

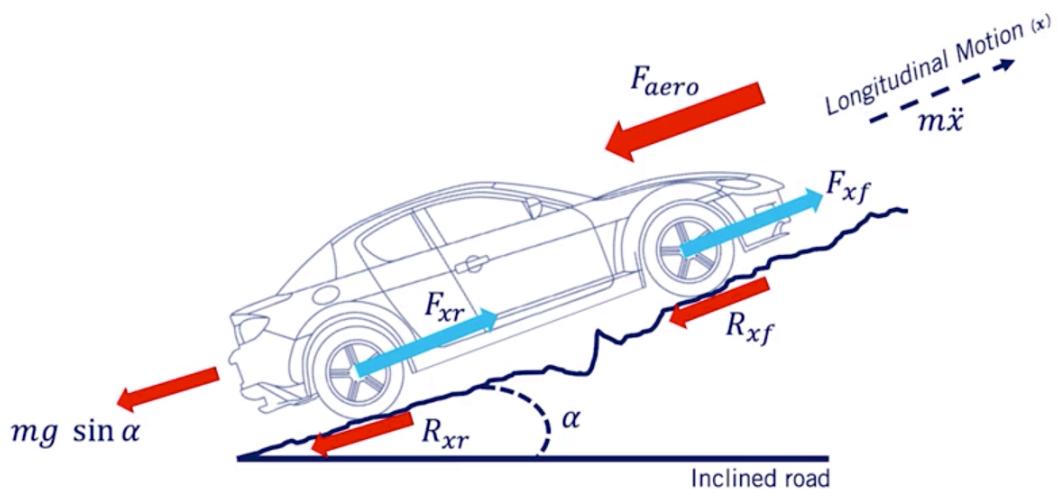
Module 4: Vehicle Dynamic Modeling

Lesson 4: Longitudinal Vehicle Modeling

単語:

1. IC Engine: internal combustion engine.
2. torque converter: a hydraulic device for the **smooth transmission** of power in which an **engine-driven impeller** transmits its momentum to a **fluid** held in a sealed container, which in turn drives a **rotor**.
3. normal force: In mechanics, the normal force F_n is the component of a contact force that is perpendicular to the surface that an object contacts.
4. diesel engine: A diesel engine is an internal combustion engine in which oil is burned by very hot air. Diesel engines are used in buses and trucks, and in some trains and cars.

Longitudinal Vehicle Model

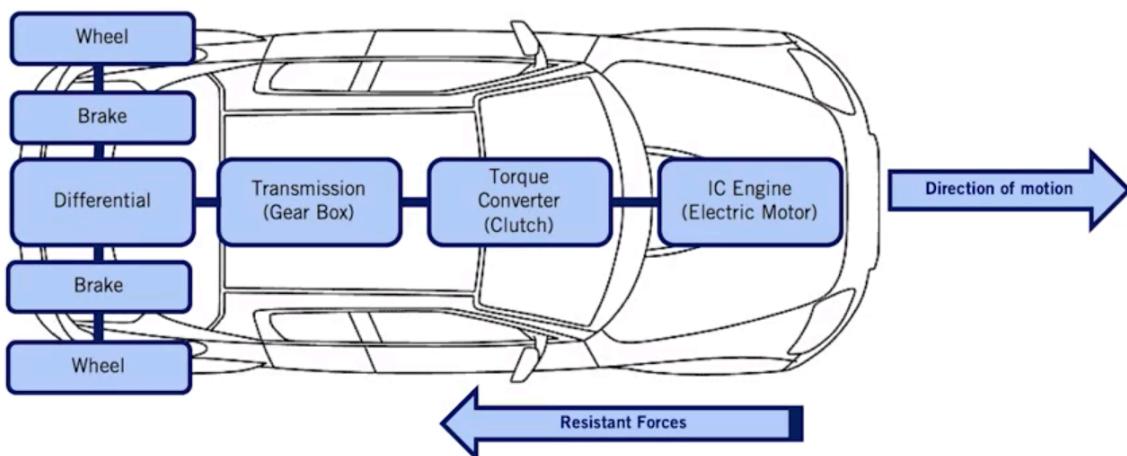


LONGITUDINAL VEHICLE MODEL

$$m\ddot{x} = F_{xf} + F_{xr} - F_{aero} - R_{xf} - R_{xr} - mg \sin \alpha$$

Simplified Longitudinal Dynamics

Let F_x - total longitudinal force: $F_x = F_{xf} + F_{xr}$ (traction force, **牽引** 「けんいん」 力, generated by **power train**)



POWERTRAIN MODELING

Let R_x - total rolling resistance: $R_x = R_{xf} + R_{xr}$

Assume α is a small angle: $\sin \alpha = \alpha$

$$m \ddot{x} = F_x - F_{aero} - R_x - mg\alpha$$

$F_{aero} + R_x + mg\alpha$: total resistant forces (F_{load})

→ develop models for each of the forces in this equation and define how they connect to the throttle and brake inputs that our autonomous system will apply. (power train, brake, tyre force modeling)

