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COOPERATIVE AND NONCOOPERATIVE R&D IN DUOPOLY MANUFACTURERS WITH A COMMON SUPPLIER

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Abstract. We consider the R&D strategy of firms under competitive environments from the supply chain perspective. Specifically, we investigate a supply chain consisting of one upstream component supplier and two downstream manufacturers, who however are the Stackelberg leader(s). At the early stage (*R&D stage*), the two manufacturers decide on whether to cooperate or not in the R&D activities and how much to invest in R&D accordingly. At the late stage (*market stage*), the component supplier decides on the uniform wholesale price and the manufacturers decide on the production quantities. Our main findings include: (i) Cooperative R&D strategy will be adopted when the technology spillover effect is either too large or too small and in contrast non-cooperative strategy will be accepted when the spillover effect is moderate. However, the underlying driving forces for coordination are different when the spillover effect is small or large, i.e., *cost reduction effect* and *sales increasing effect*. (ii) Cooperative R&D could increase the social welfare when both the technology spillover effect and the (initial) unit production cost are high. (iii) As the equilibrium under the cooperative R&D strategy is unstable, we give a coordination mechanism, to guarantee the stability of cooperative R&D investments.

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1. **Introduction.** Cooperative research and development (R&D) has many merits and hence is widely used in research-intensive industries recently. Firms are willing to conduct cooperative R&D as they can share the risk and its associated cost [6, 13, 26, 31, 27]. BMW and Mercedes-Benz, will jointly develop the next generation self-driving technology, to force to make a better and safely autonomous car [5]. Cooperative R&D could share complementary resources [29] and achieve economy of scale [17]. Ericsson and OPPO have signed a global patent license agreement, which includes a cross license covering the 2G-5G patent portfolios from both companies [16].

The cooperative R&D activities will take positive externalities to the industry due to technological spillover effect. The new knowledge generated by R&D can be easily penetrated and diffused in the industry, and in turn stimulates economic growth to some extent [3, 34]. It is quite common in auto industry to form partnership so that firms involved perfectly share knowledge considered useful to R&D investments [19, 33]. However, as pointed out by [14], if the technological spillover effect is small, non-cooperative R&D strategy dominates cooperative R&D strategy where duopoly manufactures confront a common product market. Things could become even more complicated if duopoly manufacturers face a common upstream supplier, who himself is also a profit maximizer knowing the competition behavior between the two manufacturers. For example, Vivo and Samsung, competing in the smartphone market, while cooperating in the R&D of 5G chip, make a joint push into 5G smartphone chip [10]. They both have a common upstream supplier, Qualcomm, who provides the Snapdragon865 CPU. As a result, Vivo and Samsung not only compete with each other, but also need to consider the vertical competition from the supplier Qualcomm.

We intend to investigate the R&D strategies (cooperative or non-cooperative) of competing manufacturers from a supply chain perspective. Specifically, we aim to answer the following research questions: (i) When will the duopoly manufacturers adopt the cooperative R&D strategy, facing the vertical competition from a common (key component) supplier? (ii) Given the cooperative (non-cooperative) strategy adopted, what's the performance of duopoly manufacturers and the upstream supplier, the consumers' surplus, as well as the total social welfare? (iii) Moreover, how to guarantee its stability, if both manufacturers are willing to cooperate in R&D activities?

To facilitate our study, we establish a stylized model to investigate a supply chain consisting of one upstream supplier and two downstream manufacturers, focusing on their corresponding R&D strategy (cooperative or non-cooperative). The whole decision process is divided into two stages, i.e., R&D stage and market stage sequentially. In the R&D stage, the two manufacturers choose to conduct either noncooperative or cooperative R&D activities. In the market stage, the supplier and two manufacturers engage in a Stackelberg game setting with the manufacturers acting as the Stackelberg leaders while the supplier being the follower.

Our key contributions are two-fold: Firstly, we introduce the vertical competition into the duopolistic game setting regarding the selection of R&D strategies. This significantly expands the classic model investigated by [14] (hereinafter AJ model) from a supply chain perspective. In AJ model, the wholesale price of the supplier is exogenously given. However, in our paper it is endogenously given. The interplay between the upstream supplier and downstream manufacturers plays an important role in the R&D strategy selection. Secondly, we find both the (initial) production cost and the spillover effect affect the optimal R&D strategy chosen by the manufacturers. This is a clear distinction from existing literature in which the main driver deciding whether to conduct cooperative or noncooperative R&D activities is the technological spillover effect. In particular, we find that cooperative R&D strategy will be adopted when the technological spillover effect is either too large or too small, whereas the noncooperative strategy will be accepted when the spillover effect is moderate. Further, cooperative R&D could increase the social welfare when both the technological spillover effect and the (initial) production cost are high.

The rest of the paper is organized as follows. Section 2 reviews the relevant literature. Section 3 describes the problem and presents the optimal decisions under cooperative and noncooperative model settings. Section 4 we give the optimal R&D strategy and show the stability analysis of cooperative R&D. Section 5 conducts some social comparisons regarding consumers' surplus and total social welfare. Section 6 conducts some extensions. Finally we give our conclusions in Section 7.

2. **Literature review.** Our research mainly relates to two streams of research regarding the cooperation of R&D activities, i.e., horizontal cooperation and vertical cooperation. We next give a detailed review from these two aspects.

Cooperative R&D has received much attention in extant literature. [3] first shows firm's R&D activities that aimed at reducing production cost have positive externalities to other firms, and explains the role of technology spillovers on economic growth by externalities. [15] explore the effect of horizontal technology spillover on small enterprise for international growth, offering quantitative metrics such as technology upgrading, knowledge spillover, and technology transfer. [38] examines the R&D risk choice in a duopoly market with technology spillovers. Findings indicate that, in equilibrium, the R&D risk level decreases in the spillover rate under noncooperative R&D, while it may increase under cooperative R&D. Firms are more likely to engage in higher R&D risks under cooperative R&D than they are under non-cooperative R&D. [22] examine the effects of firms' cost asymmetry on R&D investments, finding that the social preference between noncooperative and cooperative R&D investments is independent of the degree of firms' cost asymmetry. [14] (hereinafter AJ model) divide the whole decision process into two stages, namely, the R&D stage and market stage. Three different cooperation games are throughly investigated, i.e., the fully noncooperative game, the mixed game (cooperate in the R&D stage, while compete in the market stage) , and the fully cooperative game. [20] points out that, in the AJ model, when the level of technology spillovers is very small, the equilibrium R&D output levels under the fully noncooperative game are unstable. After that, scholars have expanded the AJ model in many ways, e.g., from duopoly market to oligopoly market [25, 35, 23], from homogeneous products to heterogeneous products [8, 7], from symmetric firms to asymmetric firms [18, 27], from static spillovers to random spillovers [17, 32], from process innovation to product innovation [30, 36], from complete information to incomplete information [24], etc. We, however, expand AJ model from a new angle, the supply chain perspective. Specifically, we introduce a common supplier into the model setting, whose interactions with duopoly manufacturers will affect their cooperation in R&D investments.

The vertical cooperation of R&D between upstream and downstream firms has a wide range of applications and also received lots of attention. [4] investigates four types of vertical R&D cooperation, i.e., no cooperation, horizontal cooperation, vertical cooperation, and simultaneous horizontal and vertical cooperation. None of the cooperative R&D setting dominates the others. Which type of cooperation yields more R&D output depends on types of spillovers (horizontal or vertical) and the market structure. [21, 11] analyze the effects of cooperative R&D in two vertically related duopolies, which are two final-good manufacturers and two input suppliers, with horizontal and vertical spillovers. Vertical R&D cartels yield a larger social welfare than noncooperative R&D and, if the horizontal spillover rate between the input suppliers is not sufficiently high, than horizontal R&D cartels. [19] investigate firms' R&D cooperation in a supply chain where two firms involved first cooperate in R&D investments and then decide the production quantity according to a wholesale price contract. [37] study the stability of R&D cooperation in a supply chain and find that the vertical R&D cooperation is inherently unstable, and the downstream firm is more likely to break the agreement. When the level of knowledge spillover is low or the cost of R&D input is high, mechanisms such as punishment for opportunism may be more effective to guarantee the stability of cooperation. [39] investigate the effects of asymmetric knowledge spillovers on the stability of horizontal and vertical R&D cooperation. [12] explore firms' green R&D cooperation behaviour in a two-echelon supply chain and evaluate the effects of green R&D cooperation on the economic, environmental and social performances of the supply chain while simultaneously considering the technological spillover and supply chain power relationship.

Other literature related to our work focuses on technology spillover and production cost. [43] use the Spatial Durbin Model (SDM) to examine the effect of reverse technology spillovers of outward foreign direct investment (OFDI) on total factor carbon productivity (TFCP) via a panel dataset, showing that the reverse technology spillovers of OFDI can increase the TFCP of neighboring provinces through the spatial spillover mechanism. [44] develop a theoretical model for technology spillover research, finding that vertical technology spillover effects are more significant than horizontal technology spillover effects, both within- and between-regions. [45] point out that there is a threshold for the influence of the institutional environment on the relationship between reverse technology spillover effects and green innovation efficiency and when the institutional development level surpasses the threshold value, an acceleration effect is generated. [46] show that the reverse green technology spillover brought by OFDI can be amplified through well-designed environmental regulations and improved knowledge transfer ability of OFDI. [40] show that technology spillovers shift the composition of corporate research and development by promoting innovation based on the exploitation of existing knowledge while disincentivizing innovation that explores new areas and breaks new ground. [41] show that technology spillovers can increase firm productivity, innovation output, and valuation. [42] point out that, in the face of large technology spillovers, firms will increase their cash holdings to adapt to the accelerated production caused by these spillovers. [50] find that R&D value chain spillovers took place intra-regionally but not inter-regionally which indicates that there are two-way R&D value chain spillovers in which the forward spillover effects are stronger than the backward spillover effects. [51] examine the effects of asymmetric spillovers on R&D investment in an oligopoly market and suggest that an increase in spillover asymmetry between firms deepens the difference in R&D investments between asymmetric and symmetric spillovers. [52] find that the spillover effect will discourage the R&D effort but encourage the capacity investment, when both the probability of potential shift in environmental damage and the efficacy of R&D activities are high. [53] examine whether a firm will select an overoptimistic manager when a cost-reduction investment has a spillover effect and show that only when the spillover effect is sufficiently high do firms benefit from delegation. [47], [48] and [49] explore the presence of interfirm IT spillovers between related industries.

