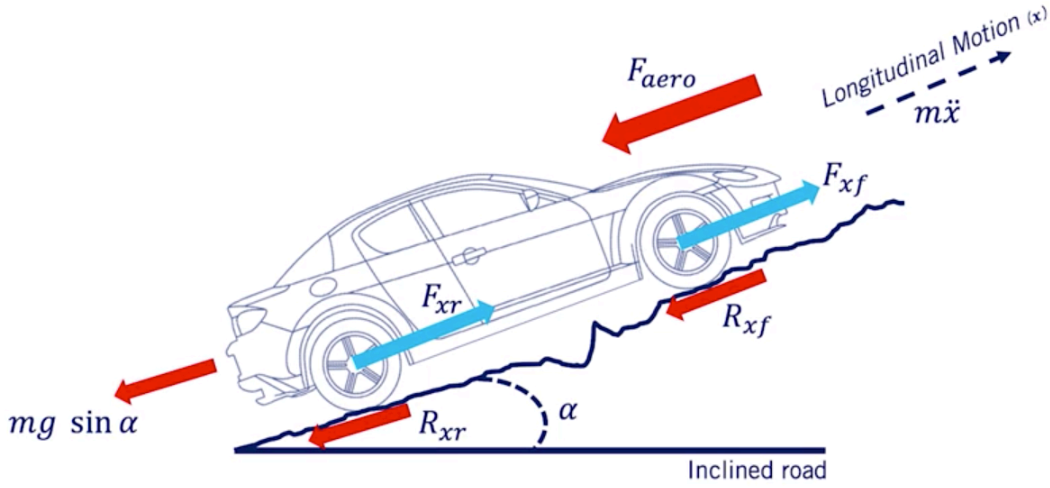
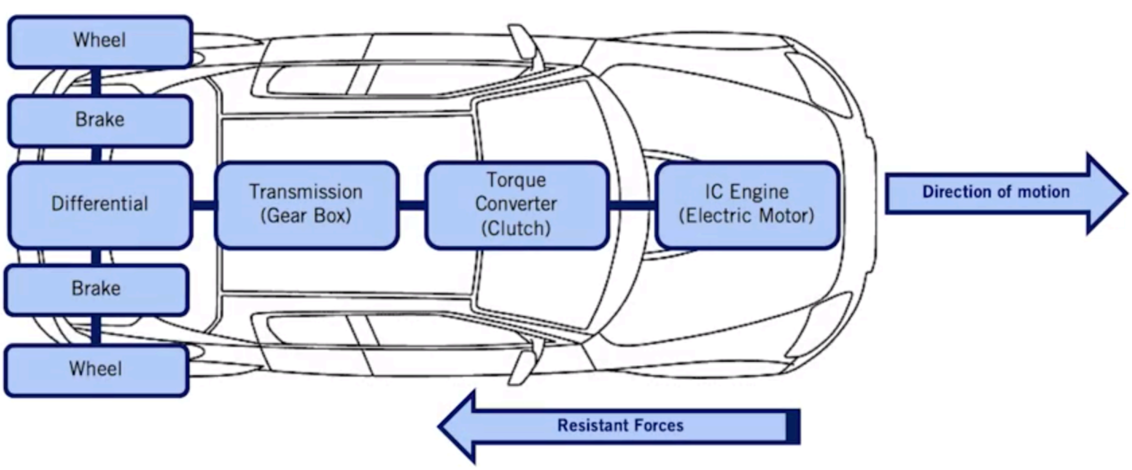
Module 4: Vehicle Dynamic Modeling
Lesson 4: Longitudinal Vehicle Modeling
Longitudinal Vehicle Model



LONGITUDINAL VEHICLE MODEL

$$m\ddot{x} = F_{xf} + F_{xr} - F_{aero} - R_{xf} - R_{xr} - mgsin\alpha$$

Simplified Longitudinal Dynamics
Let F_x - total longitudinal force: $F_x = F_{xf} + F_{xr}$ （traction force, 牽引「けんいん」力, generated by power train）
Let R_x - total rolling resistance: $R_x = R_{xf} + R_{xr}$
Assume α is a small angle: $sin\alpha = \alpha$
$m\ddot{x} = F_x - F_{aero} - R_x - mg\alpha$
$F_{aero} + R_x + mg\alpha$: total resistant forces （F_{load}）
→ develop models for each of the forces in this equation and define how they connect to the throttle and brake inputs that our autonomous system will apply. (power train, brake, tyre force modeling)