Simple Resistance Force Models

- The aerodynamic force can depend on air density, frontal area, the vehicle's coefficient of friction, the current speed of the vehicle
 - Given a fixed vehicle shape and standard atmospheric pressure. これらの因子が一つの c_α で表す (a simple lumped coefficient of the aerodynamic drag)
 - $$F_{aero} = \frac{1}{2} C_\alpha \rho A \dot{x}^2 = c_\alpha \dot{x}^2$$
- The rolling resistance can depend on the tire **normal force**, tire pressure and characteristics, and vehicle speed
 - $$R_x = N(\hat{c}_{r,o} + \hat{c}_{r,1} |\dot{x}| + \hat{c}_{r,2} \dot{x}^2) \approx c_{r,1} |\dot{x}|$$

Powertrain Modeling

drive line

- the sequence of components between the engine and the wheels
 - torque converter (clutch: In a vehicle, the clutch is the pedal that you press before you change gear.)
 - transmission (gearbox)
 - differential (差動装置)

Because of the **direct connection** between wheel and engine **when in gear** (gear: The gears on a machine or vehicle are a device for changing the rate at which energy is changed into motion.), it is possible to model the relationship between the wheel speed and the engine speed as a kinematic constraint.

Rotational Coupling

$$\omega_w = GR\omega_t = GR\omega_e \dots (1)$$

ω_w : wheel angular speed

ω_t : turbine angular speed (turbine: A turbine is a machine or engine which uses a stream of air, gas, water, or steam to turn a wheel and produce power.)

ω_e : engine angular speed

GR : **Combined gear ratios** (including the torque converter, transmission, differential)

Longitudinal Velocity

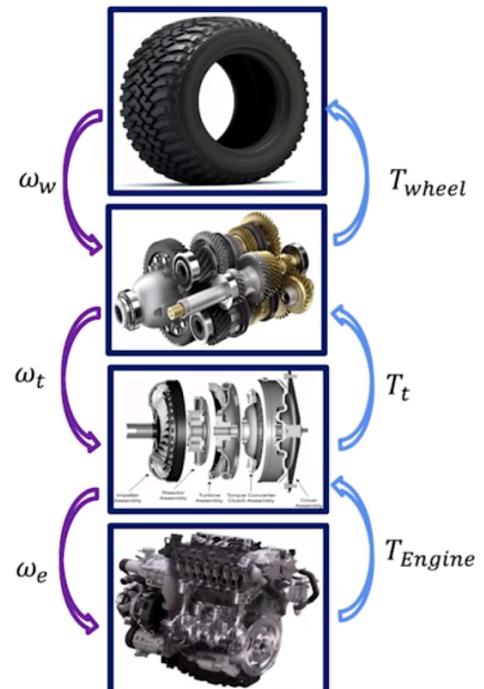
$$\dot{x} = r_{eff}\omega_w \dots (2)$$

r_{eff} : Tire effective radius

- This expression for velocity can be differentiated to give longitudinal acceleration in terms of the **engine rotational acceleration**.

Longitudinal Acceleration

$$\ddot{x} = r_{eff} GR \dot{\omega}_e$$



POWER FLOW IN POWERTRAIN

Power Flow in Powertrain

- The wheel is the intersection between the torques coming from the power train side, and the torques acting from the external resistance forces.
- Wheel
 - $I_w \dot{\omega}_w = T_{wheel} - r_{eff} F_x$
 - I : moment of inertial
 - $T_{wheel} = I_w \dot{\omega}_w + r_{eff} F_x$
- Transmission
 - $I_t \dot{\omega}_t = T_t - (GR)T_{wheel}$
 - $I_t \dot{\omega}_t = T_t - GR(I_w \dot{\omega}_w + r_{eff} F_x)$
 - T_{wheel} : actually the combination of the brake torque and the output torque of the transmission or gearbox
 - T_t : the torque applied to the transmission. still includes a dependence on the tire force F_x .
- Torque Converter
 - $\omega_t = \omega_e$ (When Coupled)
 - $T_t = (I_t + I_w GR^2) \dot{\omega}_e + GR r_{eff} F_x$: see Rotational Coupling (1)
- Engine
 - $I_e \dot{\omega}_e = T_{Engine} - T_t$
 - $I_e \dot{\omega}_e = T_{Engine} - (I_t + I_w GR^2) \dot{\omega}_e - GR r_{eff} F_x \dots (3)$

