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Downstream firm's equity financing for capacity expansion in a supply chain

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Abstract

In this research, we investigate a supply chain consisting of a downstream firm who purchases a component from an upstream firm and then transforms it into a final product. The downstream firm has a production capacity constraint and considers to raise capital from an investor through equity financing. The raised capital not only enables the downstream firm to expand the capacity but also allows the investor to hold equity shares in the downstream firm. We derive the optimal pricing and production decisions of the two firms and discuss the optimal equity financing strategy of the downstream firm. We consider two distinct models: the external equity financing model, where the capital is raised from an outside institution, and the internal equity financing model, where the capital is raised from the upstream firm. We show that because the cooperative relationship between the two firms can be improved in the internal equity financing model, the production quantity in this model may be even higher than that in a benchmark model with no capacity constraint and no equity holding by the investor in the downstream firm. In addition, the original shareholder of the downstream firm gets more benefit from the internal equity financing activity than from the external equity financing activity. We also analyze the impacts of the key model parameters on the equity financing strategy and find that the dependence of the financing strategy on the initial asset of the downstream firm is quite different in the two models. Moreover, when the production cost of the downstream firm is changed, less capital raised for expanding capacity may create more value for the original shareholder of the downstream firm in each model. Finally, we show that the key finding remains unchanged when deterministic demand is changed to stochastic demand.

Keywords: supply chain; equity financing; capacity expansion; equity holding

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1. Introduction

In reality, firms usually have limited production capacity (Chen et al., 2024), which depends on resources such as capital, energy, and workforce (Ivanov and Dolgui, 2022). It has become an increasingly common business practice that manufacturers raise capital through equity financing to expand their production capacities. For example, in September 2020, XPeng Inc., a leading Chinese smart electric vehicle company, raised 4 billion yuan from Guangzhou Kaide Investment Holdings Co. Ltd., through equity financing to establish a new production base and purchase production equipment (Liu, 2020). Additionally, in April 2021, Maxeon Solar Technologies Ltd., a famous solar panel manufacturer in Singapore, raised millions of dollars from Tianjin Zhonghuan Semiconductor Co. Ltd., through equity financing to expand its manufacturing capacity for Maxeon 5 and Maxeon 6, etc. (Shumkov, 2021).

Clearly, when the manufacturer has insufficient capital for the production activity and obtains capital from the investor through equity financing, there are both advantages and disadvantages: On the one hand, the raised capital can be used by the manufacturer to expand its production capacity, which alleviates the manufacturer's production bottleneck and thus benefits the manufacturer; On the other hand, the raised capital requires the manufacturer to reward the investor with equity shares, which enables the investor to share the manufacturer's profit and hence causes detriment to the manufacturer. Then, the following research question arises: How does the equity financing strategy of the manufacturer interact with the operational decisions (e.g., pricing and production decisions) of players in the manufacturer's supply chain, and what are the optimal strategies and decisions of supply chain players? Second, note from the previous two motivating examples that Guangzhou Kaide Investment Holdings Co. Ltd. is an outside investment institution, who does not belong to XPeng Inc.'s supply chain; but Tianjin Zhonghuan Semiconductor Co. Ltd. is a silicon wafer supplier of Maxeon Solar Technologies Ltd. This indicates that the manufacturer can raise capital through equity financing from not only the outside institution but also the upstream supplier. Thus, the following interesting question also arises immediately: What is the difference between these two types of equity financing activities, and which type of the equity financing activity is more suitable for the manufacturer to expand its production capacity?

To answer the above questions, this research considers a supply chain in which a downstream firm (or a downstream manufacturer) purchases a component from an upstream firm (or an upstream supplier) and then the downstream firm processes this component into an end product. The downstream firm has a production capacity constraint and raises production capital from an investor through equity financing. The percentage of the equity shares held by the investor in the downstream firm depends on both the raised capital and the downstream firm's initial asset. We assume that the equity holding allows the investor to share the asset of the downstream firm but does not enable the investor to influence the decision-making of the downstream firm. That is, the decisions and strategies of the supply chain firms are given by original shareholders, who only care about their own interests. We first study operational decisions of the two supply chain firms by assuming that the equity financing strategy of the downstream firm is given and obtain the dependence of the operational decisions on the equity financing strategy. Then, we use these results to investigate the equity financing strategy. We consider both the external and the internal equity financing models, in which the investors of the downstream firm's financing activity are the outside institution and the upstream firm, respectively.

In summary, the main contributions of this research are as follows:

1. We develop a game model to study the decision problem of the downstream firm's equity financing for capacity expansion in a supply chain and characterize the equilibrium decisions and strategies of chain members.
2. We analyze the impacts of the key supply chain parameters on the equity financing strategy and obtain some interesting results. We show that when the production cost decreases, less capital raised for increasing the production capacity may create more value.
3. We compare the external and the internal equity financing activities and find that the internal equity financing activity is more beneficial for the original shareholder of the downstream firm because it can improve the supply chain cooperative relationship.

The organization of this paper is as follows: Section 2 reviews the related literature. Section 3 describes the model. In Section 4, we analyze operational decisions of the two supply chain firms under any given equity financing strategy. In Section 5, we explore the downstream firm's equity financing strategy. Numerical studies are presented in Section 6 to enrich the analytical results. Conclusions are given in Section 7. All mathematical proofs are given in the Appendix.

2. Literature review

Supply chain finance is an important financing solution for supply chain firms that are struggling with access to the available cash flow (Choi and Ivanov, 2020; Olan et al., 2022). In this section, we will review existing literature on supply chain finance from the perspectives of non-equity financing and equity financing, respectively, and summarize the differences between our study and existing literature.

2.1. *Non-equity financing in supply chains*

Some literature focuses on the non-equity financing in supply chains, including supplier financing, bank financing, platform financing, etc. For example, assuming that supply chain firms are capital constrained and make financing activities to improve payment abilities, Kouvelis and Zhao (2012) compare bank financing with supplier financing and find that the retailer usually prefers supplier financing to bank financing. Wang et al. (2019) compare bank financing with platform financing and suggest that the retailer always prefers platform financing to bank financing. Jing et al. (2012) and Jing and Seidmann (2014) compare bank financing with trade credit and conclude that when the upstream supplier's production cost is relatively low, trade credit is more effective than bank financing. Bi and Yang (2022) state that the relative effectiveness between bank financing and trade credit highly depends on the monitoring cost. Yang et al. (2022a) compare bank financing with crowdfunding financing and characterize the condition of the financing preference between these two financing schemes. Huang et al. (2019) analyze bank financing with the upstream supplier's guarantee for the downstream retailer and conclude that the supplier's guarantee can well allocate the loan risk among the supplier, the retailer, and the bank. Jin et al. (2024) compare two financing schemes, i.e., guarantee for quality, guarantee for both quality and sales and find that when the loan rate is relatively high the latter financing scheme can achieve a win-win outcome. Huang et al. (2020) compare three financing schemes for the retailer, i.e., the core enterprises credit guarantee financing, the buyback guarantee financing, and the trade credit financing, and obtain the applicability condition of each financing scheme. Fu et al. (2022) identify conditions under which the trade credit between the upstream firm and the downstream firm benefits each supply chain firm. Ning (2022) derives condition under which the trade credit benefits players in a supply chain with multiple downstream firms. Yang et al. (2022b) derive the optimal interest rates for both the bank and the platform in a supply chain where the retailer adopts a mixed financing scheme combining bank financing and platform financing.

Assuming that supply chain firms are capacity constrained and make financing activities to expand their production capacities, Deng et al. (2018) compare bank financing and buyer financing for upstream suppliers in an assembly supply chain where capacity-constrained suppliers produce complementary components and deliver them to a buyer who assembles these components to produce a final product. The authors identify the conditions under which buyer financing is better than bank financing. Shen et al. (2020) compare bank financing and retailer financing in a supply chain consisting of one manufacturer and two competing retailers. Zhen et al. (2020) further compare three financing schemes, i.e., bank financing, retailer financing, and platform financing for an upstream manufacturer in a dual-channel supply chain where the manufacturer is capacity constrained and produces and sells the product through both the retailer and the platform. de Matta and Hsu (2022) explore a supply chain model where the manufacturer raises capital through inventory-based financing to expand the production capacity. They derive the optimal production and financing decisions. Guo and Liu consider (2020) bank financing in a supply chain with risk-averse players who make decisions to maximize their own mean-variance utilities.

All the above literature on non-equity financing assumes that capital-constrained firms raise capital either to improve the payment ability or to expand the production capacity. In addition, some other literature assumes that the raised capital can not only lead to payment ability improvement for downstream firms but also results in production capacity expansion for upstream firms. For example, Raghavan and Mishra (2011) consider a supply chain where a lender provides debt financing for both the upstream and downstream firms and show that if the lender knows the fact that the upstream and downstream firms are linked in a supply chain, then he can obtain more profit by making a joint supply chain financing decision. Kouvelis and Zhao (2016) develop a revenue-sharing contract to coordinate the supply chain where both the upstream and downstream firms raise capital through bank financing. Gao et al. (2018) focus on a supply chain where the capital is raised from a platform and find that the upstream firm's wholesale price and the downstream firm's order quantity both decrease as the platform's service rate increases. Jin et al. (2019) compare three financing schemes, i.e., bank financing separately, bank financing with trade credit, and bank financing with the upstream firm's guarantee. They show that the upstream firm earns more profit under the second and third schemes as compared to the first scheme, while the downstream firm obtains more profit under the first scheme as compared to the second and third schemes.

2.2. Equity financing in supply chains

Equity financing does not require financing firms to repay the principal and the interest but requires them to transfer a proportion of equity shares, which endows investors with rights to share assets of financing firms. There are several studies on asset sharing caused by equity financing or equity shares, including equity shares between vertically related firms (e.g., Chen et al., 2017; Ren et al. 2021) and equity shares between horizontally related firms (e.g., Aviv and Shamir, 2021). However, all these studies assume that equity shares held by investors in financing firms are exogenously given and do not consider the possible impacts on production and operations activities of financing firms caused by the raised capital.

Several other researchers focus on the impacts of the raised capital from equity financing activities and assume that the raised capital can be used to reduce the transportation and production costs by conducting R&D activities or improve payment abilities. For example, Fu et al. (2021) investigate a platform supply chain where a third-party logistics (3PL) firm raises capital through equity financing from outside financial institutions to reduce its transportation cost and show that the optimal equity financing strategy of the 3PL firm depends on its initial asset. Fu et al. (2018) study an assembly supply chain where multiple complementary upstream firms raise capital through equity financing from a downstream firm to reduce their component production costs. The authors derive the optimal equity investment strategy from the perspective of the investor, i.e., the downstream firm. Zhang and Lee (2022) study a supply chain where one upstream firm raises capital through equity financing from a downstream firm to reduce its production cost and then competes with the other upstream firm who sells a substitutable product to the common downstream firm.

Additionally, assuming that the raised capital is used to improve payment abilities, Zhang et al. (2021), Li et al. (2020), and Yang et al. (2017) derive optimal equity financing strategies for downstream firms in different supply chain settings, respectively. For example, in the study of Zhang et al. (2021), the supply chain is composed of one upstream firm and one capital-constrained downstream firm; in the study of Li et al. (2020), the single upstream firm sells the product not only through its own sales channel directly but also through the capital-constrained downstream firm indirectly; in the study of Yang et al. (2017), the supply chain consists of one upstream firm and two competitive downstream firms.

2.3. Gap analysis of the existing literature

Tables 1 and 2 summarize the relevant literature on non-equity financing and equity financing in supply chains, respectively.

We can see from Table 1 that although some researchers consider the problem of financing for the production capacity expansion in supply chains, they assume that the capital for achieving production capacity expansion is raised though non-equity financing, which is essentially the debt financing and requires the financing firms to pay interest costs. Our paper considers equity financing, which does not involve interest costs, but enables the investors to share the assets of the financing firms. The asset-sharing scheme establishes a close relationship between the financing firms and the investors, and the decision problem of equity financing for the production capacity expansion may be more complex. We can also see from Table 2 that the relevant literature supposes that the

Table 1
Summary of the relevant literature on non-equity financing in supply chains

	Equity financing	Payment ability improvement	Production capacity expansion
Kouvelis and Zhao (2012)	×	✓	×
Wang et al. (2019)	×	✓	×
Jing et al. (2012)	×	✓	×
Jing and Seidmann (2014)	×	✓	×
Bi and Yang (2022)	×	✓	×
Yang et al. (2022a)	×	✓	×
Huang et al. (2019)	×	✓	×
Jin et al. (2024)	×	✓	×
Huang et al. (2020)	×	✓	×
Fu et al. (2022)	×	✓	×
Ning (2022)	×	✓	×
Yang et al. (2022b)	×	✓	×
Deng et al. (2018)	×	×	✓
Shen et al. (2020)	×	×	✓
Zhen et al. (2020)	×	×	✓
de Matta and Hsu (2022)	×	×	✓
Guo and Liu (2020)	×	×	✓
Raghavan and Mishra (2011)	×	✓	✓
Kouvelis and Zhao (2016)	×	✓	✓
Gao et al. (2018)	×	✓	✓
Jin et al. (2019)	×	✓	✓
This research	✓	×	✓

raised capital is used for improving payment abilities or reducing costs. In their studies, the supply chain firms have sufficient production capacities. In our study, the downstream firm raises capital to expand its production capacity. Accordingly, the production decision in our study is a constraint optimization problem.

3. Model description

We consider a supply chain with an upstream firm (denote as firm U) and a downstream firm (denote as firm D). The upstream firm provides a component to the downstream firm, who in turn transforms the component into a final product and sells it to a market over a single selling period. We suppose that the upstream firm and the downstream firm interact based on a simple wholesale price contract because it is the most commonly used contract in practice (Tan et al., 2021). We denote w as the wholesale price of the upstream firm. The consumers in the market buy the product only when the value of the product for them is higher than the price of the product (Du et al., 2019). Thus, the higher the retail price, the less consumers will buy. Similar to Ha et al. (2017) and Hu et al. (2022), we further suppose that market demand $M(p)$ linearly depends on the retail price

Table 2
Summary of the relevant literature on equity financing in supply chains

	Production capacity expansion	Cost reduction	Payment ability improvement
Chen et al. (2017)	×	×	×
Ren et al. (2021)	×	×	×
Aviv and Shamir (2021)	×	×	×
Fu et al. (2021)	×	✓	×
Fu et al. (2018)	×	✓	×
Zhang and Lee (2022)	×	✓	×
Zhang et al. (2021)	×	×	✓
Li et al. (2020)	×	×	✓
Yang et al. (2017)	×	×	✓
This research	✓	×	×

p and takes the form of

$$M(p) = a - bp, \tag{1}$$

where a represents the potential market scale, b is the price sensitivity of the market demand, and $a > 0, b > 0$. We denote c_U and c_D as the unit production costs of the upstream firm and the downstream firm, respectively. To avoid the possibility of negative demand, we require that $a > b(c_U + c_D)$.

