Mismatch Risk Allocation in a Coproduct Supply Chain

Yangyang Peng
Nanjing University

Joint work with Xiaolin Xu, Xue Liang, Weili Xue

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Outline

- Motivation and Research Problem
- Literature Review
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- Model Analysis
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Motivation

- **Pork supply chain** (extensive vertical integration initiatives).
  Firms like Yurun Group and COFCO cover both the feeding and the pork processing activities, while firms like TechBank mainly focuses on feeding while leaves the processing activity to downstream parties

雨润集团 YURUN GROUP, 中粮 COFCO	VS	TECHBANK Create Your Value
Both feeding and processing activities		Feeding

- **Petrochemical supply chain**

ConocoPhillips	VS	Unilever, P&G
Both oil extraction and processing activities		processing activities

Motivation

➢Blood supply chain

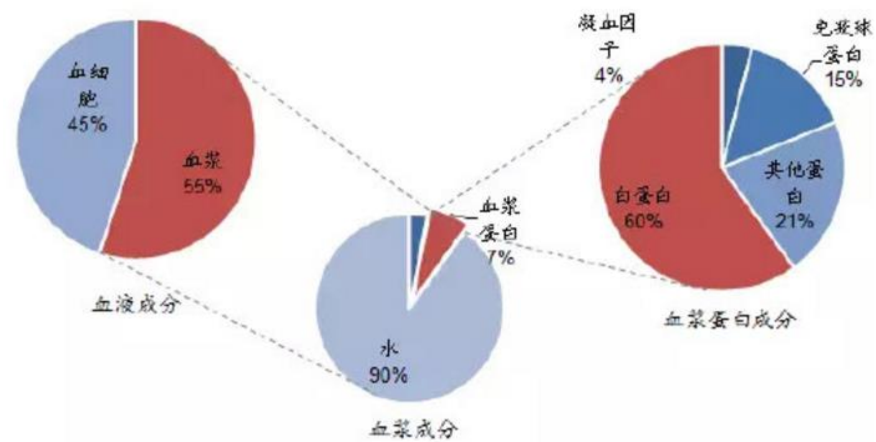
In 2016, the total demand of Plasma in China in is more than **13, 000** tons, and will continue to grow, whereas the total plasma production is less than **7, 000** tons.

Blood is also a coproduct. The demand for different parts varies very much.

Plasma
55% volume of all Blood

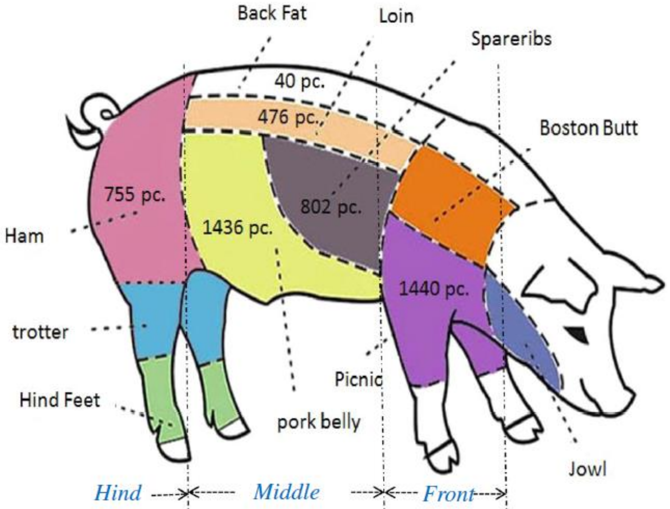
White blood cells and platelets
<1% volume of all Blood

Erythrocyte
45% volume of all Blood



Motivation

- **Co-products** (e.g., fresh goods, recycled goods, etc.) are typically obtained in uncontrollable fractions, leading to ***mismatches*** between their demand and supply.
- Mean demand for different parts varies very much while the fractions of those parts are almost fixed.



Part	Demand（unit）
Front	3,102
Middle	2,947
Hind	1,502

Middle	Demand（unit）
Pork belly	1,436
Spareribs	802
Loin	476
Back Fat	40
others	194

-----Demand in a typical day in COFCO (December 21, 2017)

Table 1 Weekly supply-demand of COFCO in Dec, 2017

Items (1 unit = half a pig)	Thu.	Fri.	Sat.	Sun.	Mon.	Tue.	Wed.
Initial inventory	3152	2582	1653	1893	2513	2574	2759
Actual processing quantity	1300	1300	1248	1300	1300	1300	1400
Demand on product itself	1900	2069	2169	1872	1881	1839	1974
Newly production quantity	2630	2440	3657	3792	3242	3324	3572
End inventory	2582	1653	1893	2513	2574	2759	2957
Forecast processing quantity	1953	1248	1445	1741	1696	1727	2064
Forecast(t − 1)-Actual(t)	–	653	0	145	441	396	327

Research Problem

Management Problems:

1.Who should bear the mismatch risk: producer vs. retailer?

2.What kinds of factors will affect the division of chain profit?

Technical Problems:

The existence and uniqueness of the equilibrium results in both cases.

Literature Review

i) Much of the works focused on the decision optimization from the perspective of a monopoly firm to control the mismatch cost for the coproducts, such as

- *Production and downward substitution decisions*

  (Bitran and Gilbert 1994, Nahmias and Moinzadeh 1997)

- *Pricing decisions*

  (Tomlin and Wang 2008, Bansal and Transchel 2014)

- *Product-line design decisions*

  (Chen et al. 2013, Transchel et al. 2016),

- *Joint procurement and production decisions*

  (Boyabatli et al. 2017; Dong et al. 2014; Boyabatli 2015)

ii) Some other works focused on operations flexibility, whereby a resource (inventory or capacity) has the ability to produce multiple products.

- *Three forms of flexibility* :

  totally flexible (Fine and Freund 1990, Gupta et al. 1992)

  mix flexible (Van Mieghem 2004, Tomlin and Wang 2005)

  dedicated (Li and Tirupati 1994, 1995, 1997, Van Mieghem 1998)

Literature Review

iii) Other works focused on inventory risk allocation by pull or push contracts

- *Pull and push contracts*

  (Cachon 2004, Wang et al. 2014)

- *Push contract*

  (Dong and Zhu 2007, Granot and Yin 2008)

- *Pull contract*

  (Davis et al. 2014)

iv) Another related direction is the product sorting under which decisions are conducted based on different attributes of products

- *Quality*

  (Yayla et al. 2013; Ferguson et al. 2010)

- *Shelflife*

  (Gilland and Heese 2013)

- *Other attributes*

  (Honhon and Pan 2017;Eric and Sabyasachi 2014)