Different from above mentioned papers, we consider the horizontal cooperation of R&D under both horizontal competition (in product market) and vertical competition (one common supplier) in a Stackelberg game setting in this paper.

3. Model descriptions.

3.1. Basic model settings.

We consider a supply chain consisting of one component supplier and two symmetric manufacturers under a Stackelberg setting with the downstream manufacturers being the leaders. The upstream supplier produces the component at unit cost c_u and sells it to the downstream at unit price w. The two manufacturers produce homogeneous products at unit cost c_d and sell them to consumers at price p. Without loss of generality, we assume that each final product consumes exactly one component. Manufacturers face an inverse demand function $p = a - q_1 - q_2$, where q_i is the production quantity of manufacturer i, $i = 1, 2$. In addition, we assume that $a > c_u + c_d$ to avoid some trivial cases.

In our stylized model, we assume only the downstream manufacturers engage in R&D activities while the upstream component supplier does not engage in any R&D activities. The efforts devoted to R&D activities by manufacturer i can reduce his marginal production cost. Moreover, there is a positive effect of technology spillover to the other manufacturer. Specifically, let x_i be the (target) output level of R&D, which reflects the R&D efforts devoted by manufacturer i and $\beta \in (0, 1)$ be the magnitude of spillover effect between the two manufacturers. As a result, the marginal production cost of manufacturer i after R&D investments is $c_d - x_i - \beta x_{3-i}$, $i = 1, 2$. We assume that R&D investment is costly and the cost incurred by manufacturer i is $\frac{1}{2}x_i^2$, $i = 1, 2$.

The sequence of events includes four stages (see Figure 1):

i. In the first stage, two downstream manufacturers decide on their R&D strategies, i.e., noncooperative R&D or cooperative R&D.
ii. In the second stage, given the R&D strategy, two manufacturers decide on their R&D output levels x_i, $i = 1, 2$. Under noncooperative R&D strategy, manufacturers decide on their own individual R&D output levels simultaneously to maximize their corresponding profits. Under cooperative R&D strategy, two manufacturers decide on their R&D output levels jointly so as to maximize their total profit.
iii. In the third stage, whether the two manufacturers choose to cooperate in R&D or not, they compete in the product market. Given the R&D output levels (x_1, x_2) of downstream manufacturers, the upstream supplier decides on the wholesale price w.
iv. In the fourth stage, given the wholesale price w of the upstream supplier, two downstream manufacturers act as Cournot duopoly competitors, who aim to maximize their profits by determining their order quantities q_i respectively. The market then clears at $p = a - q_1 - q_2$.

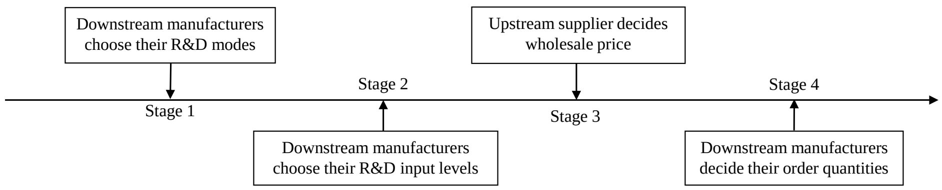

FIGURE 1. The Four-Stage Game between Upstream Supplier and Downstream Manufacturers

3.2. **Optimal decisions under noncooperative R&D.** In this subsection, we consider the case when the two manufacturers do not take cooperative R&D activities. We solve the game from Stage 2 to Stage 4 by backward induction.

We first consider Stage 4 decision. Given the wholesale price w and R&D input levels x_i, $i = 1, 2$, manufacturer i decides on his production quantity q_i to maximize his profit. Recall that $p = a - q_1 - q_2$. The manufacturer's profit can be expressed as

$$\max_{q_i \geq 0} \pi_i\big(q_1, q_2 | w, x_1, x_2\big) = \big[a - q_1 - q_2 - w - (c_d - x_i - \beta x_{3-i})\big] q_i, \text{ i=1, 2.} \quad (1)$$

Since $\frac{d^2 \pi_i}{d{q_i}^2} = -2 < 0$, π_i is strictly concave in q_i. By solving the first order conditions of equation (1), we can obtain the equilibrium order quantities $q_i^N(w, x_1, x_2)$ of downstream manufacturer i as follows:

$$q_i^N(w, x_1, x_2) = \frac{1}{3}\big[a - c_d - w + (2-\beta)x_i - (1-2\beta)x_{3-i}\big], \text{ i=1, 2.} \quad (2)$$

We next consider the component supplier's optimal wholesale price. Given the R&D output level x_i of downstream manufacturer i ($i = 1, 2$), the upstream supplier decides on the wholesale price w to maximize her profit by taking into account manufacturers' reaction on the order quantities:

$$\max_{w} \pi_u(w | x_1, x_2) = (w - c_u)\big[q_1^N(w, x_1, x_2) + q_2^N(w, x_1, x_2)\big]. \quad (3)$$

Substituting (2) into (3) and taking the second derivative w.r.t w, we have $\frac{d^2\pi_u}{dw^2} = -\frac{4}{3} < 0$, which implies that π_u is strictly concave in w and hence there exists a unique optimal solution. Based on (3), we have the equilibrium wholesale price w^N of the common component:

$$w^N(x_1, x_2) = \frac{1}{4}\big[2(a + c_u - c_d) + (1+\beta)(x_1 + x_2)\big]. \quad (4)$$

We then consider the optimal equilibrium decisions on the R&D output levels made by the two manufacturers. Substituting w^N, q_1^N, q_2^N into the profit function π_i of downstream manufacturer i, we thus obtain:

$$\max_{x_i \leq c_d - \beta x_{3-i}} \Pi_i(x_1, x_2) = \pi_i(q_1^N, q_2^N | w^N, x_1, x_2) - \frac{1}{2}x_i^2, \text{ i=1, 2.} \quad (5)$$

where the first term represents the sales revenue and the second term is the R&D investment. Since $\frac{d^2\Pi_i}{dx_i^2} = -\frac{1}{72}(23 + 70\beta - 25\beta^2) < 0$, π_i is the strict concave function in x_i. Also, as the constraint is linear, there exists a unique Nash equilibrium for the competition of the two manufacturers. Let $k = a - c_u - c_d$, $c_n(\beta) = \frac{(a-c_u)(7-5\beta)(1+\beta)}{36}$. We then have the following proposition.

Proposition 1. *(Non-cooperative R&D activities) The optimal equilibrium R&D output levels of the two manufacturers satisfy*

(i) *if* $0 < c_d \leq c_n(\beta)$, *then* $x_1^N = x_2^N = \frac{c_d}{1+\beta}$;

(ii) *if* $c_n(\beta) < c_d < a - c_u$, *then* $x_1^N = x_2^N = \frac{7-5\beta}{29-2\beta+5\beta^2}(a - c_u - c_d)$.

Proposition 1 shows that the equilibrium R&D input level is decreasing in the spillover effect in general, for any given initial marginal production cost. However, for any given spillover effect β, the sensitivity of R&D input level with regard to the initial marginal production cost could be mixed. Specifically, when the initial unit production cost is relatively low, the equilibrium output level is decreasing with magnitude of spillover effect and increases with the unit production cost. When the unit production cost is relatively high, then the equilibrium output level decreases with the production costs of upstream supplier and downstream manufacturers, moreover, the equilibrium output level decreases with the magnitude of spillover effect, which can be easily verified.

From the proposition 1 and the equations (2)-(5), we can get the equilibrium wholesale price w^N, order quantities q_i^N as well as the profits π_u^N and π_i^N, which are summarized in Table 1.

TABLE 1. The Equilibrium Outcomes in Noncooperative R&D Strategy

Scenario	$c_n(\beta) < c_d < a - c_u$	$0 < c_d \leq c_n(\beta)$
x_i^N	$\frac{(7-5\beta)k}{29-2\beta+5\beta^2}$	$\frac{c_d}{1+\beta}$
w^N	$\frac{18(a-c_d)+(11-2\beta+5\beta^2)c_u}{29-2\beta+5\beta^2}$	$\frac{a+c_u}{2}$
p^N	$\frac{a(17-2\beta+5\beta^2)+12(c_u+c_d)}{29-2\beta+5\beta^2}$	$\frac{2a+c_u}{3}$
q_i^N	$\frac{6k}{29-2\beta+5\beta^2}$	$\frac{a-c_u}{6}$
π_u^N	$\frac{216k^2}{(29-2\beta+5\beta^2)^2}$	$\frac{(a-c_u)^2}{6}$
π_i^N	$\frac{(23+70\beta-25\beta^2)k^2}{2(29-2\beta+5\beta^2)^2}$	$\frac{(a-c_u)^2(1+\beta)^2-18c_d^2}{36(1+\beta)^2}$

3.3. **Optimal decisions under cooperative R&D.** We consider the case when the two manufacturers make the decisions jointly to maximize their total profits. Since given the R&D input levels, decisions in Stages 3 and 4 are exactly the same as in the non-cooperative game, we thus focus on the joint R&D investments of manufacturers in Stage 2, the objective function of which can be expressed as follows:

$$\begin{aligned}\max_{x_1,x_2} \Pi(x_1,x_2) &= \sum_{i=1}^{2} \Pi_i(x_1,x_2)\\ &= \sum_{i=1}^{2} \left\{ \frac{1}{144}\left[2(a-c_u-c_d)+(7-5\beta)x_i-(5-7\beta)x_{3-i}\right]^2 - \frac{1}{2}x_i^2 \right\}\\ &\text{s.t. } x_1+\beta x_2 \leq c_d\end{aligned} \tag{6}$$

When $(2-\sqrt{2})/2 < \beta < 1$, $\frac{\partial^2\pi}{\partial x_1^2} = \frac{1}{36}(1-70\beta+37\beta^2) < 0$ and the determinant of Hessian matrix satisfies $\frac{1}{18}(2\beta^4-41\beta^2+70\beta-17) > 0$. As a result, the objective function is jointly concave in (x_1, x_2). Let $c_o(\beta) = \frac{(a-c_u)(1+\beta)^2}{18}$, then we have the following proposition and equilibrium results under cooperative R&D strategy (see Table 2).