Let v_U and v_D be the initial assets of the upstream firm and the downstream firm, respectively. The initial assets of firms may include but are not limited to fixed assets, such as factories and machines. The downstream firm's production capacity is limited by its production capital (or working capital) for paying wages, raw materials, and other daily operations. We assume that the downstream firm can raise production capital through equity financing activity but cannot obtain production capital by selling fixed assets because of the low liquidity of these assets. For convenience, we further assume that the downstream firm's initial production capacity is zero (see, e.g., Jing et al., 2012; Zhang and Lee, 2022) and define $x(x \geq 0)$ as the amount of the production capital raised by the downstream firm. Then, after the equity financing activity, the production capacity of the downstream firm is $\frac{x}{c_D}$. The downstream firm carries out the production activity according to the capacity $\frac{x}{c_D}$ and the demand $M(p)$. Obviously, if $\frac{x}{c_D} \leq M(p) = a - bp$, i.e., $x \leq (a - bp)c_D$, then any quantity of the product produced by the downstream firm can be sold to the market, and thus the production quantity is $\frac{x}{c_D}$. Otherwise, if $\frac{x}{c_D} > M(p) = a - bp$, i.e., $x > (a - bp)c_D$, then any additional quantity above the market demand has no chance to be sold to the market, and so the production quantity is $a - bp$. In summary, the production quantity of the downstream firm can be expressed as

$$q = \begin{cases} \dfrac{x}{c_D}, & \text{if } x \leq (a - bp)c_D, \\ a - bp, & \text{if } x > (a - bp)c_D. \end{cases} \tag{2}$$

We investigate the following two distinct types of the downstream firm's equity financing activities in practice:

- *External equity financing activity*: The production capital is raised by the downstream firm from an outside institution who does not belong to the downstream firm's supply chain. For example, XPeng Inc. raises production capital from Guangzhou Kaide Investment Holdings Co. Ltd. through equity financing (Liu, 2020).
- *Internal equity financing activity*: The production capital is raised by the downstream firm from its upstream partner. For example, Maxeon Solar Technologies Ltd. raises production capital from its upstream supplier, Tianjin Zhonghuan Semiconductor Co. Ltd., through equity financing (Shumkov, 2021).

In each of the above two equity financing activities, the raised capital can not only expand the downstream firm's production capacity but also enable the investor (i.e., the outside institution or the upstream firm) to hold equity shares of the downstream firm, and thus share the asset of the downstream firm according to the equity shares. We assume that before the equity financing activity, the upstream firm is wholly owned by Shareholder A, while the downstream firm is wholly owned by Shareholder B. Then, after the equity financing activity, on the one hand, the production capital of the downstream firm increases from 0 to x, and so the production capacity increases from 0 to $\frac{x}{c_D}$. On the other hand, the asset of the downstream firm increases from v_D to $v_D + x$ and thus the percentage of equity shares held by the investor in the downstream firm is

$$\Lambda(x) = \frac{x}{v_D + x}, \tag{3}$$

where $0 \leq \Lambda(x) < 1$, and the percentage of remaining equity shares held by the original shareholder of the downstream firm, i.e., Shareholder B, is

$$1 - \Lambda(x) = \frac{v_D}{v_D + x}. \tag{4}$$

An illustration of the impacts on the raised capital is shown in Fig. 1.

We assume that equity shares enable the investor to share the asset of the downstream firm but do not allow the investor to influence the decision-making process of the downstream firm (Salop and O'Brien, 1999; Fu et al., 2021). This assumption, in conjunction with the fact that Shareholder A is the sole shareholder of the upstream firm, and Shareholder B is the sole shareholder of the downstream firm before the equity financing activity, implies that the decisions of the upstream firm and the downstream firm are always essentially given by Shareholders A and B, respectively. Thus, it is reasonable to further assume that the upstream firm and the downstream firm make decisions to maximize the assets of Shareholders A and B, respectively.

The supply chain involves two levels of decision-making. The first level is concerned about the equity financing strategy of the downstream firm, and the second level is concerned about the operational decisions of the two chain firms. Moreover, as for the second level, the upstream firm first determines the wholesale price of the component and then the downstream firm optimizes the retail price of the final product. In the following two sections, using backward induction, we first examine the pricing decisions of the two firms and then investigate the financing strategy of the

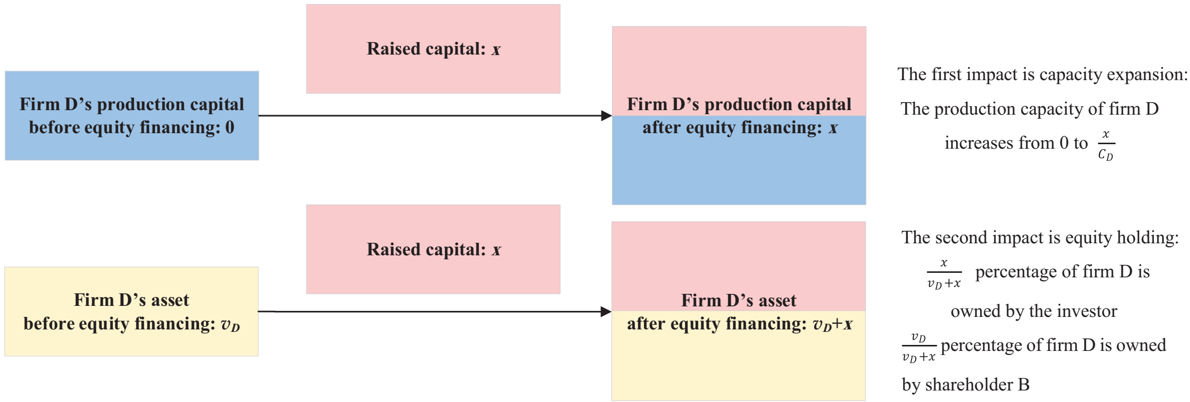


Fig. 1. Two impacts of the raised capital.

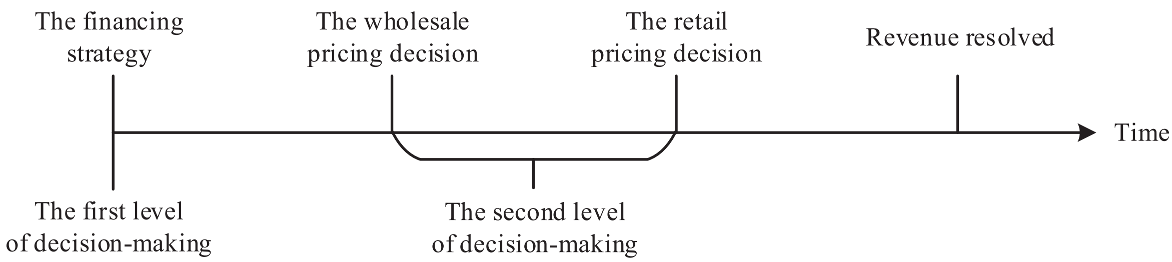


Fig. 2. Sequence of events.

downstream firm. An illustration of the sequence of events is shown in Fig. 2, and a summary of the model notation is given in Table 3.

4. Pricing decisions under given equity financing strategies

In this section, assuming that the equity financing strategy is given, we analyze pricing decisions of the upstream firm and the downstream firm in two equity financing models. For convenience, we use subscripts I and II to denote the external equity financing model and the internal equity financing model, respectively. The pricing decision problem of the two firms is modeled in a two-stage Stackelberg game, where the upstream firm chooses the wholesale price in the first stage and the downstream firm chooses the retail price in the second stage.

4.1. Benchmark model

To provide a benchmark, we first consider the following special case, where the impacts of the capacity expansion and the equity holding caused by the raised capital are ignored:

Table 3
Summary of the model notation

Notation	Description
a	Potential market scale
b	Price sensitivity of the demand
c_U	Unit production cost of the upstream firm
c_D	Unit production cost of the downstream firm
w	Wholesale price of the upstream firm
p	Retail price of the downstream firm
$M(p)$	Market demand of the product
q	Production quantity of the downstream firm
x	Amount of production capital raised by the downstream firm
$\Lambda(x)$	Percentage of equity shares held by the investor
v_U	Initial asset of the upstream firm before the equity financing activity
v_D	Initial asset of the downstream firm before the equity financing activity
V_U	Final asset of the upstream firm after the selling season
V_D	Final asset of the downstream firm after the selling season
V_A	Final asset of Shareholder A after the selling season
V_B	Final asset of Shareholder B after the selling season

- Case 1: The downstream firm's production capacity is sufficient, and neither the outside institution nor the upstream firm has equity holding in the downstream firm.

In Case 1, the sufficient production capacity implies that the production quantity of (2) reduces to $q = M(p) = a - bp$. Moreover, in Case 1, the upstream firm and the downstream firm are wholly owned by Shareholders A and B, respectively, and so the asset of each shareholder is equal to the asset of its own firm, i.e., $V_A = V_U$ and $V_B = V_D$. Accordingly, after the selling season, the final asset functions of Shareholders A and B can be expressed as follows:

$$V_A(w, p) = V_U(w, p) = v_U + (w - c_U)q = v_U + (w - c_U)(a - bp), \tag{5}$$

$$V_B(w, p) = V_D(w, p) = v_D + (p - w - c_D)q = v_D + (p - w - c_D)(a - bp). \tag{6}$$

One can see from (5) and (6) that the final asset of each shareholder in the benchmark model contains two parts: the initial asset (i.e., v_U or v_D) and the earned profit (i.e., $(w - c_U)(a - bp)$ or $(p - w - c_D)(a - bp)$). Since the initial asset is independent of the pricing decision of each shareholder, maximizing the final asset is equivalent to maximizing the earned profit. We analyze the pricing decision problem using backward induction. First, it is easy to verify from (6) that for any given wholesale price w, $V_B(w, p)$ achieves its maximum at

$$p(w) = \frac{a + b(w + c_D)}{2b}. \tag{7}$$

Second, substituting (7) into (5), yields

$$V_A(w) = v_U + \frac{(w - c_U)[a - b(w + c_D)]}{2}, \tag{8}$$

which achieves its maximum at

$$w^* = \frac{a + bc_U - bc_D}{2b}. \tag{9}$$

Therefore, the optimal retail price and production quantity are

$$p^* = p(w^*) = \frac{3a + bc_U + bc_D}{4b}, \tag{10}$$

$$q^* = a - bp^* = \frac{a - bc_U - bc_D}{4}. \tag{11}$$

For convenience, we refer to the optimal production quantity in Case 1, i.e., q^* of (11), as the *benchmark production quantity* (BPQ). The BPQ essentially represents the optimal production quantity in a model, where the production capacity constraint of the downstream firm is removed, and the equity shares held by the investor in the downstream firm are negligible and can help us to better understand the decisions of the two chain firms, which we will investigate in the following subsections.

4.2. Pricing decisions in the external equity financing model

In the external equity financing model, the production capital is raised from the outside institution who does not belong to the downstream firm's supply chain. When the raised production capital is x, the asset of the downstream firm increases from v_D to $v_D + x$, and the downstream firm is owned by the original shareholder B and the outside institution together. Moreover, the raised production capital supports the downstream firm's production activity and enables the upstream firm and the downstream firm to earn additional profits $(w - c_U)q$ and $(p - w - c_D)q$, respectively. Thus, after the selling season, the asset of the upstream firm will increase from v_U to $v_U + (w - c_U)q$, and the asset of the downstream firm will further increase from $v_D + x$ to $v_D + x + (p - w - c_D)q$. Note that after the selling season, the upstream firm is still wholly owned by Shareholder A, while the downstream firm is owned by Shareholder B and the investor together. So, for any given external equity financing strategy, the final asset functions of Shareholders A and B can be written as follows:

$$V_{I,A}(w, p|x) = V_{I,U}(w, p|x) = v_U + (w - c_U)q, \tag{12}$$

$$V_{I,B}(w, p|x) = [1 - \Lambda(x)]V_{I,D}(w, p|x) = [1 - \Lambda(x)][v_D + x + (p - w - c_D)q]. \tag{13}$$

Substituting (2) and (3) into (13), we can further rewrite the final asset function of Shareholder B as follows:

$$\begin{aligned} V_{I,B}(w, p|x) &= v_D + [1 - \Lambda(x)](p - w - c_D)q \\ &= \begin{cases} v_D + [1 - \Lambda(x)](p - w - c_D)\dfrac{x}{c_D}, & \text{if } p \le \dfrac{1}{b}\left(a - \dfrac{x}{c_D}\right), \\ v_D + [1 - \Lambda(x)](p - w - c_D)(a - bp), & \text{if } p \ge \dfrac{1}{b}\left(a - \dfrac{x}{c_D}\right). \end{cases} \end{aligned} \tag{14}$$

One can see from (12) and (14) that the final asset of Shareholder A consists of the initial asset v_U and the earned profit $(w - c_U)q$, and the final asset of Shareholder B consists of the initial asset v_D and the earned profit $[1 - \Lambda(x)](p - w - c_D)q$. Because the initial assets v_U and v_D are exogenously given, there is an equivalence between maximizing the final asset and the earned profit of each shareholder in the external equity financing model. Using backward induction, we have the following lemma.

Lemma 1. *For any given wholesale price of the upstream firm, the final asset function of Shareholder B in the external equity financing model, i.e., $V_{I,B}(w, p|x)$, reaches its maximizer at*

$$p_I^*(w|x) = \begin{cases} \dfrac{1}{b}\left(a - \dfrac{x}{c_D}\right), & if\ w \le \dfrac{1}{b}\left(a - \dfrac{2x}{c_D}\right) - c_D, \\ \dfrac{a + b(w + c_D)}{2b}, & if\ w > \dfrac{1}{b}\left(a - \dfrac{2x}{c_D}\right) - c_D. \end{cases} \tag{15}$$

Lemma 1 gives the response retail price in the external equity financing model. We can see from Lemma 1 that the response retail price $p_I^*(w|x)$ is independent of the proportion $\Lambda(x)$. The reason here is simple: For any given $\Lambda(x)$, the problem of choosing a retail price to maximize Shareholder B's final asset $V_{I,B}(w, p|x) = [1 - \Lambda(x)]V_{I,D}(w, p|x)$ is equivalent to choosing a retail price to maximize the downstream firm's final asset $V_{I,D}(w, p|x)$. We can also see from Lemma 1 that when the wholesale price is relatively low, the response retail price $p_I^*(w|x)$ is independent of the wholesale price w; when the wholesale price is relatively high, the response retail price $p_I^*(w|x)$ increases in the wholesale price w. Intuitively, the higher the wholesale price w, the higher the downstream firm's procurement cost w. The downstream firm with a higher procurement cost usually sets a higher retail price, leading to a lower market demand. However, the downstream firm with a lower procurement cost may not always choose a lower retail price to induce a higher market demand. This is because if the downstream firm is capacity constrained, the capacity is insufficient to meet the higher market demand caused by the lower retail price. Therefore, there is no need for the downstream firm to set a lower retail price to induce a higher market demand, and the downstream firm should set a retail price according to its production capacity such that $a - bp = \frac{x}{c_D}$, i.e., $p = \frac{1}{b}(a - \frac{x}{c_D})$. Thus, the response retail price $p_I^*(w|x)$ is independent of the wholesale price w.