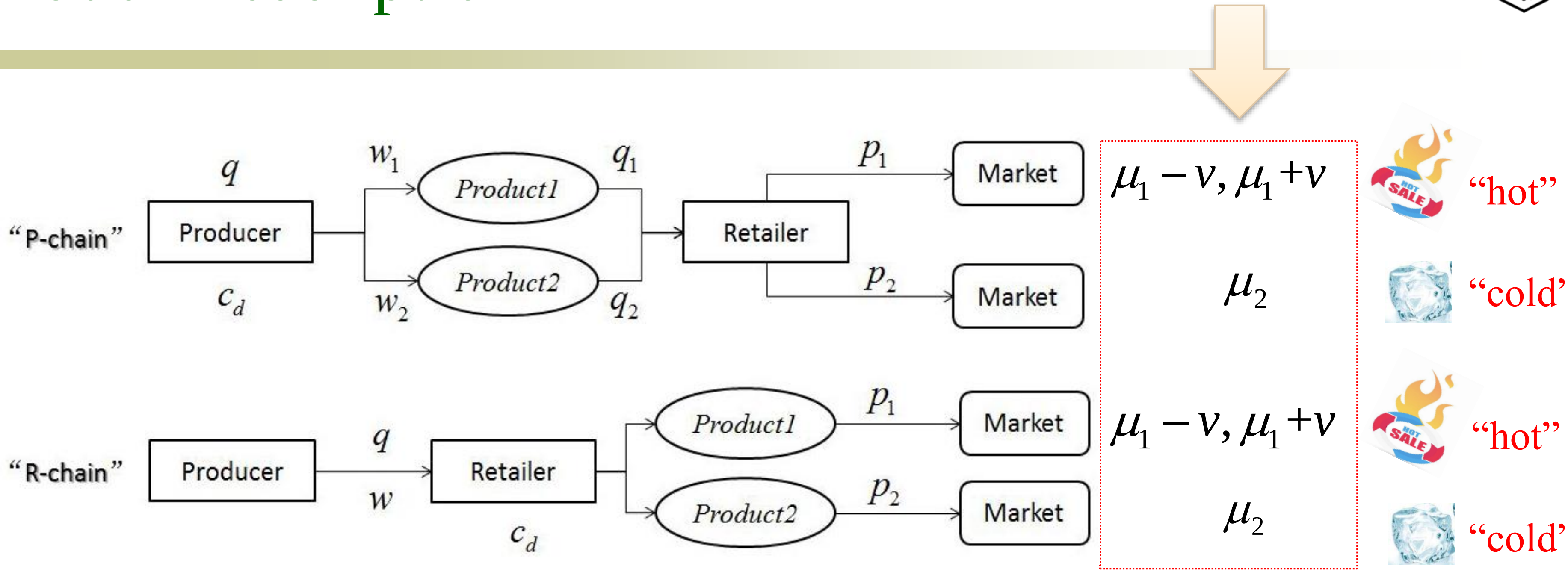
Literature Review

Different to those existing research, our paper, by allocating the processing process among supply chain parties, studies the value of different allocation of flexibility in a supply chain;

Our work focuses on the **allocation of mismatch risk** that is specific in the co-product system and we show that those traditional insights only partially hold.

Model Description

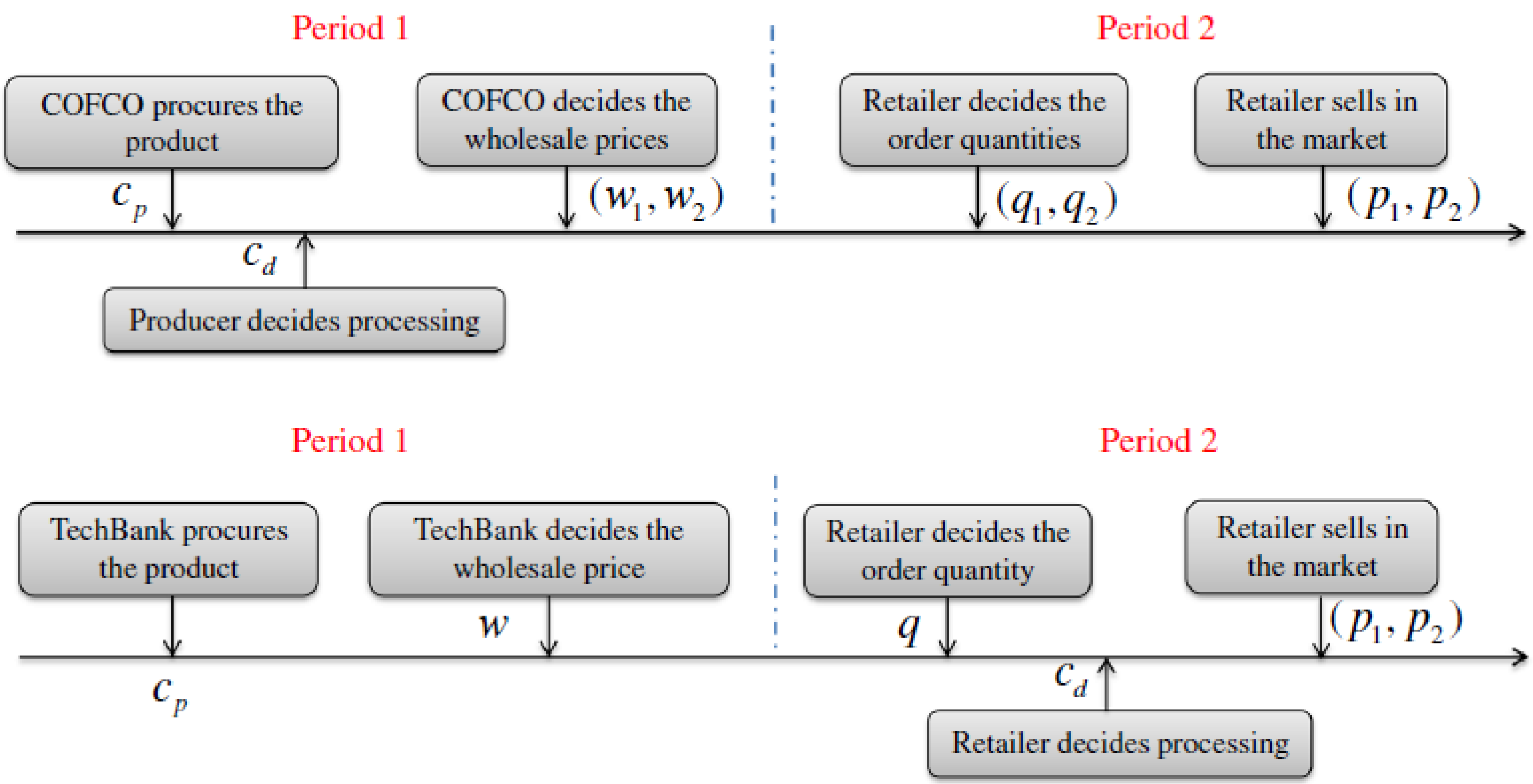
Demand distribution



- One unit product can be processed into λ unit product 1 and 1-λ unit product 2.
- Mean unbalanced supply-demand ratio $\rho := \frac{\mu_1}{\lambda} : \frac{\mu_2}{1-\lambda}$
- Demand of product 1 follows a **two-point distribution** with equal probability1/2
- Demand for product 2 is **deterministic**

Sequences

Figure below shows the decision sequences of COFCO（i.e.,P-*chain*） and TechBank（i.e.,R-*chain*）



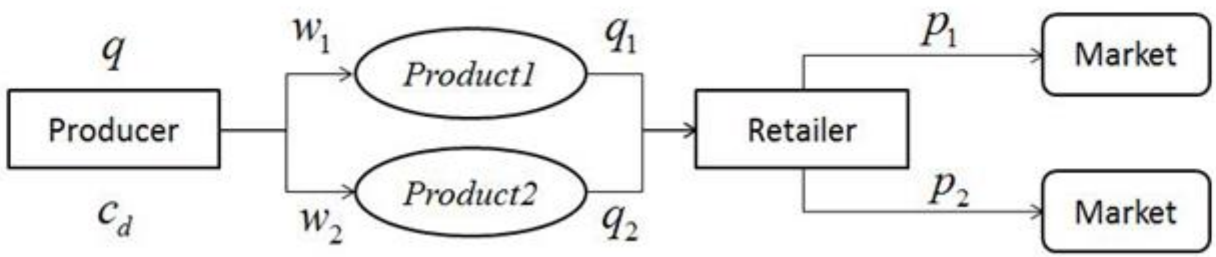
Assumption 1.

(Unbalancedness between supply and demand) $\frac{\mu_1 - v}{\lambda} > \frac{\mu_2}{1\text{-}\lambda}$

Given $v = 0$, Assumption 1 is equal to $\mu_1 : \mu_2 > \lambda_1 : (1 - \lambda)$, which means for each processed coproduct, the demand of product 1 is relatively larger than product 2. If $v > 0$, it generally reflects the unbalancedness between demand and supply among different parts of the processed coproduct. As to our case, product 1 is hot in the market whereas product 2 is cold. In reality, the producer or retailer sometimes has to stock certain amount of inventory of cold products passively for meeting the market demand of hot products.

