

Module 3: Mission Planning in Driving Environments

Lesson 1: Creating a Road Network Graph

単語

- Closure: The closure of a road or border is the blocking of it in order to prevent people from using it.

内容

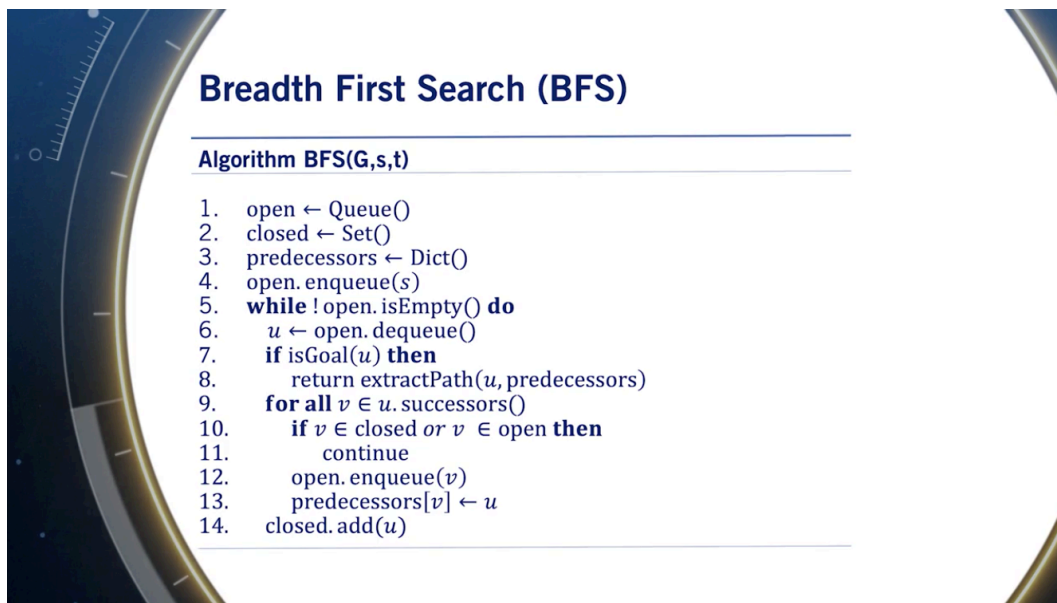
- The mathematical concept of a graph.
- Use a directed graph to represent a road network.
- Implement Breadth-First Search.

Mission Planning

- The mission planner will focus on **aspects of the road network** when planning, such as **speed limits** and **road lengths**, **traffic flow rates** and **road closures**.

Graphs

- Since our graph formulation is currently unweighted, a good candidate algorithm is the Breadth-First Search or BFS.



Breadth First Search (BFS)

Algorithm BFS(G,s,t)

```
1. open ← Queue()
2. closed ← Set()
3. predecessors ← Dict()
4. open.enqueue(s)
5. while !open.isEmpty() do
6.   u ← open.dequeue()
7.   if isGoal(u) then
8.     return extractPath(u, predecessors)
9.   for all v ∈ u.successors()
10.    if v ∈ closed or v ∈ open then
11.      continue
12.    open.enqueue(v)
13.    predecessors[v] ← u
14.    closed.add(u)
```

Breadth First Search (BFS)

- At a high level, BFS can be thought of as iterating through all of the vertices in the graph but doing so in a manner such that **all adjacent vertices are evaluated first** before proceeding deeper into the graph.
- extractPathする時、途中predecessors[u]に複数あっても、どのpredecessorを経由しても必ずsに行ける？
 - そうだろう。sから探索するから。しかし複数のpredecessorの可能性はなさそう。なぜなら、vertexがopen queueに入れられる時predecessorをつけるでしょう。つまり一回だけつけるでしょう。
 - でもloopがあったら？ (if v ∈ closed or v ∈ open then continueはこれを防ぐ)

- つまりgraphにloopがあるかもしれないが、searchはtreeの形になる。
- This prevents us from getting stuck in cycles during the graph search.
- A dictionary is an unordered set of key-value pairs and for each node in the closed set, stores a predecessor vertex that will identify momentarily.
- Because we use a queue to store open vertices, we ensure that all adjacent vertices at the current depth in the search are processed before proceeding deeper into the graph.

