Statement of Research Interests

Youngmin Park

1 Background

Generally, I am interested in learning the mathematics necessary to address problems that arise in neuroscience. In practice, my research is largely focused on the study of deterministic, smooth and non-smooth dynamical systems, especially systems that arise in mathematical neuroscience.

1.1 Calculating the Infinitesimal Phase Response Curve of Piecewise Smooth Systems

My masters work is on the infinitesimal phase response curves (iPRCs) of planar piecewise smooth dynamical systems [5]. The iPRC is a useful tool for predicting entrainment and synchronization of weakly coupled or weakly forced oscillators. In piecewise smooth systems, the iPRC may be discontinuous due to the piecewise defined vector field. My masters thesis shows how to compute the exact size of these discontinuities, provided that solutions are continuous and transversely cross boundaries of the piecewise regions.

1.2 Reducing Dynamics Through a Separation of Timescales

My doctoral work is on smooth dynamical systems, in particular the reduction of unwieldy high-dimensional systems to low-dimensional systems that are more amenable to a classic dynamical systems analysis. In the systems I have considered with my doctoral advisor Bard Ermentrout, the reduction is possible due to a natural separation of timescales.

1.2.1 Synchronization of Weakly Coupled Cortical Neurons Modulated by a Slowly Varying Concentration of Acetylcholine

The first project on this theme is on the synchronization of conductance-based neural models modulated by a slowly varying level of acetylcholine. In this system, the period of the neural spikes are orders of magnitude faster than the varying concentration of acetylcholine. This separation of timescales allows for a phase reduction and a standard weak coupling analysis [6].

1.2.2 Spatio-Temporal Dynamics of a Bump Solution of a Neural Field Model

In my second doctoral project, we analyze a particular neural field model, first studied in [8], with weak and slow adaptation. This model produces a single bump solution with a centroid that exhibits translational movement depending on the strength of a weak input current and the weak adaptation variable. We reduce the infinite dimensional partial integro-differential equation to a system of two scalar delay integro-differential equations, then analyze the many bifurcations in the movement of the bump solution on the ring and torus using classic analytical and numerical techniques. We have submitted this paper to SIAM Journal on Applied Dynamical Systems.

2 Recent Work

My doctoral work continues in the form of collaborations, and will conclude on a third paper. My masters work also continues in the form of an n-dimensional generalization of the planar result of my masters thesis.

2.1 The Connection Between the Mean-Field Description and Spiking Neurons

In the third and final paper of my Doctorate work, we seek to rigorously derive the connection between mean field neural networks and neural activity at the spiking level. In particular, we consider a system of synaptically coupled oscillators that generate a limit cycle oscillation at the mean field level in the synaptic variables. The synchronization properties at the spiking level are not well understood.

Recent works seek to address this issue using theta neuron models [4], but the general case remains unsolved. In our approach, we assume a network of heterogeneous, possibly multi-dimensional oscillators, with slow synapses. In this case, we can rephrase the problem as a weak coupling problem [9]. Preliminary results have shown a phase reduction is possible when the slow synapses vary slowly with small amplitude.

2.2 General Weakly Coupled Piecewise Smooth Oscillators

In addition to these primary studies, I have other collaborations. For instance, I have continued to build on my masters work in continued collaborations with my masters thesis advisor Peter J. Thomas (with colleagues Hillel J. Chiel and Kendrick M. Shaw). We have written a manuscript for an n-dimensional generalization of this result and have started to apply the result to a weakly coupled piecewise smooth systems [7]. Most existing literature on coupling of piecewise smooth systems are often restricted to one or a combination of linear coupling, planar systems, and piecewise linear vector fields [1, 3, 2]. Although we do require solutions to be continuous, our weak coupling analysis applies to piecewise smooth systems of arbitrary dimension with nonlinear, heterogeneous coupling. This work has been submitted to the European Journal of Applied Mathematics.

2.3 A New Way to Compute Isochrons

In another collaboration, I am working with another graduate student named Benjamin G. Letson to construct a numerical scheme for the computation of isochrons of limit cycles without using `AUTO`. The underlying analysis involves a change of coordinates to a moving frame with axes that are perpendicular and parallel to the underlying limit cycle solution. In this new frame, isochrons are relatively easy to visualize and compute. There are many numerical details yet to be ironed out, but preliminary results are promising.

3 Future Directions

Each of this work has several potential, fruitful directions. The extension of weak coupling theory to piecewise smooth oscillators opens a path to a rigorous re-rerivation of the classic weak coupling results to an extremely rich class of dynamical systems. I would re-derive and extend the classic results of weak coupling to non-smooth weak coupling as one possible research direction.

Another fruitful direction would be to continue using the techniques from my doctoral projects. As implicitly mentioned above, my doctorate thesis depends strongly on a weak and slow assumption. These assumptions are rather general and could be found in many other systems. For example, traveling waves with some sufficiently slow variables could be amenable to a multiple timescale analysis, where the reduced dynamics could describe the velocity and direction of the wave front on a two dimensional domain. In fact, any neural field model that produces a simple, structurally coherent pattern with some weak and slow dynamics are good candidates for this reduction analysis.

References

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