Engine Dynamics

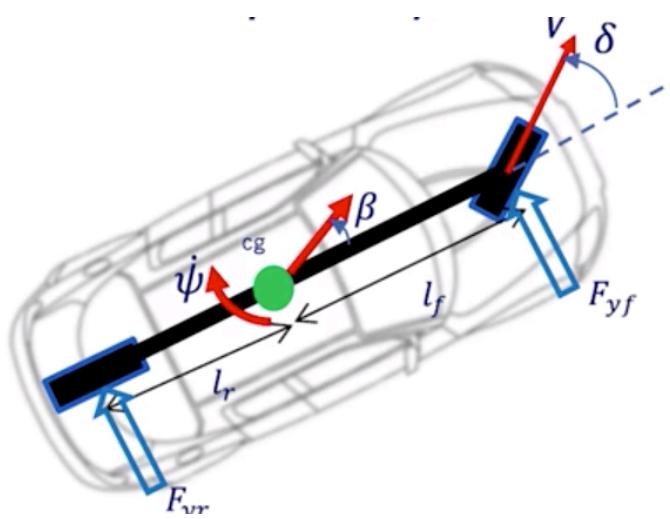
- Tire force in terms of inertia and load force:
 - $F_x = m \ddot{x} + F_{load} = mr_{eff} GR \dot{\omega}_e + F_{load} \dots (4)$: combine (1) and (2)
- Combine with our engine dynamics model yields:
 - $(I_e + I_t + I_w GR^2 + m(GR^2)r_{eff}^2) \dot{\omega}_e = T_{Engine} - (GR)(r_{eff} F_{load})$: combine (3) and (4)
 - an effective power train inertia as the sum of all the individual component inertias
 - $(I_e + I_t + I_w GR^2 + m(GR^2)r_{eff}^2) : J_e$
- Finally, the engine dynamic model simplifies to
 - $J_e \dot{\omega}_e = T_{Engine} - (GR)(r_{eff} F_{load})$
 - $(GR)(r_{eff} F_{load})$: Total Load Torque (T_{Load})
- We do still need to relate the engine torque to the **accelerator pedal position**, and the brake torque to the **brake pedal position**, which in the video on Actuator Modeling.

Lesson 5: Lateral Dynamics of Bicycle Model

単語リスト :

1. suspension : A vehicle's suspension consists of the springs and other devices attached to the wheels, which give a smooth ride over uneven ground.
2. centripetal: acting, moving, or tending to move towards a centre.
3. centrifugal: acting, moving, or tending to move away from a centre.

extend our kinematic bicycle model to a dynamic model by **relaxing the no slip condition** and force for the kinematic model.



Vehicle Model to Bicycle Model

- Assumptions

- Longitudinal velocity is constant (目的: to decouple lateral and longitudinal dynamic models. but does **lead to modeling inaccuracies** when accelerating or decelerating out of curves.)
- Left and right axle are lumped into a single wheel (bicycle model)
- Suspension movement, road inclination and aerodynamic influences are neglected

Lateral Dynamics

- use the vehicle center of gravity as the reference point.

$$a_y = \ddot{y} + \omega^2 R = V\dot{\beta} + V\dot{\psi}$$

- a_y : the total acceleration in the **inertial frame**.

- \ddot{y} : the lateral acceleration in the **body frame**.

- $\omega^2 R$: the centripetal acceleration from rotation of the vehicle

- $\dot{\beta}$: the slip angle rate of change (**side slip rate**)

- $\dot{\psi}$: the heading rate of change (psi) (**yaw rate**)

$$mV(\dot{\beta} + \dot{\psi}) = F_{yf} + F_{yr} \dots (1)$$

- $F_{yf} + F_{yr}$: front and rear tire forces

$$I_z\ddot{\psi} = l_f F_{yf} - l_r F_{yr} \dots (2)$$

- I_z : vehicle inertia

- $\ddot{\psi}$: angular acceleration

- the moments produced by the tire forces act in opposite directions.

Tire forces are generally hard to predict exactly, and tire models tend to be nonlinear and empirically identified. linear tire modelを利用する時十分注意必要!

- Many different tire slip models
- For small tire slip angles, the lateral tire forces are approximated as a linear function of tire slip angle.
 - This approximation is really **only valid for small slip angles**, and the tire forces are modeled as varying linearly with slip angles specifically.
 - As we use the linear tire model in our control design, we must make sure to not exceed this small angle assumption by **sticking to non-aggressive driving maneuvers**.
 - typically less than 8 degrees.
- Tire variables
 - Front tire slip angle, α_f
 - Rear tire slip angle, α_r

Front and Rear Tire Forces

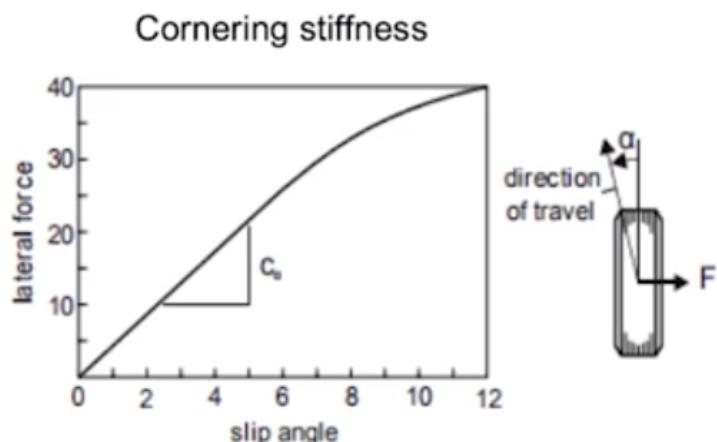
- **cornering stiffness** of a tire : the **ability to resist deformation while the vehicle corners**.