Model Analysis

Optimal Decisions under P-chain Structure



Given the wholesale price (w_1, w_2), the retailer's objective function is

$$\pi_r^P(q_1, q_2 | w_1, w_2) = p_1 E[\min(q_1, D_1)] - w_1 q_1 + (p_2 - w_2)\min(\mu_2, q_2)$$

The producer's objective function can be described as

$$\pi_p^P(w_1, w_2) = w_1 q_1^*(w_1) + w_2 q_2^*(w_2) - c_d \max\left(\frac{q_1^*(w_1)}{\lambda}, \frac{q_2^*(w_2)}{1-\lambda}\right)$$

It's straightforward that $w_2^* = p_2$. Let $\bar{v} = \frac{\lambda p_1 \mu_1}{3\lambda p_1 - 4c_d}$. We then have the following proposition.

Proposition 1 *The optimal wholesale price of product 2 satisfies $w_2^* = p_2$, while the optimal wholesale price for product 1 satisfies*

(i) if $c_d > \frac{3}{4}\lambda p_1$, then $w_1^* = p_1$;

(ii) otherwise, if $v > \bar{v}$, then $w_1^* = \frac{1}{2}p_1$; and if $v \le \bar{v}$, then $w_1^* = p_1$.

Model Analysis

THEOREM 1. *The profits of the producer, the retailer and the supply chain under P-chain structure are as follows*

(i) If $c_d > \frac{3}{4}\lambda p_1$ *or if* $c_d \leq \frac{3}{4}\lambda p_1$ *and* $v \leq \bar{v}$*, then* $\pi_r^P = 0$ *and*

$$\pi_p^P = \pi_{chain}^P = (p_1 - \frac{c_d}{\lambda})(\mu_1 - v) + p_2\mu_2;$$

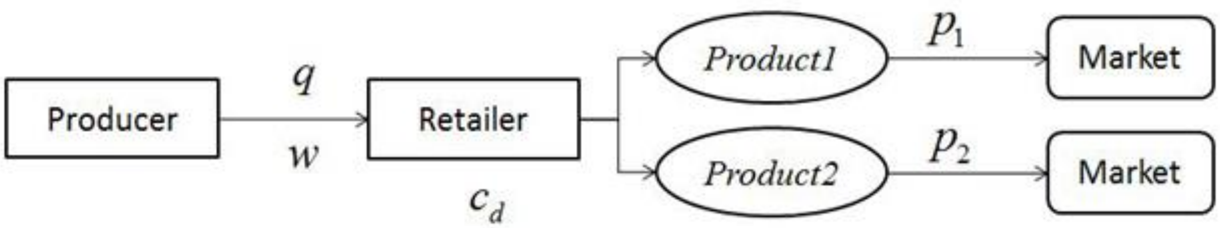
(ii) If $c_d \leq \frac{3}{4}\lambda p_1$ *and* $v > \bar{v}$*, then*

$$\pi_p^P = (\frac{p_1}{2} - \frac{c_d}{\lambda})(\mu_1 + v) + p_2\mu_2, \pi_r^P = \frac{1}{2}p_1(\mu_1 - v), \text{ and } \pi_{chain}^P = p_1\mu_1 + p_2\mu_2 - \frac{c_d}{\lambda}(\mu_1 + v).$$

- If ***the processing cost is high*** *or* ***the demand uncertainty of both products are low***, then the upstream producer will exploit all the chain profit by taking advantage of the first mover's advantage.
- By contrast, if ***the processing cost is low*** *and* ***the demand uncertainty of the hot product is high***, then the producer will let the retailer share the chain profit by adopting a low price strategy.

Model Analysis

Optimal Decisions under R-chain Structure



The retailer is responsible for processing the co-product into products 1 and 2. Her objective function can be expressed as follows:

$$\pi_r^R(q|w) = p_1 E[\min(D_1, \lambda q)] + p_2[\min(\mu_2, (1-\lambda)q)] - (w+c_d)q$$

$$\Rightarrow \pi_r^R(q|w) = \begin{cases} (\lambda p_1 - w - c_d)\frac{\mu_2}{1-\lambda} + p_2\mu_2 \text{ if } q = \frac{\mu_2}{1-\lambda}; \\ (\mu_1 - v)p_1 + p_2\mu_2 - (w+c_d)\frac{\mu_1 - v}{\lambda}, \text{ if } q = \frac{\mu_1 - v}{\lambda} \\ p_1\mu_1 + p_2\mu_2 - (w+c_d)\frac{\mu_1+v}{\lambda}, \text{ if } q = \frac{\mu_1+v}{\lambda}, \end{cases}$$

The producer's objective function can be expressed as:

$$\pi_p^R(w) = wq^*(w) = \begin{cases} \frac{\mu_2}{1-\lambda} w \text{ if } \lambda p_1 - c_d < w \leq \lambda p_1 + (1-\lambda)p_2 - c_d; \\ \frac{\mu_1 - v}{\lambda} w \text{ if } \frac{\lambda p_1}{2} - c_d < w \leq \lambda p_1 - c_d; \\ \frac{\mu_1+v}{\lambda} w \text{ if } w \leq \frac{\lambda p_1}{2} - c_d. \end{cases}$$

Model Analysis

Let $\rho_-(v) = \frac{\mu_1 - v}{\lambda} : \frac{\mu_2}{1-\lambda}$ and $\rho_+(v) = \frac{\mu_1 + v}{\lambda} : \frac{\mu_2}{1-\lambda}$. It's easy to verify that $\rho_-(v)$ is decreasing in v, while $\rho_+(v)$ is increasing in v. Then we have the following proposition.

PROPOSITION 2. *The producer's optimal wholesale price under R-chain structure satisfies*

(i) If $c_d > \frac{3}{4}\lambda p_1$ *or if* $c_d \le \frac{3}{4}\lambda p_1$ *and* $v \le \bar{v}$, *then*

(i)-1 $w^* = \lambda p_1 - c_d$ *if* $c_d \le \lambda p_1 - \frac{(1-\lambda)p_2}{\rho_-(v)-1}$,

(i)-2 $w^* = \lambda p_1 + (1-\lambda)p_2 - c_d$ *otherwise;*

(ii) If $c_d \le \frac{3}{4}\lambda p_1$ *and* $v > \bar{v}$, *then*

(ii)-1 $w^* = \frac{\lambda p_1}{2} - c_d$ *if* $c_d \le \lambda p_1 - \frac{\lambda p_1 \rho_+(v) + 2(1-\lambda)p_2}{2(\rho_+(v)-1)}$,

(ii)-2 $w^* = \lambda p_1 + (1-\lambda)p_2 - c_d$ *otherwise.*

- According to Propositions 2(i)-2 and 2(ii)-2, if the **processing cost is high**, then the producer will charge **an extremely high wholesale price** to the retailer.
- Otherwise, **a low wholesale price** strategy should be adopted, which may depends on the magnitude of **demand uncertainty**.