The upstream firm anticipates the response retail price of the downstream firm and chooses a wholesale price to maximize the final asset of Shareholder A. Substituting (15) into (12), the optimization problem of the upstream firm can be described as follows:

$$\max_w V_{I,A}(w|x) = \begin{cases} v_U + (w - c_U)\dfrac{x}{c_D}, & \text{if } w \le \dfrac{1}{b}\left(a - \dfrac{2x}{c_D}\right) - c_D, \\ v_U + (w - c_U)\dfrac{a - b(w + c_D)}{2}, & \text{if } w > \dfrac{1}{b}\left(a - \dfrac{2x}{c_D}\right) - c_D. \end{cases} \tag{16}$$

This allows us to derive the following theorem.

Theorem 1. *Suppose that the downstream firm chooses to raise capital through external equity financing. Then, the optimal wholesale price and retail price are*

$$w_I^*(x) = \begin{cases} \dfrac{1}{b}\left(a - \dfrac{2x}{c_D}\right) - c_D, & \text{if } x \le \underline{x}, \\ \dfrac{a + bc_U - bc_D}{2b}, & \text{if } x > \underline{x}, \end{cases} \tag{17}$$

$$p_I^*(x) = \begin{cases} \dfrac{1}{b}\left(a - \dfrac{x}{c_D}\right), & \text{if } x \le \underline{x}, \\ \dfrac{3a + bc_U + bc_D}{4b}, & \text{if } x > \underline{x}, \end{cases} \tag{18}$$

and the resulting production quantity is

$$q_I^*(x) = \begin{cases} \dfrac{x}{c_D}, & \text{if } x \le \underline{x}, \\ \dfrac{a - bc_U - bc_D}{4}, & \text{if } x > \underline{x}, \end{cases} \tag{19}$$

where

$$\underline{x} = \frac{(a - bc_U - bc_D)c_D}{4} > 0. \tag{20}$$

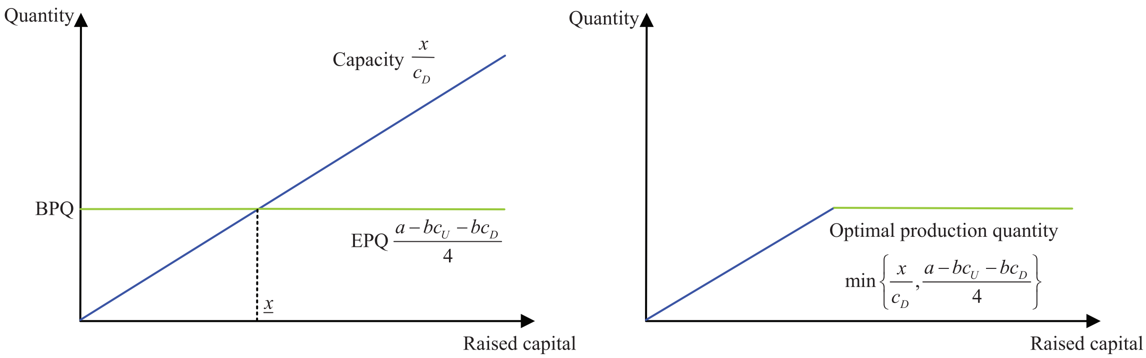
Theorem 1 completely characterizes the optimal pricing and production decisions of the two supply chain firms in the external equity financing model. We now summarize some properties of the optimal decisions as the following corollary.

Corollary 1. *When the downstream firm chooses to raise capital through external equity financing: (i) If $x \le \underline{x}$, then $w_I^*(x)$ and $p_I^*(x)$ decrease in x, while $q_I^*(x)$ increases in x; (ii) If $x > \underline{x}$, then $w_I^*(x)$, $p_I^*(x)$ and $q_I^*(x)$ are independent of x.*

In this research, the production capital raised through equity financing activity can not only result in the production capacity expansion for the downstream firm but also lead to equity holding for the investor. Corollary 1 suggests that, in the external equity financing model, there exists a threshold capital $\underline{x}$, below which the optimal pricing and production decisions highly depend on the raised capital, and above which the optimal decisions are independent of the raised capital. To better understand Corollary 1, we now turn our attention to the following particular case, which allows us to get the impact of the outside institution's equity holding in the downstream firm on the optimal pricing and the production decisions, because the production capacity constraint is removed:

- Case 2: The downstream firm's production capacity is sufficient, and the outside institution has equity holding in the downstream firm.

Similar to the discussion following Case 1, we refer to the optimal production quantity in Case 2, as the *external equity financing model's potential production quantity* (EPQ). It follows directly from (19) that if the downstream firm has sufficient production capacity, then the optimal production quantity is $q_I^*(x) = \frac{a-bc_U-bc_D}{4}$, i.e., EPQ $= \frac{a-bc_U-bc_D}{4}$. This, together with BPQ $= \frac{a-bc_U-bc_D}{4}$ in Case



(a) Capacity and EPQ (b) Optimal production quantity

Fig. 3. The capacity, EPQ and optimal production quantity in the external equity financing model.

1, indicates that the outside institution's equity holding in the downstream firm has no impact on the optimal production quantity. The reason is as follows: When the downstream firm raises production capital from the outside institution, the asset of Shareholder A and the optimal retail price do not depend on the outside institution's equity holding (this can be concluded from (12) and (15)). Therefore, the optimal wholesale price, and, hence, the corresponding retail price and production quantity are independent of the outside institution's equity holding. Consequently, when the downstream firm chooses the external equity financing activity, the raised capital can only influence the optimal pricing and production decisions through production capacity expansion.

Additionally, the EPQ cannot always be achieved because of the production capacity constraint. The downstream firm should conduct its production activity according to its production capacity and the EPQ, and the optimal production quantity is min{Capacity, EPQ}, i.e., $\min\{\frac{x}{c_D}, \frac{a-bc_U-bc_D}{4}\}$. Clearly, if $x > \underline{x}$, then $\frac{x}{c_D} > \frac{a-bc_U-bc_D}{4}$, and thus the optimal production quantity is equal to the EPQ. Therefore, any additional production capacity expanded by the raised capital above the EPQ has no practical impact on the optimal production quantity. Of course, the corresponding pricing decisions are also not influenced by the additional production capacity above the EPQ. Moreover, if $x \le \underline{x}$, then $\frac{x}{c_D} \le \frac{a-bc_U-bc_D}{4}$, and hence the optimal production quantity is equal to the production capacity. Consequently, the optimal production quantity increases as the raised capital or the production capacity increases. At the same time, a lower wholesale price is necessary to induce the downstream firm to order more components, and a lower retail price is necessary to induce customers to purchase more products. Fig. 3 gives an illustration on the impacts of the raised capital on the capacity, EPQ, and optimal production quantity in the external equity financing model. The figure shows that the optimal production quantity first increases as the raised capital increases and then is independent of the raised capital.

4.3. Pricing decisions in the internal equity financing model

The previous subsection has investigated the problem of the pricing decisions in the external equity financing model. This subsection considers the pricing decision problem in the internal equity financing model, where the production capital of the downstream firm is raised from the upstream firm through equity financing. When the production capital is raised through internal equity financing: On the one hand, the downstream firm's asset increases from v_D to $v_D + x$ and the upstream firm's asset decreases from v_U to $v_U - x$; on the other hand, the downstream firm is owned by Shareholders A and B together. In addition, the raised capital allows the downstream firm to conduct production activity, which will increase the upstream firm's asset from $v_U - x$ to $v_U - x + (w - c_U)q$ and the downstream firm's asset from $v_D + x$ to $v_D + x + (p - w - c_D)q$. Based on the analysis above, we can conclude that in the internal equity financing model, the final asset functions of two shareholders can be expressed as follows:

$$\begin{aligned} V_{II,A}(w, p|x) &= V_{II,U}(w, p|x) + \Lambda(x)V_{II,D}(w, p|x) \\ &= [v_U - x + (w - c_U)q] + \Lambda(x)[v_D + x + (p - w - c_D)q], \end{aligned} \tag{21}$$

$$V_{II,B}(w, p|x) = [1 - \Lambda(x)]V_{II,D}(w, p|x) = [1 - \Lambda(x)][v_D + x + (p - w - c_D)q]. \tag{22}$$

The first term in (21) is the asset of the upstream firm which is wholly owned by Shareholder A, and the second term represents the asset shared from the downstream firm. As a matter of fact, substituting (3) into (21), we can further show that $V_{II,A}(w, p|x) = v_U + [(w - c_U)q + \Lambda(x)(p - w - c_D)q]$, which implies that the final asset of Shareholder A is composed of the initial asset v_U and the earned profit $(w - c_U)q + \Lambda(x)(p - w - c_D)q$, including the profit shared from the upstream firm, i.e., $(w - c_U)q$, and the profit shared from the downstream firm, i.e., $\Lambda(x)[(p - w - c_D)q]$. Similarly, substituting (3) into (22), we have $V_{II,B}(w, p|x) = v_D + [1 - \Lambda(x)][(p - w - c_D)q]$, which indicates that the final asset of Shareholder B is composed of the initial asset v_D and the earned profit $[1 - \Lambda(x)][(p - w - c_D)q]$. That is, the final asset of each shareholder is composed of the initial asset and the earned profit. Therefore, maximizing the final asset is equivalent to maximizing the earned profit in the internal equity financing model. Using backward induction, the following lemma can be obtained.

Lemma 2. *For any given wholesale price of the upstream firm, the maximizer of the final asset function of Shareholder B, i.e., $V_{II,B}(w, p|x)$, in the internal equity financing model, is identical to that in the external equity financing model, which is*

$$p^*_{II}(w|x) = p^*_I(w|x) = \begin{cases} \dfrac{1}{b}\left(a - \dfrac{x}{c_D}\right), & \text{if } w \le \dfrac{1}{b}\left(a - \dfrac{2x}{c_D}\right) - c_D, \\ \dfrac{a + b(w + c_D)}{2b}, & \text{if } w > \dfrac{1}{b}\left(a - \dfrac{2x}{c_D}\right) - c_D. \end{cases} \tag{23}$$

Lemma 2 indicates that the response retail price remains unchanged when the external equity financing model is changed to the internal equity financing model. This result is intuitive since when the downstream firm makes the equity financing activity, the final asset function of Shareholder B is independent of whom the investor is.

Knowing that the downstream firm's response retail price is $p^*_{II}(w|x)$, the upstream firm selects a wholesale price to maximize Shareholder A's final asset $V_{II,A}(w, p|x)$. Substituting (23) into (21), the upstream firm's decision problem can be described as follows:

$$\max_w V_{II,A}(w|x) = \begin{cases} v_U + \dfrac{x}{c_D} \times \\ \left\{ \dfrac{\Lambda(x)}{b}\left(a - \dfrac{x}{c_D}\right) + [1-\Lambda(x)]w - c_U - \Lambda(x)c_D \right\}, & \text{if } w \le \dfrac{1}{b}\left(a - \dfrac{2x}{c_D}\right) - c_D, \\ v_U + \dfrac{[a - b(w + c_D)]}{2} \times \\ \left\{ \dfrac{\Lambda(x)}{b}\dfrac{[a + b(w + c_D)]}{2} + [1-\Lambda(x)]w - c_U - \Lambda(x)c_D \right\}, & \text{if } w > \dfrac{1}{b}\left(a - \dfrac{2x}{c_D}\right) - c_D. \end{cases} \quad (24)$$

This leads to the following theorem.

Theorem 2. *Suppose that the downstream firm raises capital through internal equity financing. Then the optimal pricing decisions of the two firms are*

$$w^*_{II}(x) = \begin{cases} \dfrac{1}{b}\left(a - \dfrac{2x}{c_D}\right) - c_D, & \textit{if } x \le \overline{x}, \\ \dfrac{[1-\Lambda(x)](a - bc_D) + bc_U}{[2-\Lambda(x)]b}, & \textit{if } x > \overline{x}, \end{cases} \quad (25)$$

$$p^*_{II}(x) = \begin{cases} \dfrac{1}{b}\left(a - \dfrac{x}{c_D}\right), & \textit{if } x \le \overline{x}, \\ \dfrac{[3 - 2\Lambda(x)]a + bc_U + bc_D}{[4 - 2\Lambda(x)]b}, & \textit{if } x > \overline{x}, \end{cases} \quad (26)$$

and the corresponding production quantity is

$$q^*_{II}(x) = \begin{cases} \dfrac{x}{c_D}, & \textit{if } x \le \overline{x}, \\ \dfrac{a - bc_U - bc_D}{4 - 2\Lambda(x)}, & \textit{if } x > \overline{x}, \end{cases} \quad (27)$$

where

$$\overline{x} = \frac{(a - bc_U - bc_D)c_D - 4v_D + \sqrt{(a - bc_U - bc_D)^2 c_D^2 + 16v_D^2}}{4} > 0. \quad (28)$$

Theorem 2 gives the optimal pricing and production decisions of the two supply chain members in the internal equity financing model and shows that the dependences of the pricing and the production decisions on the equity financing strategy in the internal equity financing model is quite different from that in the external equity financing model.

From Theorem 2, we have the following corollary.

Corollary 2. *When the downstream firm raises capital through internal equity financing, $w^*_{II}(x)$ and $p^*_I(x)$ decrease in x, while $q^*_{II}(x)$ increases in x.*

Corollary 2 suggests that in the internal equity financing model, the optimal pricing and production decisions always highly depend on the raised capital, which can lead to not only the capacity expansion but also the equity holding. To get a clear picture of the overall impacts caused by these two factors (i.e., the capacity expansion and the equity holding) on the optimal decisions in the internal equity financing model, we examine the following case, which enables us to obtain the impact of the equity holding on the optimal decisions, because the capacity constraint is relaxed:

- Case 3: The downstream firm's production capacity is sufficient, and the upstream firm has equity holding in the downstream firm.

Similar to discussions following Cases 1 and 2, we refer to the optimal production quantity in Case 3, as the *internal equity financing model's potential production quantity* (IPQ). From (27), it is clear that $\text{IPQ} = q^*_{II}(x) = \frac{a-bc_U-bc_D}{4-2\Lambda(x)}$, implying that the IPQ increases in $\Lambda(x)$. That is, if the downstream firm has sufficient production capacity, then the upstream firm's equity holding will enlarge the optimal production quantity in the internal equity financing model. In fact, besides the revenue from selling the component to the downstream firm, i.e., wq, Shareholder A can also gain additional revenue from sharing the profit of the downstream firm according to the equity shares, i.e., $\Lambda(x)(p - w - c_D)q$. That is, the equity holding can increase the revenue obtained by Shareholder A. In other words, to get the same revenue, Shareholder A incurs a relatively lower production cost. Of course, a relatively lower wholesale price ought to be chosen for the component with a relatively lower production cost, which will in turn lead to a relatively higher ordering quantity of the component, a relatively higher production quantity, and a relatively lower retail price of the product. Our analysis reveals that the equity holding by the upstream firm in the downstream firm can improve the cooperative relationship between the two firms by decreasing the retail price and increasing the production quantity. Here, it is worth pointing out that although the IPQ always increases as $\Lambda(x)$ increases, it can not exceed $\frac{a-bc_U-bc_D}{2}$. The reason is that $\Lambda(x) = \frac{x}{v_D+x} < 1$, and so $\text{IPQ} = \frac{a-bc_U-bc_D}{4-2\Lambda(x)} < \frac{a-bc_U-bc_D}{2}$.