Proposition 2. *(Cooperative R&D activities)The optimal joint R&D output levels under cooperative R&D strategy satisfy*

(i) if $0 < c_d \le c_o(\beta)$, *then* $x_1^C = x_2^C = \frac{c_d}{1+\beta}$;

(ii) if $c_o(\beta) < c_d < a - c_u$, *then* $x_1^C = x_2^C = \frac{1+\beta}{17-2\beta-\beta^2}(a-c_u-c_d)$.

According to Proposition 2(i), if the initial marginal production cost is relatively low, then the equilibrium R&D input level increases in it and deceases in the spillover effect β. In contrast, Proposition 2(ii) shows that if the initial marginal production cost is relatively low, the equilibrium R&D input level decreases in it and increases in the spillover effect, a significantly departure from the uncooperative situation (refer to Proposition 1(ii)).

TABLE 2. The Equilibrium Outcomes in Cooperative R&D Strategy

Scenario	$c_o(\beta) < c_d < a - c_u$	$0 < c_d \le c_o(\beta)$
x_i^C	$\frac{k(1+\beta)}{17-2\beta-\beta^2}$	$\frac{c_d}{1+\beta}$
w^C	$\frac{9(a-c_d)+(8-2\beta-\beta^2)c_u}{17-2\beta-\beta^2}$	$\frac{a+c_u}{2}$
p^C	$\frac{a(11-2\beta-\beta^2)+6(c_u+c_d)}{17-2\beta-\beta^2}$	$\frac{2a+c_u}{3}$
q_i^C	$\frac{3k}{17-2\beta-\beta^2}$	$\frac{a-c_u}{6}$
π_u^C	$\frac{54k^2}{(17-2\beta-\beta^2)^2}$	$\frac{(a-c_u)^2}{6}$
π_i^C	$\frac{k^2}{2(17-2\beta-\beta^2)}$	$\frac{(a-c_u)^2(1+\beta)^2-18c_d^2}{36(1+\beta)^2}$

4. R&D strategies: Cooperative vs. noncooperative.

4.1. Optimal R&D strategy.
In this subsection, we investigate the optimal R&D strategy of the two manufacturers, i.e., cooperative or noncooperative. If the profits under cooperative and noncooperative R&D strategies are the same, the two manufacturers choose the noncooperative R&D strategy in equilibrium. This can be justified as cooperation usually incurs extra costs due to pre-stage negotiation and post-stage verification.

To facilitate the investigation of optimal R&D strategies that could be adopted by the two manufacturers and their associated social welfare comparisons, we give the equilibrium results in Lemma 4.1.

Lemma 4.1. *Comparing the optimal decisions under cooperative and noncooperative R&D strategies, we have*

(i) $q_i^C < q_i^N$, $x_i^C < x_i^N$, $i = 1, 2$, *and* $w^C < w^N$, *if* $(2-\sqrt{2})/2 < \beta < 5/7$ *and* $c_d \ge c_o(\beta)$;

(ii) $q_i^C > q_i^N$, $x_i^C > x_i^N$, $i = 1, 2$, *and* $w^C > w^N$, *if* $5/7 < \beta < 1$ *and* $c_d \geq c_n(\beta)$*;*
(iii) $q_i^C = q_i^N$, $x_i^C = x_i^N$, $i = 1, 2$, *and* $w^C = w^N$, *otherwise.*

Based on Lemma 4.1, the decisions under cooperative and noncooperative R&D strategies could be different, which mainly depend on the two key factors, the magnitude of spillover effect and the initial marginal production cost.

Theorem 4.2. *(Optimal R&D Strategy)*

(i) *If* $\beta \neq 5/7$ *and* $c_d \geq min\{c_n(\beta), c_o(\beta)\}$*, then cooperative R&D strategy dominates;*
(ii) *Otherwise, noncooperative R&D strategy dominates.*

Theorem 4.2(i) shows that, when R&D spillover effect is either relatively small or relatively large while the initial marginal production cost is large, coordinative R&D takes more benefits for the two manufacturers. However, the underlying driving forces for coordination are different when the spillover effect is small or large.

(1) If the spillover effect of R&D is relatively small, according to Lemma 4.1, the manufacturers will invest less in R&D and manufacture less products in equilibrium under cooperation strategy compared with that under noncooperation strategy, i.e., $\beta < 5/7$ and $c_d > \min\{c_o(\beta), c_n(\beta)\} = c_o(\beta)$. As a consequence, the final marginal production cost after R&D stage is higher under cooperation strategy, which in turn leads to a lower profit in the product sales market. However, lower R&D input levels also mean a reduction in cost. Due to the cost reduction at R&D stage, the overall net profit of each manufacturer is larger compared with noncooperation strategy.

(2) If the spillover effect of R&D is relatively large ($5/7 < \beta < 1$) and $c_d > \min\{c_o(\beta), c_n(\beta)\} = c_n(\beta)$, based on Lemma 4.1, we know the manufacturers will invest more in R&D and manufacture more products for sale in the market. As a result, the final marginal production cost after R&D becomes lower, resulting in a larger profit at the product sales market, using noncooperative R&D strategy as a benchmark. Therefore, the overall net profit for each manufacturer will be increased under cooperation, albeit an increase in the R&D investment.

Theorem 4.2(ii) shows that if the initial marginal production cost is lower enough ($c_d < \min(c_o(\beta), c_n(\beta))$), then no matter how high the spillover effect is, there is no extra benefit for the two manufacturers to cooperate in R&D activities. This is because, the two manufacturers will adopt the same levels of R&D investment under both cooperative and noncooperative R&D strategies. Consequently, the component supplier will set up a same wholesale price, which leads to a same production quantity for each manufacturer for either strategy (collusive or competitive).

The following Corollary 1 shows the impact of spillover level on the profitability of downstream manufacturers when choose to cooperate in R&D.

Corollary 1. *Under cooperative R&D strategy, manufacturers' profits increase in the spillover effect of R&D.*

According to Theorem 4.2, there exists two scenarios when manufacturers choose to cooperate. Firstly, if the initial marginal production cost is modest ($c_n(\beta) < c_d \leq c_o(\beta)$), then the R&D decisions lie in the boundary conditions, i.e., $(1+\beta)x = c_d$. As a result, when the technology spillover effect increases, the equilibrium R&D levels will decrease, while the equilibrium profit of each manufacturer in the sales market remains unchanged as the marginal production cost reduction has achieved its maximum. Therefore, the larger the technology spillover effect, the

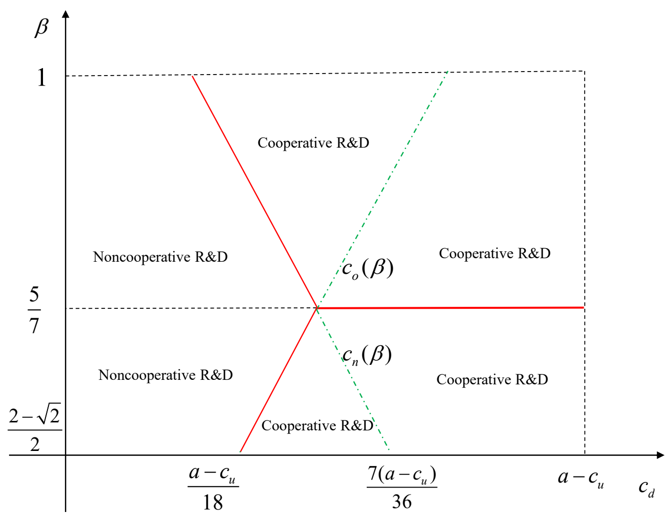

FIGURE 2. Optimal R&D Strategies for Downstream Manufacturers

lower the equilibrium R&D input, leading to a lower R&D cost and hence a larger net profit for each manufacturer. Secondly, if the initial marginal production cost is high ($c_d > c_o(\beta)$), a larger spillover effect implies a larger equilibrium R&D input level the based on Proposition 2(ii), and hence a lower marginal production cost. The equilibrium output thus becomes larger, leading to a larger gross profit in the sales market. The net profit of each manufacturer increases in the spillover effect, although he faces an increase of R&D investment, the magnitude of which, however, is lower than the increase of gross profit in the sales market.

4.2. **Stability analysis of cooperative R&D.** In this subsection, we give an analysis on the stability of cooperative R&D for the two manufacturers under equilibrium solutions. If the cooperation strategy is not stale, we further investigate what mechanism could induce a stable cooperation in their R&D activities. In the absence of other coordination mechanisms. Proposition 3 shows the results on the stability of cooperative R&D of the two manufacturers.

