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Mismatch risk allocation in a coproduct supply chain

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Abstract

Products such as cattles and pigs can be processed into several types of products (parts) targeting different segments of customers, which belong to the so called *coproducts*. Mismatch risk is a significant issue in such coproduct supply chains. Under the Stackelberg game setting, we consider a coproduct supply chain consisting of one producer acting as the leader and one retailer being the follower and establish a stylized model to study how the mismatch risk should be allocated. Two supply chain modes are considered, i.e., the *P-chain* mode under which the producer is responsible for the processing activity and hence holds the mismatch risk, and the *R-chain* under which the retailer is responsible for the processing activity. We use the unbalanced ratio to reflect the degree of mismatch between supply and demand among different parts of the coproduct and study how the tradeoff between the bargaining power and the mismatch cost, by different mismatch risk allocations, influences the optimal decisions and the performances of the two parties as well as the whole supply chain. Our main findings include: (1) P-chain dominates R-chain from the perspective of the chain performance; and (2) the upstream producer is not always better off in the P-chain under which he bears more mismatch risk. Numerical study shows the robustness of our main results and further studies the effect of demand uncertainty and the processing cost on the performance of P-chain as compared to R-chain.

Keywords Coproduct · Supply chain design · Mismatch risk management

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1 Introduction

Coproducts are those differentiated products being produced simultaneously (e.g., fresh produce, recycled goods, etc.) to meet the needs of different types of customers. In reality, some firms use one kind of raw material to produce many kinds of products and the production quantity of each kind will affect the ordering policy of the raw material, as well as firms' profits. These products are typically obtained in uncontrollable fractions, leading to mismatches between their demand and supply. Moreover, the demand uncertainty in the consumer market will amplify the mismatch risk. Therefore, one of the main challenges facing those industries is to balance the mismatch risk and its corresponding revenue, in addition to the production cost widely considered in traditional industries.

1.1 Practical cases: COFCO versus Techbank

We investigate two firms (COFCO and Techbank) in the pork supply chain. Basically, the two firms have different supply chain structures. COFCO covers both the pig breeding stage and the pork processing activities, while TechBank mainly focuses on the breeding but leaves the processing activities to downstream players. Figure 1 below shows the decision sequences of both firms. Using a simple case (the pig is only processed into two different final products), we illustrate how the two firms make decisions. For COFCO, it breeds the pigs and processes them into two coproducts, and then decides on the wholesale prices w_1 and w_2 for each coproduct. The retailer thus decides on the order quantities q_1 and q_2 for each coproduct separately. In contrast, for Techbank, it only decides on the unit wholesale price w for the pork as a whole. As a consequence, the retail decides on the total order quantity q and processes the pork into two coproducts targeting differentiated markets.

Figure 2 above illustrates the unbalancedness between supply and demand among different parts of a pig. According to the statistics of a typical day in December 2017, we can see the demand of the hind part (1502 units) is only roughly half of the demand of the front and middle parts (3102 units and 2947 units). As a result, it is a big challenge for COFCO to decide how many pigs should be slaughtered and processed into different final products to

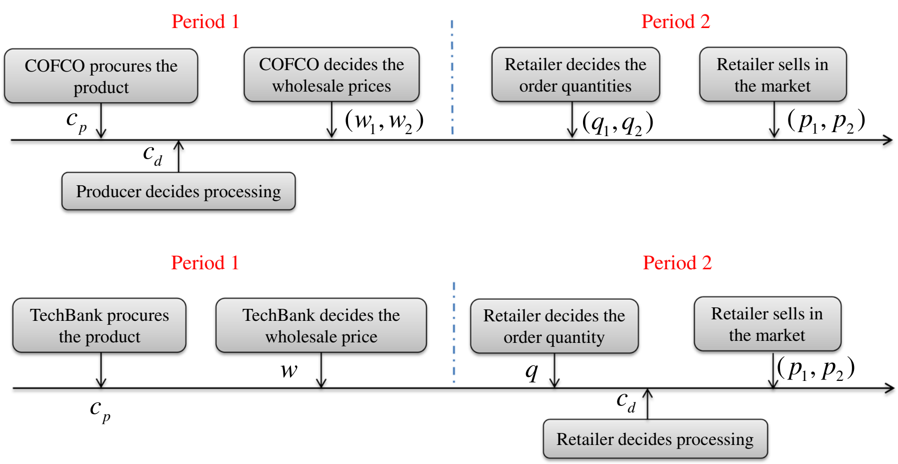
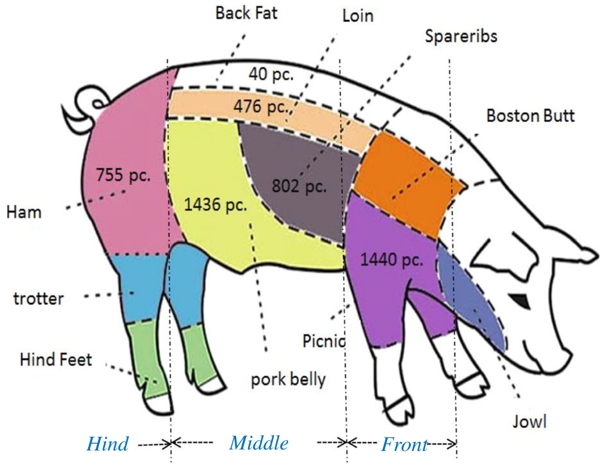


Fig. 1 Sequences of event: COFCO versus TechBank



Part	Demand（unit）
Front	3,102
Middle	2,947
Hind	1,502

Middle	Demand（unit）
Pork belly	1,436
Spareribs	802
Loin	476
Back Fat	40
others	194

Order in December 21, 2017——(data from: COFCO)

Fig. 2 Demand of different parts of the pig

Table 1 Weekly supply-demand of COFCO in Dec, 2017

Items (1 unit = half a pig)	Thu.	Fri.	Sat.	Sun.	Mon.	Tue.	Wed.
Initial inventory	3152	2582	1653	1893	2513	2574	2759
Actual processing quantity	1300	1300	1248	1300	1300	1300	1400
Demand on product itself	1900	2069	2169	1872	1881	1839	1974
Newly production quantity	2630	2440	3657	3792	3242	3324	3572
End inventory	2582	1653	1893	2513	2574	2759	2957
Forecast processing quantity	1953	1248	1445	1741	1696	1727	2064
Forecast(t − 1)-Actual(t)	–	653	0	145	441	396	327

meet the customers' need with different tastes. As can be seen from Table 1, the gap between the planned (forecast) processing quantity and actual processing quantity varies from 0 to 653, which in turn incurs a big loss due to the mismatch risk between the supply and demand among different coproducts.

1.2 Main findings

As a typical coproduct supply chain, the pork supply chain has been subjected to extensive vertical integration initiatives. Firms like Yurun Group and COFCO cover both the feeding activity and the pork processing activities, while firms like TechBank mainly focuses on feeding while leaves the processing activity to downstream parties. From the perspective of mismatch risk management, when a firm covers both the feeding and processing activities, it takes the mismatch risk itself, while when a firm focuses on feeding, it leaves the mismatch risk to the downstream parties. However, which party in the supply chain should and when it is valuable for him to take the mismatch risk, and how the demand uncertainty influences the optimal mismatch allocation still remain unclear. This paper will try to fill this gap by investigating the mismatch risk allocation issue in a coproduct supply chain and provide some preliminary answers to those questions thus raised.

Specifically, we build a stylized model by considering a coproduct supply chain consisting of one producer and one retailer, under a Stackelberg game setting in which the producer is the leader and the retailer is the follower. Two supply chain modes, depending on who takes the mismatch risk by functioning the processing activity, are considered: the *P-chain* when the producer takes the risk and the *R-chain* when the retailer bears the risk, respectively. To simplify the analysis, in the basic setting we assume that only two types of products are produced with a pre-determined fixed ratio, and one product has a deterministic demand while the other faces an uncertainty market demand. We derive the equilibrium decisions for those two supply chains and study the preferences of both parties (i.e., the producer and the retailer), as well as the whole supply chain. We also conduct a thorough check on the robustness of our managerial insights under more general settings through either numerical examples or theoretical analysis.

Basically, we find that P-chain dominates R-chain in terms of the chain's total expected profit. This is consistent with our conventional wisdom that in a supply chain with the supplier bearing more risk, the retailer's profit margin becomes thinner, resulting in a reduction of the double marginalization effect. Moreover, we show that the advantage of P-chain is more significant when the processing cost is higher, which implies a a larger mismatch risk. However, the upstream producer is not always better off in the P-chain although he bears more mismatch risk, a stark contrast to the traditional single product supply chain. Specifically, we show that when the unbalanced ratio or the demand uncertainty is higher, the producer is worse off while the retailer is better off under the P-chain structure. The underlying reason lies in the tradeoff between the mismatch cost and the bargaining power. That is, when the demand uncertainty or the unbalanced ratio is high, the mismatch cost exceeds the benefit taken by the bargaining power as the producer bears more risk. Numerical study further shows how the demand uncertainty and the processing cost influence the relative advantage of P-chain as compared to R-chain.