Now, we examine the influence of the raised capital on the optimal decisions by exploring the overall impacts of both the capacity expansion and the equity holding. The IPQ is constrained by the capacity, so the downstream firm will produce min{Capacity, IPQ}, i.e., $\min\{\frac{x}{c_D}, \frac{a-bc_U-bc_D}{4-2\Lambda(x)}\}$. From Theorem 2, we have that if $x \le \overline{x}$, then $\frac{x}{c_D} \le \frac{a-bc_U-bc_D}{4-2\Lambda(x)}$; if $x > \overline{x}$, then $\frac{x}{c_D} > \frac{a-bc_U-bc_D}{4-2\Lambda(x)}$. The intuition is that if $x \le \overline{x}$, then the capacity expanded by the raised capital is relatively low and is insufficient to meet the IPQ. Therefore, the downstream firm conducts its production activity according to the capacity, which increases as the raised capital increases. If $x > \overline{x}$, then the capacity expanded by the raised capital is relatively high and is sufficient to meet the IPQ. Consequently, the downstream firm makes its production activity according to the IPQ, which also increases as the raised capital increases. Therefore, the optimal production quantity always increases as the raised capital increases. In addition, a higher production quantity requires the upstream firm to set a lower wholesale price and the downstream firm to set a lower retail price, in order to induce a higher market demand. Hence, the optimal wholesale price and retail price always decrease as the

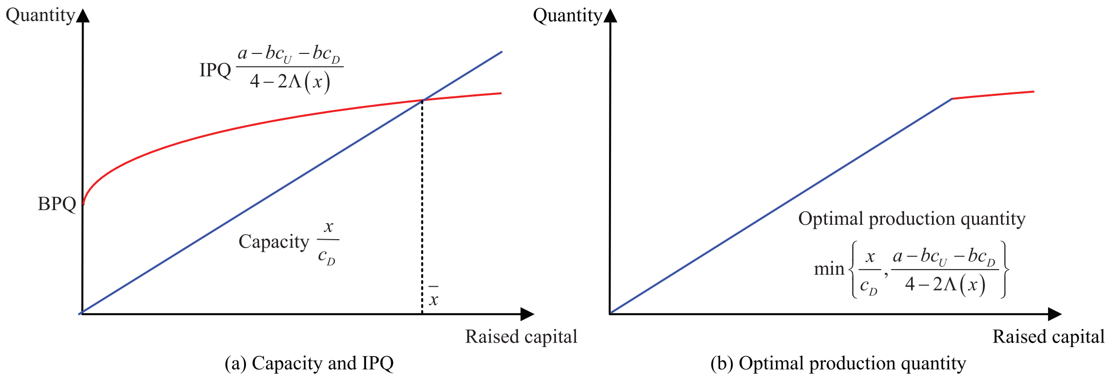


Fig. 4. The capacity, IPQ, and optimal production quantity in the internal equity financing model

raised capital increases. Fig. 4 illustrates the impacts of raised capital on the capacity, IPQ, and optimal production quantity in the internal equity financing model.

Comparing Theorem 1 with Theorem 2, we have the following corollary.

Corollary 3. *When the downstream firm conducts the equity financing activity: (i) The threshold capitals $\underline{x}$ and $\overline{x}$ satisfy $0 < \underline{x} < \overline{x}$; (ii) If $x \leq \underline{x}$, then $w_I^*(x) = w_{II}^*(x)$, $p_I^*(x) = p_{II}^*(x)$, and $q_I^*(x) = q_{II}^*(x)$; if $x > \underline{x}$, then $w_I^*(x) > w_{II}^*(x)$, $p_I^*(x) > p_{II}^*(x)$, and $q_I^*(x) < q_{II}^*(x)$.*

From the discussions following Corollaries 1 and 2, it is clear that the threshold capitals $\underline{x}$ and $\overline{x}$ are essentially the capitals which ensure that the capacities are increased to the EPQ and the IPQ, respectively, i.e., $\frac{\underline{x}}{c_D} = \frac{a-bc_U-bc_D}{4}$ and $\frac{\overline{x}}{c_D} = \frac{a-bc_U-bc_D}{4-2\Lambda(\overline{x})}$. Corollary 3(i) is an intuitive result due to the fact that EPQ $= \frac{a-bc_U-bc_D}{4} < \frac{a-bc_U-bc_D}{4-2\Lambda(\overline{x})} =$ IPQ, and more capital is required to increase the capacity to the IPQ, as compared with the EPQ.

Corollary 3(ii) can be explained as follows: If the capital is below the low threshold $\underline{x}$, then the capacity is not only lower than the EPQ but also lower than the IPQ. Thus, the capacity in each of the two equity financing models will be used for the production, implying that $q_I^*(x) = q_{II}^*(x) = \frac{x}{c_D}$. If the capital is above the high threshold $\overline{x}$, then the capacity is not only higher than the EPQ but also higher than the IPQ. So the optimal production quantities in the two equity financing models are equal to the EPQ and the IPQ, respectively, i.e., $q_I^*(x) =$ EPQ $<$ IPQ $= q_{II}^*(x)$. If the capital is between the two thresholds, then the capacity is higher than the EPQ but lower than the IPQ. Thus, only partial capacity in the external equity financing model but all capacity in the internal equity financing model will be used for the production, which indicates that $q_I^*(x) < \frac{x}{c_D} = q_{II}^*(x)$. Therefore, $q_I^*(x) = q_{II}^*(x)$ for $x \leq \underline{x}$, and $q_I^*(x) < q_{II}^*(x)$ for $x > \underline{x}$. Together with the fact that a higher production quantity requires a lower wholesale price and a lower retail price to induce a higher market demand, we know that $w_I^*(x) = w_{II}^*(x)$, $p_I^*(x) = p_{II}^*(x)$ for $x \leq \underline{x}$ and $w_I^*(x) > w_{II}^*(x)$, $p_I^*(x) > p_{II}^*(x)$ for $x > \underline{x}$.

5. Equity financing strategy

In the previous section, we have studied the pricing decisions of the two firms under given equity financing strategies. In this section, we investigate the financing problem for the downstream firm. By substituting (3), (17), (18), and (19) into (13) and (3), (25), (26), and (27) into (22), we can formulate the financing problems for the downstream firm in the external financing model and the internal financing model as follows:

$$\max_{x\geq 0} V_{I,B}(x) = \begin{cases} v_D + \dfrac{v_D x^2}{bc_D^2(v_D + x)}, & \text{if } x \leq \underline{x}, \\ v_D + \dfrac{v_D(a - bc_U - bc_D)^2}{16b(v_D + x)}, & \text{if } x > \underline{x}, \end{cases} \tag{29}$$

$$\max_{x\geq 0} V_{II,B}(x) = \begin{cases} v_D + \dfrac{v_D x^2}{bc_D^2(v_D + x)}, & \text{if } x \leq \overline{x}, \\ v_D + \dfrac{v_D(a - bc_U - bc_D)^2}{b\left(4 - \dfrac{2x}{v_D + x}\right)^2 (v_D + x)}, & \text{if } x > \overline{x}. \end{cases} \tag{30}$$

From (29) and (30), we have the following theorem.

Theorem 3. *(i) In the external equity financing model, the asset function $V_{I,B}(x)$ is continuous and unimodal in x, and the optimal financing strategy is $x_I^* = \underline{x}$, where $\underline{x}$ is given by (20), i.e., $x_I^* = \frac{(a-bc_U-bc_D)c_D}{4}$. (ii) In the internal equity financing model, the asset function $V_{II,B}(x)$ is also continuous and unimodal in x, and the optimal financing strategy is $x_{II}^* = \overline{x}$, where $\overline{x}$ is characterized by (28), i.e., $x_{II}^* = \frac{(a-bc_U-bc_D)c_D-4v_D+\sqrt{(a-bc_U-bc_D)^2c_D^2+16v_D^2}}{4}$.*

Theorem 3 provides the optimal equity financing strategy in each of the two models. Note that the raised capital can lead to not only the capacity expansion for the downstream firm but also the equity holding for the investor. Thus, when the original shareholder of the downstream firm chooses an equity financing strategy to maximize its own asset, he must balance the benefit from the capacity expansion and the loss in the asset which is allocated to the investor.

Corollary 4. *In the two equity financing models, the optimal equity financing strategies satisfy $x_{II}^* > x_I^* > 0$.*

First, the result that $x_I^* > 0$ and $x_{II}^* > 0$ suggests that Shareholder *B* can always benefit from an appropriate non-zero capital raised through either the external equity financing activity or the internal equity financing activity. The managerial insight behind this result is that the benefit from the capacity expanded by the appropriate non-zero capital outweighs the loss in the downstream firm's asset that is allocated to the investor according to the equity shares caused by the appropriate non-zero capital. This implies that equity financing activities are effective ways of raising capital for capacity expansion.

Second, the result that $x_{II}^* > x_I^*$ shows that more capital is raised by the downstream firm from the upstream firm, as compared to the outside institution. This is mainly due to the fact that equity shares held by different investors in the downstream firm have different impacts on the optimal pricing and production decisions and can be explained as follows:

- In the external equity financing model, because the potential output (i.e., EPQ) is independent of the equity shares held by the outside institution and is equal to the BPQ, the downstream firm will produce at most $\frac{a-bc_U-bc_D}{4}$ units of the product even if the product capacity is infinite (see the discussion following Case 2). Accordingly, any additional capacity above the BPQ has no impact on the production activity of the downstream firm and cannot yield any benefit for Shareholder B, but requires more capital at the expense of allowing the outside institution to hold more equity shares in the downstream firm and share a higher percentage of the downstream firm's asset. So, the raised capital will not exceed $\frac{(a-bc_U-bc_D)c_D}{4}$, i.e., x_I^*. Moreover, when the capacity is lower than the BPQ, the capacity expansion can yield benefit for Shareholder B by mitigating the production bottleneck of the downstream firm. Theorem 3(i) confirms that when the capacity is lower than the BPQ, the benefit gained from capacity expansion always outweighs the loss in the asset shared by the outside institution. So it is optimal to raise capital such that the capacity is equal to the BPQ.
- In the internal equity financing model, since the potential output (i.e., IPQ) increases in the percentage of equity shares held by the upstream firm, the raised capital not only expands the capacity but also leads to a higher potential output via the upstream firm's equity holding (see the discussion following Case 3). Thus, the additional capacity above the BPQ can yield benefit for Shareholder B because the more capacity is useful to meet the higher potential output. Theorem 3(ii) demonstrates that the benefit from the appropriate capacity above the BPQ outweighs the loss in the asset shared by the upstream firm.

From Theorem 3, we can also get the following corollary.

Corollary 5. *(i) In the external equity financing model, x_I^* increases in a, decreases in b and c_U, but is independent of v_D. Moreover, if $c_D \leq \frac{a-bc_U}{2b}$, then x_I^* increases in c_D; if $c_D > \frac{a-bc_U}{2b}$, then x_I^* decreases in c_D. (ii) In the internal equity financing model, x_{II}^* increases in a, decreases in b, c_U, and v_D. In addition, if $c_D \leq \frac{a-bc_U}{2b}$, then x_{II}^* increases in c_D; if $c_D > \frac{a-bc_U}{2b}$, then x_{II}^* decreases in c_D.*

In reality, the exogenous operating environments of firms usually vary in different stages of their life cycles, and the supply chain parameters a, b, c_U, c_D, and v_D essentially represent the exogenous operating environments of firms. As a matter of fact, when the potential market scale a increases or the price sensitivity b decreases, the market demand $M(p) = a - bp$ increases, and thus more capital is required to establish more capacity to meet a higher market demand. Moreover, a higher production cost indicates a more cost inefficient supply chain, which needs less capacity for the production activity. Nevertheless, the need for less capacity (i.e., $\frac{x_I^*}{c_D}$ or $\frac{x_{II}^*}{c_D}$) does not always mean the need for less capital (i.e., $\frac{x_I^*}{c_D} \cdot c_D$ or $\frac{x_{II}}{c_D} \cdot c_D$), especially when the production cost c_D is below the threshold $\frac{a-bc_U}{2b}$.

Although the impacts of the parameters a, b, c_U, and c_D on the equity financing strategies in the two models are similar, the influences of the parameter v_D are different. The explanation is as follows: Recall that the potential output in the external financing model is independent of the percentage of the equity shares held by the outside institution, i.e., the EPQ is independent of $\Lambda(x)$ but the potential output in the internal financing model increases as the percentage of the equity shares held by the upstream firm increases, i.e., the IPQ increases with $\Lambda(x)$. This, in conjunction with the fact that the percentage of the equity shares held by the investor decreases as the downstream firm's initial asset increases, i.e., $\Lambda(x)$ decreases as v_D increases, implies that when v_D increases, the EPQ remains unchanged, but the IPQ decreases. Accordingly, when v_D increases, no changes will be made in the production capacity and the production capital to meet the unchanged EPQ, but less production capacity and less production capital are required to meet a lower IPQ.

Additionally, Fu et al. (2021) study a platform supply chain and find that when the 3PL firm raises capital through equity financing from an outside institution to conduct the cost reduction innovation activity, the optimal equity financing strategy highly depends on the initial asset of the 3PL firm. Our result suggests that when the downstream firm raises capital through equity financing from an outside institution to conduct the capacity expansion activity, it is optimal to expand the capacity to the EPQ, and so the optimal equity financing strategy is independent of the initial asset of the downstream firm. The intuition behind this is that, when a firm raises capital from an outside institution through equity financing, the impact of the financing firm's initial asset on the financing strategy is related to the financing motivations, e.g., cost reduction, capacity expansion, etc.