Proposition 3. *Suppose $\beta \neq 5/7$ and $c_d > min\{c_o(\beta), c_n(\beta)\}$ hold. The cooperative R&D strategy in equilibrium is not stable.*

Proposition 3 shows that if the initial marginal production cost is high, although cooperative *R&D* investments of the two manufacturers can improve the profitability of each, the cooperative *R&D* strategy itself is unstable. For convenience of description and without loss of generality, assumed that downstream manufacturer 2 adheres to the level of cooperative R&D output, and then we consider whether downstream manufacturer 1 will unilaterally choose to betray. In the absence of other coordination mechanisms, each manufacturer will not adhere to the equilibrium (R&D input levels under cooperation) and choose to betray. In other words, the cooperative R&D of downstream manufacturers are more likely to fall into the *prisoners' dilemma*.

Proposition 3 shows that two downstream manufacturers can improve their profitability by cooperating in R&D. When $\beta \in ((2-\sqrt{2})/2\ ,\ 5/7) \cup (5/7\ ,\ 1)$ and

R&D Decisions		Manufacturer 2	
		Betray	Cooperate
Manufacturer 1	Betray	$\pi_{10}^d(x_{10}^d), \pi_2^n(x_1^n)$	$\pi_{10}^d(x_{10}^d), \pi_{10}^d(x_1^c)$
	Cooperate	$\pi_1^c(x_1^n), \pi_2^n(x_2^n)$	$\pi_{10}^c(x_1^c), \pi_2^c(x_2^c)$

FIGURE 3. The R&D Decision Matrix

$\min\{c_h\ ,\ c_n\} < c_d < a - c_u$. Moreover, when $\beta \in ((2-\sqrt{2})/2\ ,\ 5/7) \cup (5/7\ ,\ 1)$ and $c_h < c_d < a - c_u$, the equilibrium R&D output level of each downstream manufacturer is x^c; Accordingly, when $\beta \in (5/7\ ,\ 1)$ and $c_n < c_d \leq c_h$, the equilibrium R&D output level of each downstream manufacturer is x^b.

Hence, the objective is to find whether there is a downstream manufacturer that will choose to betray in the cooperative R&D stage. Next, we give the two situations as below.

- When $\beta \in ((2-\sqrt{2})/2\ ,\ 5/7) \cup (5/7\ ,\ 1)$ and $c_h < c_d < a - c_u$, for convenience and without loss of generality, we assume that the R&D output level of downstream manufacturer 2 is x^c. Let π_{10}^d represent the profit made by downstream manufacturer 1 when it unilaterally chooses to betray. Then we consider whether downstream manufacturer 1 will unilaterally choose betrayal. Under the above assumptions, the problem of downstream manufacturer 1 is

$$\max_{x_1} \pi_{10}^d = \frac{[(7-5\beta)(17-2\beta-\beta^2)x_1+(29-2\beta+5\beta^2)k]^2}{144(17-2\beta-\beta^2)^2} - \frac{1}{2}x_1^2$$
$$s.t.\ \ x_1 \leq c_d - \beta x^c.$$

  Since $\frac{\partial^2 \pi_{10}^d}{\partial x_1^2} = -\frac{1}{72}(23+70\beta-25\beta^2) < 0$ and the constraints are linear inequalities, the formula above is a strict concave programming. Therefore, there exists a unique global optimal solution. Let x_{10}^d indicate the R&D output level of downstream manufacturer 1 when it chooses to betray unilaterally, $d_1 = \frac{(17-\beta)c_d-(a-c_u)\beta(1+\beta)}{17-2\beta-\beta^2}$, $d_2 = \frac{(7-5\beta)(29-2\beta+5\beta^2)k}{(17-2\beta-\beta^2)(23+70\beta-25\beta^2)}$ and $c_s = \frac{(a-c_u)(203-136\beta+138\beta^2+20\beta^3-25\beta^4)}{18(33+56\beta-25\beta^2)}$ respectively. Then we can obtain:

  i) When $\beta \in ((2-\sqrt{2})/2\ ,\ 5/7)$ and $c_h < c_d \leq c_s$, $x_{10}^d = d_2 \neq x^c$, that is, downstream manufacturer 1 will unilaterally choose betrayal.

  ii) When $\beta \in (5/7\ ,\ 1)$ and $c_s < c_d < a - c_u$ or when $\beta \in (5/7\ ,\ 1)$ and $c_h < c_d < a - c_u$, $x_{10}^d = d_2 \neq x^c$, that is, downstream manufacturer 1 will unilaterally choose betrayal.
- When $\beta \in (5/7\ ,\ 1)$ and $c_n < c_d \leq c_h$, without loss of generality, we assume that downstream manufacturer 2 insists output level x^b. Let π_{11}^d represent the profit made by downstream manufacturer 1 when it unilaterally chooses to betray. Then we consider whether downstream manufacturer 1 will unilaterally choose betrayal. Under the above assumptions, the problem of downstream

manufacturer 1 is

$$\max_{x_1} \pi_{11}^d = \frac{1}{144(1+\beta)^2}[2(a-c_u)(1+\beta)-(7-5\beta)c_d+1+\beta)(7-5\beta)x_1]^2-\frac{1}{2}x_1^2$$
$$s.t.\ x_1 \le c_d - \beta x^b.$$

Since $\frac{\partial^2 \pi_{11}^d}{\partial x_1^2} = -\frac{1}{72}(23+70\beta-25\beta^2) < 0$ and the constraints are linear inequalities, the formula above is a strict concave programming. Therefore, there exists a unique global optimal solution. Let $d_3 = \frac{(7-5\beta)[2(a-c_u)(1+\beta)-c_d(7-5\beta)]}{(1+\beta)(23+70\beta-25\beta^2)}$ and x_{11}^d indicates the R&D output level of downstream manufacturer 1 when it chooses to betray unilaterally. From the formula above, we have $x_{11}^d = d_3 \neq x^b$, that is, the downstream manufacturer 1 will unilaterally choose betrayal

The following Proposition 4 gives the coordination mechanism of cooperative R&D between the two downstream firms. Under the coordination mechanism, each firm will not choose to betray in the cooperative R&D stage. We adopt the Pareto equilibrium. That is if the profit of the downstream firm under cooperative R&D is the same as that under unilateral betrayal, he will choose cooperative R&D. F denotes the penalty paid by the defaulting firm to the non-defaulting firm in the cooperative R&D stage.

Let $c_s(\beta) = \frac{(a-c_u)(203-136\beta+138\beta^2+20\beta^3-25\beta^4)}{18(33+56\beta-25\beta^2)}$, $s = \frac{1}{144(17-2\beta-\beta^2)^2}$, $t_1 = s[36(a-c_u)c_d(213-130\beta+120\beta^2+6\beta^3-25\beta^4)-(a-c_u)^2(1+\beta)^2(383-388\beta+138\beta^2+20\beta^3-25\beta^4)-324c_d^2(43+42\beta-25\beta^2)]$, $t_2 = \frac{9k^2(5-7\beta)^2}{(17-2\beta-\beta^2)^2(23+70\beta-25\beta^2)}$, $t_3 = \frac{[(a-c_u)(7-5\beta)(1+\beta)-36c_d]^2}{36(1+\beta^2)(23+70\beta-25\beta^2)}$. Then we have the following proposition.

Proposition 4. *(Stability of Cooperative R&D strategy). Regarding the stability of cooperative R&D activities, we have*

(i) when $\beta \in ((2-\sqrt{2})/2, 5/7)$ and $c_o(\beta) < c_d \le c_s(\beta)$, if $F > t_1$, then each downstream firm would prefer to cooperate rather than betray in the cooperative R&D stage;

(ii) when $\beta \in ((2-\sqrt{2})/2, 5/7)$ and $c_s(\beta) < c_d < a-c_u$, or $\beta \in (5/7, 1)$ and $c_o(\beta) < c_d < a-c_u$, if $F \ge t_2$ then each downstream firm would prefer to cooperate rather than betray in the cooperative R&D stage;

(iii) when $\beta \in (5/7, 1)$ and $c_n(\beta) < c_d \le c_o(\beta)$, if $F \ge t_3$ then each downstream firm would prefer to cooperate rather than betray in the cooperative R&D stage.

The coordination mechanism of cooperative R&D in Proposition 4 is actually a punishment mechanism, which can make it unprofitable for both downstream enterprises if either of them choose betrayal unilaterally at the stage of cooperative R&D. It can ensure that both downstream firms will adhere to the level of balanced R&D output and avoid the presence of "prisoner's dilemma" in cooperative R&D.

5. Social welfare comparisons: Cooperative vs. noncooperative R&D strategy.

Let CS^N and CS^C denote the consumer surplus under noncooperative and cooperative R&D respectively. Accordingly, we have

$$CS^m = \int_0^{q_1^m+q_2^m} (a-Q-p^m)dQ, \quad m \in \{N, C\}. \tag{7}$$

Then we have the following proposition.

Proposition 5. *Comparing the consumer surplus under noncooperative and cooperative R&D strategies, we obtain*

(i) if $\beta \in ((2-\sqrt{2})/2, 5/7]$ *and* $c_d > c_o(\beta)$*, or* $\beta \in (5/7, 1)$*,* $c_d \leq min\{c_o(\beta), c_n(\beta)\}$*, then* $CS^N \geq CS^C$*;*

(ii) if $\beta \in (5/7, 1)$ *and* $c_d > c_n(\beta)$*, then* $CS^N < CS^C$*.*

Proposition 5(i) shows that, either a combination of a low technological spillover effect with a high (initial) production cost or a high technological spillover effect but with a low production cost can lead to a lower consumer surplus for cooperative R&D strategy between the duopoly manufacturers. This is because, for both cases the cooperation in R&D activities can result in a higher selling price at the market stage (see Lemma 4.1), which in turn exploits more surplus from consumers. In contrast, if both the level of technology spillover and (initial) production cost are high, then the cooperative R&D will take a huge leap of production cost reduction, which thus incurs a more fierce competition at the market stage. As a result, the selling price will become lower, leaving more surplus for consumers.