POWERTRAIN MODELING

Simple Resistance Force Models

- The aerodynamic force can depend on air density, frontal area, the vehicle's coefficient of friction, the current speed of the vehicle
  - Given a fixed vehicle shape and standard atmospheric pressure. これらの因子が一つのc_αで表す（a simple lumped coefficient of the aerodynamic drag）
  - $F_{aero} = \frac{1}{2}C_\alpha \rho A \dot{x}^2 = c_\alpha \dot{x}^2$
- The rolling resistance can depend on the tire normal force, tire pressures and vehicle speed
  - $R_x = N(\hat{c}_{r,o} + \hat{c}_{r,1}|\dot{x}| + \hat{c}_{r,2}\dot{x}^2) \approx c_{r,1}|\dot{x}|$

Powertrain Modeling

drive line

- the sequence of components between the engine and the wheels
  - torque converter (clutch: In a vehicle, the clutch is the pedal that you press before you change gear.)
  - transmission (gearbox)
  - differential（差動装置）

Because of the direct connection between wheel and engine when in gear (gear: The gears on a machine or vehicle are a device for changing the rate at which energy is changed into motion.), it is possible to model the relationship between the wheel speed and the engine speed as a kinematic constraint.

Rotational Coupling

$\omega_w = GR\omega_t = GR\omega_e \ldots (1)$

ω_w : wheel angular speed

ω_t : turbine angular speed (turbine: A turbine is a machine or engine which uses a stream of air, gas, water, or steam to turn a wheel and produce power.)

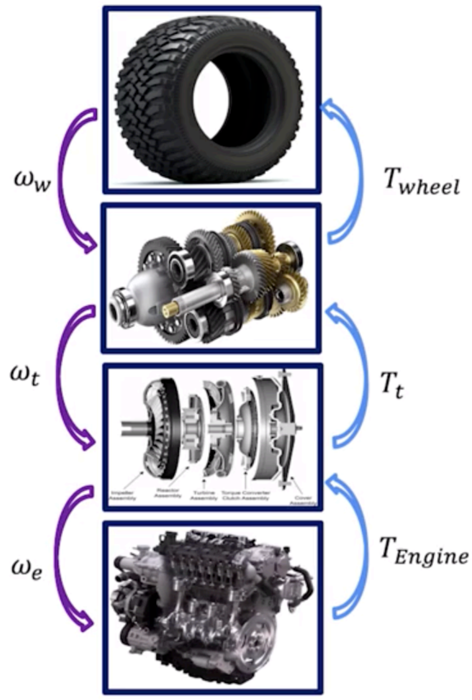
ω_e : engine angular speed

GR : Combined gear ratios (including the torque converter, transmission, differential)

Longitudinal Velocity

$\dot{x} = r_{eff}\omega_w \ldots (2)$

r_{eff} : Tire effective radius



POWER FLOW IN POWERTRAIN

Power Flow in Powertrain

- The wheel is the intersection between the torques coming from the power train side, and the torques acting from the external resistance forces.
- Wheel
  - $I_w\dot{\omega}_w = T_{wheel} - r_{eff}F_x$
  - I : moment of inertial
  - $T_{wheel} = I_w\dot{\omega}_w + r_{eff}F_x$
- Transmission
  - $I_t\dot{\omega}_t = T_t - (GR)T_{wheel}$
  - $I_t\dot{\omega}_t = T_t - GR(I_w\dot{\omega}_w + r_{eff}F_x)$
  - T_{wheel} : actually the combination of the brake torque and the output torque of the transmission or gearbox
  - T_t : the torque applied to the transmission. still includes a dependence on the tire force F_x.