The rest of the paper is organized as follows. Section 2 reviews the relevant literature. Section 3 describes the problem and develops models. Section 4 presents the comparison results between different supply chain modes. Some extensions are made in Sect. 5. We also study the effect of system parameters and make a comparison through numerical experiment in Sect. 6. Section 7 concludes the paper.

2 Literature review

Our work is most related to those research studying the operations management issues in a coproduction system. Much of the works in this stream focused on the decision optimization from the perspective of a monopoly firm to control the mismatch cost for the coproducts, by the production and downward substitution decisions (Bitran and Gilbert 1994; Nahmias and Moinzadeh 1997), the pricing decisions (Tomlin and Wang 2008; Bansal and Transchel 2014), the product-line design decision (Chen et al. 2013; Transchel et al. 2016), the joint procurement and production decisions in the presence of input and/or output spot markets (Boyabatli et al. 2017; Dong et al. 2014; Boyabatli 2015). Sunar and Plambeck (2016) focus on how process emissions should be allocated among coproducts when emissions are taxed for environmental reasons. Chen et al. (2013) study the optimal adoption-and-usage strategy of a firm with one incumbent technology considering a new technology. However, few work has considered operations management issues in a coproduction system from the perspective of supply chain. Thus, our work contributes to this stream by extending existing works to a supply chain consisting an upstream producer and a downstream retailer and by studying the mismatch risk allocation issues other than mismatch risk control issues.

The processing activity in our paper is assumed to coproduce multiple products at fixed fractions and thus is a kind of partially flexible resources. Thus, our work is also related to research on operations flexibility, whereby a resource (inventory or capacity) has the ability to produce multiple products. Three forms of flexibility are considered in literature, namely, totally flexible, mix flexible and dedicated. Fine and Freund (1990), Gupta et al. (1992), Li and Tirupati (1994, 1995, 1997), Van Mieghem (1998) mainly focus on the investment decisions in dedicated or totally flexible resources. Jordan and Graves (1995) and Graves and Tomlin (2003) investigate how the structure of partial flexibility in supply chains influences chain performance. Van Mieghem (2004) investigates a mix flexible resource investment problem in perfectly reliable newsvendor networks. Tomlin and Wang (2005) study the value of mix flexibility and dual sourcing in an unreliable supply chains that produce multiple products. Different from those existing researches, our paper, by allocating the processing activity among supply chain parties, studies the value of different allocation policies in a supply chain. However, to the best of our knowledge, few extant research has been devoted to that issue, and our work is pioneer in shedding some light on it in a coproduct supply chain.

The third stream related to our work is those studying inventory risk allocation by pull or push contract. Cachon (2004) studies the allocation of inventory risk in the supply chain under pull and push contracts. Dong and Zhu (2007) further study a two-wholesale-price contract and show that retailer can benefit under the push contract. Granot and Yin (2008) show that the value of push contract is decreasing in the number of alliances of the suppliers in an assembly system. Yin (2010) examines the complementary suppliers' incentives to coordinate prices via alliance formation when selling their products to a downstream buyer who faces price-sensitive demand in an assembly system. Wang et al. (2014) compare two outsourcing structures under push and pull contracts in a three-tie supply chain. Davis et al. (2014) employ behavior experiment to show that a pull contract achieves higher channel efficiency. The traditional insight from this literature is that the pull contract may lead to higher supply chain efficiency, larger suppier's profit and lower retailer's profit. However, our work focus on the allocation of mismatch risk that is specific in the coproduct system and we will show that those traditional insights only partially hold.

Another related direction is the product sorting, under which decisions are conducted based on different attributes of products, e.g., quality (Yayla et al. 2013; Ferguson et al. 2010), shelf-life (Gilland and Heese 2013) and other attributes (Honhon and Pan 2017; Eric and Sabyasachi 2014). The complexity of managing a category assortment has grown tremendously due to the increased product turnover and proliferation rates in most categories(Chong et al. 2001). In the assortment planning problem, a firm chooses the products from a discrete set of potential products to offer to consumers with heterogeneous preferences for the products (Pan and Honhon 2012). Galbreth and Blackburn (2006) develop a model explicitly considering variable used product condition and examine how acquisition and sorting decisions affect remanufacturing costs. However, our model is mainly to consider product processing activity to investigate the supply and demand mismatch risk allocation, rather than make optimal decisions according to product sorting attributes.

To conclude, our work contributes to those mentioned works above by investigating the mismatch risk allocation issues in a supply chain. Different from those existing research, our paper, by allocating the processing activity among different supply chain parties, studies the value of different allocations of flexibility in a supply chain. Our work is pioneer in shedding some light on mismatch risk allocation in a coproduct supply chain.

3 Model descriptions

We consider a coproduct supply chain consisting of a single producer and a single retailer. The producer procures the product (e.g., a cattle) at unit cost c_p. The product is then decomposed and processed into two types of products either by the producer or the retailer. The processing cost is c_d for each product. One unit product can be processed into λ unit product 1 and $(1-\lambda)$ unit product 2. The demand of product 1 is uncertain following a two-point distribution. Specifically, the demand for product 1 could be either $\mu_1 - v$ or $\mu_1 + v$ with equal probability $\frac{1}{2}$. Clearly, the average demand for product 1 is μ_1, while the variance is v^2. To keep our model parsimonious, we assume that the demand for product 2 is μ_2, which is deterministic. Let $\rho := \frac{\mu_1}{\lambda} : \frac{\mu_2}{1-\lambda}$. ρ is actually the mean unbalanced supply-demand ratio. The larger the ρ, the more unbalanced between supply and demand.

The selling prices of products 1 and 2 are p_1 and p_2, which are exogenously determined by the market. For the tractability of our model, we give the following assumption.

Assumption 1 (*Unbalancedness between supply and demand*) $\frac{\mu_1 - v}{\lambda} > \frac{\mu_2}{1-\lambda}$.

Given $v = 0$, Assumption 1 is equal to $\mu_1 : \mu_2 > \lambda : (1-\lambda)$ which means for each processed coproduct, the demand of product 1 is relatively larger than product 2. If $v > 0$, it generally reflects the unbalancedness between demand and supply among different parts of the processed coproduct. As to our case, product 1 is hot in the market whereas product 2 is cold. In reality, the producer or retailer sometimes has to stock certain amount of inventory of cold products passively for meeting the market demand of hot products.

3.1 Model settings

This paper mainly considers two kinds of supply chain structure, namely P-chain structure and R-chain structure. The main different decision sequences between the two models are as follows (see Fig. 3).

P-chain: The producer is responsible for processing the product. As a result, he will undertake the unbalanced risk between supply and demand and the downstream retailer will undertake the demand risk due to uncertainty. The producer decides on the selling prices w_1 and w_2 of products 1 and 2 jointly. The retailer then decides on the order quantities q_1 and q_2 for the two products, to be sold at price p_1 and p_2 in the market.

R-chain: The retailer is responsible for processing the product and hence bears the decomposing cost.The producer decides on the wholesale price w for each coproduct. As a sequel, the retailer decides on the order quantity q. Therefore both the unbalanced risk between supply and demand among products and the risk of demand uncertainty (recall that the demand of product 1 is random) are undertaken by the retailer.

3.2 Optimal decisions under P-chain structure

We use the backward induction to derive the optimal decisions made by the producer and the retailer. For any given wholesale price (w_1, w_2) provided by the producer, the retailer's objective function is

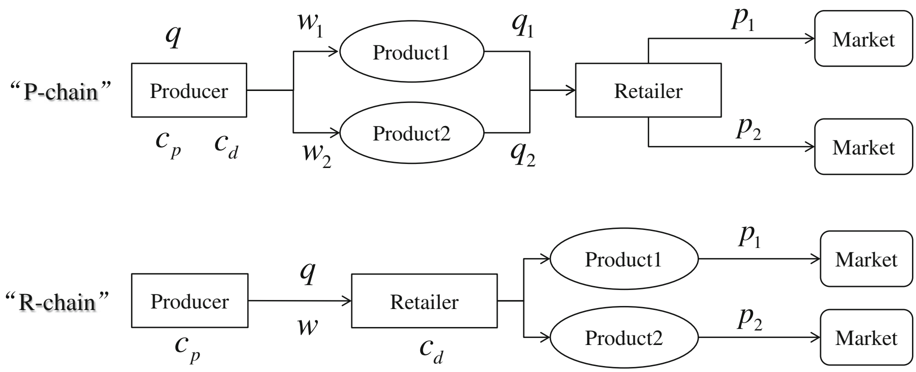


Fig. 3 P-chain and R-chain structure

$$\pi_r^P(q_1, q_2|w_1, w_2) = p_1 E[\min(q_1, D_1)] - w_1 q_1 + (p_2 - w_2)\min(\mu_2, q_2), \tag{1}$$

where the first two terms represent the net profit of product 1 and the third represents the net profit from product 2. As the demand for product 2 is deterministic, we have $q_2^*(w_2) = \mu_2$ if $w_2 \le p_2$. The objective function in (1) can thus be simplified as

$$\pi_r^P(q_1|w_1) = p_1 E[\min(q_1, D_1)] - w_1 q_1.$$

By comparing $\pi_r^P(\mu_1 - v|w_1)$ and $\pi_r^P(\mu_1 + v|w_1)$, we have the following result:

$$q_1^*(w_1) = \begin{cases} \mu_1 - v \text{ if } \frac{p_1}{2} < w_1 \le p_1; \\ \mu_1 + v \text{ if } w_1 \le \frac{p_1}{2}. \end{cases}$$