We next do some comparisons on the social welfare under the two R&D strategies. Denote SW^N and SW^C the corresponding social welfares under cooperative and non-cooperative R&D strategies, respectively. Accordingly, we have

$$SW^m = \pi_u^m + \sum_{i=1}^{2} \pi_i^m + CS^m, m \in \{N, C\}, \tag{8}$$

where π_u is the supplier's profit and π_i represents manufacturer i's profit. We have the following comparative results.

Proposition 6. *Comparing the social welfare under noncooperative and cooperative R&D strategies, we have*

(i) if $\beta \in ((2-\sqrt{2})/2, 5/7]$ *and* $c_d > c_o(\beta)$*, or* $\beta \in (5/7, 1)$*,* $c_d \leq min\{c_o(\beta), c_n(\beta)\}$ *then* $SW^N \geq SW^C$*;*

(ii) if $\beta \in (5/7, 1)$ *and* $c_d > c_n(\beta)$*, then* $SW^N < SW^C$*.*

Proposition 6(i) shows that non-cooperative R&D strategy will lead to a higher social welfare with either a high technological spillover effect with a low production cost or a low level of spillover effect but with a high production cost. If the duopoly manufacturers choose not to coordinate in R&D stage, then the supplier will set up a higher wholesale price, which makes the manufacturers achieve a lower selling price in equilibrium. As a result, both the supplier's profit and the consumers' surplus become larger, and thus the overall social welfare is larger compared to the cooperative R&D strategy, albeit lower manufacturers' profit. However, according to Proposition 6(ii), the cooperative R&D will lead to a larger social welfare with both a high technological spillover effect and a high (initial) production cost. As Lemma 1 and Proposition 5 illustrate, the cooperation in R&D activity will be conducive for manufacturers to achieve a higher selling price in equilibrium, which in turn results in a larger profit for both manufacturers, albeit with the sacrifice from both the supplier and consumers. Nevertheless, the overall social welfare is better off under the cooperative R&D strategy.

6. **Model extension.** In the above noncooperative and cooperative R&D model, we assume that the initial marginal production costs of the two downstream firms are equal. In this part, we relax the assumption and suppose that the initial marginal production costs of the two downstream firms are not equal. Let c_i denote

the initial marginal production cost of the downstream manufacturers i, $i = 1, 2$. Without loss of generality, c_1 and c_2 are assumed to satisfy $c_2 = 2c_1$. We assume that $\beta > \frac{1}{10}(17 - \sqrt{129})$. The remaining assumptions of extended model are the same as that of the basic assumptions.

Let x_i^{en}, q_i^{en}, π_i^{en} denote the equilibrium R&D levels, quantities and profits of downstream manufacturers i under noncooperative R&D respectively. x_i^{eo}, q_i^{eo}, π_i^{eo} denote the equilibrium R&D levels, quantities and profits of downstream manufacturers i under cooperative R&D respectively. Let $l_1 = \frac{(a-c_u)(1+\beta)(7-5\beta)(1-12\beta+5\beta^2)}{3(47-218\beta+95\beta^2)}$, $l_2 = \frac{2(a-c_u)(1+\beta)(2\beta^2-4\beta+1)}{4\beta^3-8\beta^2+29\beta-31}$, $l_3 = \frac{(a-c_u)(5\beta^2-12\beta+1)}{5\beta^2-17\beta-13}$, and we can easily verify that

$S = \{(\beta, c_1)\,|\,\frac{1}{10}(17 - \sqrt{129}) < \beta < \frac{5}{7},\ \max\{l_1,\ l_2\} < c_1 < l_3\}$. Then, we have the following corollary.

Corollary 2. *$\pi_1^{eo} > \pi_1^{en}$, $\pi_2^{eo} < \pi_2^{en}$, when $(\beta, c_1) \in S$.*

Corollary 2 shows that, for $(\beta, c_1) \in S$, downstream manufacturer 1 is willing to conduct cooperative R&D, while downstream manufacturer 2 is reluctant to cooperative R&D.

Corollary 3. *$\pi_1^{eo} + \pi_2^{eo} > \pi_1^{en} + \pi_2^{en}$, when $(\beta, c_1) \in S$.*

The intuitive implication of Corollary 3 is as follows: compared with cooperative R&D, for $(\beta, c_1) \in S$, the noncooperative R&D results in excessive R&D of the two downstream manufacturers aiming at maximizing their own profit. Furthermore, this results in fierce competition between two downstream manufacturers in the market stage and reduces their total profits.

It can be seen from Corollary 2-3 that, for $(\beta, c_1) \in S$, although the cooperative R&D between two downstream manufacturers can improve the total profit of both sides, in the absence of joint profit distribution mechanism, the downstream manufacturer 2 will not choose cooperative R&D.

Let λ^* denote the proportion of the profit allocated to downstream manufacturer 1 to the largest joint profit in R&D cooperation, $1 - \lambda^*$ the proportion of the profit allocated to downstream manufacturer 2 to the largest joint profit in R&D cooperation and

$$\Lambda = \{\lambda\,|0 < \lambda < 1,\ \lambda(\pi_1^{eo} + \pi_2^{eo}) > \pi_1^{en}\ ,\ (1-\lambda)(\pi_1^{eo} + \pi_2^{eo}) > \pi_2^{en}\}.$$

Then we have the following proposition.

Proposition 7. *When $(\beta, c_1) \in S$ and $\lambda^* \in \Lambda$, each downstream manufacturer will choose cooperative R&D. Under cooperative R&D, the equilibrium R&D level of downstream manufacturer 1 is $x_1^{eo} = \frac{2(a-c_u)(1+\beta)+(33-39\beta)c_1}{70\beta-37\beta^2-1}$, and the equilibrium R&D level of downstream manufacturer 2 is $x_2^{eo} = 0$.*

Proposition 7 shows that, for $(\beta, c_1) \in S$ and $\lambda^* \in \Lambda$, two downstream firms will choose cooperative R&D. However, the equilibrium R&D output level of downstream firm 2 is 0, that is, downstream firm 2 enjoys free ride in R&D.

We consider the the vertical cooperation of R&D. We assume the downstream manufacturers engage in R&D activities and the upstream component supplier partially engages in R&D activities. The efforts devoted to R&D activities by supplier and manufacturer i can reduce marginal production cost. Moreover, there is a positive effect of technology spillover to the other manufacturer.

Specifically, let x_i be the (target) output level of R&D, which reflects the R&D efforts devoted by manufacturer i and $\beta \in (0, 1)$ be the magnitude of spillover effect between the two manufacturers. Let $\theta \in [1, \theta_0)$ represent the upstream supplier's willingness to cooperate with downstream manufacturers in R&D activities. As a result, the marginal production cost of manufacturer i after R&D investments is $c_d - x_i - \beta(1+\theta)x_{3-i}$, $i = 1, 2$. We assume that R&D investment is costly and the cost incurred by manufacturer i is $\frac{1}{2}x_i^2$, $i = 1, 2$. Some other assumptions are the same as before. Thus, we can verify the equilibrium outcomes in cooperative and Noncooperative R&D Strategy (see figure 4).

TABLE 3. The Equilibrium Outcomes in Cooperative R&D Strategy

Scenario	$c_o(\beta) < c_d < a - c_u$	$0 < c_d \le c_o(\beta)$
x_i^C	$\frac{k(1+\beta(1+\theta))}{17-2\beta(1+\theta)-(\beta(1+\theta))^2}$	$\frac{c_d}{1+\beta(1+\theta)}$
w^C	$\frac{9(a-c_d)+(8-2\beta(1+\theta)-(\beta(1+\theta))^2)c_u}{17-2\beta(1+\theta)-(\beta(1+\theta))^2}$	$\frac{a+c_u}{2}$
p^C	$\frac{a(11-2\beta(1+\theta)-(\beta(1+\theta))^2)+6(c_u+c_d)}{17-2\beta(1+\theta)-(\beta(1+\theta))^2}$	$\frac{2a+c_u}{3}$
q_i^C	$\frac{3k}{17-2\beta(1+\theta)-(\beta(1+\theta))^2}$	$\frac{a-c_u}{6}$
π_u^C	$\frac{54k^2}{(17-2\beta(1+\theta)-(\beta(1+\theta))^2)^2}$	$\frac{(a-c_u)^2}{6}$
π_i^C	$\frac{k^2}{2(17-2\beta(1+\theta)-(\beta(1+\theta))^2)}$	$\frac{(a-c_u)^2(1+\beta(1+\theta))^2-18c_d^2}{36(1+\beta(1+\theta))^2}$

TABLE 4. The Equilibrium Outcomes in Noncooperative R&D Strategy

Scenario	$c_n(\beta) < c_d < a - c_u$	$0 < c_d \le c_n(\beta)$
x_i^N	$\frac{(7-5\beta(1+\theta))k}{29-2\beta(1+\theta)+5(\beta(1+\theta))^2}$	$\frac{c_d}{1+\beta(1+\theta)}$
w^N	$\frac{18(a-c_d)+(11-2\beta(1+\theta)+5(\beta(1+\theta))^2)c_u}{29-2\beta(1+\theta)+5(\beta(1+\theta))^2}$	$\frac{a+c_u}{2}$
p^N	$\frac{a(17-2\beta(1+\theta)+5(\beta(1+\theta))^2)+12(c_u+c_d)}{29-2\beta(1+\theta)+5(\beta(1+\theta))^2}$	$\frac{2a+c_u}{3}$
q_i^N	$\frac{6k}{29-2\beta(1+\theta)+5(\beta(1+\theta))^2}$	$\frac{a-c_u}{6}$
π_u^N	$\frac{216k^2}{(29-2\beta(1+\theta)+5(\beta(1+\theta))^2)^2}$	$\frac{(a-c_u)^2}{6}$
π_i^N	$\frac{(23+70\beta(1+\theta)-25(\beta(1+\theta))^2)k^2}{2(29-2\beta(1+\theta)+5(\beta(1+\theta))^2)^2}$	$\frac{(a-c_u)^2(1+\beta(1+\theta))^2-18c_d^2}{36(1+\beta(1+\theta))^2}$

By comparing the situation of cooperative and non-cooperative R&D, it is not difficult to find that when the production cost of downstream manufacturers is relatively small, and upstream suppliers are less willing to cooperate with downstream manufacturers in R&D activities, the downstream manufacturers' optimal strategy is non-cooperative R&D, which is consistent with previous research findings.