- c_y : cornering stiffness coefficients, the slope of the line at zero

- c_f : **linearized** cornering stiffness of the front wheel

$$- F_{yf} = C_f \alpha_f = C_f (\delta - \beta - \frac{l_f \dot{\psi}}{V})$$

- α : the tire slip angle



LATERAL FORCE & SLIP ANGLE

- β : the vehicle slip angle
- δ : the steering angle
- c_r : linearized cornering stiffness of the rear wheel
 - $F_{yr} = C_r \alpha_r = C_r(-\beta + \frac{l_r \dot{\psi}}{V})$
 - (1) (2) に代入する

Lateral and Yaw Dynamics (linear because of assumptions)

$$\begin{aligned}\dot{\beta} &= \frac{-(C_r + C_f)}{mV} \beta + \left(\frac{C_r l_r - C_f l_f}{mV^2} - 1 \right) \dot{\psi} + \frac{C_f}{mV} \delta \\ \ddot{\psi} &= \frac{C_r l_r - C_f l_f}{I_z} \beta - \frac{C_r l_r^2 + C_f l_f^2}{I_z V} \dot{\psi} + \frac{C_f l_f}{I_z} \delta\end{aligned}$$

- $\dot{\beta}$: the rate of change of the vehicle slip angle
- $\dot{\psi}$: the rate of change of the yaw rate

Standard State Space Representation

- State Vector : $X_{lat} = [y \ \beta \ \psi \ \dot{\psi}]^T$
- y : lateral position
- β : side slip angle
- ψ : yaw angle
- $\dot{\psi}$: yaw rate

$$\dot{X}_{lat} = A_{lat} X_{lat} + B_{lat} \delta$$

$$A_{lat} = \begin{bmatrix} 0 & V & V & 0 \\ 0 & -\frac{C_r + C_f}{mV} & 0 & \frac{C_r l_r - C_f l_f}{mV^2} - 1 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{C_r l_r - C_f l_f}{I_z} & 0 & -\frac{C_r l_r^2 + C_f l_f^2}{I_z V} \end{bmatrix}$$

$$B_{lat} = \begin{bmatrix} 0 \\ \frac{C_f}{mV} \\ 0 \\ \frac{C_f l_f}{I_z} \end{bmatrix}$$

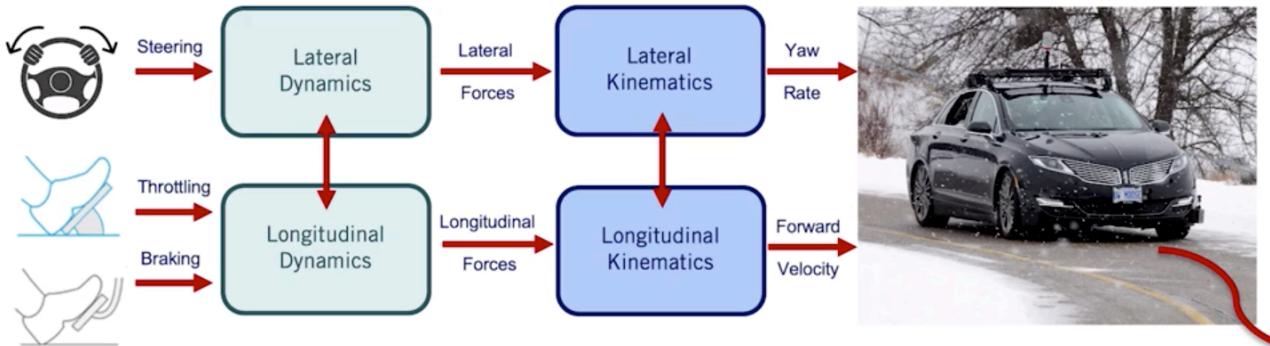
- A_{lat}, B_{lat} are time-invariant if the forward speed V is kept constant.
- The main input to the system is the driver steering angle command, δ .
- This state space representation is very useful when we are designing different control strategies such as PID or MPC for lateral control.
- The linearity of this model also makes it suitable for state estimation with Kalman filters.

Lesson 6: Vehicle Actuation

単語:

1. consistent: Someone who is consistent always behaves in the same way, has the same attitudes towards people or things, or achieves the same level of success in something.
2. pinion: a cogwheel that engages with a larger wheel or rack, which it drives or by which it is driven.

- build models for the main vehicle actuation system such as throttling, braking, and steering.
- connect these models to longitudinal and lateral vehicle dynamic models.
- the lateral dynamics and the longitudinal dynamics can affect each other.
- The **main task of vehicle control** is to **provide suitable steering, throttle and brake commands** to keep the vehicle **on the desired path** and following a **desired speed profile**.
- These desired elements are **provided by the motion planning system**.