- Torque Converter
  - $\omega_t = \omega_e$
  - $T_t = (I_t + I_w GR^2)\dot{\omega}_e + GR r_{eff} F_x$: see Rotational Coupling (1)
- Engine
  - $I_e \dot{\omega}_e = T_{Engine} - T_t$
  - $I_e \dot{\omega}_e = T_{Engine} - (I_t + I_w GR^2)\dot{\omega}_e - GR r_{eff} F_x \ldots (3)$

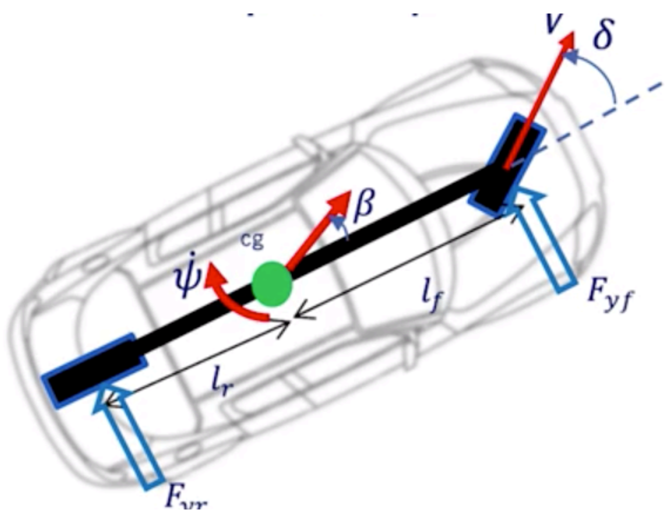
Engine Dynamics
- Tire force in terms of inertia and load force:
  - $F_x = m\ddot{x} + F_{load} = m r_{eff} GR \dot{\omega}_e + F_{load} \ldots (4)$: combine (1) and (2)
- Combine with our engine dynamics model yields:
  - $(I_e + I_t + I_w GR^2 + m(GR^2) r_{eff}^2)\dot{\omega}_e = T_{Engine} - (GR)(r_{eff} F_{load})$: combine (3) and (4)
  - an effective power train inertia as the sum of all the individual component inertias
    - $(I_e + I_t + I_w GR^2 + m(GR^2) r_{eff}^2)$: J_e
- Finally, the engine dynamic model simplifies to
  - $J_e \dot{\omega}_e = T_{Engine} - (GR)(r_{eff} F_{load})$
  - $(GR)(r_{eff} F_{load})$: Total Load Torque (T_{Load})
- We do still need to relate the engine torque to the accelerator pedal position, and the brake torque to the brake pedal position, which in the video on Actuator Modeling.

Lesson 5: Lateral Dynamics of Bicycle Model

単語リスト:
1. suspension : A vehicle's suspension consists of the springs and other devices attached to the wheels, which give a smooth ride over uneven ground.

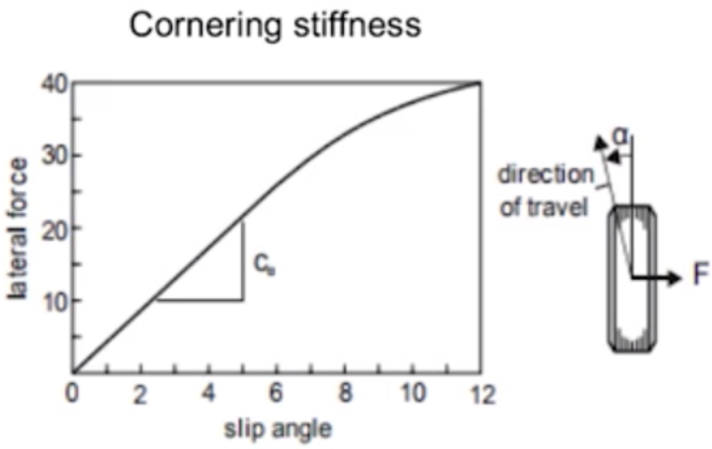
extend our kinematic bicycle model to a dynamic model by relaxing the no slip condition and force for the kinematic model.
Vehicle Model to Bicycle Model
- Assumptions
  - Longitudinal velocity is constant（目的：to decouple lateral and longitudinal dynamic models. but does lead to modeling inaccuracies when accelerating or decelerating out of curves.）
  - Left and right axle are lumped into a single wheel (bicycle model)
  - Suspension movement, road inclination and aerodynamic influences are neglected

Lateral Dynamics
- use the vehicle center of gravity as the reference point.
- Lateral acceleration
  - $a_y = \ddot{y} + \omega^2 R = V\dot{\beta} + V\dot{\psi}$
  - $\dot{\beta}$: the slip angle rate of change (side slip rate)
  - $\dot{\psi}$: the heading rate of change (psi) (yaw rate)
- $mV(\dot{\beta} + \dot{\psi}) = F_{yf} + F_{yr} \ldots (1)$
  - $F_{yf} + F_{yr}$: front and rear tire forces
- $I_z \ddot{\psi} = l_f F_{yf} - l_r F_{yr} \ldots (2)$
  - I_z : vehicle inertia
  - $\ddot{\psi}$: angular acceleration
  - the moments produced by the tire forces



LATERAL DYNAMICS

act in opposite directions.