Not only that, when the production cost of downstream manufacturers is relatively large, and the willingness of upstream supplier to cooperate with downstream manufacturers in R&D activities is relatively large, it can be seen that the benefits brought by cooperation between downstream manufacturers increase with the

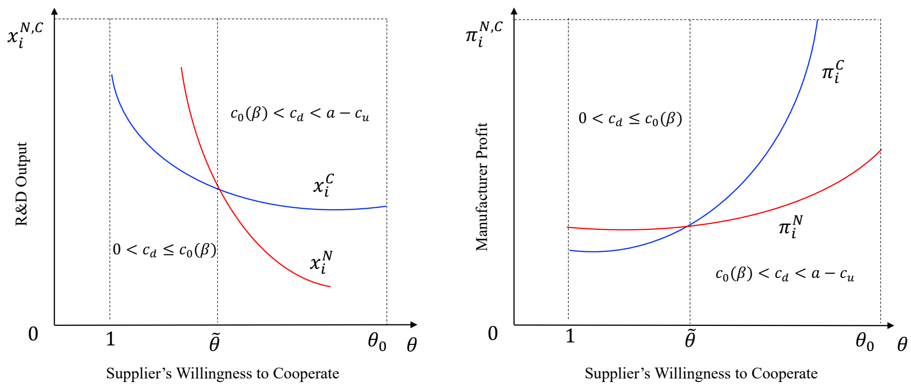

FIGURE 4. The Effect of Supplier's Willingness to Cooperate

willingness of cooperation in research and development. It means that, from the perspective of the supply chain, the participation of upstream enterprises on the R&D process of downstream enterprises will help enhance the benefits of the entire supply chain and help downstream manufacturers. It can be seen that the effective implementation of supply chain integration requires better willingness between vertical enterprises and the establishment of stable partnerships between horizontal enterprises.

7. **Conclusions.** We investigate the R&D strategies of duopoly manufacturers from a supply chain perspective. Specifically, we consider a supply chain consisting of one upstream (common component) supplier and two downstream manufacturers, who may conduct R&D activities cooperatively or independently, while facing competition in the same product market. The whole decision process can be divided into two stages, namely R&D stage and market stage sequentially. At the R&D stage, the two manufacturers decide whether to conduct R&D activities in a cooperative or non-cooperative way. At the market stage, the two manufacturers and the supplier engages in a Stackelberg game, with the manufacturers being the leader(s) while the supplier being the follower.

Whether the manufacturers should adopt a cooperative R&D strategy or not depends on two main factors, the technology spillover effect and the (initial) production cost. If the technology spillover effect is moderate or the production cost of manufacturers is rather low, noncooperative R&D strategy is beneficial to the manufacturers. In contrast, if the technology spillover effect is either too large or too small, and the production cost is very large, cooperative R&D strategy will be adopted by the two manufacturers. Given the production cost being small (large), the equilibrium R&D inputs of manufacturers decrease (increase) in the technology spillover effect and correspondingly their profits increase (decrease) in it. In terms of social welfare comparisons, interestingly, we find that cooperative R&D strategy of manufacturers may reduce the consumer surplus and the total social welfare. We also investigate the stability of the cooperative R&D strategies, which in general is unstable, and thus propose a coordination mechanism to guarantee the stability of cooperation of duopoly manufacturers.

Possible future directions include: (i) We consider deterministic demand in this paper. Incorporating demand uncertainty into our modeling may generate some other managerial insights, e.g., how will the demand uncertainty affect the R&D strategy of the duopoly manufacturers. (ii) The supply chain we consider consists of one upstream supplier and two downstream manufacturers, who are the Stackelberg leaders. How about the situation with the supplier being the leader? How about the supply chain structure with two upstream manufacturers and one downstream customer (retailer)? (iii) We consider Cournot competition in the product market. In reality, the competition could be of Bertrand style (i.e., price competition).

Appendix. Proofs.

A: Proofs for main results

Proof. Proof of Proposition 1.

In noncooperative R&D, we get the equilibrium solutions from the second to fourth stages by reverse induction. Since $\frac{d^2\pi_i}{d{q_i}^2} = -2 < 0$, π_i is strictly concave in q_i. Hence, there exists a unique optimal solution to the profit function (see Equation (1)). Similarly, since $\frac{d^2\pi_u}{dw^2} = -\frac{4}{3} < 0$, π_u is strictly concave in w, which implies a unique optimal solution to Equation (3).

Substitute w^N, q_1^N and q_2^N into the profit function of downstream manufacturer i. Since $\frac{d^2\pi_i}{dx_i^2} = -\frac{1}{72}(23+70\beta-25\beta^2) < 0$, π_i is strictly concave in x_i. Let $x_i^*(x_{3-i})$ be the unique solution of $\frac{d\pi_i}{dx_i} = 0$. Then we have $x_i^*(x_{3-i}) = \frac{(7-5\beta)[2(a-c_u-c_d)-(5-7\beta)x_{3-i}]}{23+70\beta-25\beta^2}$, $i = 1\ ,\ 2$.

Let $R_i(x_{3-i})$ denote the response function of manufacturer i w.r.t the R&D input level. Then we have $R_i(x_{3-i}) = \min\{x_i^*(x_{3-i})\ ,\ c_d - \beta x_{3-i}\}$, $i = 1\ ,\ 2$. Let $k = a - c_u - c_d$, $x^b = \frac{c_d}{1+\beta}$, $x^n = \frac{(7-5\beta)k}{29-2\beta+5\beta^2}$ and $c_n = \frac{(a-c_u)(7-5\beta)(1+\beta)}{36}$, where x_i^N represents the equilibrium R&D output level of downstream manufacturer i under non-cooperative R&D strategy. By solving Equation (5), we then obtain the equilibrium R&D output level of downstream manufacturer i. As a result, if $0 < c_d \le c_n$, $x_i^N = x^b$, $i = 1\ ,\ 2$; and if $c_n < c_d < a - c_u$, $x_i^N = x^n$, $i = 1\ ,\ 2$.

Proof. Proof of Proposition 2. We conduct the equilibrium analysis by backward induction under the cooperative R&D strategy. In the games of the third and fourth stages, for given R&D input levels, the decisions are the same as those under non-cooperative R&D strategy. Therefore, we next focus on the game of the second stage, under which the manufacturers' joint objective function can be expressed as follows:

$$\max_{x_1,x_2} \pi = \sum_{i=1}^{2} \{\tfrac{1}{144}[2(a-c_u-c_d) + (7-5\beta)x_i - (5-7\beta)x_{3-i}]^2 - \tfrac{1}{2}x_i^2\}$$
$$s.t.\ x_i + \beta x_{3-i} \le c_d\ ,\ i = 1\ ,\ 2.$$

When $(2-\sqrt{2})/2 < \beta < 1$, we have $\frac{\partial^2\pi}{\partial x_1^2} = \frac{1}{36}(1 - 70\beta + 37\beta^2) < 0$ and $\frac{\partial^2\pi}{\partial x_1^2} \times \frac{\partial^2\pi}{\partial x_2^2} - (\frac{\partial^2\pi}{\partial x_1 \partial x_2})^2 = \frac{1}{18}(2\beta^4 - 41\beta^2 + 70\beta - 17) > 0$, which implies the strict concavity of the objective function. As the constraint is linear, there exists a unique optimal solution to the objective function.

Let $c_h = \frac{(a-c_u)(1+\beta)^2}{18}$ and $x^c = \frac{(1+\beta)k}{17-2\beta-\beta^2}$, which is the equilibrium output level of downstream manufacturer i under the cooperative R&D strategy. The equilibrium R&D output level of downstream manufacturer i can be obtained by solving the equation satisfying the first-order condition. Thus, we have: when $0 < c_d \le c_h$, $x_i^C = x^b, i = 1\ ,\ 2$; and when $c_h < c_d < a - c_u$, $x_i^C = x^c, i = 1\ ,\ 2$. □

Proof. Proof of Lemma 4.1.

Firstly, comparing c_h and c_n, we can easily obtain

$$c_h \begin{cases} < c_n \ , \ if \beta < \ 5/7; \\ = c_n \ , \ if \beta = 5/7; \\ > c_n \ , \ if \beta > 5/7. \end{cases}$$

As a consequence, next we have following cases.