Steering

- The steering angle is translated into a wheel angle through a special mechanism and gear ratios that provide the lateral forces to keep the vehicle on a curved path.
- Simple Steering Model
 - $\delta = c\delta_s$
 - δ : wheel angle
 - δ_s : steering angle
- A **fully dynamic model** may be needed if steering commands are very near the bandwidth of the steering assembly. (steering assemblyの限界という意味?)
- steering assemblyのmain subsystem: steering wheel, steering column, steering gears and linkages that connect the steering column to the left and right wheels.
- **rack and pinion gears** which translate the **rotational motion** of the steering wheel **to the linear motions** needed to move the wheel linkages.

Powertrain System (Driveline)

- Throttle and brake commands affect torque balance
- Gear one and two are torque modes and the higher gears are called speed modes, referring to the different modes of operation in the torque converter.

Accelerating Model

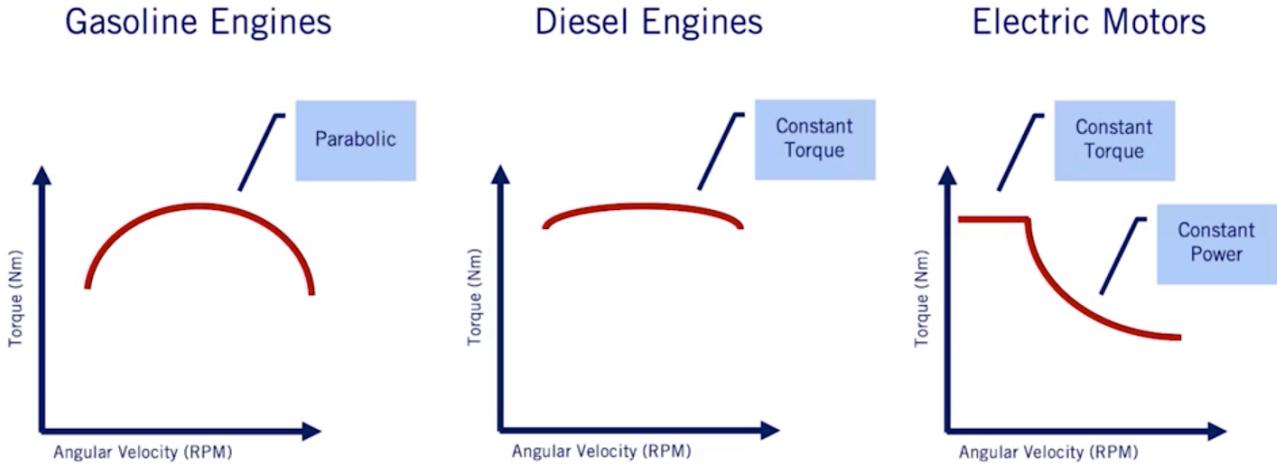
x_θ : accelerator pedal position (Throttle)

$x_\theta \rightarrow$ Power Unit (Engine) $\rightarrow \omega_e, T_e$

- The characteristics of the internal combustion engine, diesel engines, and electric motors are all different.
- The torque-speed diagram is used as a diagram to represent these characteristics for power units.

Characteristics Plots

- Gasoline Engines
 - The highest torque operating point for a gas engine tends to be in the middle RPMs typically 2,000 or 3,000 RPM for a passenger vehicle engine.
 - At low and high speeds, the ability of the engine to produce torque falls off.
- Diesel Engines
 - show more **consistent torque** generation over a wider range of speeds.



- This is why **diesel engines** are more suitable for **heavy-duty vehicles** and the gasoline engines are best suited for small and city cars.
- Electric Motors
 - more efficient at lower RPMs such as 1,500.
 - not very efficient at higher RPM.
 - to **compensate** the lack of torque characteristics at the higher RPM in electric motors, the **hybrid electric** vehicles use the **internal combustion engine** to improve performance over electric motors at higher speeds.

Typical Torque Curves for Gasoline Engines

- a second-order polynomial
- $T_{e_{max}}(\omega_e) = A_0 + A_1\omega_e + A_2\omega_e^2$
- $T_e(\omega_e, x_\theta) \approx x_\theta(A_0 + A_1\omega_e + A_2\omega_e^2)$
 - Given the engine speed and throttle position, it is possible to compute the engine torque produced based on this semi-empirical map, which then feeds into the longitudinal dynamics model for the vehicle.
- x_θ : Throttle position (percentage)
- A_0, A_1, A_2 are identified and tuned for different engines
- a semi-empirical model
 - more detailed models can include the dynamics of fluids, heat transfer, combustion and many other phenomena.