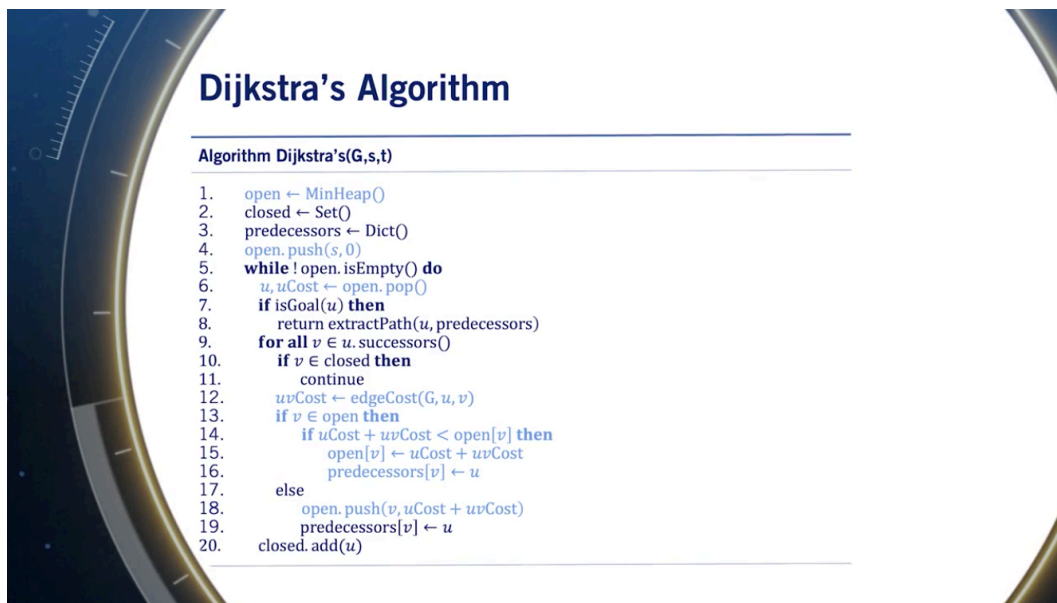
Lesson 2: Dijkstra's Shortest Path Search

内容

- The difference between weighted and unweighted graphs.
- Recognize the value of weighted graphs to the mission planning problem.
- Be able to implement Dijkstra's algorithm in a mission planning context to find the shortest path to a destination in a graph.

Weighted Graph

- The units of the weights are arbitrary, as long as they are common to all edges.



Dijkstra's Algorithm

Algorithm Dijkstra's(G, s, t)

1. $open \leftarrow \text{MinHeap}()$
2. $closed \leftarrow \text{Set}()$
3. $predecessors \leftarrow \text{Dict}()$
4. $open.push(s, 0)$
5. **while** ! $open.isEmpty()$ **do**
6. $u, uCost \leftarrow open.pop()$
7. **if** isGoal(u) **then**
8. **return** extractPath($u, predecessors$)
9. **for all** $v \in u.successors()$
10. **if** $v \in closed$ **then**
11. **continue**
12. $uvCost \leftarrow \text{edgeCost}(G, u, v)$
13. **if** $v \in open$ **then**
14. **if** $uCost + uvCost < open[v]$ **then**
15. $open[v] \leftarrow uCost + uvCost$
16. $predecessors[v] \leftarrow u$
17. **else**
18. $open.push(v, uCost + uvCost)$
19. $predecessors[v] \leftarrow u$
20. $closed.add(u)$

Dijkstra's Algorithm (大事、先生の説明が凄い)

- The main difference is in the **order we process the vertices**.
- A MinHeap is a data structure that stores keys and values, and sort the keys in terms of their associated values from smallest to largest.
- In our case, **the values of each key vertex in the graph will correspond to the distance it takes to reach that vertex**, along the shortest path to that vertex we've found so far.
 - In this sense, Dijkstra's algorithm processes vertices with a **lower accumulated cost** before other ones.
 - Thus unlike BFS, a vertex that was added later in the search can be processed before when that was added earlier, so long as its accumulated cost is lower.
 - 処理順番はaccumulated costです。挿入順番じゃない。
- One interesting case however, is if we **find a new path to a vertex** that is **already in the open heap but has not been processed yet**.
 - In this case, we have to check if the newly found path to this vertex is cheaper than the older path.
- Once we process the goal vertex, we must have necessarily processed all possible predecessor vertices of the goal node.

- Since a predecessor must have accumulated distance less than or equal to the goal vertex.
- この保証は処理順番によるんだ。つまりもしあるnodeが本当に最短パスに位置したら、このnodeはgoal nodeの処理前必ず既に処理された。このnodeまでのcostがgoal nodeまでのcostより絶対低いから。つまり先に処理される。
- Since all predecessor vertices have been processed, we will have found the shortest path to the goal vertex.