LATERAL FORCE & SLIP ANGLE

Tire Slip Angles
- Many different tire slip angles
- For small tire slip angles, the lateral tire forces are approximated as a linear function of tire slip angle.
- Tire variables
  - Front tire slip angle, α_f
  - Rear tire slip angle, α_r

Front and Rear Tire Forces
- cornering stiffness of a tire : the ability to resist deformation while the vehicle corners.
- c_y : cornering stiffness coefficients, the slope of the line at zero
- c_f : linearized cornering stiffness of the front wheel
  - $F_{yf} = C_f \alpha_f = C_f(\delta - \beta - \frac{l_f \dot{\psi}}{V})$
  - α : the tire slip angle
  - β : the vehicle slip angle
  - δ : the steering angle
- c_r : linearized cornering stiffness of the rear wheel
  - $F_{yr} = C_r \alpha_r = C_r(-\beta + \frac{l_r \dot{\psi}}{V})$
  - (1) (2) に代入する

Lateral and Yaw Dynamics (linear because of assumptions)

$$\dot{\beta} = \frac{-(C_r + C_f)}{mV}\beta + (\frac{C_r l_r - C_f l_f}{mV^2} - 1)\dot{\psi} + \frac{C_f}{mV}\delta$$

$$\ddot{\psi} = \frac{C_r l_r - C_f l_f}{I_z}\beta - \frac{C_r l_r^2 + C_f l_f^2}{I_z V}\dot{\psi} + \frac{C_f l_f}{I_z}\delta$$

Standard State Space Representation
- State Vector : $X_{lat} = \begin{bmatrix} y & \beta & \psi & \dot{\psi} \end{bmatrix}^T$
- y : lateral position
- β : side slip angle
- ψ : yaw angle
- $\dot{\psi}$: yaw rate

$$\dot{X}_{lat} = A_{lat} X_{lat} + B_{lat}\delta$$

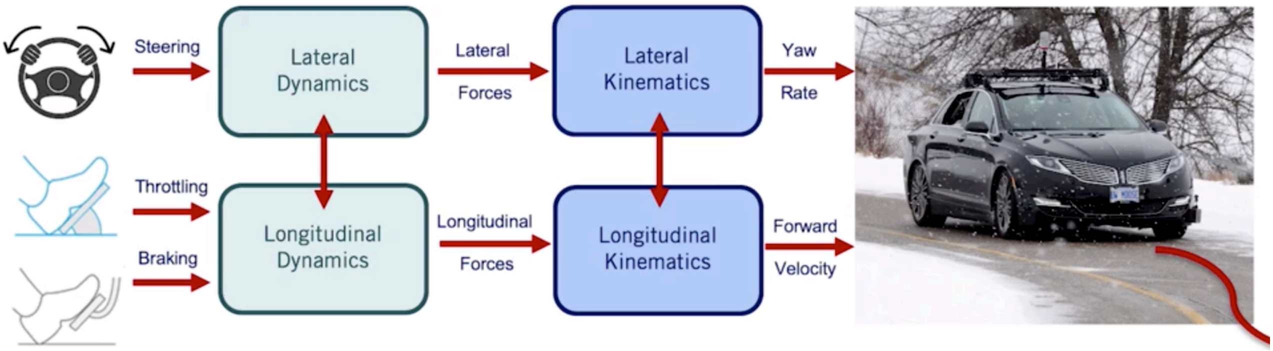
$$A_{lat} = \begin{bmatrix} 0 & V & V & 0 \\ 0 & -\frac{C_r + C_f}{mV} & 0 & \frac{C_r l_r - C_f l_f}{mV^2} - 1 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{C_r l_r - C_f l_f}{I_z} & 0 & -\frac{C_r l_r^2 + C_f l_f^2}{I_z V} \end{bmatrix}$$

$$B_{lat} = \begin{bmatrix} 0 \\ \frac{C_f}{mV} \\ 0 \\ \frac{C_f l_f}{I_z} \end{bmatrix}$$

- A_{lat}, B_{lat} are time-invariant if the forward speed V is kept constant.
- The main input to the system is the driver steering angle command, δ.
- This state space representation is very useful when we are designing different control strategies such as PID or MPC for lateral control.
- The linearity of this model also makes it suitable for state estimation with Kalman filters.