We then consider the producer's wholesale price decision. The total cost of the producer c incorporates the manufacturing cost c_p and the processing cost c_d. The producer's objective function can be described as

$$\pi_p^P(w_1, w_2) = w_1 q_1^*(w_1) + w_2 q_2^*(w_2) - (c_p + c_d)\max\left(\frac{q_1^*(w_1)}{\lambda}, \frac{q_2^*(w_2)}{1-\lambda}\right) \tag{2}$$

where the first two terms mean the profits generated by products 1 and 2, while the third means the total cost thus incurred. For our model follows the Stackelberg game, cost c_p and c_d always appear together in the equilibrium solutions, without loss of generality, we normalize the c_p to zero, i.e. $c = c_d$. Substituting $q_1^*(w_1)$ and $q_2^*(w_2)$ into (2), we obtain

$$\pi_p^P(w_1, w_2) = \begin{cases} \left(w_1 - \frac{c_d}{\lambda}\right)(\mu_1 - v) + w_2\mu_2 \text{ if } \frac{p_1}{2} < w_1 \le p_1, w_2 \le p_2; \\ \left(w_1 - \frac{c_d}{\lambda}\right)(\mu_1 + v) + w_2\mu_2 \text{ if } w_1 \le \frac{p_1}{2}, w_2 \le p_2. \end{cases}$$

It's straightforward that $w_2^* = p_2$. Let $\bar{v} = \frac{\lambda p_1 \mu_1}{3\lambda p_1 - 4c_d}$. We then have the following proposition.

Proposition 1 *The optimal wholesale price of product 2 satisfies $w_2^* = p_2$, while the optimal wholesale price for product 1 satisfies*

(i) *if $c_d > \frac{3}{4}\lambda p_1$, then $w_1^* = p_1$;*
(ii) *otherwise, if $v > \bar{v}$, then $w_1^* = \frac{1}{2}p_1$; and if $v \le \bar{v}$, then $w_1^* = p_1$.*

For cold product (product 2), the producer will charge a high price, which extracts all the chain profit to compensate his loss due to the oversupply if he fully satisfies the corresponding demand. However, for hot product (product 1), he can either chooses a high price strategy ($w_1^* = p_1$) or a low price strategy ($w_1^* = \frac{1}{2}p_1$) depending on the processing cost and the magnitude of demand uncertainty. Specifically, if the processing cost is high or the demand uncertainty is low, he should adopt a high price strategy to extract all the chain profit as the Stackelberg leader. In contrast, if the processing cost is relatively low whereas the demand uncertainty is high, he should adopt a low price strategy to maximize his profit by inducing the retailer to order more.

Based on Proposition 1, we can obtain the producer's profit, the retailer's profit as well as the chain profit under different circumstances. We summarize the results in Theorem 1.

Theorem 1 *The profits of the producer and retailer as well as the chain under P-chain structure are as follows*

(i) *If* $c_d > \frac{3}{4}\lambda p_1$ *or if* $c_d \le \frac{3}{4}\lambda p_1$ *and* $v \le \bar{v}$, *then* $\pi_r^P = 0$ *and*

$$\pi_p^P = \pi_{chain}^P = \big(p_1 - \frac{c_d}{\lambda}\big)(\mu_1 - v) + p_2\mu_2;$$

(ii) *If* $c_d \le \frac{3}{4}\lambda p_1$ *and* $v > \bar{v}$, *then*

$$\pi_p^P = \big(\frac{p_1}{2} - \frac{c_d}{\lambda}\big)(\mu_1 + v) + p_2\mu_2, \pi_r^P = \frac{1}{2}p_1(\mu_1 - v), \text{ and } \pi_{chain}^P = p_1\mu_1 + p_2\mu_2 - \frac{c_d}{\lambda}(\mu_1 + v).$$

Theorem 1 reveals that if the processing cost is high or the demand uncertainty of both products are low, then the upstream producer will exploit all the chain profit by taking advantage of the first mover's advantage. By contrast, if the processing cost is low and the demand uncertainty of the hot product is high, then the producer will let the retailer share the chain profit by adopting a low price strategy.

3.3 Optimal decisions under R-chain structure

We conduct the backward induction. For any given wholesale price of the coproduct, the retailer decides how much to order from the producer. She is also responsible for processing the coproduct into products 1 and 2 to satisfy the market need, partially or fully. Her objective function can be expressed as follows:

$$\pi_r^R(q|w) = p_1E[\min(D_1, \lambda q)] + p_2[\min(\mu_2, (1-\lambda)q)] - (w + c_d)q, \tag{3}$$

where the first two represent the total revenue from selling products 1 and 2, whereas the third means the total procurement and processing cost. Recall we assume the demand of product 1 follows two point distribution. As a result, (3) can be re-expressed as follows:

$$\pi_r^R(q|w) = \begin{cases} (\lambda p_1 - w - c_d)\frac{\mu_2}{1-\lambda} + p_2\mu_2 \text{ if } q = \frac{\mu_2}{1-\lambda}; \\ (\mu_1 - v)p_1 + p_2\mu_2 - (w + c_d)\frac{\mu_1 - v}{\lambda}, \text{ if } q = \frac{\mu_1 - v}{\lambda}; \\ p_1\mu_1 + p_2\mu_2 - (w + c_d)\frac{\mu_1 + v}{\lambda}, \text{ if } q = \frac{\mu_1 + v}{\lambda}. \end{cases}$$

Consequently, we have $q^*(w) = \arg\max\left(\pi_r^R\left(\frac{\mu_2}{1-\lambda}\right), \pi_r^R\left(\frac{\mu_1-v}{\lambda}\right), \pi_r^R\left(\frac{\mu_1+v}{\lambda}\right)\right)$, or equivalently,

$$q^*(w) = \begin{cases} \frac{\mu_2}{1-\lambda}, \text{ if } \lambda p_1 - c_d < w \le \lambda p_1 + (1-\lambda)p_2 - c_d; \\ \frac{\mu_1 - v}{\lambda}, \text{ if } \frac{\lambda p_1}{2} - c_d < w \le \lambda p_1 - c_d; \\ \frac{\mu_1+v}{\lambda}, \text{ if } w \le \frac{\lambda p_1}{2} - c_d. \end{cases}$$

We next consider the producer's wholesale price decision. The producer's objective function can be expressed as:

$$\pi_p^R(w) = wq^*(w) = \begin{cases} \frac{\mu_2}{1-\lambda} w \text{ if } \lambda p_1 - c_d < w \le \lambda p_1 + (1-\lambda)p_2 - c_d; \\ \frac{\mu_1 - v}{\lambda} w \text{ if } \frac{\lambda p_1}{2} - c_d < w \le \lambda p_1 - c_d; \\ \frac{\mu_1+v}{\lambda} w \text{ if } w \le \frac{\lambda p_1}{2} - c_d. \end{cases} \tag{4}$$

Let $\rho_-(v) = \frac{\mu_1 - v}{\lambda} : \frac{\mu_2}{1-\lambda}$ and $\rho_+(v) = \frac{\mu_1+v}{\lambda} : \frac{\mu_2}{1-\lambda}$. It's easy to verify that $\rho_-(v)$ is decreasing in v, while $\rho_+(v)$ is increasing in v. Then we have the following proposition.

Proposition 2 *The producer's optimal wholesale price under R-chain structure satisfies*

(i) *If* $c_d > \frac{3}{4}\lambda p_1$ *or if* $c_d \le \frac{3}{4}\lambda p_1$ *and* $v \le \bar{v}$, *then*

(i)-1 $w^* = \lambda p_1 - c_d$ *if* $c_d \le \lambda p_1 - \frac{(1-\lambda)p_2}{\rho_-(v)-1}$,
(i)-2 $w^* = \lambda p_1 + (1-\lambda)p_2 - c_d$ *otherwise;*

(ii) *If* $c_d \le \frac{3}{4}\lambda p_1$ *and* $v > \bar{v}$, *then*

(ii)-1 $w^* = \frac{\lambda p_1}{2} - c_d$ *if* $c_d \le \lambda p_1 - \frac{\lambda p_1 \rho_+(v) + 2(1-\lambda)p_2}{2(\rho_+(v)-1)}$,
(ii)-2 $w^* = \lambda p_1 + (1-\lambda)p_2 - c_d$ *otherwise.*

According to Propositions 2(i)-2 and 2(ii)-2, if the processing cost is high, then the producer will charge a very high wholesale price to the retailer such that the retailer's profit for each procured coproduct becomes zero (recall $\lambda p_1 + (1-\lambda)p_2 - c_d$ is the maximum net profit that could be possibly generated for each coproduct after processing). Otherwise, a low wholesale price strategy ($p_1 - c_d$ or $\frac{\lambda p_1}{2} - c_d$) should be adopted, which may depends on the magnitude of demand uncertainty.