Substituting the optimal financing strategies x_I^* and x_{II}^* specified in Theorem 3, the optimal pricing and production decisions $w_I^*(x)$, $w_{II}^*(x)$, $p_I^*(x)$, $p_{II}^*(x)$, $q_I^*(x)$, and $q_{II}^*(x)$ specified in Theorems 1 and 2, into (12), (13), (21), and (22), we can get the optimal final assets of Shareholders A and B in each of the two models, denoted by $V_{I,A}(x_I^*)$, $V_{II,A}(x_{II}^*)$, $V_{I,B}(x_I^*)$, and $V_{II,B}(x_{II}^*)$. In addition, since the supply chain contains two firms, and so the final asset of the entire supply chain is equal to the total final asset of them. We denote the final asset of the entire supply chain by V_{SC}, then it follows from (5) and (6) that $V_{SC} = V_U(w, p) + V_D(w, p) = v_U + v_D + (p - c_U - c_D)(a - bp)$. This together with the optimal strategies and decisions specified in Theorems 1–3, allows us to derive the optimal final assets of the entire supply chain in the two models, denoted by $V_{I,SC}(x_I^*)$ and $V_{II,SC}(x_{II}^*)$. The optimal final assets of each shareholder and the entire supply chain are summarized as follows:

$$V_{I,A}(x_I^*) = v_U + \frac{\Theta^2}{8b}, \tag{31}$$

$$V_{II,A}(x_{II}^*) = v_U + \frac{\Theta\left(\Theta c_D - 4v_D + \sqrt{\Theta^2 c_D^2 + 16v_D^2}\right)}{8bc_D}, \tag{32}$$

$$V_{I,B}(x_I^*) = \left[1 + \frac{\Theta^2}{4(4v_D + \Theta c_D)b}\right] v_D, \tag{33}$$

$$V_{II,B}(x_{II}^*) = \left[1 + \frac{\left(\Theta c_D + \sqrt{\Theta^2 c_D^2 + 16v_D^2}\right)\Theta^2}{b\left(\Theta c_D + 4v_D + \sqrt{\Theta^2 c_D^2 + 16v_D^2}\right)^2}\right] v_D, \tag{34}$$

$$V_{I,SC}(x_I^*) = v_U + v_D + \frac{3\Theta^2}{16b}, \tag{35}$$

$$V_{II,SC}(x_{II}^*) = v_U + v_D + \frac{\left(3\Theta + \frac{4v_D}{c_D} - \sqrt{\Theta^2 + \frac{16v_D^2}{c_D^2}}\right)\left(\Theta - \frac{4v_D}{c_D} + \sqrt{\Theta^2 + \frac{16v_D^2}{c_D^2}}\right)}{16b}, \quad (36)$$

where $\Theta = a - bc_U - bc_D > 0$.

From (31)–(36), we can get the following corollary.

Corollary 6. *The optimal final assets of Shareholder A, Shareholder B, and the entire supply chain in the internal equity financing model, are higher than that in the external equity financing model, i.e., $V_{II,A}(x_{II}^*) > V_{I,A}(x_I^*)$, $V_{II,B}(x_{II}^*) > V_{I,B}(x_I^*)$, and $V_{II,SC}(x_{II}^*) > V_{I,SC}(x_I^*)$.*

Corollary 6 states that the internal equity financing activity is more beneficial for the entire supply chain, and original shareholders of the two supply chain firms, as compared to the external equity financing activity. As a matter of fact, when the downstream firm conducts the internal equity financing activity, the cooperative relationship between the two supply chain firms is improved, and so the supply chain efficiency is improved (see the discussion following Corollary 2). Of course, original shareholders of supply chain firms can benefit from a more efficient supply chain.

From (33), we can also obtain the following corollary.

Corollary 7. *In the external equity financing model, $V_{I,B}(x_I^*)$ increases in a and v_D and decreases in b, c_U, and c_D.*

Corollary 7 characterizes the impacts of the supply chain parameters a, b, c_U, c_D, and v_D on the final asset of Shareholder B, i.e., $V_{I,B}(x_I^*)$, in the external equity financing model. However, due to the complexity of $V_{II,B}(x_{II}^*)$ in the internal equity financing model, it is quite challenging to obtain analytical results regarding the impacts of these parameters on the final asset of Shareholder B. We therefore conduct numerical studies to explore the impacts of these parameters on $V_{II,B}(x_{II}^*)$ in the next section. Comparisons and explanations of the impacts on the two equity financing models will also be given thereafter.

From Corollaries 5 and 7, we can see that when $c_D \leq \frac{a-bc_U}{2b}$, x_I^* decreases as c_D decreases. This together with the result that $V_{I,B}(x_I^*)$ always increases as c_D decreases, implies that when the production cost of the downstream firm is lower than $\frac{a-bc_U}{2b}$, if the production cost decreases, the original shareholder will raise less capital, but get more assets. Second, we can also see that when $c_D > \frac{a-bc_U}{2b}$, x_I^* first increases as c_D decreases to $\frac{a-bc_U}{2b}$, and then decreases as c_D further decreases to a lower value. In conjunction with the dependence of x_I^* on c_D, as characterized by Theorem 3, we can show that when $c_D > \frac{a-bc_U}{2b}$, if the production cost decreases to a sufficiently low-value $c_D - \delta$ such that

$$x_I^*(c_D - \delta) = \frac{[a - bc_U - b(c_D - \delta)](c_D - \delta)}{4} < \frac{(a - bc_U - bc_D)c_D}{4} = x_I^*(c_D),$$

where δ represents the decreased value of the production cost, then less capital will be raised by the original shareholder. Note that $V_{I,B}(x_I^*)$ always increases as c_D decreases. We can conclude that when the production cost of the downstream firm is higher than $\frac{a-bc_U}{2b}$, if the decreased value of the production cost is sufficiently high such that $x_I^*(c_D - \delta) < x_I^*(c_D)$, then the original shareholder

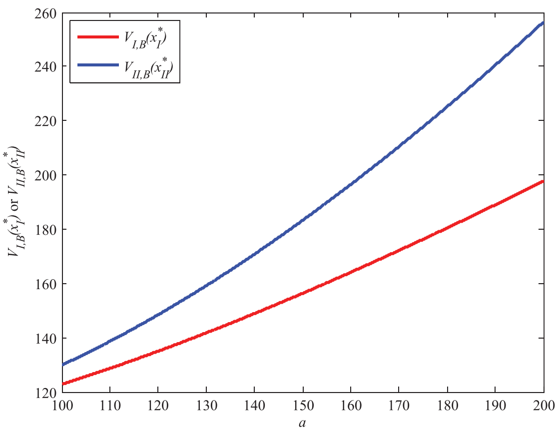


Fig. 5. Impacts of a on $V_{I,B}(x_I^*)$ and $V_{II,B}(x_{II}^*)$.

will raise less capital but get more asset. The intuition here is that, in the external equity financing model, when the operating environment regarding the production cost of the downstream firm changes, less investment for capacity expansion may create more value for Shareholder B.

6. Numerical examples

In Section 5, we have obtained the analytical results of Shareholder B's final assets in both the external and the internal equity financing models, i.e., $V_{I,B}(x_I^*)$ and $V_{II,B}(x_{II}^*)$, and characterized the impacts of the key supply chain parameters a, b, c_U, c_D, and v_D on $V_{I,B}(x_I^*)$. In this section, we conduct numerical studies to enrich the analytical results by comparing the impacts of the key parameters on $V_{I,B}(x_I^*)$ and $V_{II,B}(x_{II}^*)$. We consider the following five numerical examples:

- Example 1. $100 \leq a \leq 200$, $b = 10$, $c_U = 1$, $c_D = 2$, and $v_D = 100$.
- Example 2. $a = 200$, $1 \leq b \leq 10$, $c_U = 1$, $c_D = 2$, and $v_D = 100$.
- Example 3. $a = 200$, $b = 10$, $1 \leq c_U \leq 10$, $c_D = 2$, and $v_D = 100$.
- Example 4. $a = 200$, $b = 10$, $c_U = 1$, $1 \leq c_D \leq 10$, and $v_D = 100$.
- Example 5. $a = 200$, $b = 10$, $c_U = 1$, $c_D = 2$, and $1 \leq v_D \leq 100$.

Substituting the values of the supply chain parameters in the examples above into (33) and (34), respectively, we can get the results of Shareholder B's final asset in the two models, which are summarized in Figs. 5–9. Figs. 5–9 show that the final asset of Shareholder B in the internal equity financing model is higher than that in the external equity financing model. This numerical result is consistent with Corollary 6.

Figs. 5–9 also suggest that the final asset of Shareholder B increases in a and v_D but decreases in b, c_U, and c_D, in each of the two equity financing models. These results not only partially coincide with Corollary 7 but also show that the impacts of the key supply chain parameters on the final asset of Shareholder B are identical in the two equity financing models. The reason can be explained from the following aspects: First, the market demand function $M(p) = a - bp$ implies that the higher

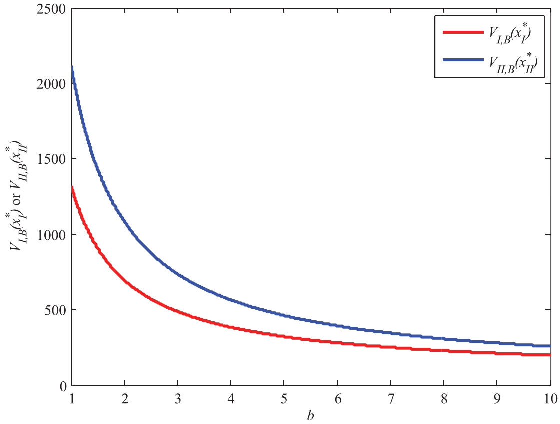


Fig. 6. Impacts of b on $V_{I,B}(x_I^*)$ and $V_{II,B}(x_{II}^*)$.

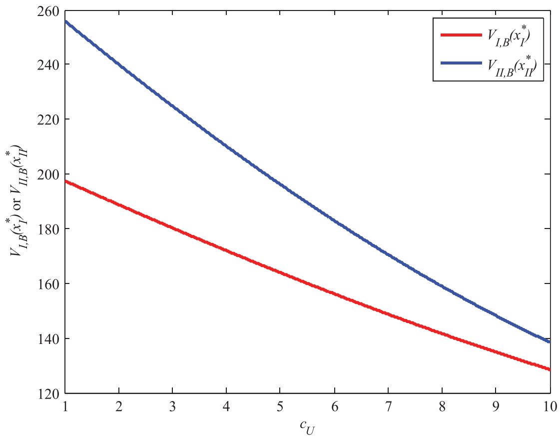


Fig. 7. Impacts of c_U on $V_{I,B}(x_I^*)$ and $V_{II,B}(x_{II}^*)$.

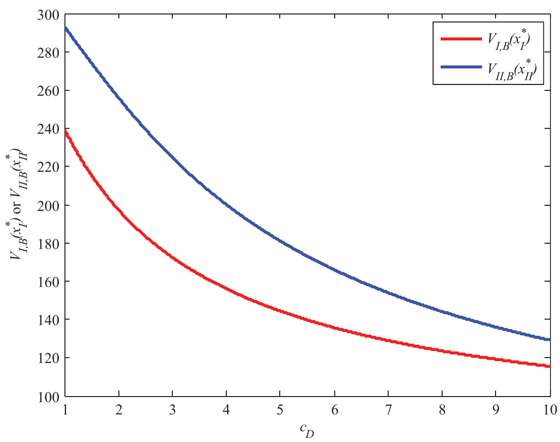


Fig. 8. Impacts of c_D on $V_{I,B}(x_I^*)$ and $V_{II,B}(x_{II}^*)$.

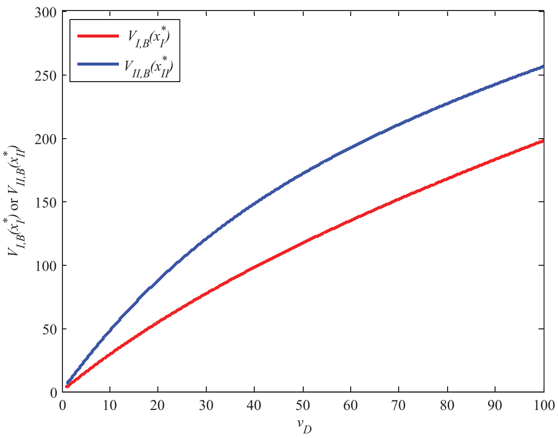


Fig. 9. Impacts of v_D on $V_{I,B}(x_I^*)$ and $V_{II,B}(x_{II}^*)$.

the potential market scale a or the lower the price sensitivity b is, the larger the market demand $M(p)$ is. Of course, the downstream firm can earn more profit in the market with a larger demand, and Shareholder B can get more assets from sharing the profit of the downstream firm. Thus, the final asset of Shareholder B will increase in a but decrease in b, i.e., $\frac{dV_{I,B}(x_I^*)}{da} > 0$, $\frac{dV_{II,B}(x_{II}^*)}{da} > 0$, $\frac{dV_{I,B}(x_I^*)}{db} < 0$ and $\frac{dV_{II,B}(x_{II}^*)}{\mathrm{d}b} < 0$. Second, the higher the production costs of the upstream firm and the downstream firm, the lower is the supply chain efficiency. Because the supply chain inefficiency is detrimental to each supply chain firm, the final assets of the downstream firm and its shareholder decrease in the production costs of the upstream firm and the downstream firm. Therefore, the final asset of Shareholder B decreases in c_U and c_D, i.e., $\frac{dV_{I,B}(x_I^*)}{dc_U} < 0$, $\frac{dV_{II,B}(x_{II}^*)}{dc_U} < 0$, $\frac{dV_{I,B}(x_I^*)}{dc_D} < 0$, and $\frac{dV_{II,B}(x_{II}^*)}{dc_D} < 0$. Third, one can easily see from (13) and (22) that the final asset of Shareholder B can be expressed as

$$V_B = [1 - \Lambda(x)][v_D + x + (p - w - c_D)q] = v_D + \frac{v_D}{v_D + x}(p - w - c_D)q.$$

This, in conjunction with the fact that the initial assets of the downstream firm and Shareholder B are identical and equal to v_D, implies that Shareholder B's final asset includes two parts: (i) Shareholder B's initial asset, i.e., v_D, and (ii) $\frac{v_D}{v_D+x}$ percentage of the downstream firm's profit, i.e., $\frac{v_D}{v_D+x}(p - w - c_D)q$, which essentially represents the increased asset of Shareholder B, i.e., $\frac{v_D}{v_D+x}(p - w - c_D)q = V_B - v_D$. Intuitively, the second part increases in v_D because a higher percentage of the downstream firm's profit should be allocated to Shareholder B if v_D increases. Such an intuition is confirmed by Fig. 10. Accordingly, if v_D increases, the first and second parts both increase. Consequently, Shareholder B's final asset increases in v_D, i.e., $\frac{dV_{I,B}(x_I^*)}{dv_D} > 0$ and $\frac{dV_{II,B}(x_{II}^*)}{dv_D} > 0$.