Lesson 6: Vehicle Actuation

- build models for the main vehicle actuation system such as throttling, braking, and steering.
- connect these models to longitudinal and lateral vehicle dynamic models.
- the lateral dynamics and the longitudinal dynamics can affect each other.
- The main task of vehicle control is to provide suitable steering, throttle and brake commands to keep the vehicle on the desired path and following a desired speed profile.
- These desired elements are provided by the motion planning system.



COUPLED LATERAL & LONGITUDINAL

Steering

- The steering angle is translated into a wheel angle through a special mechanism and gear ratios that provide the lateral forces to keep the vehicle on a curved path.
- Simple Steering Model
  - $\delta = c\delta_s$
  - δ : wheel angle
  - δ_s : steering angle
- a fully dynamic model may be needed if steering commands are very near the bandwidth of the steering assembly. （steering assemblyの限界という意味？）

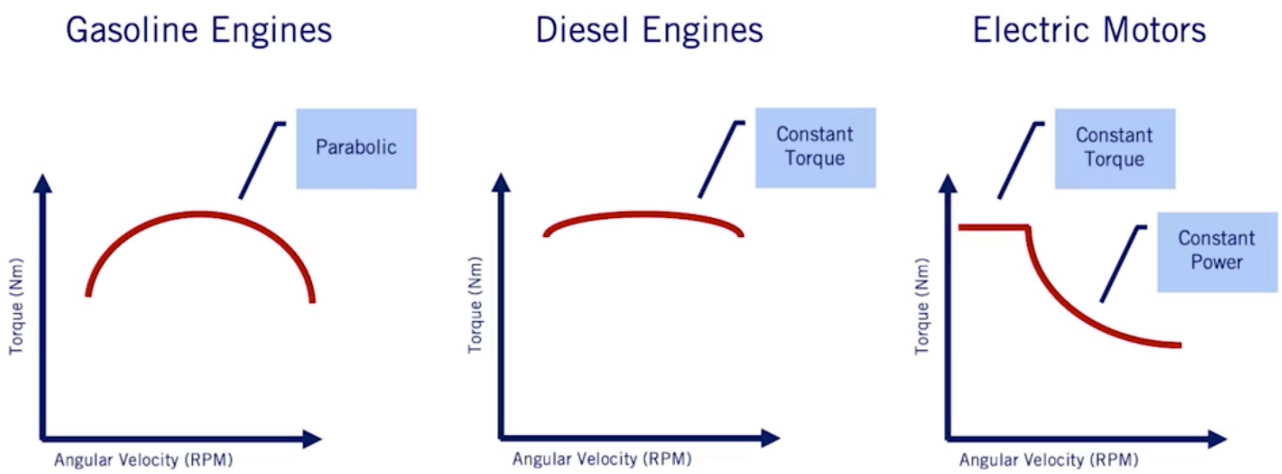
Powertrain System (Driveline)

- Throttle and brake commands affect torque balance
- Gear one and two are torque modes and the higher gears are called speed modes, referring to the different modes of operation in the torque converter.

Accelerating Model

x_θ : accelerator pedal position (Throttle)

x_θ → Power Unit (Engine) → ω_e, T_e
- The characteristics of the internal combustion engine, diesel engines, and electric motors are all different.
- The torque-speed diagram is used as a diagram to represent these characteristics for power units.



CHARACTERISTICS PLOTS OF ENGINES

Characteristics Plots
- Gasoline Engines
  - The highest torque operating point for a gas engine tends to be in the middle RPMs typically 2,000 or 3,000 RPM for a passenger vehicle engine.
- Diesel Engines
  - show more consistent torque generation over a wider range of speeds.
  - This is why diesel engines are more suitable for heavy-duty vehicles and the gasoline engines are best suited for small and city cars.
- Electric Motors
  - more efficient at lower RPMs such as 1,500.
  - not very efficient at higher RPM.
  - to compensate the lack of torque characteristics at the higher RPM in electric motors, the hybrid electric vehicles use the internal combustion engine to improve performance over electric motors at higher speeds.