Theorem 2 *The profits of the producer and retailer as well as the chain under R-chain structure are as follows*

(i) *For case (i)-1, we have*

$$\pi_p^R = (\lambda p_1 - c_d)\frac{\mu_1 - v}{\lambda}, \pi_r^R = p_2\mu_2, \text{ and } \pi_{chain}^R = \left(p_1 - \frac{c_d}{\lambda}\right)(\mu_1 - v) + p_2\mu_2;$$

(ii) *For case (ii)-1, we have*

$$\pi_p^R = \left(\frac{\lambda p_1}{2} - c_d\right)\frac{\mu_1+v}{\lambda}, \pi_r^R = p_1\mu_1 + p_2\mu_2 \\ -\frac{(\mu_1+v)p_1}{2}, \text{ and } \pi_{chain}^R = p_1\mu_1 + p_2\mu_2 - \frac{c_d}{\lambda}(\mu_1 + v);$$

Table 2 Optimal decision under P-chain and R-chain structures

Scenario	$c_d > \frac{3}{4}\lambda p_1$ or $c_d \le \frac{3}{4}\lambda p_1$ and $v \le \bar{v}$		$c_d \le \frac{3}{4}\lambda p_1$ and $v > \bar{v}$	
Cost	$c_d \le \beta_-(v)$	$c_d > \beta_-(v)$	$c_d \le \beta_+(v)$	$c_d > \beta_+(v)$
P-chain				
q	$\frac{\mu_1 - v}{\lambda}$	$\frac{\mu_2}{1-\lambda}$	$\frac{\mu_1 + v}{\lambda}$	$\frac{\mu_1 + v}{\lambda}$
q_1	$\mu_1 - v$	$\mu_1 - v$	$\mu_1 + v$	$\mu_1 + v$
q_2	μ_2	μ_2	μ_2	μ_2
w_1	p_1	p_1	$\frac{1}{2}p_1$	$\frac{1}{2}p_1$
w_2	p_2	p_2	p_2	p_2
R-chain				
q	$\frac{\mu_1 - v}{\lambda}$	$\frac{\mu_2}{1-\lambda}$	$\frac{\mu_1 + v}{\lambda}$	$\frac{\mu_2}{1-\lambda}$
q_1	$\mu_1 - v$	$\frac{\lambda}{1-\lambda}\mu_2$	$\mu_1 + v$	$\frac{\lambda}{1-\lambda}\mu_2$
q_2	$\frac{(1-\lambda)(\mu_1 - v)}{\lambda}$	μ_2	$\frac{(1-\lambda)(\mu_1 + v)}{\lambda}$	μ_2
w	$\lambda p_1 - c_d$	$\lambda p_1 + (1-\lambda)p_2 - c_d$	$\frac{1}{2}\lambda p_1 - c_d$	$\lambda p_1 + (1-\lambda)p_2 - c_d$

(iii) *For cases (i)-2 & (ii)-2, we have* $\pi_r^R = 0$ *and*

$$\pi_p^R = \pi_{chain}^R = (\lambda p_1 + (1-\lambda)p_2 - c_d)\frac{\mu_2}{1-\lambda}.$$

According to Theorem 2, if the processing cost is very high, then the producer will exploit all the chain profit by setting a high price. In response, the retailer will be very conservative in the ordering decision, which is based on the demand of the cold product (product 2). If the processing cost is very low while the demand uncertainty is high, then the produce will charge a rather low wholesale price to incentive the retailer to order more, which is based on the highest possible demand of the hot product (product 1). By contrast, if the processing cost is medium and the demand uncertainty is relatively low, then the producer will charge a relatively high price anchoring the net profit of the hot product ($w^* = p_1 - c_d$), which implies after processing the coproduct the retailer uses the hot product to cover her cost while obtains her net profit via the cold product.

4 Comparisons between P-chain and R-chain

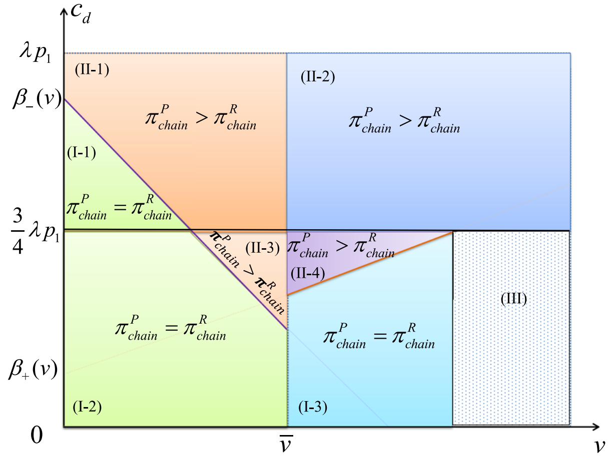
Let $\beta_-(v) = \lambda p_1 - \frac{(1-\lambda)p_2}{\rho_-(v)-1}$ and $\beta_+(v) = \lambda p_1 - \frac{\lambda p_1 \rho_+(v) + 2(1-\lambda)p_2}{2(\rho_+(v)-1)}$, $\delta = \frac{(1-\lambda)p_2}{\rho_+(v)-1}$, $(\delta > 0)$. As $\rho_-(v)$ is decreasing in v, and $\rho_+(v)$ is increasing in v, it's easy to verify that $\beta_-(v)$ is decreasing in v, while $\beta_+(v)$ is increasing in v. We summarize the optimal decisions under both the P-chain and R-chain structures in Table 2 and the profits of both chains in Table 3 accordingly.

Based on the results listed above, we next give a detailed comparisons between the two chain structures (see Figs. 4, 5, 6). Specifically, we consider the following two cases, based on a separation of scope with regard to the processing cost and the magnitude of demand uncertainty. We exclude the trial cases Areas III in Figs. 4, 5 and Area IV in Fig. 6, under which the retailer will order nothing from the producer.

Table 3 The profits of P-chain and R-chain

Scenario	$c_d > \frac{3}{4}\lambda p_1$ or $c_d \le \frac{3}{4}\lambda p_1$ and $v \le \bar{v}$		$c_d \le \frac{3}{4}\lambda p_1$ and $v > \bar{v}$	
Cost	$c_d \le \beta_-(v)$	$c_d > \beta_-(v)$	$c_d \le \beta_+(v)$	$c_d > \beta_+(v)$
P-chain				
π_p^P	$\left(p_1 - \frac{c_d}{\lambda}\right)(\mu_1 - v) + p_2\mu_2$	$\left(p_1 - \frac{c_d}{\lambda}\right)(\mu_1 - v) + p_2\mu_2$	$\left(\frac{p_1}{2} - \frac{c_d}{\lambda}\right)(\mu_1 + v) + p_2\mu_2$	$\left(\frac{p_1}{2} - \frac{c_d}{\lambda}\right)(\mu_1 + v) + p_2\mu_2$
π_r^P	0	0	$\frac{1}{2}p_1(\mu_1 - v)$	$\frac{1}{2}p_1(\mu_1 - v)$
π_{chain}^P	$\left(p_1 - \frac{c_d}{\lambda}\right)(\mu_1 - v) + p_2\mu_2$	$\left(p_1 - \frac{c_d}{\lambda}\right)(\mu_1 - v) + p_2\mu_2$	$p_1\mu_1 + p_2\mu_2 - \frac{c_d}{\lambda}(\mu_1 + v)$	$p_1\mu_1 + p_2\mu_2 - \frac{c_d}{\lambda}(\mu_1 + v)$
R-chain				
π_p^R	$(\lambda p_1 - c_d)\frac{\mu_1 - v}{\lambda}$	$(\lambda p_1 + (1-\lambda)p_2 - c_d)\frac{\mu_2}{1-\lambda}$	$\left(\frac{\lambda p_1}{2} - c_d\right)\frac{\mu_1 + v}{\lambda}$	$(\lambda p_1 + (1-\lambda)p_2 - c_d)\frac{\mu_2}{1-\lambda}$
π_r^R	$\mu_2 p_2$	0	$p_1\mu_1 + p_2\mu_2 - \frac{(\mu_1+v)p_1}{2}$	0
π_{chain}^R	$(p_1 - \frac{c_d}{\lambda})(\mu_1 - v) + \mu_2 p_2$	$(\lambda p_1 + (1-\lambda)p_2 - c_d)\frac{\mu_2}{1-\lambda}$	$p_1\mu_1 + p_2\mu_2 - \frac{c_d}{\lambda}(\mu_1 + v)$	$(\lambda p_1 + (1-\lambda)p_2 - c_d)\frac{\mu_2}{1-\lambda}$

Fig. 4 Chain performance with c_d and v



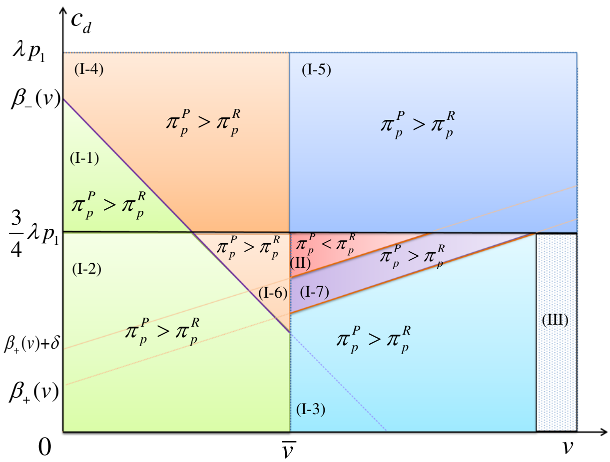


Fig. 5 Producer performance with c_d and v

4.1 Case (A): High processing cost or low demand uncertainty

We consider the case satisfying $c_d > \frac{3}{4}\lambda p_1$ or $c_d \le \frac{3}{4}\lambda p_1$ and $v \le \bar{v}$. In this case, we compare the producer's performance, the retailer's performance as well as the chain's performance under both the P-chain and R-chain structures.