Braking (Decelerating)

- brake pedal position → translated into a brake pressure by the electronic control unit.
- The brake pressure results in a braking force on the brake disc or drum, which becomes a **braking wheel torque** at the wheel.
 - The braking torque on the wheels creates a negative longitudinal force to decelerate the vehicle.
 - Because of the control unit management of the braking torque, the response to the braking system is very predictable.
- $T_{brake} = k\Delta P$
- ΔP : Brake Pressure

Braking System

- Basic functionality of braking includes:
 - Shorten stopping distance through hard braking

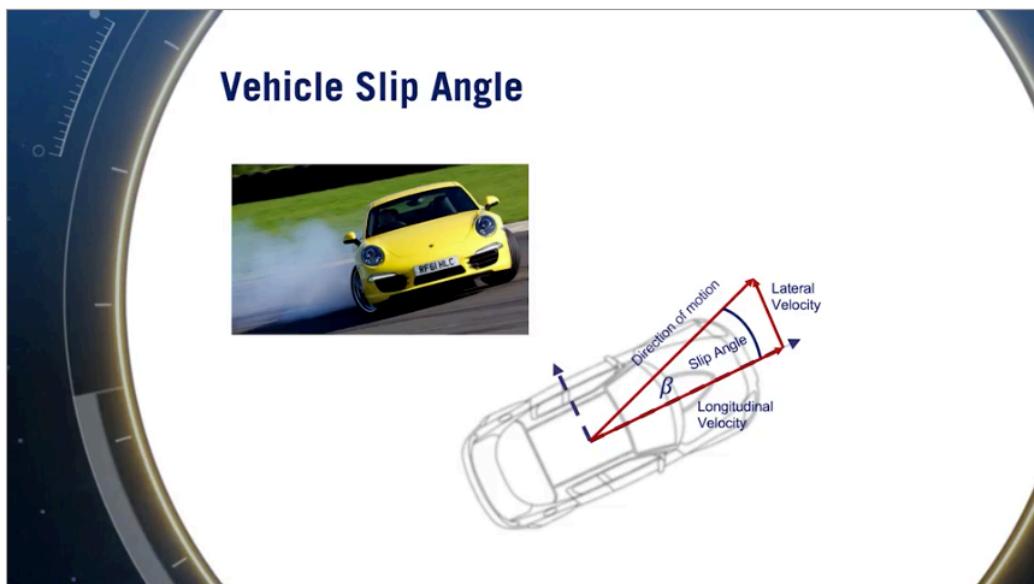
- Maintain steerability during braking through ABS systems (ABS : Anti-skid Brake System), anti-lock braking
- Maintain stability during braking to avoid overturning

In the autonomous car, these actuation signals will be the main output of the vehicle control module.

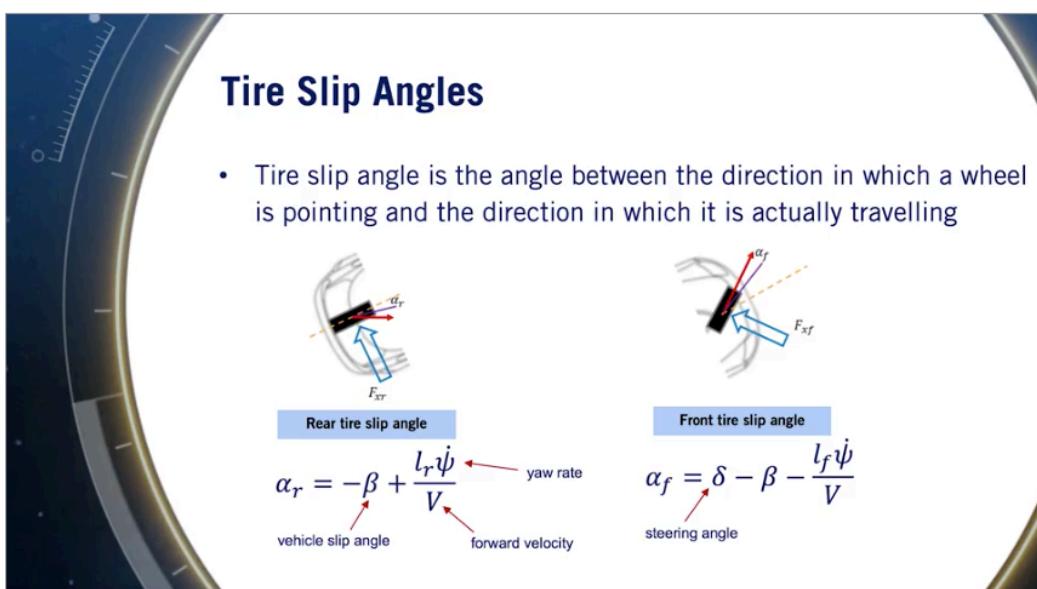
Lesson 7: Tire Slip and Modeling

単語:

1. camber angle : 車体に取り付けられている車輪を正面から見たときに、車輪縦方向の中心線が垂直直線となす角度をいう。the angle between the tire rotation plane and the road.
2. skid: If a vehicle skids, it slides sideways or forward while moving, for example, when you are trying to stop it suddenly on a wet road.



Importance of Tire Modeling



- The tire is the interface between the vehicle and road

Vehicle Slip Angle

- Slip Angle : the angle between the forward direction of the vehicle and the **actual direction of its motion**.

- Slip Angle

$$- \beta = \tan^{-1} \frac{V_y}{V_x} = \tan^{-1} \frac{\dot{y}}{\dot{x}}$$

- Using small angle assumption

$$- \beta \approx \frac{\dot{y}}{\dot{x}}$$

Tire Slip Angles

- Tire slip angle is the angle between the **direction** in which a wheel is **pointing** and the **direction** in which it is **actually traveling**.