THEOREM 2. *The profits of the producer and retailer as well as the chain under R-chain structure are as follows*

(a) For cases (i)-2 & (ii)-2, we have $\pi_r^R = 0$ *and*

$$\pi_p^R = \pi_{chain}^R = (\lambda p_1 + (1-\lambda)p_2 - c_d)\frac{\mu_2}{1-\lambda}.$$

(b) For case (ii)-1, we have

$$\pi_p^R = (\frac{\lambda p_1}{2} - c_d)\frac{\mu_1 + v}{\lambda}, \pi_r^R = p_1\mu_1 + p_2\mu_2 - \frac{(\mu_1+v)p_1}{2}, \text{ and } \pi_{chain}^R = p_1\mu_1 + p_2\mu_2 - \frac{c_d}{\lambda}(\mu_1+v);$$

(c) For case (i)-1, we have

$$\pi_p^R = (\lambda p_1 - c_d)\frac{\mu_1 - v}{\lambda}, \pi_r^R = p_2\mu_2, \text{ and } \pi_{chain}^R = (p_1 - \frac{c_d}{\lambda})(\mu_1 - v) + p_2\mu_2.$$

- If the processing cost is very high, then the producer will exploit all the chain profit by setting a high price.
- If the **processing cost is extremely low while the demand uncertainty is high**, then the produce will charge a low price to incentivize the retailer to order more.
- If the processing cost is medium and the demand uncertainty is relatively low, then the producer will charge a relatively high price anchoring the net profit of the hot product.

Model Analysis

Comparisons between P-chain and R-chain

Let $\beta_-(v)=\lambda p_1-\frac{(1-\lambda)p_2}{\rho_-(v)-1}$, $\beta_+(v)=\lambda p_1-\frac{\lambda p_1\rho_+(v)+2(1-\lambda)p_2}{2(\rho_+(v)-1)}$ and $\delta=\frac{(1-\lambda)p_2}{\rho_+(v)-1}$, $(\delta>0)$.

$\boxed{\beta_-(v) \text{ is decreasing in v}}$

$\boxed{\beta_+(v) \text{ is increasing in v}}$

Table 2 Optimal decision under P-chain and R-chain structures

Scenario	$c_d>\frac{3}{4}\lambda p_1$ or $c_d\le\frac{3}{4}\lambda p_1$ and $v\le\bar{v}$		$c_d\le\frac{3}{4}\lambda p_1$ and $v>\bar{v}$	
Cost	$c_d\le\beta_-(v)$	$c_d>\beta_-(v)$	$c_d\le\beta_+(v)$	$c_d>\beta_+(v)$
P-chain				
q	$\frac{\mu_1-v}{\lambda}$	$\frac{\mu_2}{1-\lambda}$	$\frac{\mu_1+v}{\lambda}$	$\frac{\mu_1+v}{\lambda}$
q_1	μ_1-v	μ_1-v	μ_1+v	μ_1+v
q_2	μ_2	μ_2	μ_2	μ_2
w_1	p_1	p_1	$\frac{1}{2}p_1$	$\frac{1}{2}p_1$
w_2	p_2	p_2	p_2	p_2
R-chain				
q	$\frac{\mu_1-v}{\lambda}$	$\frac{\mu_2}{1-\lambda}$	$\frac{\mu_1+v}{\lambda}$	$\frac{\mu_2}{1-\lambda}$
q_1	μ_1-v	$\frac{\lambda}{1-\lambda}\mu_2$	μ_1+v	$\frac{\lambda}{1-\lambda}\mu_2$
q_2	$\frac{(1-\lambda)(\mu_1-v)}{\lambda}$	μ_2	$\frac{(1-\lambda)(\mu_1+v)}{\lambda}$	μ_2
w	λp_1-c_d	$\lambda p_1+(1-\lambda)p_2-c_d$	$\frac{1}{2}\lambda p_1-c_d$	$\lambda p_1+(1-\lambda)p_2-c_d$

Model Analysis

Comparisons between P-chain and R-chain

Table 3 The profits of P-chain and R-chain

Scenario	$c_d > \frac{3}{4}\lambda p_1$ or $c_d \le \frac{3}{4}\lambda p_1$ and $v \le \bar{v}$		$c_d \le \frac{3}{4}\lambda p_1$ and $v > \bar{v}$	
Cost	$c_d \le \beta_-(v)$	$c_d > \beta_-(v)$	$c_d \le \beta_+(v)$	$c_d > \beta_+(v)$
P-chain				
π_p^P	$(p_1 - \frac{c_d}{\lambda})(\mu_1 - v) + p_2\mu_2$	$(p_1 - \frac{c_d}{\lambda})(\mu_1 - v) + p_2\mu_2$	$(\frac{p_1}{2} - \frac{c_d}{\lambda})(\mu_1 + v) + p_2\mu_2$	$(\frac{p_1}{2} - \frac{c_d}{\lambda})(\mu_1 + v) + p_2\mu_2$
π_r^P	0	0	$\frac{1}{2}p_1(\mu_1 - v)$	$\frac{1}{2}p_1(\mu_1 - v)$
π_{chain}^P	$(p_1 - \frac{c_d}{\lambda})(\mu_1 - v) + p_2\mu_2$	$(p_1 - \frac{c_d}{\lambda})(\mu_1 - v) + p_2\mu_2$	$p_1\mu_1 + p_2\mu_2 - \frac{c_d}{\lambda}(\mu_1 + v)$	$p_1\mu_1 + p_2\mu_2 - \frac{c_d}{\lambda}(\mu_1 + v)$
R-chain				
π_p^R	$(\lambda p_1 - c_d)\frac{\mu_1 - v}{\lambda}$	$(\lambda p_1 + (1-\lambda)p_2 - c_d)\frac{\mu_2}{1-\lambda}$	$(\frac{\lambda p_1}{2} - c_d)\frac{\mu_1 + v}{\lambda}$	$(\lambda p_1 + (1-\lambda)p_2 - c_d)\frac{\mu_2}{1-\lambda}$
π_r^R	$\mu_2 p_2$	0	$p_1\mu_1 + p_2\mu_2 - \frac{(\mu_1+v)p_1}{2}$	0
π_{chain}^R	$(p_1 - \frac{c_d}{\lambda})(\mu_1 - v) + \mu_2 p_2$	$(\lambda p_1 + (1-\lambda)p_2 - c_d)\frac{\mu_2}{1-\lambda}$	$p_1\mu_1 + p_2\mu_2 - \frac{c_d}{\lambda}(\mu_1 + v)$	$(\lambda p_1 + (1-\lambda)p_2 - c_d)\frac{\mu_2}{1-\lambda}$

Model Analysis

Comparisons between P-chain and R-chain

Case (A): High Processing Cost or Low Demand Uncertainty

THEOREM 3. *If* $c_d > \frac{3}{4}\lambda p_1$ *or if* $c_d \leq \frac{3}{4}\lambda p_1$ *and* $v \leq \bar{v}$, *then we have*

(i) $\pi_p^P > \pi_p^R$;

(ii) $\pi_r^P < \pi_r^R$ *if* $c_d \leq \beta_-(v)$, *whereas* $\pi_r^P = \pi_r^R$ *if* $c_d > \beta_-(v)$;

(iii) $\pi_{chain}^P = \pi_{chain}^R$ *if* $c_d \leq \beta_-(v)$, *while* $\pi_{chain}^P > \pi_{chain}^R$ *if* $c_d > \beta_-(v)$

Case (B): Low Processing Cost and High Demand Uncertainty

THEOREM 4. *If* $c_d \leq \frac{3}{4}\lambda p_1$ *and* $v > \bar{v}$, *then*

(i) $\pi_p^P > \pi_p^R$ *when* $c_d < \beta_+(v) + \delta$ *whereas* $\pi_p^P < \pi_p^R$ *when* $c_d > \beta_+(v) + \delta$;

(ii) $\pi_r^P < \pi_r^R$ *when* $c_d \leq \beta_+(v)$ *whereas* $\pi_r^P > \pi_r^R$ *when* $c_d > \beta_+(v)$;

(iii) $\pi_{chain}^P = \pi_{chain}^R$ *when* $c_d \leq \beta_+(v)$ *whereas* $\pi_{chain}^P > \pi_{chain}^R$ *when* $c_d > \beta_+(v)$.