Theorem 3 *If $c_d > \frac{3}{4}\lambda p_1$ or if $c_d \le \frac{3}{4}\lambda p_1$ and $v \le \bar{v}$, then we have*

(i) $\pi^P_p > \pi^R_p$;
(ii) $\pi^P_r < \pi^R_r$, *if* $c_d \le \beta_-(v)$; *whereas* $\pi^P_r = \pi^R_r$, *if* $c_d > \beta_-(v)$;
(iii) $\pi^P_{chain} = \pi^R_{chain}$, *if* $c_d \le \beta_-(v)$; *while* $\pi^P_{chain} > \pi^R_{chain}$, *if* $c_d > \beta_-(v)$.

For the producer, his performance under the P-chain structure dominates that under the R-chain structure if he faces a relatively stable market demand ($v < \bar{v}$) or a high processing cost (see Areas I-1, I-2, I-4, I-6 in Fig. 5). The underlying reason is the producer can obtain a larger

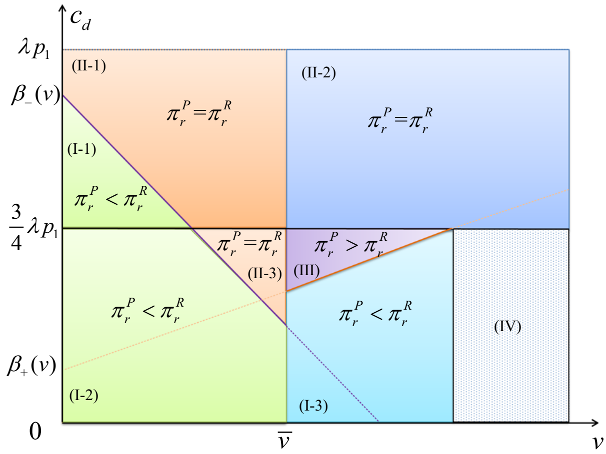


Fig. 6 Retailer performance with c_d and v

first-mover advantage by setting up the wholesale prices for products 1 and 2 simultaneously, which results in a lower mismatch cost between supply and demand.

For the retailer, when the processing cost c_d is relatively low, e.g., $c_d \leq \beta_-(v)$, her performance is better under the R-chain structure (see Areas I-1 and I-2 in Fig. 6). This is mainly due to the low price strategy adopted by the producer. As a result, it will result in a lower mismatch risk between supply and demand among the different products processed by the coproduct.

In terms of the chain's performance, the P-chain structure in general dominates the R-chain structure (see Areas I-1, I-2, II-1 and II-3 in Fig. 4). In other words, if the producer is responsible for processing the coproduct, the supply chain efficiency is higher compared with the situation when the retailer undertakes the processing functionality. This is because the producer as the Stackelberg leader could help reduce the double marginalization effect due to the mismatch between supply and demand of the two products. However, it is worth noting that the P-chain structure is not always strictly better than the R-chain structure. When the processing cost c_d is extremely high, then the producer will adopt a high price strategy under both structures, which will exploit all the chain's profit. As a consequence, the chain's performance is equal under both structures.

4.2 Case (B): Low processing cost and high demand uncertainty

In this subsection, we consider the case satisfying $c_d \leq \frac{3}{4}\lambda p_1$ and $v > \bar{v}$. The comparison results are summarized in the following theorem.

Theorem 4 *If $c_d \leq \frac{3}{4}\lambda p_1$ and $v > \bar{v}$, then we have*

(i) $\pi_p^P > \pi_p^R$, *when* $\beta_+(v) < c_d < \beta_+(v) + \delta$; *whereas* $\pi_p^P < \pi_p^R$, *when* $c_d > \beta_+(v) + \delta$;
(ii) $\pi_r^P < \pi_r^R$, *when* $c_d \leq \beta_+(v)$; *whereas* $\pi_r^P > \pi_r^R$, *when* $c_d > \beta_+(v)$;
(iii) $\pi_{chain}^P = \pi_{chain}^R$, *when* $c_d \leq \beta_+(v)$; *whereas* $\pi_{chain}^P > \pi_{chain}^R$, *when* $c_d > \beta_+(v)$.

Under this case, if the processing cost is extremely low ($c_d \leq \beta_+(v)$), the chain profit remains unchanged. However, the producer performs better under the P-chain structure, while the retailer is better off under the R-chain structure. As the processing cost is very low while the demand uncertainty is rather high, the producer has to adopt a low wholesale price strategy under the R-chain structure, which benefits more to the retailer (note that a low wholesale price means a low mismatch cost between supply and demand).

If the processing cost is extremely high (say $c_d > \beta_+(v) + \delta$), both the retailer's profit and the chain's profit under the P-chain structure dominate that under the R-chain structure (see Area III in Fig. 6), whereas the producer's profit is worse off under the P-chain structure (see Area II in Fig. 5). This conclusion seems counter intuitive as the producer can decide on the prices of both products 1 and 2 under the P-chain structure, which could help mitigate more mismatch cost between supply and demand. One of the possible explanations could be attributed to the high demand uncertainty (recall that $v > \bar{v}$ under this case), which could result in a extremely high mismatch cost as the unbalanced ratio could be amplified by the demand uncertainty.

5 Model extensions and discussions

In this section, we extend our basic model settings to more general settings. In Sect. 5.1, we consider the setting with general demand distributions and investigate the existence of optimal decisions under two chain structures. More specifically, we study the situation of uniform demand distribution, to illustrate the robustness of our main results under the basic setting. In Sect. 5.2, we consider the situation that the product (raw material) can be processed into k coproducts and investigate the optimal decisions under some special conditions.

5.1 General demand distributions

In this subsection, we first give the existence conditions of the optimal order quantity q_1 and q_2 as well as q, when the two final products face demands D_1 and D_2. Let $F_i(\cdot)$ be the cumulative distribution function of part i and $f_i(\cdot)$ be the probability density function, $i \in \{1, 2\}$, respectively. Define $h_i(x) = \frac{x f_i(x)}{1-F_i(x)}$, the general failure rate function.

In the P-chain structure, the expected profit of the retailer is

$$\begin{aligned}\max \pi_r^P(q_1, q_2|w_1, w_2) &= p_1 E\min(q_1, D_1) + p_2 E\min(q_2, D_2) - w_1q_1 - w_2q_2\\ &= -p_1\int_0^{q_1} F_1(x)dx - p_2\int_0^{q_2} F_2(x)dx + p_1q_1 + p_2q_2\\ &\quad -w_1q_1 - w_2q_2, \end{aligned} \tag{5}$$

while the expected profit of the producer is $\max \pi_p^P(w_1, w_2) = w_1q_1 + w_2q_2 - (c_p + c_d)\max\{\frac{q_1}{\lambda}, \frac{q_2}{1-\lambda}\}$.

Correspondingly, in the R-chain structure, the expected profit of the retailer is

$$\begin{aligned}\max \pi_r^R(q|w) &= p_1 E\min(q_1, D_1) + p_2 E\min(q_2, D_2) - c_dq - wq\\ &= -p_1\int_0^{\lambda q} F_1(x)dx - p_2\int_0^{(1-\lambda)q} F_2(x)dx + p_1\lambda q + p_2(1-\lambda)q\\ &\quad -c_dq - wq, \end{aligned} \tag{6}$$

while the expected profit of the producer is $\max \pi_p^R(w) = wq - c_pq$.

Assumption 2 $h_i(x)$ is increasing in $(0, \infty)$ (IGFR), and $G_i(x) = \bar{F}_i(x) - x f_i(x)$ is strictly decreasing in x.

IGFR is widely assumed in recent pricing, revenue, and supply chain management literature (Lariviere and Porteus 2001). The deceasing feature of $G_i(x)$ is relatively restrictive, but is satisfied by uniform, exponential, and other distributions in certain parameter domains.