Finally, we can see from Fig. 8 and Corollary 5 that, $V_{II,B}(x_{II}^*)$ always decreases in c_D; while x_{II}^* increases in c_D when $c_D \leq \frac{a-bc_U}{2b}$, but decreases in c_D when $c_D > \frac{a-bc_U}{2b}$. Then, similar to the discussion at the end of Section 5, we can conclude that when the production cost c_D is lower than $\frac{a-bc_U}{2b}$, if the production cost decreases, then the original shareholder B raises less capital, but

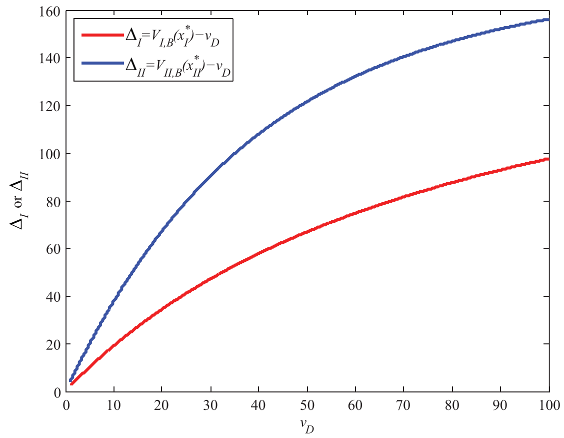


Fig. 10. Impacts of v_D on Δ_I and Δ_{II}

obtains more assets. In addition, when the production cost c_D is higher than $\frac{a-bc_U}{2b}$, if the production cost c_D decreases to a sufficiently low-value $c_D - \delta$ such that

$$x_{II}^*(c_D - \delta) < x_{II}^*(c_D),$$

where δ represents the decreased value of the production cost and the dependence of x_{II}^* on c_D is characterized by Theorem 3, then the original shareholder B raises less capital but obtains more assets. That is, in the internal equity financing model, when the operating production cost changes, less production capital raised from the upstream firm may create more value for Shareholder B.

7. Extension: stochastic demand

The previous study has focused on the price-sensitive linear deterministic demand. In reality, the demand is usually stochastic. In this section, we check the robustness of the previous key result by considering stochastic demand ξ, with the probability density function $f(\cdot)$ and the cumulative distribution function $F(\cdot)$. Here, it is worth to point out that although the price-sensitive linear stochastic demand has been widely studied, the supply chain under the wholesale price contract raises a technical challenge with respect to identifying the concavity of the upstream supply chain player's profit function (see Theorem 4 in the study of Liu et al., 2006). In this section, we assume similar to Cachon (2004), etc., that the retail price is fixed to ensure the concavity of the profit function and the existence of the optimal decision.

When facing stochastic demand ξ, the supply chain players make decisions in three stages: In stage 1, Shareholder B chooses the production capital x; In stage 2, Shareholder A decides the wholesale price w; In stage 3, Shareholder B determines the production quantity q. Similar to the analysis of (12) and (13) in Subsection 4.2, we can express the expected final asset functions of Shareholders A and B in the external financing model as

$$V_{I,A}(x, w, q) = v_U + (w - c_U)q, \tag{37}$$

$$V_{I,B}(x, w, q) = [1 - \Lambda(x)]\{v_D + x + pE[\min(q, \xi)] - (w + c_D)q\}. \tag{38}$$

Additionally, similar to the analysis of (21) and (22) in Subsection 4.3, we can write the expected final asset functions of Shareholders A and B as

$$V_{II,A}(x, w, q) = [v_U - x + (w - c_U)q] + \Lambda(x)\{v_D + x + pE[\min(q, \xi)] - (w + c_D)q\}, \tag{39}$$

$$V_{II,B}(x, w, q) = [1 - \Lambda(x)]\{v_D + x + pE[\min(q, \xi)] - (w + c_D)q\}. \tag{40}$$

Using backward induction, we can get the following theorem.

Theorem 4. *(i) Suppose that* $\frac{xf(x)}{1-F(x)}$ *is increasing in x, then in the external equity financing model, the optimal financing strategy* x_I^* *is uniquely determined by*

$$p\left[1 - F\left(\frac{x_I^*}{c_D}\right)\right]\left[1 - \frac{f\left(\frac{x_I^*}{c_D}\right)\frac{x_I^*}{c_D}}{1 - F\left(\frac{x_I^*}{c_D}\right)}\right] = c_U + c_D, \tag{41}$$

the optimal wholesale price and the optimal production quantity are

$$w_I^* = p\left[1 - F\left(\frac{x_I^*}{c_D}\right)\right] - c_D, \tag{42}$$

$$q_I^* = \frac{x_I^*}{c_D}. \tag{43}$$

(ii) Suppose that $\frac{1}{v_D + c_D x} \cdot \frac{xf(x)}{1-F(x)}$ *is increasing in x, then in the internal equity financing model, the optimal financing strategy* x_{II}^* *is uniquely characterized by*

$$p\left[1 - F\left(\frac{x_{II}^*}{c_D}\right)\right]\left[1 - \frac{v_D}{v_D + x_{II}^*} \cdot \frac{f\left(\frac{x_{II}^*}{c_D}\right)\frac{x_{II}^*}{c_D}}{1 - F\left(\frac{x_{II}^*}{c_D}\right)}\right] = c_U + c_D, \tag{44}$$

the optimal pricing and production decisions are

$$w_{II}^* = p\left[1 - F\left(\frac{x_{II}^*}{c_D}\right)\right] - c_D, \tag{45}$$

$$q_{II}^* = \frac{x_{II}^*}{c_D}. \tag{46}$$

(iii) The optimal expected final asset of Shareholder B in the internal equity financing model is higher than that in the external equity financing model, i.e., $V_{II,B}(x_{II}^*, w_{II}^*, q_{II}^*) > V_{I,B}(x_I^*, w_I^*, q_I^*)$.

Theorem 4(i) supposes that $\frac{xf(x)}{1-F(x)}$ is increasing in x, which is a very weak assumption because exponential and uniform distributions, as well as normal, gamma, and Weibull distributions, etc., satisfy this assumption (see Cachon, 2004). In addition, Theorem 4(ii) supposes that $\frac{1}{v_D+c_Dx} \cdot \frac{xf(x)}{1-F(x)}$ is increasing in x. As a matter of fact, we can verify that exponential and uniform distributions, etc., satisfy this assumption.

Theorem 4 characterizes the optimal decisions of supply chain players. We can see from Theorem 4 that Shareholder B gets more benefit from the internal equity financing activity than from the external equity financing activity. That is, the previous key result remains unchanged when facing stochastic demand. The intuition behind this theorem is as follows: When Shareholder B conducts the internal equity financing activity, Shareholder A can share the asset of the downstream firm and will set a lower wholesale price to induce Shareholder B to produce more products, which in turn requires Shareholder B to raise more production capital and produce more product to meet the stochastic demand. That is, the internal equity financing activity can improve the cooperative relationship between the two supply chain players. Of course, Shareholder B can benefit from the improved cooperative relationship.

8. Concluding remarks

In practice, it is very common for downstream manufacturers (or downstream firms) to raise capital to expand their production capacities through equity financing. However, existing studies on supply chain finance assume that production capacities are expanded by the capital raised through non-equity financing and neglect the possibility that production capacities can also be expanded by the capital raised through equity financing. Motivated by the gap between the practical activities and theoretical studies, this research explores the problem of a downstream manufacturer's equity financing for production capacity expansion in a supply chain, where the manufacturer purchases a component from the supplier and needs capital to expand the production capacity for transforming the component into the final product.

Two different equity financing models are considered and compared: the external equity financing model and the internal equity financing model. In the former model, the investor is the outside institution who does not belong to the manufacturer's supply chain; while in the latter model, the investor is the upstream supplier. By assuming that the raised capital not only enables the manufacturer to expand the production capacity but also allows the investor to hold equity shares in the manufacturer, our research derives the optimal pricing decisions of the two supply chain members and analyzes the optimal equity financing strategy of the manufacturer. In summary, the findings are as follows:

1. When the raised capital increases, the retail price always decreases and the production quantity always increases in the internal equity financing model, which may not be the case in the external equity financing model.
2. The production quantity in the internal equity financing model may be higher than that in the benchmark model with no capacity constraint and no equity shares held by the investor.

However, the production quantity in the external financing model cannot exceed that in the benchmark model.

3. For the original shareholder of the manufacturer, the benefit from the capacity expansion caused by an appropriate non-zero capital outweighs the loss in the asset shared by the investor according to equity shares. Thus, equity financing activities are effective ways to expand production capacities. In addition, the internal equity financing activity is more beneficial for the original shareholders of supply chain firms and the entire supply chain, as compared to the external equity financing activity.
4. The optimal equity financing strategy highly depends on the supply chain parameters, such as the potential market scale, the price sensitivity of the market demand, the initial asset of the downstream firm, and production costs of the two firms. However, the dependence of the equity financing strategy on these parameters may be different in the two financing models. Moreover, when the production cost of the manufacturer is changed, less capital raised for the capacity expansion may create more value for the original shareholder of the manufacturer.

The major finding still holds when the demand is uncertain, and hence the robustness of the basic model can be demonstrated. This research also has some limitations. For example, we assume that the capital is raised through equity financing. In reality, the capital can also be obtained through non-equity financing, such as banking financing, etc. Therefore, it is interesting to investigate the downstream manufacturer's financing problem under the assumption that the capital is raised through a hybrid equity and non-equity financing. Additionally, we suppose that the upstream supplier has sufficient capital. It is worth considering a setting where the capital constraint supplier raises capital through equity financing to expand the component capacity. Finally, we focus on capacity expansion investment. Future researchers can focus on adaptation investment (Ivanov, 2021) and recovery investment (Aldrighetti et al., 2023).

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Appendix: Mathematical proofs

Proof of Lemma 1

First, we define

$$V_{I,B1}(w, p|x) = v_D + [1 - \Lambda(x)](p - w - c_D)\frac{x}{c_D},$$

$$V_{I,B2}(w, p|x) = v_D + [1 - \Lambda(x)](p - w - c_D)(a - bp).$$

It is easy to verify that $V_{I,B1}(w, p|x)$ increases in p and achieves its maximum at $\frac{1}{b}(a - \frac{x}{c_D})$ in the range $p \leq \frac{1}{b}(a - \frac{x}{c_D})$. Further, $\lim_{p \to \frac{1}{b}(a - \frac{x}{c_D})^-} V_{I,B1}(w, p|x) = \lim_{p \to \frac{1}{b}(a - \frac{x}{c_D})^+} V_{I,B2}(w, p|x) = v_D + [1 - \Lambda(x)][\frac{1}{b}(a - \frac{x}{c_D}) - w - c_D]\frac{x}{c_D}$, which implies that $V_{I,B}(w, p|x)$ is continuous in p. Thus, choosing a value for p to maximize $V_{I,B}(w, p|x)$ is equivalent to choosing a value for p in the range $p > \frac{1}{b}(a - \frac{x}{c_D})$ to maximize $V_{I,B2}(w, p|x)$. Therefore, the optimization problem of (14) can be equivalently expressed as follows:

$$\max_{p \geq \frac{1}{b}\left(a - \frac{x}{c_D}\right)} V_{I,B2}(w, p|x) = v_D + [1 - \Lambda(x)](p - w - c_D)(a - bp).$$

Second, by taking the first and second derivatives of $V_{I,B2}(w, p|x)$ with respect to p, we have $\frac{dV_{I,B2}(w,p|x)}{dp} = [1 - \Lambda(x)][a - 2bp + b(w + c_D)]$ and $\frac{d^2 V_{I,B2}(w,p|x)}{dp^2} = -2b[1 - \Lambda(x)] < 0$. That is, $V_{I,B2}(w, p|x)$ is concave in p and achieves its maximum at $p = \frac{a+b(w+c_D)}{2b}$. In conjunction with the constraint $p > \frac{1}{b}(a - \frac{x}{c_D})$, we know that if $w > \frac{1}{b}(a - \frac{2x}{c_D}) - c_D$, then $\frac{a+b(w+c_D)}{2b} > \frac{1}{b}(a - \frac{x}{c_D})$, and hence the optimal solution is $p_I^*(w|x) = \frac{a+b(w+c_D)}{2b}$; if $w \leq \frac{1}{b}(a - \frac{2x}{c_D}) - c_D$, then $\frac{a+b(w+c_D)}{2b} \leq \frac{1}{b}(a - \frac{x}{c_D})$, and so the optimal solution is $p_I^*(w|x) = \frac{1}{b}(a - \frac{x}{c_D})$. That is, (15) is obtained.

Proof of Theorem 1

(i) First, we define

$$V_{I,A1}(w|x) = v_U + (w - c_U)\frac{x}{c_D},$$

$$V_{I,A2}(w|x) = v_U + (w - c_U)\frac{a - b(w + c_D)}{2}.$$

It is easy to show that $V_{I,A1}(w|x)$ increases in w and achieves its maximum at $\frac{1}{b}(a - \frac{2x}{c_D}) - c_D$ in the range $w \leq \frac{1}{b}(a - \frac{2x}{c_D}) - c_D$ and $\lim_{w \to [\frac{1}{b}(a - \frac{2x}{c_D}) - c_D]^-} V_{I,A1}(w|x) = \lim_{w \to [\frac{1}{b}(a - \frac{2x}{c_D}) - c_D]^+} V_{I,A2}(w|x) = v_U + [\frac{1}{b}(a - \frac{2x}{c_D}) - c_U - c_D]\frac{x}{c_D}$. Hence, choosing a value for w to maximize $V_{I,A}(w|x)$ is equivalent to choosing a value for w in the range $w > \frac{1}{b}(a - \frac{2x}{c_D}) - c_D$ to maximize $V_{I,A2}(w|x)$. Accordingly, the optimization problem of (16) can be equivalently written as follows:

$$\max_{w \geq \frac{1}{b}\left(a - \frac{2x}{c_D}\right) - c_D} V_{I,A2}(w|x) = v_U + (w - c_U)\frac{a - b(w + c_D)}{2}.$$

Second, by taking the first and second derivatives of $V_{I,A2}(w|x)$ with respect to w, we have $\frac{dV_{I,A2}(w|x)}{dw} = \frac{a - 2bw + bc_U - bc_D}{2}$ and $\frac{d^2 V_{I,A2}(w|x)}{dw^2} = -b < 0$. That is, $V_{I,A2}(w|x)$ is concave in w and achieves its maximum at $w = \frac{a + bc_U - bc_D}{2b}$. Together with the constraint $w > \frac{1}{b}(a - \frac{2x}{c_D}) - c_D$, we can conclude that if $x > \underline{x}$, then $\frac{a + bc_U - bc_D}{2b} \geq \frac{1}{b}(a - \frac{2x}{c_D}) - c_D$, and hence the optimal wholesale price is $w_I^*(x) = \frac{a+bc_U-bc_D}{2b}$; if $x \leq \underline{x}$, then $\frac{a+bc_U-bc_D}{2b} \leq \frac{1}{b}(a-\frac{2x}{c_D}) - c_D$, and so the optimal wholesale price is $w_I^*(x) = \frac{1}{b}(a-\frac{2x}{c_D}) - c_D$. That is, (17) is obtained.

(ii) Substituting (17) into (15), we have (18); and substituting (18) into (2), we have (19).

Proof of Corollary 1

The result can be derived directly from (17), (18), (19), and (20).

Proof of Lemma 2

From (13) and (22), it is clear that the final asset functions of Shareholder B in the external equity financing model and the internal equity financing model are identical, i.e., $V_{I,B}(w,p|x) = V_{II,B}(w,p|x)$. This indicates that the maximizers of two asset functions are identical, i.e., $p_{II}^*(w|x) = p_I^*(w|x)$.