Model Analysis

Comparisons between P-chain and R-chain

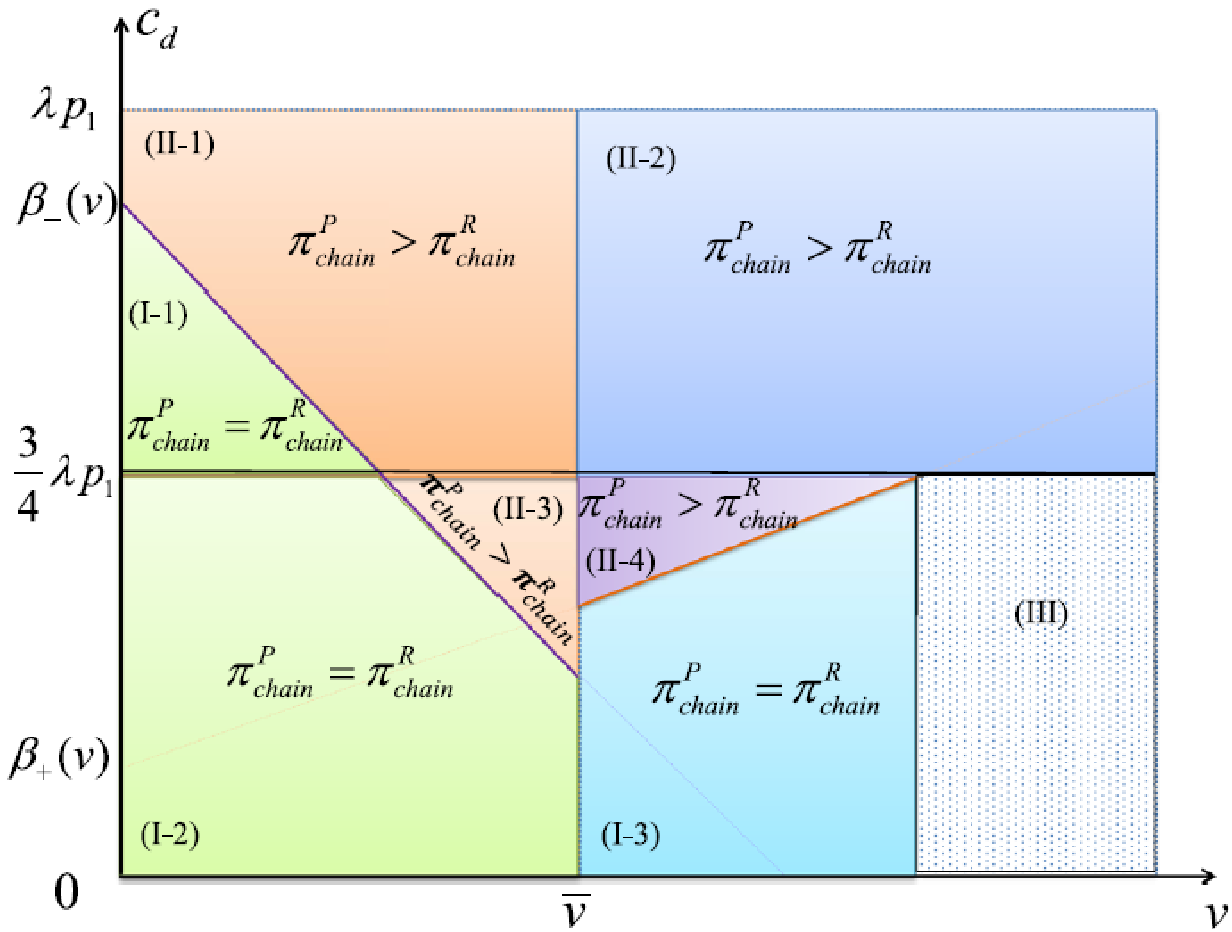


Figure 3: Chain performance comparisons

Model Analysis

Comparisons between P-chain and R-chain

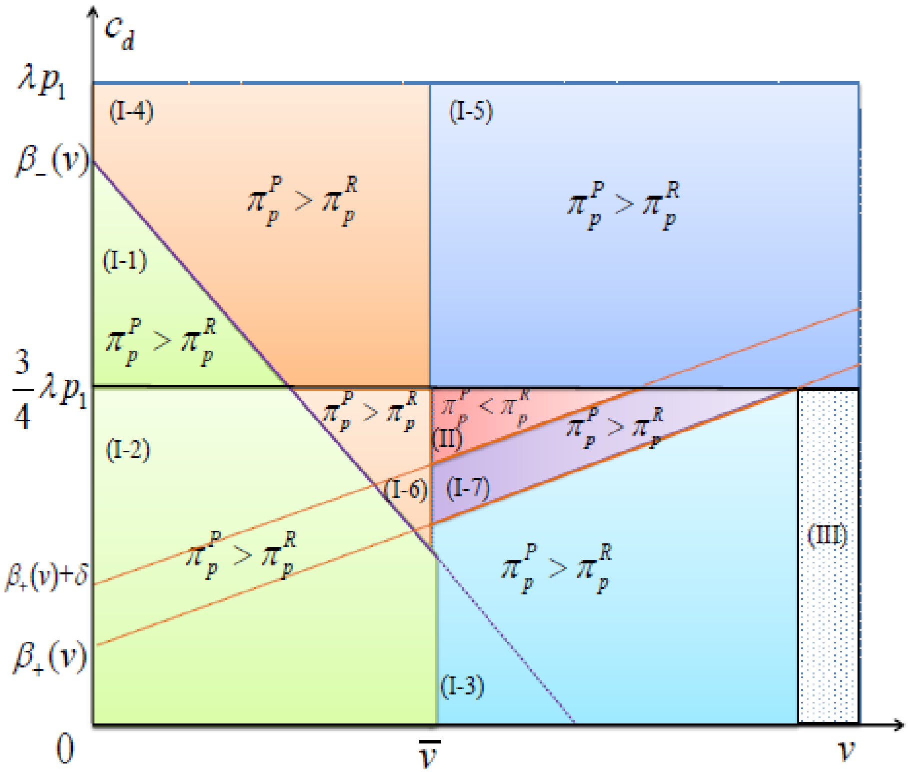


Figure 4: Producer's performance comparisons

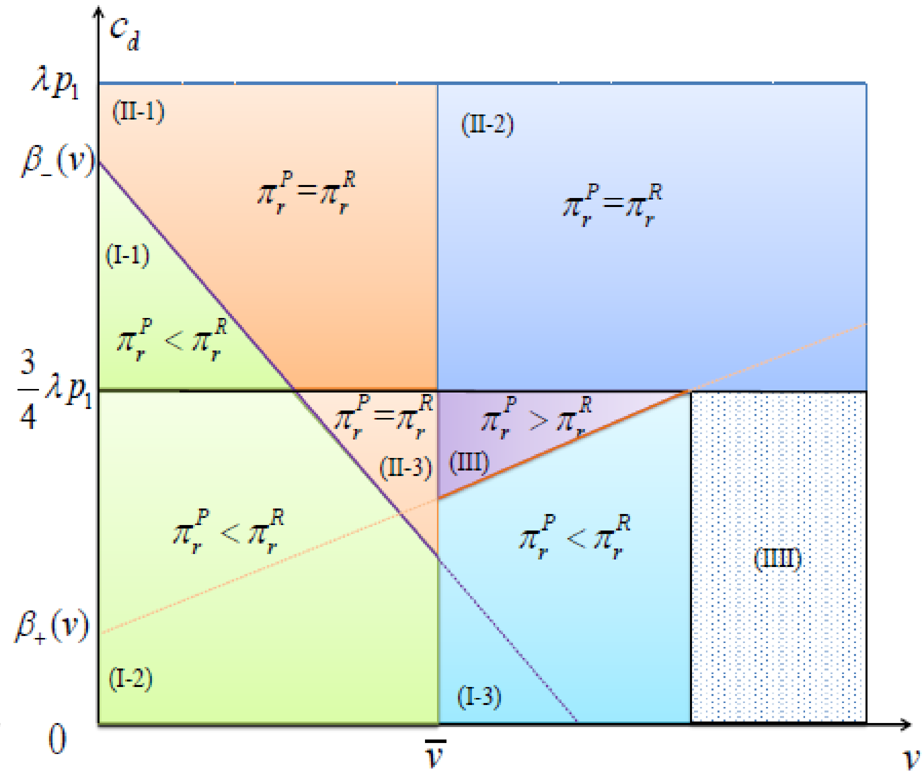


Figure 5: Retailer's performance comparisons

Model Extensions

➢General demand distributions

In this subsection, we first give the existence conditions of the optimal order quantity q_1 and q_2 as well as q, when the two final products face demands D_1 and D_2. Let $F_i(\cdot)$ be the cumulative distribution function of part i and $f_i(\cdot)$ be the probability density function, $i \in \{1, 2\}$, respectively. Define $h_i(x) = \frac{x f_i(x)}{1-F_i(x)}$, the general failure rate function.