Search on a Map

- Example - map of Berkeley, California.
 - 2097 vertices.
 - 5740 edges.
- Example - map of New York City, New York.
 - 54,837 vertices.
 - 140,497 edges.
- Vertices of the graph correspond to **intersections**, and the edges correspond to road segments.
- Dijkstra's algorithm is quite efficient, which allows it to scale really well to real-world problems such as these two maps.
- While Dijkstra's is an efficient algorithm, we can **leverage certain heuristics** to make it even faster in practice.

Lesson 3: A* Shortest Path Search

内容

- What **admissible** heuristics are in the context of graph search.
- How to use the Euclidean heuristic to improve our mission planning speed in practice.
- Implement the A* search algorithm, leveraging the Euclidean heuristic.
- How to apply A* search to variants on the mission planning problem involving **time** instead of distance.

Recall: Dijkstra's for Weighted Graph

- Dijkstra's algorithm required us to search almost all of the edges present in the graph, even though only a few of them were actually useful for constructing the optimal path.

Euclidean Heuristic

- Search Heuristicの意味: In this context, a search heuristic is an **estimate** of the **remaining cost** to reach the destination vertex from any given vertex in the graph.
- Exploits structure of the problem.
- Fast to calculate.
- Straight-line distance between two vertices is a useful estimate of true distance along the graph.
 - このestimateが可能な理由は、graphは地図だから。
 - $h(v) = \|t - v\|$.
- This estimate is always an **underestimate** of the true distance to reach the goal. (大事)
 - This is an **important requirement for A* search**, and heuristics that satisfy this requirement are called **admissible heuristics**.

A* Algorithm

- The main difference between Dijkstra's algorithm and A* is that instead of using the accumulated cost, we use the **accumulated cost plus $h(v)$** (つまり**estimated total cost to the goal**), as the value we push onto the MinHeap.
 - In this sense, A* biases the search towards vertices that are **likely to be part of the optimal path** according to our search heuristic.
 - **accumulated cost + $h(v)$** の意味は、もし始点からvまで最短パスにあったら、goalまでの最低コストだよ。
 - この最低コストが小さい方が最短パスになりそうだよ、というバイアス。

A* Algorithm

Algorithm A*(G,s,t)

```
1. open ← MinHeap()
2. closed ← Set()
3. predecessors ← Dict()
4. open.push(s, 0)
5. while !open.isEmpty() do
6.   u, uCost ← open.pop()
7.   if isGoal(u) then
8.     return extractPath(u, predecessors)
9.   for all v ∈ u.successors()
10.    if v ∈ closed then
11.      continue
12.    uvCost ← edgeCost(G, u, v)
13.    if v ∈ open then
14.      if uCost + uvCost + h(v) < open[v] then
15.        open[v] ← uCost + uvCost + h(v)
16.        costs[v] ← uCost + uvCost
17.        predecessors[v] ← u
18.    else
19.      open.push(v, uCost + uvCost)
20.      costs[v] ← uCost + uvCost
21.      predecessors[v] ← u
22.    closed.add(u)
```

間違っている、修正は下記

- Since we are storing a heuristic based total cost in the MinHeap, we also need to keep track of the true cost of each vertex as well, which we store in the *cost* structure.
- An interesting thing to note is that if we take our heuristic to be zero for all vertices which is still an admissible heuristic, we then end up with Dijkstra's algorithm.
- スライドのアルゴリズムは間違っている。
 - 6行目: *uCost*は*costs[v]*から取るべき! なぜなら*open*には*h*が入るので、本当のコストじゃない!
 - そういえば*costs* dictionaryの初期化も欠いている。
 - 14行目: スライドの判定もいいけど、*if uCost + uvCost < costs[v] then*の方がいいでしょう。
 - 19行目: *open.push(v, uCost + uvCost + h(v))*.
 - 参考ページ:
 - https://en.wikipedia.org/wiki/A*_search_algorithm
 - <http://theory.stanford.edu/~amitp/GameProgramming/ImplementationNotes.html>

Extension to Other Factors

- Traffic, speed limits, and weather affect mission planning.
- Time rather than distance is better at capturing these factors.
- Replace distance edge weights with time estimates.

- use $\frac{h(v)}{v_{max}}$ as the new heuristic.

単語

- Decimal place: the position of a digit **after the decimal point**, each successive position to the right having a denominator of an increased power of ten.