Theorem 5 *Let* $m = \frac{c_p+c_d}{\lambda p_1}$ *and* $n = \frac{c_p+c_d}{(1-\lambda)p_2}$. *Suppose Assumption 2 holds. Then, in P-chain, we can obtain the optimal* q_1^* *and* q_2^* *as follows:*

(i) $q_1^* = G_1^{-1}(0)$, $q_2^* = G_2^{-1}(n)$, *when* $\lambda G_2^{-1}(n) \geq (1-\lambda)G_1^{-1}(0)$;
(ii) $q_1^* = G_1^{-1}(m)$, $q_2^* = G_2^{-1}(0)$, *when* $(1-\lambda)G_1^{-1}(m) \geq \lambda G_2^{-1}(0)$;
(iii) $q_1^* = \lambda q^*$, $q_2^* = (1-\lambda)q^*$, *when* q^* *satisfying* $\lambda p_1 G_1(\lambda q) + (1-\lambda)p_2 G_2[(1-\lambda)q]{=}c_d + c_p$;
(iv) $q_1^* = G_1^{-1}(0)$, $q_2^* = G_2^{-1}(0)$, *otherwise.*

Theorem 5 shows the retailer's optimal order quantity decisions in different situations. Specifically, case (i) indicates that the processing is based on product 1's demand, with oversupply for product 2. In contrast, case (ii) shows processing is meeting product 2's demand with oversupply for product 1. There is a perfect match for the processing since the demand ratio is equal to the fix fraction between the supplies of product 1 and product 2. Case (iv) shows the firm should oversupply both products, which is the best choice of the producer.

Similarly, we give the conditions for the existence of the retailer's optimal order quantity q in the R-chain structure (see Theorem 6 below).

Theorem 6 *Suppose Assumption 2 holds. Then, in the R-chain,* $\pi_p^R(q)$ *is concave in* q *and there exists a unique optimal order quantity* q^* *satisfying* $\lambda p_1 G_1(\lambda q) + (1-\lambda)p_2 G_2[(1-\lambda)q]{=}c_d + c_p$.

In general, it's very difficult, if not impossible, for us to give a detailed comparison for the two chain structures. We next consider a special case, the uniform distribution, as an example to illustrate the robustness of our main results thus obtained in our basic setting. Specifically, the demand for product 1 is $D_1 \sim U(\mu_1 - v, u_1 + v)$. Clearly, the average demand for product 1 is μ_1, while the variance is $\frac{1}{3}v^2$. Also, we assume that the demand for product 2 is μ_2, which is deterministic. In the P-chain, it's straightforward that $w_2^* = p_2$. Let $\bar{v}_l = \frac{\mu_1\lambda p_1}{3\lambda p_1 - 2c_d - 4\lambda\varepsilon}$ and $\bar{v}_h = \frac{\mu_1\lambda p_1}{\lambda p_1 - 2c_d}$.

Define $\varphi = \frac{p_1}{4v}(\frac{\mu_1+v}{2} + \frac{vc_d}{\lambda p_1})^2$ and $\psi(c_d, \lambda, \mu_1) = \frac{v(\lambda p_1 - 2c_d) - \mu_1\lambda p_1}{2\lambda}$. We then have the following theorem.

Theorem 7 *By comparing the P-chain and the R-chain, we have*

(i) $\pi_p^P \geq \pi_p^R$, $\pi_r^P \geq \pi_r^R$, $\pi_{chain}^P \geq \pi_{chain}^R$, *when* $v \leq \bar{v}_l$;
(ii) $\pi_p^P > \pi_p^R$, $\pi_{chain}^P \geq \pi_{chain}^R$, *when* $v > \bar{v}_l$; $\pi_r^P \geq \pi_r^R$, *if* $\bar{v}_l < v < \bar{v}_h$ *and* $\psi(c_d, \lambda, \mu_1) > \mu_2 p_2$ *and* $\pi_r^P < \pi_r^R$ *otherwise.*

According to Theorem 7, we have: (1) in general, P-chain dominates R-chain in terms of the chain performance; and (2) the producer is not always better off under the P-chain structure although he bears more risks when the demand uncertainty is relatively high. These two findings are consistent with what we have obtained in Theorems 3 and 4.

5.2 With *k* coproducts

In this section, we investigate the situation that the product can be processed into k coproducts, $k \in N^+$. Suppose demand for product i is D_i with selling price set up at p_i. In the P-chain structure, the producer processes the product and decides on the wholesale price w_i for product i, $i = 1, \ldots, k$, then the downstream retailer makes the order quantity q_i for product i.

The expected profit of the retailer can be expressed as

$$\max_{i=1,2,\ldots,k} \pi_r^P(q_i|w_i) = \sum_{i=1}^{k} p_i E \min(q_i, D_i) - \sum_{i=1}^{k} w_i q_i = \\ -\sum_{i=1}^{k} p_i \int_0^{q_i} F_i(x)dx + \sum_{i=1}^{k} p_i q_i - \sum_{i=1}^{k} w_i q_i, \tag{7}$$

while the excepted profit of the producer is

$$\max_{i=1,2,\ldots,k} \pi_p^P(w_i) = \sum_{i=1}^{k} w_i q_i - (c_p + c_d) \max_{i=1,2,\ldots,k} \left\{ \frac{q_i}{\lambda_i} \right\}. \tag{8}$$

In contrast, in the R-chain structure, the expected profit of the retailer is

$$\max \pi_r^R(q|w) = \sum_{i=1}^{k} p_i E \min(\lambda_i q, D_i) - (c_d + w)q \\ = -\sum_{i=1}^{k} p_i \int_0^{\lambda_i q} F_i(x)dx + \sum_{i=1}^{k} p_i \lambda_i q - (c_d + w)q, \tag{9}$$

and correspondingly the expected profit of the producer is $\max \pi_p^R(w) = wq - c_p q$. Without loss of generality, we suppose $c_p = 0$, and demand of product 1 follows a two-point distribution. Specifically, $D_1 \sim \{u_1 - v, u_1 + v\}$, with euqal probability $\frac{1}{2}$. For product j, $j > 1$, its demand is deterministic (i.e., $D_j = u_j$).

Assumption 3 (*Unbalancedness between supply and demand*) Suppose $\frac{\mu_1 - v}{\lambda_1} > \frac{\mu_2}{\lambda_2} > \cdots > \frac{\mu_k}{\lambda_k}$, and $\sum_{i=1}^{k} \lambda_i = 1$.

For any $k \geq j > 1$, we have $q_j^*(w_j) = u_j$ if $w_j \leq p_j$ since demand for product j is deterministic. As a result, in the P-chain, Eqs. (7) and (8) can be simplified as

$$\max \pi_r^P(q_1|w_1) = p_1 E \min(q_1, D_1) - \sum_{i=2}^{k} w_i q_i,$$

$$\text{and } \max \pi_p^P(w_1) = w_1 q_1 + \sum_{i=2}^{k} w_i q_i - c_d \frac{q_1}{\lambda}.$$

Similarly, in the R-chain, the Eq. (9) can be simplified as

$$\max \pi_r^R(q|w) = p_1 E[D_1 \wedge (\lambda_1 q)] + \sum_{i=2}^{k} p_i E \min(\lambda_i q, D_i) - (c_d + w)q.$$

Based on Assumption 3, the main results in Propositions 1 and 2 and Theorems 1 and 2 hold. This is because, while product 1 faces the uncertain demand with a two-point distribution, the other $k-1$ products can be regarded as a whole with a deterministic demand.

6 Numerical examples

In this section, we conduct a numerical experiment by the sensitivity analysis on the producer's profit, the retailer's profit as well as the chain's profit with regard to two key parameters, the processing cost c_d and the standard deviation of the uncertain demand v under both the P-chain and R-chain structures. For the key parameters c_d and v, we assume that c_d varies from 1 to 20 and v varies from 0.01 to 5. Other parameters are set as follows: $c_p = 1$, $p_1 = 10$; $p_2 = 6$; $\mu_1 = 5$; $\mu_2 = 3$; and $\lambda = 0.3$.

Firstly, we conduct the sensitivity analysis on the chain's profits regarding c_d and v (see Figs. 7, 8). Our observations include: (1) The chain profit in general under P-chain structure dominates the one under R-chain structure. This implies, for the coproduct chain, the chain efficiency is higher if the upstream producer undertakes the mismatch risk between supply and demand. This conforms with Theorems 3 and 4. (2) The profit difference with regard to the processing cost between the P-chain structure and the R-chain structure follows the U-shape, or in other words, the loss of efficiency under the R-chain structure is low when the processing cost is medium. (3) The profit difference between the two chain structures decreases in the magnitude of demand uncertainty.