P-chain:

$$\begin{aligned}\max \pi_r^P(q_1, q_2|w_1, w_2) &= p_1 E\min(q_1, D_1) + p_2 E\min(q_2, D_2) - w_1 q_1 - w_2 q_2 \\ &= -p_1 \int_0^{q_1} F_1(x)dx - p_2 \int_0^{q_2} F_2(x)dx + p_1 q_1 + p_2 q_2 \\ &\quad - w_1 q_1 - w_2 q_2,\end{aligned}$$

R-chain:

$$\begin{aligned}\max \pi_r^R(q|w) &= p_1 E\min(q_1, D_1) + p_2 E\min(q_2, D_2) - c_d q - wq \\ &= -p_1 \int_0^{\lambda q} F_1(x)dx - p_2 \int_0^{(1-\lambda)q} F_2(x)dx + p_1 \lambda q + p_2 (1-\lambda) q \\ &\quad - c_d q - wq,\end{aligned}$$

Model Extensions ------General demand distributions

Theorem 6 *Suppose Assumption 2 holds. Then, in the R-chain, $\pi_p^R(q)$ is concave in q and there exists a unique optimal order quantity q^* satisfying $\lambda p_1 G_1(\lambda q) + (1-\lambda) p_2 G_2[(1-\lambda)q] = c_d + c_p$.*

Specifically, the demand for product 1 is $D_1 \sim U(\mu_1 - v, u_1 + v)$. Clearly, the average demand for product 1 is μ_1, while the variance is $\frac{1}{3}v^2$. Also, we assume that the demand for product 2 is μ_2, which is deterministic. In the P-chain, it's straightforward that $w_2^* = p_2$. Let $\bar{v}_l = \frac{\mu_1 \lambda p_1}{3\lambda p_1 - 2c_d - 4\lambda\varepsilon}$ and $\bar{v}_h = \frac{\mu_1 \lambda p_1}{\lambda p_1 - 2c_d}$.

Define $\varphi = \frac{p_1}{4v}(\frac{\mu_1 + v}{2} + \frac{v c_d}{\lambda p_1})^2$ and $\psi(c_d, \lambda, \mu_1) = \frac{v(\lambda p_1 - 2c_d) - \mu_1 \lambda p_1}{2\lambda}$. We then have the following theorem.

Theorem 7 *By comparing the P-chain and the R-chain, we have*

(i) $\pi_p^P \geq \pi_p^R$, $\pi_r^P \geq \pi_r^R$, $\pi_{chain}^P \geq \pi_{chain}^R$, *when* $v \leq \bar{v}_l$;

(ii) $\pi_p^P > \pi_p^R$, $\pi_{chain}^P \geq \pi_{chain}^R$, *when* $v > \bar{v}_l$; $\pi_r^P \geq \pi_r^R$, *if* $\bar{v}_l < v < \bar{v}_h$ *and* $\psi(c_d, \lambda, \mu_1) > \mu_2 p_2$ *and* $\pi_r^P < \pi_r^R$ *otherwise.*

Model Extensions ------General demand distributions

$$h_i(x) = \frac{x f_i(x)}{1 - F_i(x)}$$

Assumption 2 $h_i(x)$ is increasing in $(0, \infty)$ (IGFR), and $G_i(x) = \bar{F}_i(x) - x f_i(x)$ is strictly decreasing in x.

Theorem 5 *Let $m = \frac{c_p + c_d}{\lambda p_1}$ and $n = \frac{c_p + c_d}{(1-\lambda) p_2}$. Suppose Assumption 2 holds. Then, in P-chain, we can obtain the optimal q_1^* and q_2^* as follows:*

(i) $q_1^* = G_1^{-1}(0)$, $q_2^* = G_2^{-1}(n)$, *when* $\lambda G_2^{-1}(n) \geq (1-\lambda) G_1^{-1}(0)$;
(ii) $q_1^* = G_1^{-1}(m)$, $q_2^* = G_2^{-1}(0)$, *when* $(1-\lambda) G_1^{-1}(m) \geq \lambda G_2^{-1}(0)$;
(iii) $q_1^* = \lambda q^*$, $q_2^* = (1-\lambda) q^*$, *when* q^* *satisfying* $\lambda p_1 G_1(\lambda q) + (1-\lambda) p_2 G_2[(1-\lambda) q] = c_d + c_p$;
(iv) $q_1^* = G_1^{-1}(0)$, $q_2^* = G_2^{-1}(0)$, *otherwise.*

Theorem 5 shows the retailer's optimal order quantity decisions in different situations. Specifically, case (i) indicates that the processing is based on product 1's demand, with oversupply for product 2. In contrast, case (ii) shows processing is meeting product 2's demand with oversupply for product 1. There is a perfect match for the processing since the demand ratio is equal to the fix fraction between the supplies of product 1 and product 2. Case (iv) shows the firm should oversupply both products, which is the best choice of the producer.

Model Extensions

➤**With κ coproducts**

P-chain

Retailer:

$$\max_{i=1,2,\ldots,k} \pi_r^P(q_i|w_i) = \sum_{i=1}^{k} p_i E\min(q_i, D_i) - \sum_{i=1}^{k} w_i q_i =$$

$$-\sum_{i=1}^{k} p_i \int_0^{q_i} F_i(x)dx + \sum_{i=1}^{k} p_i q_i - \sum_{i=1}^{k} w_i q_i$$

Producer:

$$\max_{i=1,2,\ldots,k} \pi_p^P(w_i) = \sum_{i=1}^{k} w_i q_i - (c_p + c_d) \max_{i=1,2,\ldots,k} \left\{ \frac{q_i}{\lambda_i} \right\}$$

R-chain

Retailer:

$$\max \pi_r^R(q|w) = \sum_{i=1}^{k} p_i E\min(\lambda_i q, D_i) - (c_d + w)q$$

$$= -\sum_{i=1}^{k} p_i \int_0^{\lambda_i q} F_i(x)dx + \sum_{i=1}^{k} p_i \lambda_i q - (c_d + w)q$$

Producer:

$$\max \pi_p^R(w) = wq - c_p q$$

Model Extensions ------With k coproducts

Assumption 3 (*Unbalancedness between supply and demand*) Suppose $\frac{\mu_1 - v}{\lambda_1} > \frac{\mu_2}{\lambda_2} > \cdots > \frac{\mu_k}{\lambda_k}$, and $\sum_{i=1}^{k} \lambda_i = 1$.