Typical Torque Curves for Gasoline Engines
- a second-order polynomial
- $T_{e_{max}}(\omega_e) = A_0 + A_1\omega_e + A_2\omega_e^2$
- $T_e(\omega_e, x_\theta) \approx x_\theta(A_0 + A_1\omega_e + A_2\omega_e^2)$
- x_θ : Throttle position (percentage)
- A0, A1, A2 are identified and tuned for different engines
- a semi-empirical model
- feed into the longitudinal dynamics model for the vehicle

Braking (Decelerating)
- brake pedal position → translated into a brake pressure by the electronic control unit.
- The brake pressure results in a braking force on the brake disc or drum, which becomes a braking wheel torque at the wheel.

- $T_{brake} = k\Delta P$
- ΔP : Brake Pressure

Braking System
- Basic functionality of braking includes:
  - Shorten stopping distance
  - Steerability during braking through ABS systems (ABS : Anti-skid Brake System), anti-lock braking
  - Stability during braking to avoid overturning

Lesson 7: Tire Slip and Modeling

単語:

camber angle : 車体に取り付けらている車輪を正面から見たときに、車輪縦方向の中心線が垂直線となす角度をいう。the angle between the tire rotation plane and the road.

Importance of Tire Modeling
- The tire is the interface between the vehicle and road

Vehicle Slip Angle
- Slip Angle : the angle between the forward direction of the vehicle and the actual direction of its motion.
- Slip Angle
  - $\beta = tan^{-1}\frac{V_y}{V_x} = tan^{-1}\frac{\dot{y}}{\dot{x}}$
- Using small angle assumption
  - $\beta \approx \frac{\dot{y}}{\dot{x}}$

Tire Slip Angles
- Tire slip angle is the angle between the direction in which a wheel is pointing and the direction in which it is actually traveling.
- Rear tire slip angle
  - $\alpha_r = -\beta + \frac{l_r\dot{\psi}}{V}$
- Front tire slip angle
  - $\alpha_f = \delta - \beta - \frac{l_f\dot{\psi}}{V}$

Slip Ratios
- Longitudinal slip (also called slip ratio)
- The slip ratio captures the relationship between the deformation of the tire and the longitudinal forces acting upon it.
- When accelerating or braking, the observed angular velocity of the tire does not match the expected velocity for the pure rolling motion, which means there is sliding between the tire and the road in addition to rolling.
- The difference between the rotation speed of the tire and the longitudinal speed of the car can be expressed as a ratio relative to the pure rolling speed, and it's called the slip ratio.
- $s = \frac{wr_e - V}{V}$
  - $wr_e < V$: Wheels are skidding, this happens during deceleration of the vehicle, during normal braking.
  - $wr_e > V$: Wheels are spinning, this happens during acceleration, especially in low friction driving (icy road).

- $wr_e = 0$: Wheels are locked, this happens during heavy or panic braking where the vehicle loses its desired traction.
  - modern anti-lock braking systems seek to avoid this regime due to its poor stopping performance and loss of steering control.

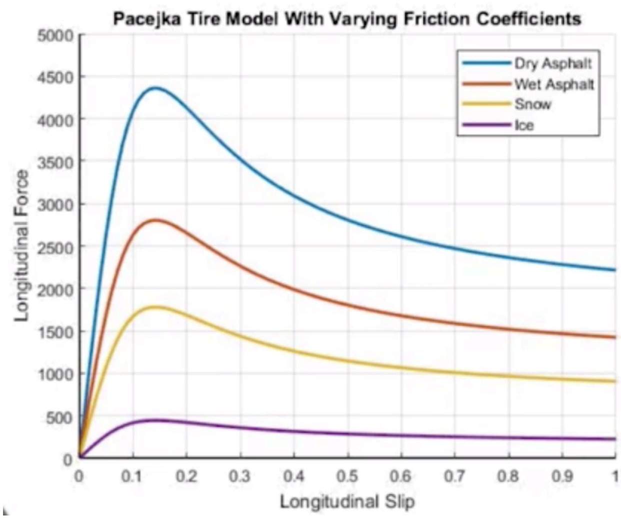
Tire Modeling
- Inputs to the tire model
  - Tire Slip Angle
  - Slip Ratio
  - Normal Force
  - Friction Coefficient
  - Camber Angle
  - Tire Properties
- Outputs of the tire model
  - Lateral Force
  - Longitudinal Force
  - Self-Aligning Moment (the moment about the steering axis. self aligns a tire with the direction of travel.よく分からない)
  - Rolling Resistance Moment
  - Overturning Moment