- Rear tire slip angle

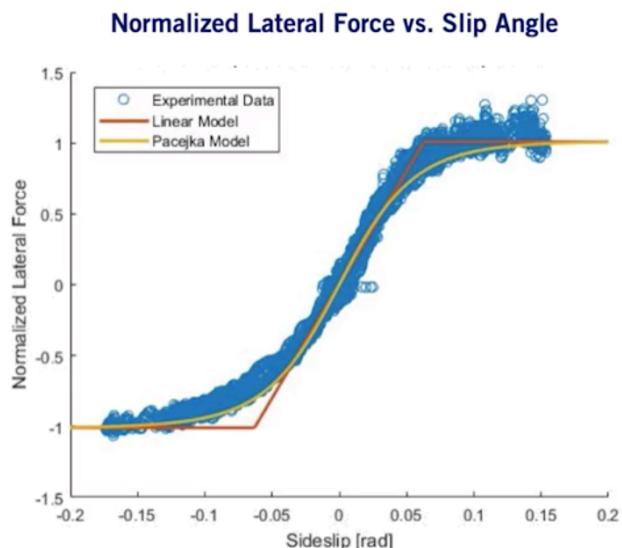
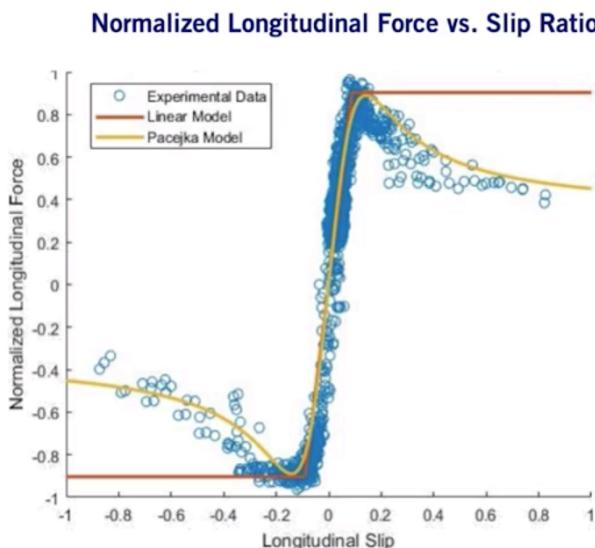
$$- \alpha_r = -\beta + \frac{l_r \dot{\psi}}{V}$$

- Front tire slip angle

$$- \alpha_f = \delta - \beta - \frac{l_f \dot{\psi}}{V}$$

Slip Ratios

- Longitudinal slip (also called slip ratio)
- The slip ratio captures the relationship between the deformation of the tire and the longitudinal forces acting upon it.
- When accelerating or braking, the observed angular velocity of the tire does not match the expected velocity for the pure rolling motion, which means there is sliding between the tire and the road in addition to rolling.
- The **difference** between the **rotation speed** of the **tire** and the **longitudinal speed** of the **car** can be expressed as a ratio relative to the pure rolling speed, and it's called the slip ratio.



$$- s = \frac{wr_e - V}{V}$$

- $wr_e < V$: Wheels are **skidding**, this happens during **deceleration** of the vehicle, during **normal braking**.
- $wr_e > V$: Wheels are **spinning**, this happens during **acceleration**, especially in **low friction** driving (icy road).
- $wr_e = 0$: Wheels are **locked**, this happens during heavy or panic braking where the vehicle loses its desired traction.
 - modern anti-lock braking systems seek to avoid this regime due to its **poor stopping performance** and **loss of steering control**.

Tire Modeling

- Inputs to the tire model
 - Tire Slip Angle
 - Slip Ratio
 - Normal Force
 - Friction Coefficient
 - Camber Angle
 - Tire Properties
- Outputs of the tire model
 - Lateral Force
 - Longitudinal Force
 - Self-Aligning Moment (the moment about the steering axis. self aligns a tire with the direction of travel. よく分からぬ)
 - Rolling Resistance Moment
 - Overturning Moment

3つ種類のTire Model : analytical, numerical, parameterized models.

- Analytical : Brush, Fiala
 - Tire physical parameters are explicitly employed
 - Low precision, but simple
- Numerical
 - Look up tables instead of mathematical equations
 - No explicit mathematical form
 - Geometry and material property of tire are considered
 - difficult to use for model-based control development
- Parameterized : Linear, Pacejka, Dugoff
 - **Need experiments for each specific tire**
 - Formed by fitting model with experimental data (parameterized function)
 - Match experimental data very well
 - Used widely for vehicle dynamics simulation studies and control design