- when $\beta \in ((2-\sqrt{2})/2 \ , \ 5/7)$ and $0 < c_d \leq c_h$ or when $\beta \in (5/7 \ , \ 1)$ and $0 < c_d \leq c_n$, from the 3rd column of Table 1 and 3rd column of Table 2, we can justify that conclusion (i) of Lemma 1 is valid.
- When $\beta \in ((2-\sqrt{2})/2 \ , \ 5/7)$ and $c_h < c_d \leq c_n$, as can be seen from 3rd column of Table 1 and 2nd column of Table 2, we have

$$w^C - w^N = \frac{9(c_h - c_d)}{17 - 2\beta - \beta^2} < 0;$$

$$p^C - p^N = \frac{6(c_d - c_h)}{17 - 2\beta - \beta^2} > 0;$$

$$q_i^C - q_i^N = \frac{3(c_h - c_d)}{17 - 2\beta - \beta^2} < 0;$$

$$x_i^C - x_i^N = \frac{18(c_h - c_d)}{(1+\beta)(17 - 2\beta - \beta^2)} < 0;$$

$$\pi_u^C - \pi_u^N < \frac{54(a - c_u - c_h)^2}{(17 - 2\beta - \beta^2)^2} - \frac{(a - c_u)^2}{6} = 0;$$

and

$$\pi_i^C - \pi_i^N = \frac{9(c_h - c_d)^2}{(1+\beta)^2(17 - 2\beta - \beta^2)} > 0.$$

- when $\max\{c_n \ , \ c_h\} < c_d < a - c_u$, in contrast, as can be seen from 2nd column of Table 1 and 2nd column of Table 2, we have

$$w^C - w^N = \frac{9k(1+\beta)(7\beta - 5)}{(17 - 2\beta - \beta^2)(29 - 2\beta + 5\beta^2)} \begin{cases} < 0 \ , \ \beta \in ((2-\sqrt{2})/2, \ 5/7); \\ = 0 \ , \ \beta = 5/7; \\ > 0 \ , \ \beta \in (5/7 \ , \ 1), \end{cases}$$

$$p^C - p^N = \frac{6k(1+\beta)(5 - 7\beta)}{(17 - 2\beta - \beta^2)(29 - 2\beta + 5\beta^2)} \begin{cases} > 0 \ , \ \beta \in ((2-\sqrt{2})/2 \ , \ 5/7); \\ = 0 \ , \ \beta = 5/7; \\ < 0 \ , \ \beta \in (5/7 \ , \ 1), \end{cases}$$

$$q_i^C - q_i^N = \frac{3k(1+\beta)(7\beta-5)}{(17-2\beta-\beta^2)(29-2\beta+5\beta^2)} \begin{cases} <0 \ , \ \beta \in ((2-\sqrt{2})/2 \ , \ 5/7); \\ =0 \ , \ \beta = 5/7; \\ >0 \ , \ 5/7 < \beta < 1, \end{cases}$$

$$x_i^C - x_i^N = \frac{18k(7\beta-5)}{(17-2\beta-\beta^2)(29-2\beta+5\beta^2)} \begin{cases} <0 \ , \ \beta \in ((2-\sqrt{2})/2 \ , \ 5/7); \\ =0 \ , \ \beta = 5/7; \\ >0 \ , \ \beta \in (5/7 \ , \ 1), \end{cases}$$

$$\pi_u^C - \pi_u^N = \frac{162k^2(21-2\beta+\beta^2)(1+\beta)(7\beta-5)}{[(17-2\beta-\beta^2)(29-2\beta+5\beta^2)]^2} \begin{cases} <0 \ , \ \beta \in ((2-\sqrt{2})/2 \ , \ 5/7); \\ =0 \ , \ \beta = 5/7; \\ >0 \ , \ \beta \in (5/7 \ , \ 1), \end{cases}$$

$$\pi_i^C - \pi_i^N = \frac{[3k(5-7\beta)]^2}{(17-2\beta-\beta^2)(29-2\beta+5\beta^2)^2} \begin{cases} >0 \ , \ \beta \neq 5/7; \\ =0 \ , \ \beta = 5/7. \end{cases}$$

- When $\beta \in (5/7 \ , \ 1)$ and $c_n < c_d \leq c_h$, as can be seen from the 2nd column of Table 1 and the 3rd column of Table 2, we have

$$w^C - w^N = \frac{18(c_d - c_n)}{29-2\beta+5\beta^2} > 0;$$

$$p^C - p^N = \frac{12(c_n - c_d)}{29-2\beta+5\beta^2} < 0;$$

$$q_i^C - q_i^N = \frac{6(c_d - c_n)}{29-2\beta+5\beta^2} > 0;$$

$$x_i^C - x_i^N = \frac{36(c_d - c_n)}{(1+\beta)(29-2\beta+5\beta^2)} > 0;$$

$$\pi_u^C - \pi_u^N > \frac{(a-c_u)^2}{6} - \frac{216(a-c_u-c_n)^2}{(29-2\beta+5\beta^2)^2} = 0;$$

$$\begin{aligned} \pi_i^C - \pi_i^N &= A(c_d - c_n)[B - 216(2+\beta^2)c_d] \geq A(c_d - c_n)[B - 216(2+\beta^2)c_h] \\ &= A(c_d - c_n)(a-c_u)(1+\beta)(7\beta-5)(17-2\beta-\beta^2) > 0. \end{aligned}$$

where $A = \frac{1}{(1+\beta)^2(29-2\beta+5\beta^2)^2}$ and $B = (a-c_u)(1+\beta)(5\beta^3+3\beta^2+153\beta-61)$.

- When $\beta \in ((2-\sqrt{2})/2 \ , \ 5/7)$, we have $\max\{c_n \ , \ c_h\} = c_n$. Notice that, when $\beta \in ((2-\sqrt{2})/2 \ , \ 5/7)$ and $c_h < c_d \leq c_n$, it's easy to verify that conclusion (ii) of Lemma 1 is valid. When $\beta = 5/7$, we have $c_h = c_n$. From the case of $\beta = 5/7$ and the 3rd column of Table 1 and the 3rd column of Table 2, it can be seen that conclusion (iii) of Lemma 1 is valid.
- When $\beta \in (5/7 \ , \ 1)$, finally, we have $\max\{c_n \ , \ c_h\} = c_h$. From the equation of case of $\beta \in (5/7 \ , \ 1)$, we can see that conclusion (ii) of Lemma 1 holds.

□

Proof. Proof of Corollary 1. As it can be seen from the Proposition 1 and 2, there exists two cases as follows

- When $0 < c_d \leq c_h$, Proposition 1 shows that the equilibrium R&D output level x^b of downstream manufacturer i under the optimal R&D strategy is obviously decreasing with β. From Table 1 and Table 2, we can see that π^b is the equilibrium profit of downstream manufacturer i under the optimal R&D strategy (see Table 1, column 3). Since $\frac{d\pi^b}{d\beta} = \frac{c_d^2}{(1+\beta)^3} > 0$, π^b increases with β.
- When $c_h < c_d < a - c_u$, from Proposition 1 and table 2, it can be seen that x^c is the equilibrium R&D output level of downstream manufacturer i under the optimal R&D strategy and $\pi_i^C = \frac{k^2}{2(17-2\beta-\beta^2)}$ is the equilibrium profit, accordingly. Since $\frac{dx^c}{d\beta} = \frac{k(19+2\beta+\beta^2)}{(17-2\beta-\beta^2)^2} > 0$ and $\frac{d\pi_i^C}{d\beta} = \frac{k^2(1+\beta)}{(17-2\beta-\beta^2)^2} > 0$, the equilibrium R&D output level and the equilibrium profit of downstream manufacturer i increase with β.

□

Proof. Proof of Proposition 3. When $\beta \in ((2-\sqrt{2})/2\ ,\ 5/7) \cup (5/7\ ,\ 1)$ and $\min\{c_h\ ,\ c_n\} < c_d < a - c_u$, Proposition 1 shows that two downstream manufacturers can improve their profitability by cooperating in R&D. Moreover, when $\beta \in ((2-\sqrt{2})/2\ ,\ 5/7) \cup (5/7\ ,\ 1)$ and $c_h < c_d < a - c_u$, the equilibrium R&D output level of each downstream manufacturer is x^c; Accordingly, when $\beta \in (5/7\ ,\ 1)$ and $c_n < c_d \leq c_h$, the equilibrium R&D output level of each downstream manufacturer is x^b.

Hence, the objective is to find whether there is a downstream manufacturer that will choose to betray in the cooperative R&D stage. Next, we give the two situations as below.

- When $\beta \in ((2-\sqrt{2})/2\ ,\ 5/7) \cup (5/7\ ,\ 1)$ and $c_h < c_d < a - c_u$, for convenience and without loss of generality, we assume that the R&D output level of downstream manufacturer 2 is x^c. Let π_{10}^d represent the profit made by downstream manufacturer 1 when it unilaterally chooses to betray. Then we consider whether downstream manufacturer 1 will unilaterally choose betrayal. Under the above assumptions, the problem of downstream manufacturer 1 is

$$\max_{x_1} \pi_{10}^d = \frac{[(7-5\beta)(17-2\beta-\beta^2)x_1 + (29-2\beta+5\beta^2)k]^2}{144(17-2\beta-\beta^2)^2} - \frac{1}{2}x_1^2$$
$$s.t.\ \ x_1 \leq c_d - \beta x^c.$$

Since $\frac{\partial^2 \pi_{10}^d}{\partial x_1^2} = -\frac{1}{72}(23 + 70\beta - 25\beta^2) < 0$ and the constraints are linear inequalities, the formula above is a strict concave programming. Therefore, there exists a unique global optimal solution. Let x_{10}^d indicate the R&D output level of downstream manufacturer 1 when it chooses to betray unilaterally, $d_1 = \frac{(17-\beta)c_d-(a-c_u)\beta(1+\beta)}{17-2\beta-\beta^2}$, $d_2 = \frac{(7-5\beta)(29-2\beta+5\beta^2)k}{(17-2\beta-\beta^2)(23+70\beta-25\beta^2)}$ and $c_s = \frac{(a-c_u)(203-136\beta+138\beta^2+20\beta^3-25\beta^4)}{18(33+56\beta-25\beta^2)}$ respectively. Then we can obtain:

i) When $\beta \in ((2-\sqrt{2})/2\ ,\ 5/7)$ and $c_h < c_d \leq c_s$, $x_{10}^d = d_2 \neq x^c$, that is, downstream manufacturer 1 will unilaterally choose betrayal.

ii) When $\beta \in (5/7\ ,\ 1)$ and $c_s < c_d < a - c_u$ or when $\beta \in (5/7\ ,\ 1)$ and $c_h < c_d < a - c_u$, $x_{10}^d = d_2 \neq x^c$, that is, downstream manufacturer 1 will unilaterally choose betrayal.

