(a, b)$$

$$P(B) = P(\{-2 \leq x < 1\}) + P(\{2 \leq x < 3\}) = P(-2, 1) + P(2, 3)$$

$$P(B) = P(\{-2 \leq x < 3\}) - P(\{1 \leq x < 2\}) = P(-2, 3) - P(1, 2)$$

=====

Cumulative distribution function:

From above, you used 2 numbers to define "interval" or "simple interval event A"

To use only one number, you can use negative infinity

$$S_{-1} = \{-\infty \leq X < -1\}$$

$$S_0 = \{-\infty \leq X < 0\}$$

$$S_1 = \{-\infty \leq X < 1\}$$

$$S_2 = \{-\infty \leq X < 2\}$$

$$\vdots$$

$$S_x = \{-\infty \leq X < x\}$$

=====

In interval  $\{a \leq x < b\}$

simple interval probability  $P(a, b) = P(-\infty, b) - P(-\infty, a)$

=====

Probability density function:

derivative of Cumulative distribution function

=====

Probability distribution function:

Probability mass function

Cumulative distribution function

Probability density function

=====