2つParameterized Modelの紹介

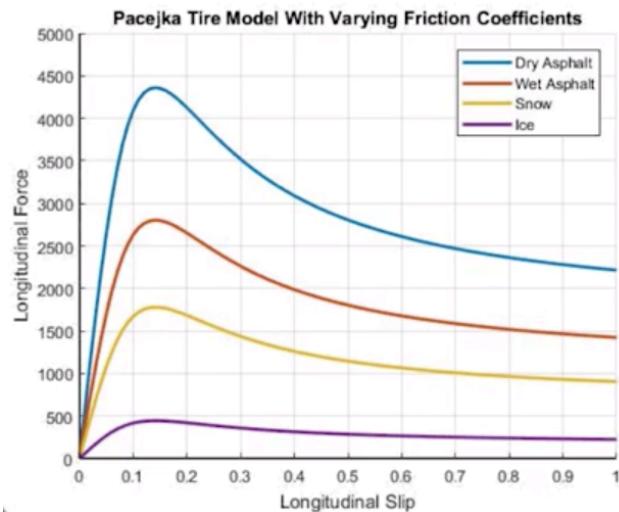
Linear Tire Model

- Assumption : the relationship between slip angle and force is linear
- Piecewise linear curves : $F(x) = \begin{cases} Cx & \text{if } |x| < x_{max} \\ F_{max} & \text{if } |x| \geq x_{max} \end{cases}$ 、以下の2つ部分がある
- S_{max} : largest slip ratio $\rightarrow F_{xmax}$: maximal longitudinal tire force
- α_{max} : largest slip angle $\rightarrow F_{ymax}$: maximal lateral tire force
- The Linear tire model is a good approximation for a significant portion of the linear region, but drops in accuracy as we enter saturation.

Pacejka Tire Model

- also called the **magic formula** because of how well it represents longitudinal and lateral tire forces.

- Widely used in model-based control development
-



$$F(x, F_z) = D \sin(C \tan^{-1}(Bx - E(Bx - \tan^{-1}(Bx)))) \mu F_z$$

- x could be either slip ratio or slip angle (in tire modeling)
- μ : road friction coefficient
- F_z : tire vertical force
- define system parameters B, C, D, E from experiments, and these will differ from one tire to another.
- B : Stiffness Factor
- C : shape factor
- D : peak factor
- E : curvature factor

Torque Converter

参考: How Torque Converters Work!

https://www.youtube.com/watch?v=z5G2zQ_3xTc

Impeller

Turbine

Blades of Impeller and Blades of Turbine

Stator及び液体の流れ（特にturbine->impellerの液体）に関して、まだ分かっていない。

Phase 1: Stall

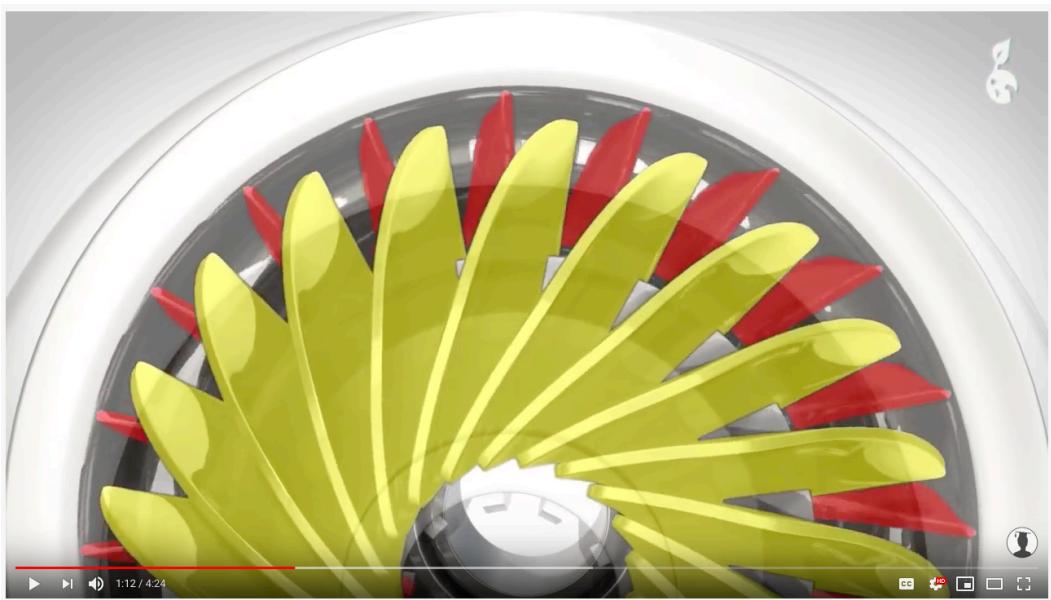
- Impeller rotates, but the turbine does not rotate
- Driver applies brakes, and the engine still runs

Phase 2: Acceleration

- High impeller speed
- RPM (revolutions per minute) impeller > turbine
- Torque impeller < turbine
- RPM impellerやTorque impellerの意味はまだ分かっていない。

Phase 3: Coupling

- Driving at high speeds
- RPM and torque almost equal
- Clutch connects impeller + turbine
- RPM and torque equal



BLADES OF IMPELLER (RED) & BLADES OF TURBINE (YELLOW)