For any $k \geq j > 1$, we have $q_j^*(w_j) = u_j$ if $w_j \leq p_j$ since demand for product j is deterministic. As a result, in the P-chain, Eqs. (7) and (8) can be simplified as

$$\max \pi_r^P(q_1|w_1) = p_1 E \min(q_1, D_1) - \sum_{i=2}^{k} w_i q_i,$$

$$\text{and } \max \pi_p^P(w_1) = w_1 q_1 + \sum_{i=2}^{k} w_i q_i - c_d \frac{q_1}{\lambda}.$$

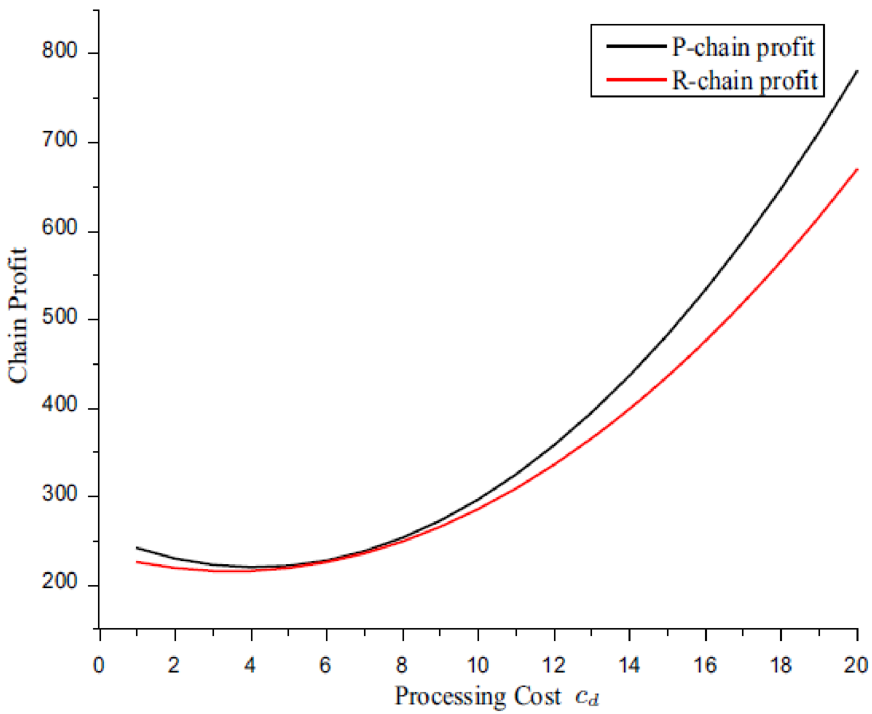
Similarly, in the R-chain, the Eq. (9) can be simplified as

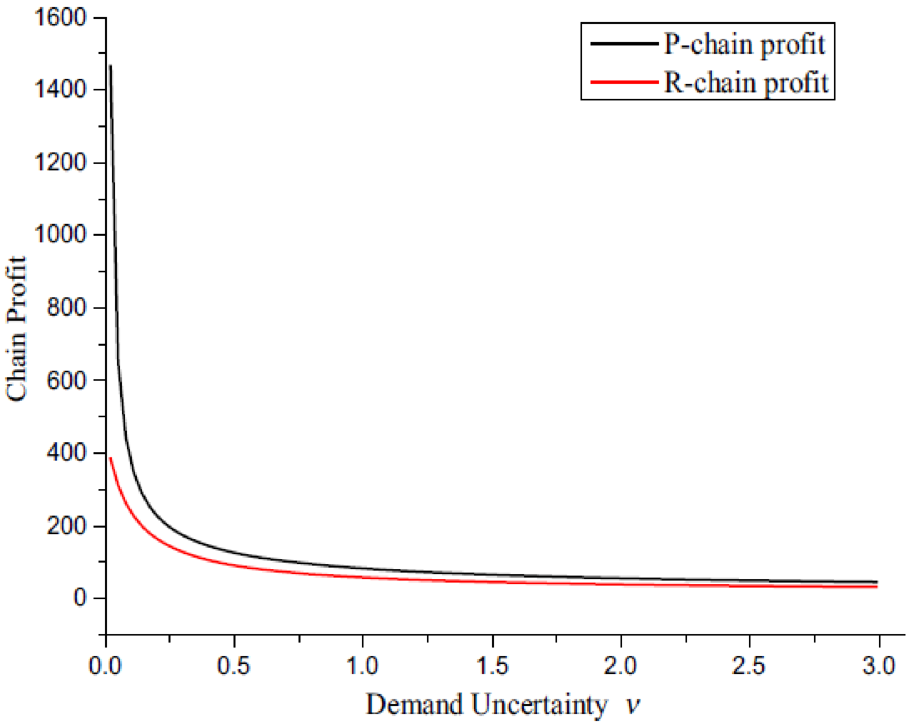
$$\max \pi_r^R(q|w) = p_1 E[D_1 \wedge (\lambda_1 q)] + \sum_{i=2}^{k} p_i E \min(\lambda_i q, D_i) - (c_d + w)q.$$

Based on Assumption 3, the main results in Propositions 1 and 2 and Theorems 1 and 2 hold. This is because, while product 1 faces the uncertain demand with a two-point distribution, the other $k-1$ products can be regarded as a whole with a deterministic demand.

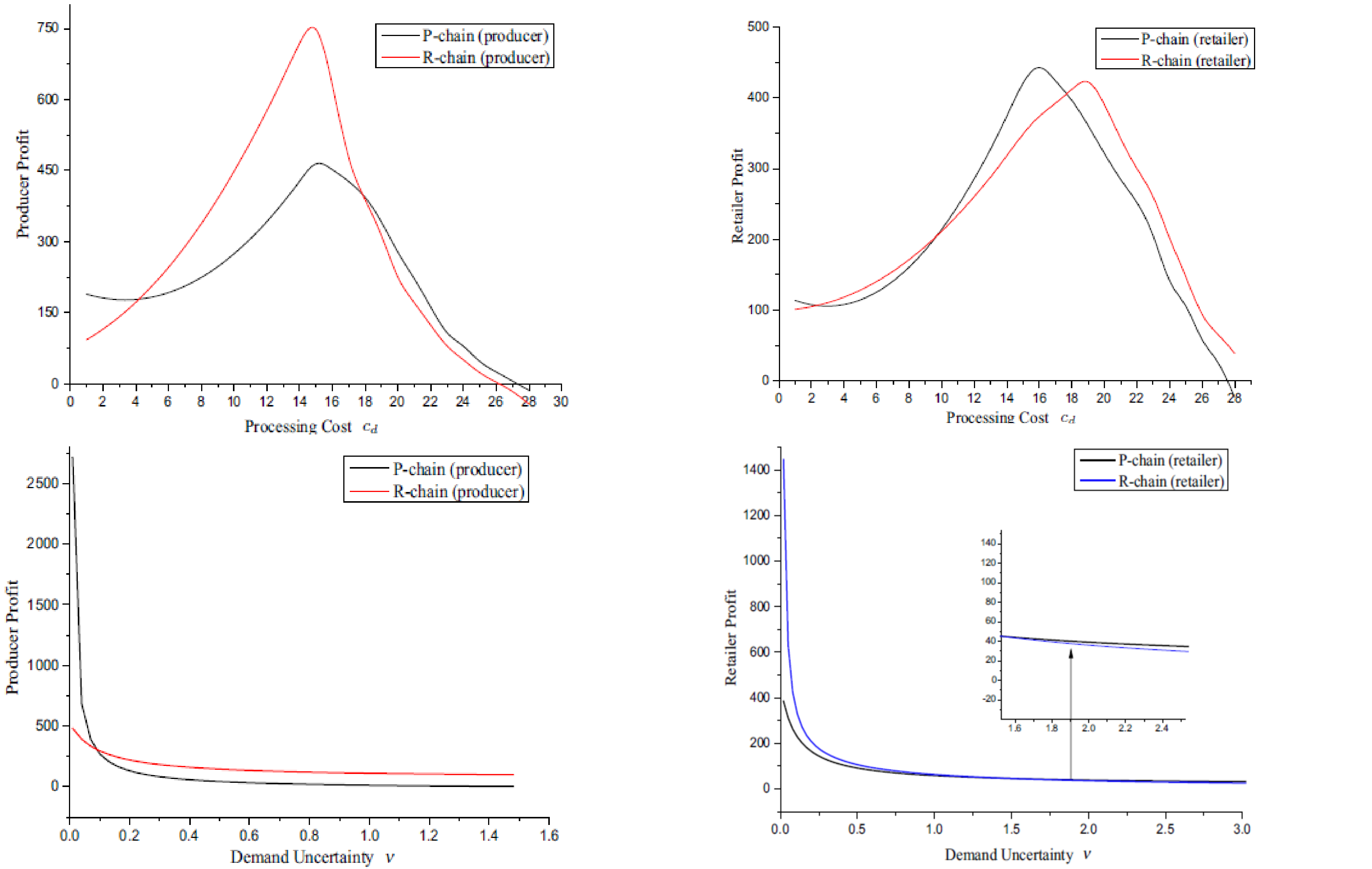
Numerical Examples

For the key parameters c_d and v, we assume that c_d varies from 1 to 20 and v varies from 0.01 to 5. Other parameters are set as follows: $p_1 = 10$; $p_2 = 6$; $u_1 = 5$; $u_2 = 3$; and λ=0.3