- When $\beta \in (5/7\ ,\ 1)$ and $c_n < c_d \leq c_h$, without loss of generality, we assume that downstream manufacturer 2 insists output level x^b. Let π_{11}^d represent the profit made by downstream manufacturer 1 when it unilaterally chooses to betray. Then we consider whether downstream manufacturer 1 will unilaterally choose betrayal. Under the above assumptions, the problem of downstream manufacturer 1 is

$$\max_{x_1} \pi_{11}^d = \frac{1}{144(1+\beta)^2}[2(a-c_u)(1+\beta) - (7-5\beta)c_d + 1 + \beta)(7-5\beta)x_1]^2 - \frac{1}{2}x_1^2$$

$$s.t.\ \ x_1 \leq c_d - \beta x^b.$$

Since $\frac{\partial^2 \pi_{11}^d}{\partial x_1^2} = -\frac{1}{72}(23 + 70\beta - 25\beta^2) < 0$ and the constraints are linear inequalities, the formula above is a strict concave programming. Therefore, there exists a unique global optimal solution. Let $d_3 = \frac{(7-5\beta)[2(a-c_u)(1+\beta)-c_d(7-5\beta)]}{(1+\beta)(23+70\beta-25\beta^2)}$ and x_{11}^d indicates the R&D output level of downstream manufacturer 1 when it chooses to betray unilaterally. From the formula above, we have $x_{11}^d = d_3 \neq x^b$, that is, the downstream manufacturer 1 will unilaterally choose betrayal.

□

Proof. Proof of Proposition 4. In the stage of cooperative R&D, every downstream manufacturer will choose cooperation rather than betrayal unilaterally if and only if the penalty for breach of contract is greater than the profit increment. Without loss of generality, we analyze the performance of downstream manufacturer 1 only. First of all, we analyze the condition of the penalty F, specifically, we have:

- When $\beta \in ((2-\sqrt{2})/2\ ,\ 5/7)$ and $c_h < c_d \leq c_s$, from the proof of Proposition 4, it can be seen that if the downstream manufacturer 1 unilaterally chooses betrayal, then $x_{10}^d = d_1$. Therefore, when $F \geq \pi_{10}^d|_{x_1=d_1} - \pi_1^C = t_1$, the downstream manufacturer 1 will choose to cooperate rather than unilaterally choosing to betray.
- When $\beta \in ((2-\sqrt{2})/2\ ,\ 5/7)$ and $c_s < c_d < a - c_u$ or $\beta \in (5/7\ ,\ 1)$, from the proof of Proposition 4, it can be seen that if the downstream manufacturer 1 unilaterally chooses betrayal, then $x_{10}^d = d_2$. Therefore, when $F \geq \pi_{10}^d|_{x_1=d_2} - \pi_1^C = t_2$, the downstream manufacturer 1 will choose to cooperate rather than unilaterally choosing to betray.
- When $\beta \in (5/7\ ,\ 1)$ and $c_n < c_d \leq c_h$, it can be seen from the proof of Proposition 4 that if the downstream manufacturer 1 unilaterally chooses betrayal, then $x_{11}^d = d_3$. Therefore, when $F \geq \pi_{11}^d|_{x_1=d_3} - \pi_1^C = t_3$, the downstream manufacturer 1 will choose to cooperate rather than unilaterally choosing to betray.

□

Proof. Proof of Proposition 5. With the conclusions of Lemma 1 and the Equation 7 and the relations of the final product prices under non-cooperative R&D and cooperative R&D, we can easily justify that Proposition 5 is valid. □

Proof. Proof of Proposition 6. Recall that $SW^N = \pi_u^N + \sum_{i=1}^{2} \pi_i^N + CS^N$ and $SW^C = \pi_u^C + \sum_{i=1}^{2} \pi_i^C + CS^C$. As a consequence, we have the following results.

- If $0 < c_d \leq c_n$, based on Column 3 of Table 1, then we have $SW^N = \frac{5(a-c_u)^2(1+\beta)^2-18c_d^2}{18(1+\beta)^2}$;
- If $c_n < c_d < a - c_u$, based on Column 2 of Table 1, then the we have $SW^N = \frac{(311+70\beta-25\beta^2)k^2}{(29-2\beta+5\beta^2)^2}$;
- If $0 < c_d \leq c_h$, based on Column 3 of Table 2, then we have $SW^C = \frac{5(a-c_u)^2(1+\beta)^2-18c_d^2}{18(1+\beta)^2}$;
- If $c_h < c_d < a - c_u$, based on Column 2 of Table 2, then we have $SW^C = \frac{(89-2\beta-\beta^2)k^2}{(17-2\beta-\beta^2)^2}$.

Next, we analyze the following three cases:

(i) When $\beta \in ((2-\sqrt{2})/2\ ,\ 5/7) \cup (5/7\ ,\ 1)$ and $0 < c_d \leq \min\{c_h\ ,\ c_n\}$, based on the Column 3 of Table 1 and Column 3 of Table 2, we have $SW^N = SW^C$; When $\beta = 5/7$ and $0 < c_d \leq c_n$, notice that $c_h = c_n$, based on the Column 3 of Table 1 and Column 3 of Table 2, we can obtain $SW^N = SW^C$;

(ii) When $\beta \in ((2-\sqrt{2})/2\ ,\ 5/7)$ and $c_h < c_d \leq c_n$, we have

$$\begin{aligned} SW^N - SW^C &= A_1(c_d - c_h)(B_1 - c_d) \\ &\geq A_1(c_d - c_h)(B_1 - c_n) \\ &= \frac{A_1(c_d - c_h)(a - c_u)(1+\beta)(167 + 357\beta - 21\beta^2 + 5\beta^3)}{108(7 + 2\beta + \beta^2)} \\ &> 0, \end{aligned}$$

where $A_1 = \frac{54(7+2\beta+\beta^2)}{(1+\beta)^2(17-2\beta-\beta^2)^2}$ and $B_1 = \frac{(a-c_u)(1+\beta)^2(157-10\beta-5\beta^2)}{54(7+2\beta+\beta^2)}$;

(iii) When $\beta \in ((2-\sqrt{2})/2\ ,\ 5/7)$ and $c_n < c_d < a - c_u$, we have

$$SW^N - SW^C = \frac{18k^2(5-7\beta)(167 + 357\beta - 21\beta^2 + 5\beta^3)}{(17 - 2\beta - \beta^2)^2(29 - 2\beta + 5\beta^2)^2} > 0.$$

Notice that $\min\{c_h\ ,\ c_n\} = c_n$. When $\beta \in (5/7, 1)$ and $c_n < c_d < a - c_u$, according to the Lemma 1(ii) and Proposition 2(ii), we can see that, $\pi_u^N < \pi_u^C$, $\pi_i^N < \pi_i^C (i = 1\ ,\ 2)$, and $CS^N < CS^C$. Therefore, we have $SW^N < SW^C$.

Proof. Proof of Corollary 2. Similar to the model establishment and solution method when the marginal production costs of the two downstream firms are equal, we have the following results:

In noncooperative R&D, when $(\beta, c_1) \in S$, we have:

$$x_1^{en} = \frac{(7-5\beta)[(a-c_u)(1-12\beta+5\beta^2) - (16-19\beta+10\beta^2)c_1]}{(1-12\beta+5\beta^2)(29-2\beta+5\beta^2)},$$

$$x_2^{en} = \frac{(7-5\beta)[(a-c_u)(1-12\beta+5\beta^2) + (13+17\beta-5\beta^2)c_1]}{(1-12\beta+5\beta^2)(29-2\beta+5\beta^2)},$$

$$q_1^{en} = \frac{6[(a-c_u)(1-12\beta+5\beta^2) - (16-19\beta+10\beta^2)c_1]}{(1-12\beta+5\beta^2)(29-2\beta+5\beta^2)},$$

$$q_2^{en} = \frac{6[(a-c_u)(1-12\beta+5\beta^2) + (13+17\beta-5\beta^2)c_1]}{(1-12\beta+5\beta^2)(29-2\beta+5\beta^2)},$$

$$\pi_1^{en} = \frac{(23+70\beta-25\beta^2)\left[(a-c_u)(1-12\beta+5\beta^2)-(16-19\beta+10\beta^2)c_1\right]^2}{2(1-12\beta+5\beta^2)^2(29-2\beta+5\beta^2)^2},$$

$$\pi_2^{en} = \frac{(23+70\beta-25\beta^2)\left[(a-c_u)(1-12\beta+5\beta^2)+(13+17\beta-5\beta^2)c_1\right]^2}{2(1-12\beta+5\beta^2)^2(29-2\beta+5\beta^2)^2}.$$

In cooperative R&D, when $(\beta, c_1) \in S$, we have: $x_1^{eo} = \frac{2(a-c_u)(1+\beta)+(33-39\beta)c_1}{70\beta-37\beta^2-1}$, $x_2^{eo} = 0$, and we have the optimal order quantities and profits of downstream manufacturers, i.e.,

$$q_1^{eo} = \frac{(a-c_u)(1+12\beta-7\beta^2)+(19-19\beta+7\beta^2)c_1}{70\beta-37\beta^2-1},$$

$q_2^{eo} = \frac{(a-c_u)(1-12\beta+5\beta^2)+(13+17\beta-5\beta^2)c_1}{1-70\beta+37\beta^2}$, $\pi_2^{eo} = \frac{[(a-c_u)(1-12\beta+5\beta^2)+(13+17\beta-5\beta^2)c_1]^2}{(1-70\beta+37\beta^2)^2}$,

$$\begin{aligned}\pi_1^{eo} = \tfrac{1}{2(1-70\beta+37\beta^2)^2}\{&2(2a-c_u)c_u(1-20\beta-128\beta^2+168\beta^3-49\beta^4)- \\ &4(a-c_u)c_1(14-215\beta+315\beta^2-217\beta^3+49\beta^4)-c_1^2(367-1130\beta+ \\ &267\beta^2+532\beta^3-98\beta^4)-a^2(2-40\beta-256\beta^2+336\beta^3-98\beta^4).\end{aligned}$$

□

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