Proof of Theorem 2

(i) First, we define

$$V_{II,A1}(w|x) = v_U + \frac{x}{c_D} \cdot \left\{ \frac{\Lambda(x)}{b}\left(a - \frac{x}{c_D}\right) + [1-\Lambda(x)]w - c_U - \Lambda(x)c_D \right\},$$

$$V_{II,A2}(w|x) = v_U + \frac{[a-b(w+c_D)]}{2} \cdot \left\{ \frac{\Lambda(x)}{b}\frac{[a+b(w+c_D)]}{2} + [1-\Lambda(x)]w - c_U - \Lambda(x)c_D \right\}.$$

It is easy to show that $V_{II,A1}(w|x)$ increases in w and achieves its maximum at $\frac{1}{b}(a-\frac{2x}{c_D}) - c_D$ in the range $w \leq \frac{1}{b}(a-\frac{2x}{c_D}) - c_D$, and $\lim_{w \to [\frac{1}{b}(a-\frac{2x}{c_D})-c_D]^-} V_{II,A1}(w|x) = \lim_{w \to [\frac{1}{b}(a-\frac{2x}{c_D})-c_D]^+} V_{II,A2}(w|x)$. Thus, choosing a value for w to maximize $V_{II,A}(w|x)$ is equivalent to choosing a value for w in the range $w > \frac{1}{b}(a-\frac{2x}{c_D}) - c_D$ to maximize $V_{II,A2}(w|x)$. Therefore, the optimization problem of (24) can be equivalently written as follows:

$$\max_{w \geq \frac{1}{b}\left(a-\frac{2x}{c_D}\right)-c_D} V_{II,A2}(w|x) = v_U + \frac{[a-b(w+c_D)]}{2} \cdot \left\{ \frac{\Lambda(x)}{b}\frac{[a-b(w+c_D)]}{2} + [1-\Lambda(x)]w - c_U - \Lambda(x)c_D \right\}.$$

Second, by taking the first and second derivatives of $V_{II,A2}(w|x)$ with respect to w, we can get $\frac{dV_{II,A2}(w|x)}{dw} = \frac{[1-\Lambda(x)](a-bc_D)+bc_U-[2-\Lambda(x)]bw}{2}$ and $\frac{d^2V_{II,A2}(w|x)}{dw^2} = -\frac{[2-\Lambda(x)]b}{2} < 0$. That is, $V_{II,A2}(w|x)$ is concave in w and achieves its maximum at $w = \frac{[1-\Lambda(x)](a-bc_D)+bc_U}{[2-\Lambda(x)]b}$. Together with the

constraint $w > \frac{1}{b}(a - \frac{2x}{c_D}) - c_D$, we can conclude that if $\frac{[1-\Lambda(x)](a-bc_D)+bc_U}{[2-\Lambda(x)]b} > \frac{1}{b}(a - \frac{2x}{c_D}) - c_D$, then the optimal wholesale price is $w^*_{II}(x) = \frac{[1-\Lambda(x)](a-bc_D)+bc_U}{[2-\Lambda(x)]b}$; otherwise, if $\frac{[1-\Lambda(x)](a-bc_D)+bc_U}{[2-\Lambda(x)]b} \leq \frac{1}{b}(a - \frac{2x}{c_D}) - c_D$, then the optimal wholesale price is $w^*_{II}(x) = \frac{1}{b}(a - \frac{2x}{c_D}) - c_D$.
Third, we define

$$Y(x) = \frac{[1-\Lambda(x)](a-bc_D)+bc_U}{[2-\Lambda(x)]b} - \left[\frac{1}{b}\left(a - \frac{2x}{c_D}\right) - c_D\right].$$

By substituting $\Lambda(x) = \frac{x}{v_D+x}$ into the above expression, we have, after some algebra, that

$$Y(x) = \frac{1}{[2-\Lambda(x)](v_D+x)bc_D} \cdot \phi(x),$$

where

$$\phi(x) = 2x^2 - [(a - bc_U - bc_D)c_D - 4v_D]x - (a - bc_U - bc_D)c_D v_D.$$

Further, we can show that the quadratic equation $\phi(x) = 0$ has two solutions:

$$x_1 = \frac{(a - bc_U - bc_D)c_D - 4v_D - \sqrt{(a - bc_U - bc_D)^2 c_D^2 + 16v_D^2}}{4},$$

$$x_2 = \frac{(a - bc_U - bc_D)c_D - 4v_D + \sqrt{(a - bc_U - bc_D)^2 c_D^2 + 16v_D^2}}{4}.$$

It is easy to verify that $x_1 < 0$ and $x_2 = \overline{x} > 0$. This, in conjunction with $\frac{1}{[2-\Lambda(x)](v_D+x)bc_D} > 0$, indicates that if $0 \leq x \leq \overline{x}$, then $Y(x) \leq 0$, and so the optimal wholesale price is $w^*_{II}(x) = \frac{1}{b}(a - \frac{2x}{c_D}) - c_D$; if $x > \overline{x}$, then $Y(x) \geq 0$, and hence the optimal wholesale price is $w^*_{II}(x) = \frac{[1-\Lambda(x)](a-bc_D)+bc_U}{[2-\Lambda(x)]b}$. That is, (25) is obtained.

(ii) Substituting (25) into (23), we have (26); and substituting (26) into (2), we have (27).

Proof of Corollary 2

First, from (25), (26), (27), and (28), it is easy to show that if $x \leq \overline{x}$, then $w^*_{II}(x)$ and $p^*_I(x)$ decrease in x, while $q^*_{II}(x)$ increases in x.
Second, from (3), (25), (26), and (27), we can show that if $x > \overline{x}$, then

$$\frac{dw^*_{II}(x)}{dx} = \frac{dw^*_{II}(x)}{d\Lambda(x)} \cdot \frac{d\Lambda(x)}{dx} = -\frac{(a - bc_U - bc_D)v_D}{[2-\Lambda(x)]^2(v_D+x)^2 b} < 0,$$

$$\frac{dp^*_{II}(x)}{dx} = \frac{dp^*_{II}(x)}{d\Lambda(x)} \cdot \frac{d\Lambda(x)}{dx} = -\frac{2(a - bc_U - bc_D)v_D}{[4-\Lambda(x)]^2(v_D+x)^2 b} < 0,$$

$$\frac{dq^*_{II}(x)}{dx} = \frac{dq^*_{II}(x)}{d\Lambda(x)} \cdot \frac{d\Lambda(x)}{dx} = \frac{2(a - bc_U - bc_D)v_D}{[4-2\Lambda(x)]^2(v_D+x)^2} > 0,$$

indicating that if $x > \overline{x}$, then $w_{II}^*(x)$ and $p_I^*(x)$ decrease in x, while $q_{II}^*(x)$ increases in x.

Third, from (25), (26), and (27), we have that $\lim_{x\to\overline{x}^-} w_{II}^*(x) = \lim_{x\to\overline{x}^+} w_{II}^*(x)$, $\lim_{x\to\overline{x}^-} p_{II}^*(x) = \lim_{x\to\overline{x}^+} p_{II}^*(x)$ and $\lim_{x\to\overline{x}^-} q_{II}^*(x) = \lim_{x\to\overline{x}^+} q_{II}^*(x)$. This implies that $w_{II}^*(x)$, $p_I^*(x)$, and $q_{II}^*(x)$ are all continuous in x. Accordingly, $w_{II}^*(x)$ and $p_I^*(x)$ always decrease in x, while $q_{II}^*(x)$ always increases in x for $x \geq 0$. That is, Corollary 2 is obtained.

Proof of Corollary 3

First, it follows from $a > b(c_U + c_D)$, (20) and (28) that $\underline{x} > 0$ and

$$\overline{x} - \underline{x} = \frac{\sqrt{(a - bc_U - bc_D)^2 c_D^2 + 16v_D^2} - 4v_D}{4} > 0.$$

Thus, we have $0 < \underline{x} < \overline{x}$.

Second, it follows from $0 < \underline{x} < \overline{x}$, (19) and (27) that

$$q_I^*(x) = \begin{cases} \dfrac{x}{c_D}, & \text{if } x \leq \underline{x}, \\ \dfrac{a - bc_U - bc_D}{4}, & \text{if } \underline{x} < x < \overline{x}, \\ \dfrac{a - bc_U - bc_D}{4}, & \text{if } x \geq \overline{x}, \end{cases}$$

$$q_{II}^*(x) = \begin{cases} \dfrac{x}{c_D}, & \text{if } x \leq \underline{x}, \\ \dfrac{x}{c_D}, & \text{if } \underline{x} < x < \overline{x}, \\ \dfrac{a - bc_U - bc_D}{4 - 2\Lambda(x)}, & \text{if } x \geq \overline{x}. \end{cases}$$

Clearly, if $x \leq \underline{x}$, then $q_I^*(x) - q_{II}^*(x) = \frac{x}{c_D} - \frac{x}{c_D} = 0$; if $\underline{x} < x < \overline{x}$, then $q_I^*(x) - q_{II}^*(x) = \frac{a-bc_U-bc_D}{4} - \frac{x}{c_D} < 0$; if $x \geq \overline{x}$, then $q_I^*(x) - q_{II}^*(x) = \frac{a-bc_U-bc_D}{4} - \frac{a-bc_U-bc_D}{4-2\Lambda(x)} < 0$. That is, if $x \leq \underline{x}$, then $q_I^*(x) = q_{II}^*(x)$; if $x > \underline{x}$, then $q_I^*(x) < q_{II}^*(x)$. Using similar techniques as in the analysis above, we can demonstrate that if $x \leq \underline{x}$, then $w_I^*(x) = w_{II}^*(x)$ and $p_I^*(x) = p_{II}^*(x)$; if $x > \underline{x}$, then $w_I^*(x) > w_{II}^*(x)$ and $p_I^*(x) > p_{II}^*(x)$.

Proof of Theorem 3

We define

$$V_{I,B1}(x) = v_D + \frac{v_D x^2}{bc_D^2(v_D + x)},$$

$$V_{I,B2}(x) = v_D + \frac{v_D(a - bc_U - bc_D)^2}{16b(v_D + x)},$$

$$V_{II,B2}(x) = v_D + \frac{v_D(a - bc_U - bc_D)^2}{b\left(4 - \frac{2x}{v_D+x}\right)^2 (v_D + x)}.$$

First, taking derivative on $V_{I,B1}(x)$ with respect to x, we have that

$$\frac{dV_{I,B1}(x)}{dx} = \frac{v_D}{bc_D^2} \cdot \frac{2v_D x + x^2}{(v_D + x)^2} > 0,$$

which implies that $V_{I,B}(x) = V_{I,B1}(x)$ increases in x and achieves its maximum at $\underline{x}$ in the range $x \leq \underline{x}$. In addition, it is easy to show that $V_{I,B}(x) = V_{I,B2}(x)$ decreases in x and achieves its maximum at $\underline{x}$ in the range $x > \underline{x}$. This, in conjunction with the fact that $\lim_{x \to \underline{x}^-} V_{I,B1}(x) = \lim_{x \to \underline{x}^+} V_{I,B2}(x)$ means that $V_{I,B}(x)$ is continuous and unimodal in x and achieves its maximum at $\underline{x}$.

Second, since $\frac{dV_{I,B1}(x)}{dx} > 0$, we know that $V_{II,B}(x) = V_{I,B1}(x)$ increases in x and achieves its maximum at $\overline{x}$ in the range $x \leq \overline{x}$. Moreover, after some algebraic simplifications, we have that

$$\frac{dV_{II,B2}(x)}{dx} = -\frac{v_D(a - bc_U - bc_D)^2 x}{4b(2v_D + x)^3},$$

which indicates that $V_{II,B}(x) = V_{II,B2}(x)$ decreases in x and achieves its maximum at $\overline{x}$ in the range $x > \overline{x}$. This, together with the fact that $\lim_{x \to \overline{x}^-} V_{I,B1}(x) = \lim_{x \to \overline{x}^+} V_{II,B2}(x)$ implies that $V_{II,B}(x)$ is continuous and unimodal in x and achieves its maximum at $\overline{x}$.

Proof of Corollary 4

Note that $x_I^* = \underline{x}$ and $x_{II}^* = \overline{x}$. Then Corollary 4 can be derived directly from Corollary 3(i).

Proof of Corollary 5

(i) The results that x_I^* increases in a, decreases in b and c_U, but is independent of v_D can be derived directly from the expression of x_I^*. Moreover, from the expression of x_I^*, we have that

$$\frac{dx_I^*}{dc_D} = \frac{a - bc_U - 2bc_D}{4},$$

which indicates that if $c_D \leq \frac{a-bc_U}{2b}$, then $\frac{dx_I^*}{dc_D} \geq 0$; if $c_D > \frac{a-bc_U}{2b}$, then $\frac{dx_I^*}{dc_D} < 0$. That is, Corollary 5(i) is obtained.

(ii) The results that x_{II}^* increases in a, decreases in b and c_U, can be derived directly from the expression of x_{II}^*. In addition, from the expression of x_{II}^*, we have that

$$\frac{dx_{II}^*}{dv_D} = \frac{4v_D - \sqrt{(a - bc_U - bc_D)^2 c_D^2 + 16v_D^2}}{\sqrt{(a - bc_U - bc_D)^2 c_D^2 + 16v_D^2}},$$

$$\frac{dx_{II}^*}{dc_D} = \frac{a - bc_U - 2bc_D}{4}\left[1 + \frac{(a - bc_U - bc_D)c_D}{\sqrt{(a - bc_U - bc_D)^2 c_D^2 + 16v_D^2}}\right].$$

Since $4v_D < \sqrt{(a - bc_U - bc_D)^2 c_D^2 + 16v_D^2}$, we have $\frac{dx_{II}^*}{dv_D} < 0$. Moreover, since $a > b(c_U + c_D)$, we can conclude that $\frac{dx_{II}^*}{dc_D} \geq 0$ if and only if $a - bc_U - 2bc_D \geq 0$. Thus, if $c_D \leq \frac{a - bc_U}{2b}$, then $\frac{dx_{II}^*}{dc_D} \geq 0$; if $c_D > \frac{a - bc_U}{2b}$, then $\frac{dx_{II}^*}{dc_D} < 0$. That is, Corollary 5(ii) is obtained.

Proof of Corollary 6

First, it follows from (31) and (32) that

$$V_{II,A}(x_{II}^*) - V_{I,A}(x_I^*) = \frac{\Theta\left(-4v_D + \sqrt{\Theta^2 c_D^2 + 16v_D^2}\right)}{8bc_D},$$

implying that $V_{II,A}(x_{II}^*) > V_{I,A}(x_I^*)$.