Secondly, we investigate how the producer's profit and the retailer's profit vary with the processing cost c_d (see Figs. 9, 10). If the processing cost is low, it is more beneficial for the producer himself to process the product into different coproducts and wholesale them to the

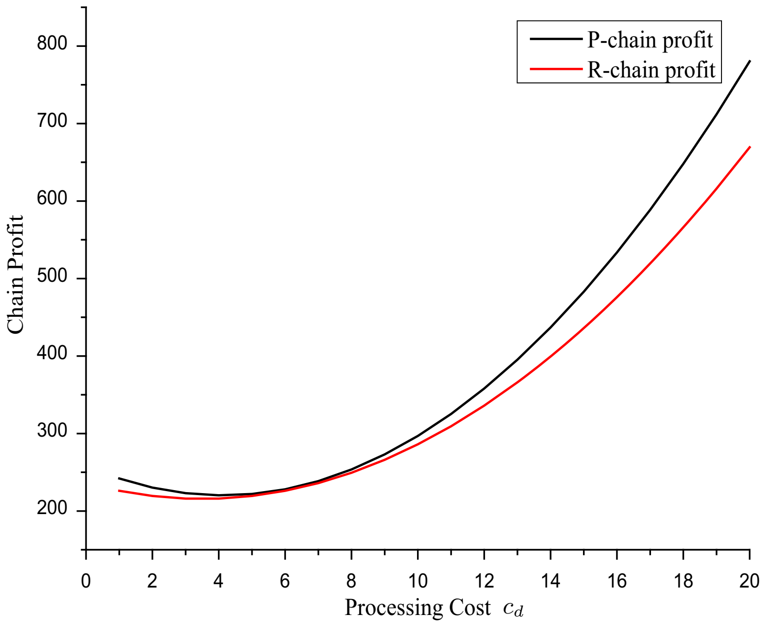


Fig. 7 P-chain vs R-chain profit with c_d

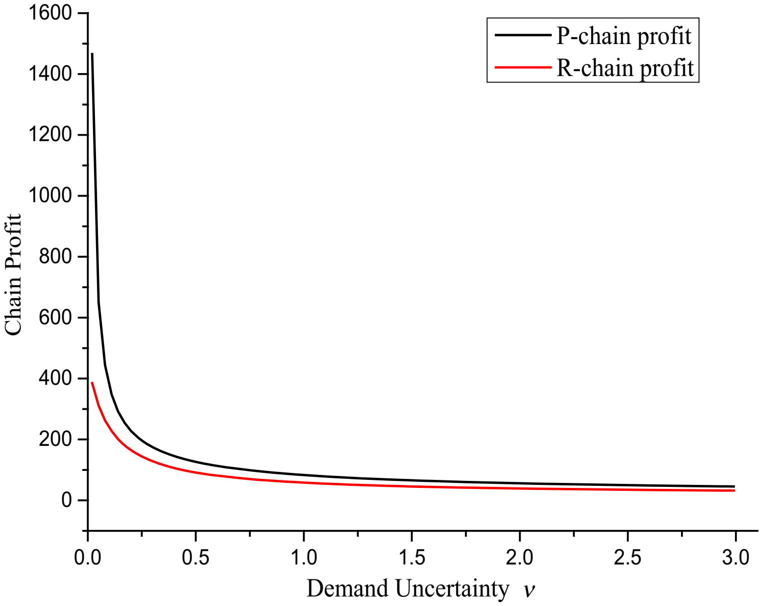


Fig. 8 P-chain vs R-chain profit with v

retailer. In contrast, if the processing cost is relatively high ($c_d \in [4; 18]$ in our numerical example in Fig. 9), then the producer prefers to move the processing task to the downstream retailer. For the retailer's side, if the processing cost is extremely high or low, it is beneficial to the retailer. However, if the processing cost lies in the medium interval ($c_d \in [2; 10]$ in our numerical example in Fig. 10), the retailer's profit becomes lower under the P-chain structure relative to the R-chain structure.

Finally, we conduct an analysis on the profits of the producer and the retailer with regard to the demand uncertainty parameter v (see Figs. 11, 12). Our findings include: (1) If the demand is relatively stable (a small v), the producer is more beneficial under P-chain, whereas the retailer becomes worst off. On the contrary, if the demand is volatile (a large v), then the producer prefers to the R-chain structure and the retailer prefers to the P-chain structure. This completely conforms with our Theorems 3, 4. (2) There exists a threshold of the demand uncertainty for the comparisons of the two chain structures. Specifically, from the producer's side, the larger the demand uncertainty deviates from the threshold, the higher the profit gap between the two chain structures. The retailer's profit gap has similar pattern (e.g., refer to Fig. 12).

We also conduct an analysis of the impact of unbalancedness on the performances of the producer and the retailer as well as the whole chain under two supply chain structures. Specifically, according to Fig. 13, the profit gap between the P-chain and the R-chain becomes larger with the increase of unbalancedness ρ. According to Fig. 14, when the supply-demand is relatively balanced (i.e., a small ρ), the retailer is better off while the producer is worse off under the P-chain. On the contrary, when the supply-demand is more unbalanced, the performance of the retailer becomes worse while the producer is better off under the P-chain.

Last, but not the least, we conduct a numerical experiment when both demands of products 1 and 2 face uncertainty. Specifically, we assume that demands D_1 and D_2 satisfy normal distributions with $N(u_1, v_1)$ and $N(u_2, v_2)$ respectively. Other parameters are set as follows:

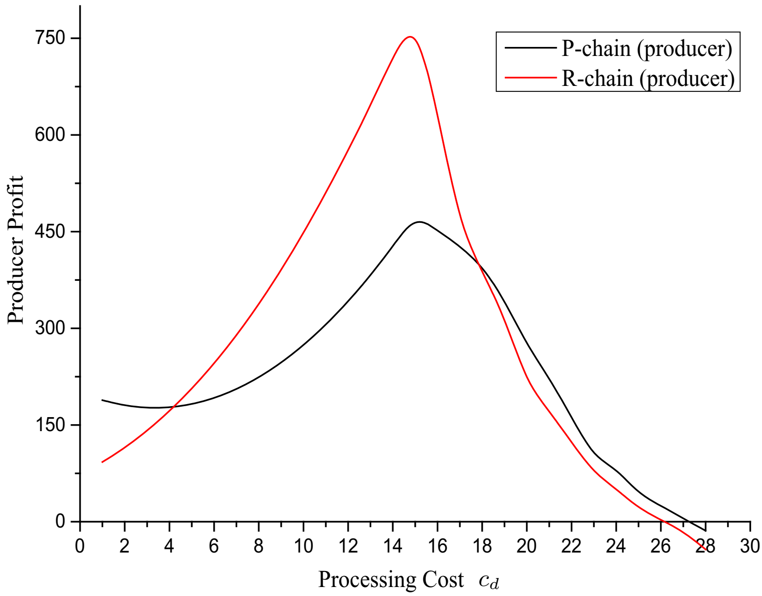


Fig. 9 Producer profit with c_d

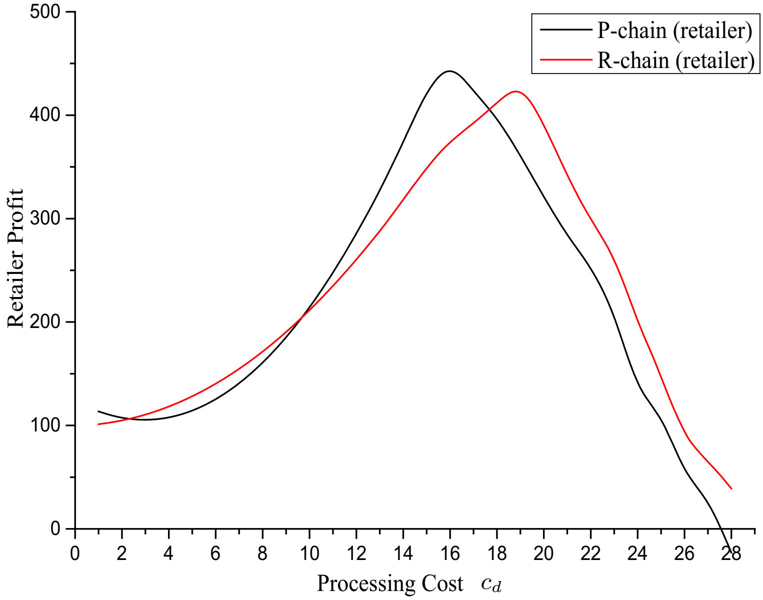


Fig. 10 Retailer profit with c_d

$c_p = 0.2, c_d = 0.1, p_1 = 4, p_2 = 2$, and $\lambda = 0.45$. The main results are summarized in Table 4. Our observations include: (1) the P-chain structure dominates the R-chain structure in terms of total chain profit, no matter how high the demand uncertainties are. (2) For the producer,