3つ種類のTire Model : analytical, numerical, parameterized models.
- Analytical : Brush, Fail, Linear
  - Tire physical parameters are explicitly employed
  - Low precision, but simple
- Numerical
  - Look up tables instead of mathematical equations
  - No explicit mathematical form
  - Geometry and material property of tire are considered
  - difficult to use for model-based control development
- Parameterized : Linear, Pacejka, Dugoff
  - Need experiments for each specific tire
  - Formed by fitting model with experimental data (parameterized function)
  - Match experimental data very well
  - Used widely for vehicle dynamics simulation studies and control design

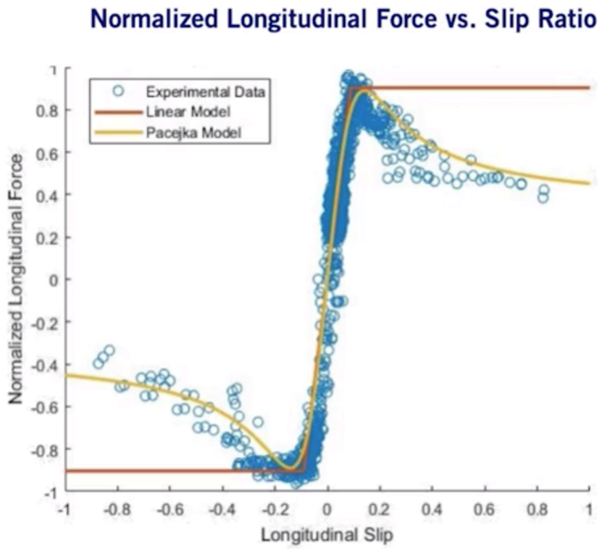
2つParameterized Modelの紹介
Linear Tire Model
- Assumption : the relationship between slip angle and force is linear
  - Piecewise linear curves : $F(x) = \begin{cases} Cx & if\,|x| < x_{max} \\ F_{max} & if\,|x| \geq x_{max} \end{cases}$ 、以下の2つ部分がある
  - S_{max} : largest slip ratio → F_{xmax} : maximal longitudinal tire force
  - α_{max} : largest slip angle → F_{ymax} : maximal lateral tire force
  - The Linear tire model is a good approximation for a significant portion of the linear region, but drops in accuracy as we enter saturation.

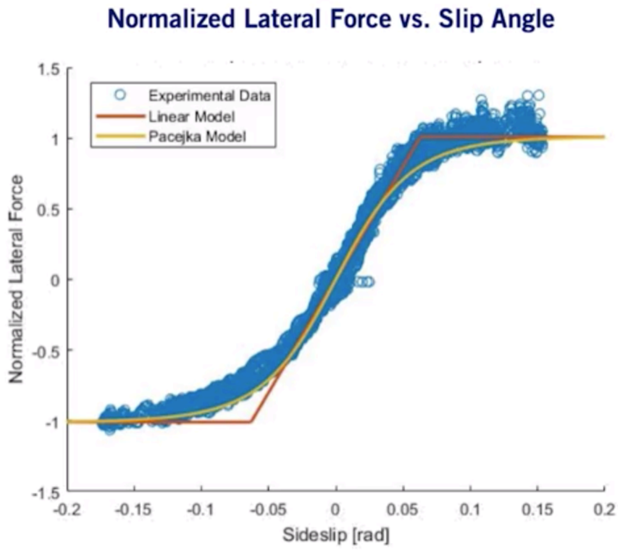
Pacejka Tire Model
- also called the magic formula because of how well it represents longitudinal and lateral tire forces.
- Widely used in model-based control development
- $F(x, F_z) = Dsin(Ctan^{-1}(Bx - E(Bx - tan^{-1}(Bx))))\mu F_z$
- x could be either slip ratio or slip angle (in tire modeling)
- μ : road friction coefficient
- F_z : tire vertical force

- define system parameters B, C, D, E from experiments, and these will differ from one tire to another.
- B : Stiffness Factor
- C : shape factor
- D : peak factor
- E : curvature factor



PACEJKA TIRE MODEL





EXPERIMENTAL DATA & LINEAR MODEL & PACEJKA MODEL