Second, it follows from (33) and (34) that

$$V_{II,B}(x_{II}^*) - V_{I,B}(x_I^*)$$
$$= \left[\frac{(4v_D + \Theta c_D)(3\Theta c_D - 4v_D) + 2(4v_D + \Theta c_D)\sqrt{\Theta^2 c_D^2 + 16v_D^2} - 16v_D^2 - \Theta^2 c_D^2}{4(4v_D + \Theta c_D)\left(\Theta c_D + 4v_D + \sqrt{\Theta^2 c_D^2 + 16v_D^2}\right)^2}\right]\frac{\Theta^2 v_D}{b}.$$

This together with the fact that $4v_D + \Theta c_D > \sqrt{\Theta^2 c_D^2 + 16v_D^2}$, indicates that

$$V_{II,B}(x_{II}^*) - V_{I,B}(x_I^*) > \left[\frac{(4v_D + \Theta c_D)(3\Theta c_D - 4v_D) + 2(\Theta^2 c_D^2 + 16v_D^2) - 16v_D^2 - \Theta^2 c_D^2}{4(4v_D + \Theta c_D)\left(\Theta c_D + 4v_D + \sqrt{\Theta^2 c_D^2 + 16v_D^2}\right)^2}\right]\frac{\Theta^2 v_D}{b},$$
$$= \left[\frac{4\Theta^2 c_D^2 + 8\Theta c_D v_D}{4(4v_D + \Theta c_D)\left(\Theta c_D + 4v_D + \sqrt{\Theta^2 c_D^2 + 16v_D^2}\right)^2}\right]\frac{\Theta^2 v_D}{b},$$

implying that $V_{II,B}(x_{II}^*) > V_{I,B}(x_I^*)$.

Third, it follows from (35) and (36) that

$$V_{II,SC}(x_{II}^*) - V_{I,SC}(x_I^*) = \frac{\left(\sqrt{\Theta^2 + \frac{16v_D^2}{c_D^2}} - \frac{4v_D}{c_D}\right)\left[\Theta + \left(\Theta + \frac{4v_D}{c_D}\right) - \sqrt{\Theta^2 + \frac{16v_D^2}{c_D^2}}\right]}{16b}.$$

This in conjunction with the fact that $\frac{4v_D}{c_D} < \sqrt{\Theta^2 + \frac{16v_D^2}{c_D^2}} < \Theta + \frac{4v_D}{c_D}$ indicates that $V_{II,SC}(x_{II}^*) > V_{I,SC}(x_I^*)$.

Proof of Corollary 7

Taking the first derivatives of $V_{I,B}(x_I^*)$ with respect to a, v_D, b, c_U, and c_D, respectively, and after some algebraic simplifications, we have

$$\frac{dV_{I,B}(x_I^*)}{da} = \frac{bv_D(a - bc_U - bc_D)[8v_D + (a - bc_U - bc_D)c_D]}{4[4bv_D + (a - bc_U - bc_D)bc_D]^2},$$

$$\frac{dV_{I,B}(x_I^*)}{dv_D} = 1 + \frac{4bc_D(a - bc_U - bc_D)^3}{[16bv_D + 4(a - bc_U - bc_D)bc_D]^2},$$

$$\frac{dV_{I,B}(x_I^*)}{db} = -\frac{v_D(a - bc_U - bc_D)[8bv_D(c_U + c_D) + (a - bc_U - bc_D)(ac_D + 4v_D)]}{4[4bv_D + (a - bc_U - bc_D)bc_D]^2},$$

$$\frac{dV_{I,B}(x_I^*)}{dc_U} = -\frac{b^2v_D(a - bc_U - bc_D)[8v_D + (a - bc_U - bc_D)c_D]}{4[4bv_D + (a - bc_U - bc_D)bc_D]^2},$$

$$\frac{dV_{I,B}(x_I^*)}{dc_D} = -\frac{v_D(a - bc_U - bc_D)[8b^2v_D + (a - bc_U - bc_D)b^2c_D + (a - bc_U - bc_D)^2b]}{4[4bv_D + (a - bc_U - bc_D)bc_D]^2}.$$

This, in conjunction with $a > b(c_U + c_D)$, indicates that $\frac{dV_{I,B}(x_I^*)}{da} > 0$, $\frac{dV_{I,B}(x_I^*)}{dv_D} > 0$, $\frac{dV_{I,B}(x_I^*)}{db} < 0$, $\frac{dV_{I,B}(x_I^*)}{dc_U} < 0$ and $\frac{dV_{I,B}(x_I^*)}{dc_D} < 0$.

Proof of Theorem 4

(i) We derive the optimal decisions in the external equity financing model. First, it follows from (38) that

$$V_{I,B}(x, w, q) = [1 - \Lambda(x)]\left\{v_D + x + p\int_0^q 1 - F(x)dx - (w + c_D)q\right\}.$$

$$\frac{dV_{I,B}(x,w,q)}{dq} = [1-\Lambda(x)]\{p[1-F(q)]-(w+c_D)\},$$

$$\frac{d^2V_{I,B}(x,w,q)}{dq^2} = -[1-\Lambda(x)]pf(q),$$

implying that $V_{I,B}(x,w,q)$ is concave in q. Thus, the maximizer of $V_{I,B}(x,w,q)$ is uniquely determined by $\frac{dV_{I,B}(x,w,q)}{dq}=0$ and $V_{I,B}(x,w,q)$ achieves its maximum at

$$q_I(w,x) = F^{-1}\left(1-\frac{w+c_D}{p}\right).$$

This, together with the production capacity constraint $q \le \frac{x}{c_D}$ and the concavity of $V_{I,B}(x,w,q)$, indicates that the optimal response production quantity in stage 3 of the external equity financing model is

$$q_I^*(w,x) = \min\left\{F^{-1}\left(1-\frac{w+c_D}{p}\right), \frac{x}{c_D}\right\}.$$

Second, substituting $q_I^*(w,x)$ into (37), we have

$$V_{I,A}(x,w) = \begin{cases} V_{I,A1}(x,w) = v_U + (w-c_U)\frac{x}{c_D}, & if\ w \le p\left[1-F(\frac{x}{c_D})\right]-c_D, \\ V_{I,A2}(x,w) = v_U + (w-c_U)F^{-1}\left(1-\frac{w+c_D}{p}\right), & if\ w \ge p\left[1-F(\frac{x}{c_D})\right]-c_D. \end{cases}$$

One can easily verify that $V_{I,A1}(x,w)$ increases in w and $V_{I,A}(x,w)$ is continuous. Thus,

$$\max_w V_{I,A}(x,w) = \max_{w\ge p\left[1-F(\frac{x}{c_D})\right]-c_D} V_{I,A2}(x,w) = v_U + (w-c_U)F^{-1}\left(1-\frac{w+c_D}{p}\right).$$

That is, the decision problem of Shareholder A in stage 2 can be reduced to choosing a value for w in the range of $w \ge p[1-F(\frac{x}{c_D})]-c_D$ to maximize $V_{I,A2}(x,w)$. For convenience, we denote $Q = F^{-1}(1-\frac{w+c_D}{p})$. Since there is a one-to-one correspondence between the w and Q. Consequently, choosing a value for w in stage 2 is equivalent to choosing a corresponding value for Q (see Cachon, 2004, for a similar transformation). That is, the decision problem of shareholder A can be further reduced to

$$\max_{Q\le\frac{x}{c_D}} V_{I,A2}(x,Q) = v_U + \{p[1-F(Q)]-c_U-c_D\}Q.$$

Furthermore, taking the first derivative of $V_{I,A2}(x,Q)$ with respect to Q, we have

$$\frac{dV_{I,A2}(x,Q)}{dQ} = p[1-F(Q)]\left[1-\frac{f(Q)Q}{1-F(Q)}\right]-c_U-c_D.$$

Since $\frac{xf(x)}{1-F(x)}$ increases in x, we know that $V_{I,A2}(x, Q)$ is unimodal in Q and achieves its maximum at Q_I^*, which is uniquely determined by

$$p\left[1 - F(Q_I^*)\right]\left[1 - \frac{f(Q_I^*)Q_I^*}{1 - F(Q_I^*)}\right] = c_U + c_D.$$

This together with the together with the production capacity constraint $q \le \frac{x}{c_D}$, indicates that Shareholder A in stage 2 will choose a wholesale price w to induce Shareholder B to produce

$$q_I^*(x) = \min\left\{Q_I^*, \frac{x}{c_D}\right\} = \begin{cases} \dfrac{x}{c_D}, & if\ x \le c_D Q_I^*, \\ Q_I^*, & if\ x \ge c_D Q_I^*. \end{cases}$$

This in conjunction with the optimal response production quantity in stage 3, i.e., $q_I^*(w, x)$, implies that the optimal response wholesale price w in stage 2 is

$$w_I^*(x) = \begin{cases} p\left[1 - F\left(\dfrac{x}{c_D}\right)\right] - c_D, & \text{if } x \le c_D Q_I^*, \\ p[1 - F(Q_I^*)] - c_D, & \text{if } x \ge c_D Q_I^*. \end{cases}$$

Third, substituting $w_I^*(x)$ and $q_I^*(x)$ into $V_{I,B}(x, w, q)$, we have

$$V_{I,B}(x) = \begin{cases} V_{I,B1}(x) = v_D + \dfrac{v_D}{v_D + x} \cdot \left[pF\left(\dfrac{x}{c_D}\right)\dfrac{x}{c_D} - p\displaystyle\int_0^{\frac{x}{c_D}} F(x)dx\right], & \text{if } x \le c_D Q_I^*, \\ V_{I,B2}(x) = v_D + \dfrac{v_D}{v_D + x} \cdot \left[pF(Q_I^*)Q_I^* - p\displaystyle\int_0^{Q_I^*} F(x)dx\right], & \text{if } x \ge c_D Q_I^*. \end{cases}$$

One can easily see that $V_{I,B2}(x)$ decreases in x, and $V_{I,B}(x)$ is continuous. Thus,

$$\max_x V_{I,B}(x) = \max_{x \le c_D Q_I^*} V_{I,B1}(x) = v_D + \frac{v_D}{v_D + x} \cdot \left[pF\left(\frac{x}{c_D}\right)\frac{x}{c_D} - p\int_0^{\frac{x}{c_D}} F(x)dx\right].$$

Furthermore, taking the first derivative of $V_{I,B1}(x)$ with respect to x, we have

$$\frac{dV_{I,B1}(x)}{dx} = \frac{v_D p\left[1 - F\left(\frac{x}{c_D}\right)\right]}{(v_D + x)^2} \cdot \frac{\frac{(v_D+x)}{c_D} f\left(\frac{x}{c_D}\right)\frac{x}{c_D} + \frac{x}{c_D}F\left(\frac{x}{c_D}\right) - \int_0^{\frac{x}{c_D}} F(x)dx}{1 - F\left(\frac{x}{c_D}\right)} > 0,$$

implying that $V_{I,B1}(x)$ increases in x and so the optimal solution to the problem $\max_{x \le c_D Q_I^*} V_{I,B1}(x)$ is $x_I^* = c_D Q_I^*$. That is, (41) is obtained. Substituting $x_I^* = c_D Q_I^*$ into $q_I^*(x)$ and $w_I^*(x)$, we have (42) and (43).

(ii) The proof for Theorem 4(ii) is similar to the proof of Theorem 4(i). First, it follows from (38) and (40) that the final asset functions of shareholder B in the two financing models are identical, i.e., the optimal response production quantity in stage 3 of the internal equity financing model is $q_{II}^*(w,x) = q_I^*(w,x) = \min\{F^{-1}(1-\frac{w+c_D}{p}), \frac{x}{c_D}\}$.
Second, substituting $q_{II}^*(w,x)$ into (40), we can show that Shareholder A in stage 2 will choose a wholesale price w to induce Shareholder A to produce

$$q_{II}^*(x) = \min\left\{Q_{II}^*, \frac{x}{c_D}\right\} = \begin{cases} \dfrac{x}{c_D}, & \text{if } x \le c_D Q_{II}^*, \\ Q_{II}^*, & \text{if } x \ge c_D Q_{II}^*, \end{cases}$$

where Q_{II}^* is uniquely determined by

$$p\big[1 - F\big(Q_{II}^*\big)\big]\left[1 - \frac{v_D}{v_D + x} \cdot \frac{f\big(Q_{II}^*\big)Q_{II}^*}{1 - F\big(Q_{II}^*\big)}\right] = c_U + c_D.$$

Third, in conjunction with the fact that $\frac{1}{v_D + c_D x} \cdot \frac{x f(x)}{1-F(x)}$ increases in x, we can conclude that the optimal financing strategy x_{II}^* is (44) and the optimal production quantity is (46). Substituting (44) into $q_{II}^*(w,x)$, we have (45).

(iii) First, we prove that $x_{II}^* > x_I^*$. If $x_{II}^* < x_I^*$, then $1 - F(\frac{x_{II}^*}{c_D}) > 1 - F(\frac{x_I^*}{c_D})$ and $\frac{v_D}{v_D + x_{II}^*} \cdot \frac{f(\frac{x_{II}^*}{c_D})\frac{x_{II}^*}{c_D}}{1-F(\frac{x_{II}^*}{c_D})} < \frac{f(\frac{x_{II}^*}{c_D})\frac{x_{II}^*}{c_D}}{1-F(\frac{x_{II}^*}{c_D})} < \frac{f(\frac{x_I^*}{c_D})\frac{x_I^*}{c_D}}{1-F(\frac{x_I^*}{c_D})}$, and thus

$$p\left[1 - F\left(\frac{x_{II}^*}{c_D}\right)\right]\left[1 - \frac{v_D}{v_D + x_{II}^*} \cdot \frac{f\left(\frac{x_{II}^*}{c_D}\right)\frac{x_{II}^*}{c_D}}{1 - F\left(\frac{x_{II}^*}{c_D}\right)}\right] > p\left[1 - F\left(\frac{x_I^*}{c_D}\right)\right]\left[1 - \frac{f\left(\frac{x_I^*}{c_D}\right)\frac{x_I^*}{c_D}}{1 - F\left(\frac{x_I^*}{c_D}\right)}\right].$$

This is contradictory to (41) and (44). Thus, we know that $x_{II}^* > x_I^*$.
Second, substituting (41), (42), and (43) into (38), we have

$$V_{I,B}(x_I^*, w_I^*, q_I^*) = v_D + \frac{v_D}{v_D + x_I^*} \cdot \left[pF\left(\frac{x_I^*}{c_D}\right)\frac{x_I^*}{c_D} - p\int_0^{\frac{x_I^*}{c_D}} F(x)dx\right],$$

Similarly, substituting (44), (45), and (46) into (40), we have

$$V_{II,B}(x_{II}^*, w_{II}^*, q_{II}^*) = v_D + \frac{v_D}{v_D + x_{II}^*} \cdot \left[pF\left(\frac{x_{II}^*}{c_D}\right)\frac{x_{II}^*}{c_D} - p\int_0^{\frac{x_{II}^*}{c_D}} F(x)dx\right].$$

Third, in conjunction with $x_{II}^* > x_I^*$ and $\frac{dV_{I,B1}(x)}{dx} > 0$ (see the proof of Theorem 4(i)), we can conclude that $V_{II,B}(x_{II}^*, w_{II}^*, q_{II}^*) > V_{I,B}(x_I^*, w_I^*, q_I^*)$.