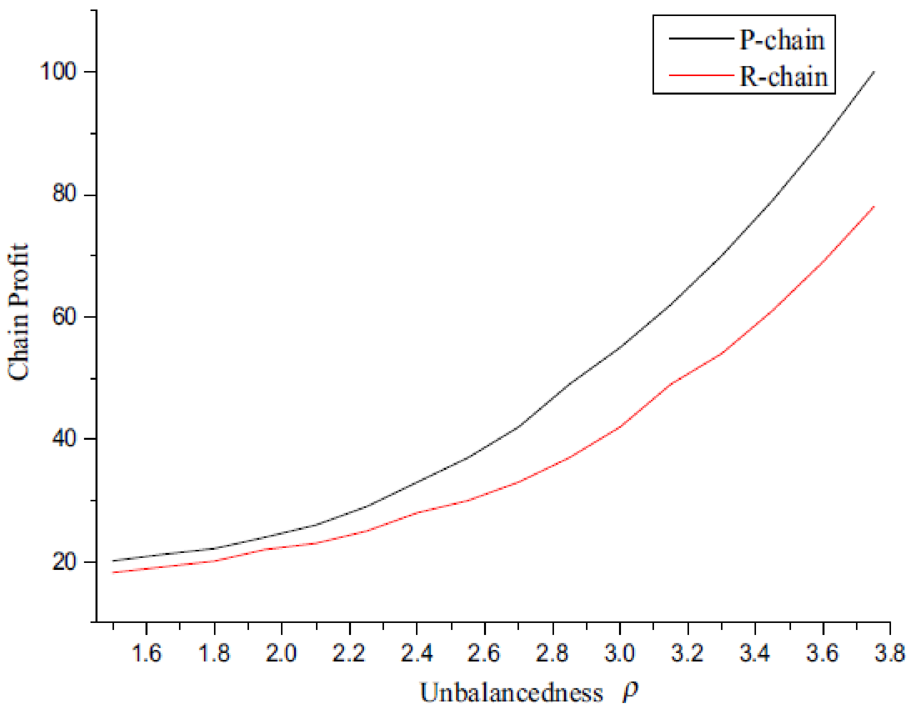


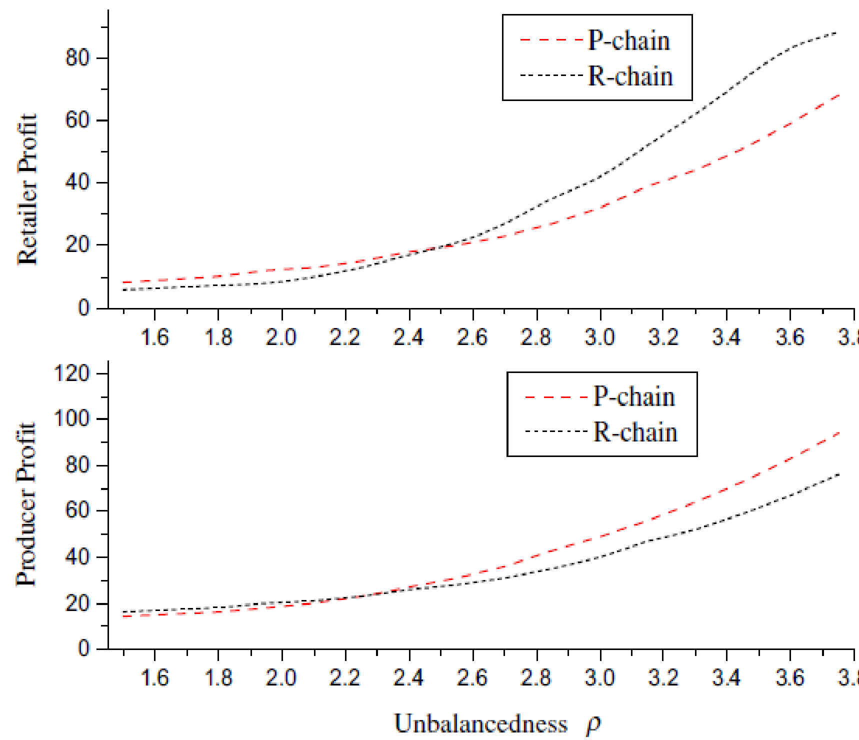


Numerical Examples



Numerical Examples





Numerical Examples

Last, but not the least, we conduct a numerical experiment when both demands of products 1 and 2 face uncertainty. Specifically, we assume that demands D_1 and D_2 satisfy normal distributions with $N(u_1, v_1)$ and $N(u_2, v_2)$ respectively. Other parameters are set as follows: $c_p = 0.2, c_d = 0.1, p_1 = 4, p_2 = 2$, and $\lambda = 0.45$.

Table 4 Profits under the P-chain and R-chain structure

D_1	D_2	P-chain			R-chain			Difference		
$N(u_1, v_1)$	$N(u_2, v_2)$	π_p^P	π_r^P	π_{chain}^P	π_p^R	π_r^R	π_{chain}^R	Δ_p	Δ_r	Δ_{chain}
(1.5, 0.2)	(0.8, 0.5)	12.74	7.37	20.11	11.45	7.57	19.02	1.29	−0.2	1.09
(1.5, 0.4)	(0.8, 0.7)	11.32	6.85	18.17	10.87	7.05	17.97	0.45	−0.2	0.20
(1.5, 0.6)	(0.8, 0.9)	10.15	7.47	17.62	10.02	6.47	16.49	0.13	1.00	1.13
(1.5, 0.8)	(0.8, 1.1)	9.42	7.08	16.50	9.51	4.70	14.21	−0.09	2.38	2.29
(1.5, 1.0)	(0.8, 1.3)	8.21	6.71	14.92	9.12	1.96	11.08	−0.91	4.75	3.84
(1.2, 1.2)	(1.6, 1.5)	7.98	5.23	13.21	8.46	1.06	9.52	−0.48	4.17	3.69
(1.2, 1.4)	(1.6, 1.7)	6.47	5.61	12.08	6.91	2.03	8.94	−0.44	3.58	3.14
(1.2, 1.6)	(1.6, 1.9)	4.62	6.31	10.93	5.37	2.36	7.73	−0.75	3.95	3.20
(1.2, 1.8)	(1.6, 2.1)	3.99	4.97	8.96	4.12	2.06	6.18	−0.13	2.91	2.78

Conclusion

We use the **unbalanced ratio** to measure the basic level of mismatch and study how the **tradeoff between bargaining power** and **the mismatch cost**, by different mismatch risk allocations, influences the optimal decisions and the performances.

- *P-chain* dominates *R-chain* from the perspective of the whole supply chain;
- Upstream producer is not always better off in the *P-chain* even he bears more mismatch risks;

Q&A

Thank You