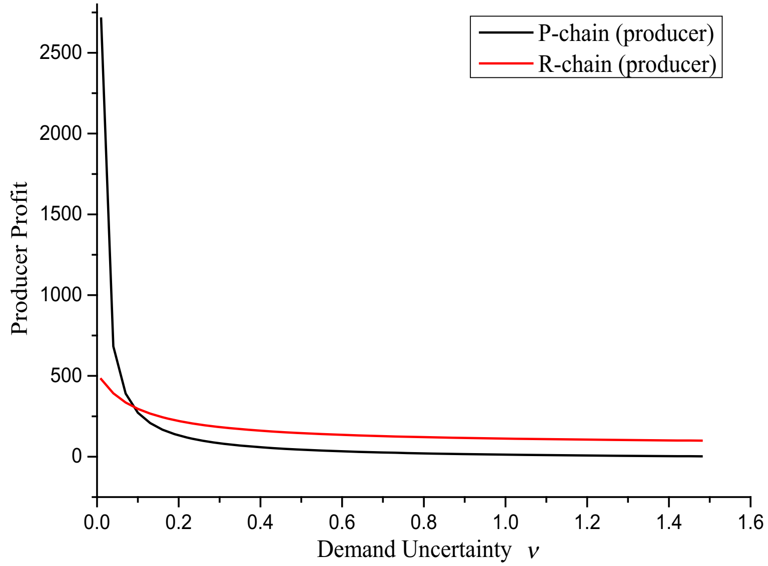


Fig. 11 Producer profit with v

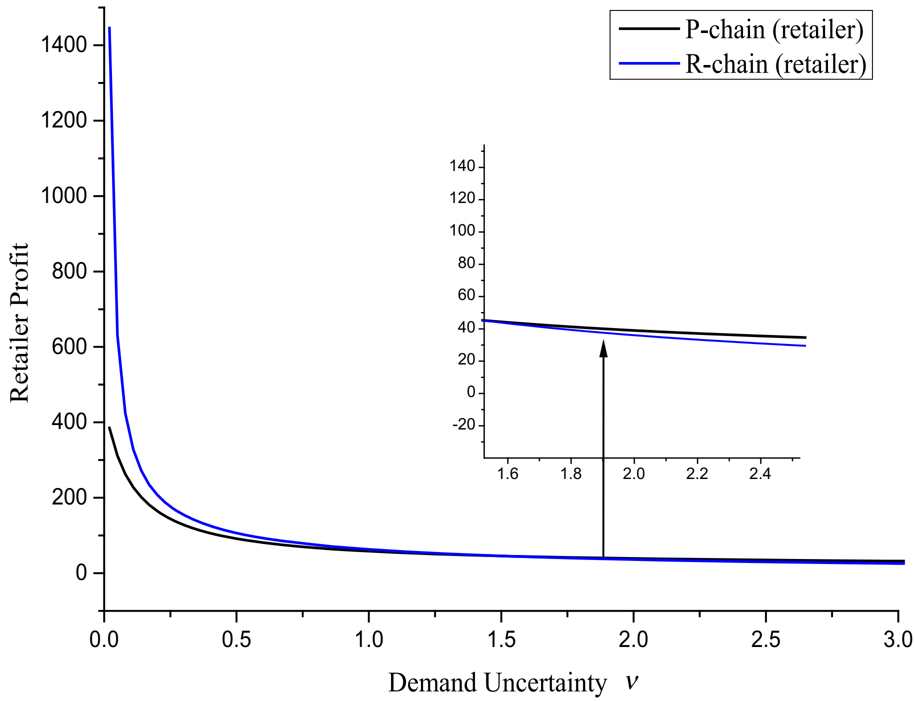


Fig. 12 Retailer profit with v

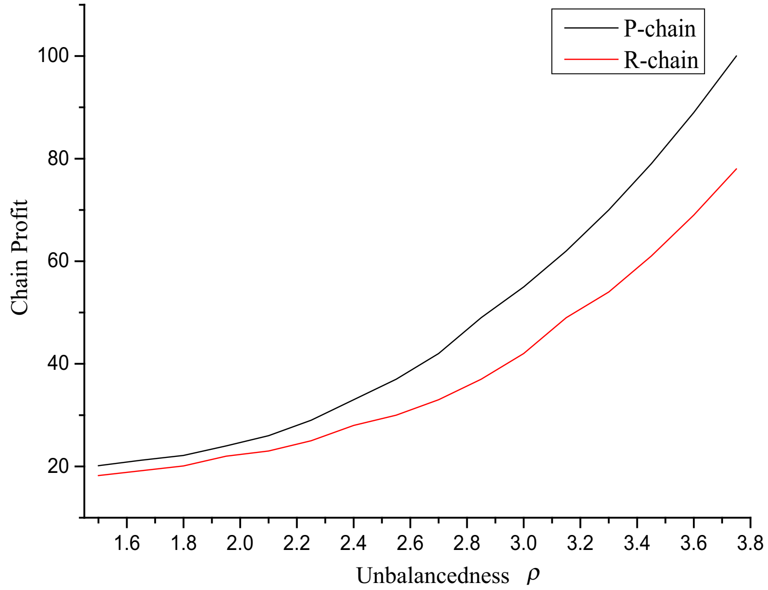


Fig. 13 Chains' profits with respect to the unbalancedness ρ

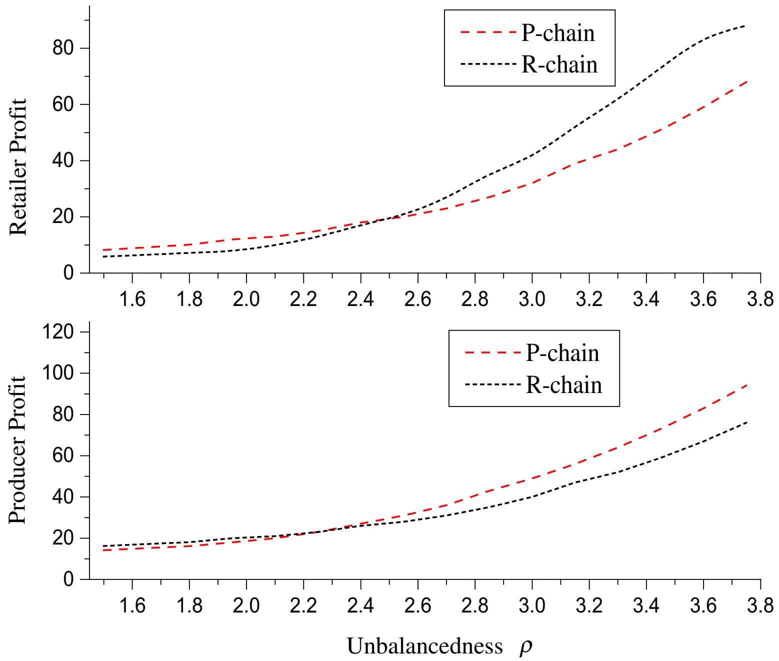


Fig. 14 Profits of producer and retailer with respect to the unbalancedness ρ

Table 4 Profits under the P-chain and R-chain structure

D_1	D_2	P-chain			R-chain			Difference		
$N(u_1, v_1)$	$N(u_2, v_2)$	π_p^P	π_r^P	π_{chain}^P	π_p^R	π_r^R	π_{chain}^R	Δ_p	Δ_r	Δ_{chain}
(1.5, 0.2)	(0.8, 0.5)	12.74	7.37	20.11	11.45	7.57	19.02	1.29	– 0.2	1.09
(1.5, 0.4)	(0.8, 0.7)	11.32	6.85	18.17	10.87	7.05	17.97	0.45	– 0.2	0.20
(1.5, 0.6)	(0.8, 0.9)	10.15	7.47	17.62	10.02	6.47	16.49	0.13	1.00	1.13
(1.5, 0.8)	(0.8, 1.1)	9.42	7.08	16.50	9.51	4.70	14.21	– 0.09	2.38	2.29
(1.5, 1.0)	(0.8, 1.3)	8.21	6.71	14.92	9.12	1.96	11.08	– 0.91	4.75	3.84
(1.2, 1.2)	(1.6, 1.5)	7.98	5.23	13.21	8.46	1.06	9.52	– 0.48	4.17	3.69
(1.2, 1.4)	(1.6, 1.7)	6.47	5.61	12.08	6.91	2.03	8.94	– 0.44	3.58	3.14
(1.2, 1.6)	(1.6, 1.9)	4.62	6.31	10.93	5.37	2.36	7.73	– 0.75	3.95	3.20
(1.2, 1.8)	(1.6, 2.1)	3.99	4.97	8.96	4.12	2.06	6.18	– 0.13	2.91	2.78

his performance is better under P-chain when the demand uncertainties of both products are relatively low. These observations conform to our theoretical results in Theorems 1 and 2.

7 Conclusions

As a typical coproduct supply chain, the pork supply chain has been subjected to extensive vertical integration initiatives. To understand such activities, it is valuable to investigate the underlying reason and the implications of the allocation of processing activities among supply chain parties. In this paper, we consider a coproduct supply chain consisting of one producer and one retailer and establish a stylized model to study how the processing activities should be allocated. Two supply chain modes, i.e., P-chain and R-chain, are considered. We use the unbalanced ratio to measure the basic level of mismatch and study how the tradeoff between bargaining power and the mismatch cost, by different mismatch risk allocations, influences the optimal decisions and the performances. We find that P-chain dominates R-chain from the perspective of the whole supply chain's performance. However, the upstream producer is not always better off in the P-chain even he bears more mismatch risk. Numerical study further shows the effect of demand uncertainty and the processing cost on the relative advantage of P-chain as compared to R-chain.

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