Module 4: Vehicle Dynamic Modeling

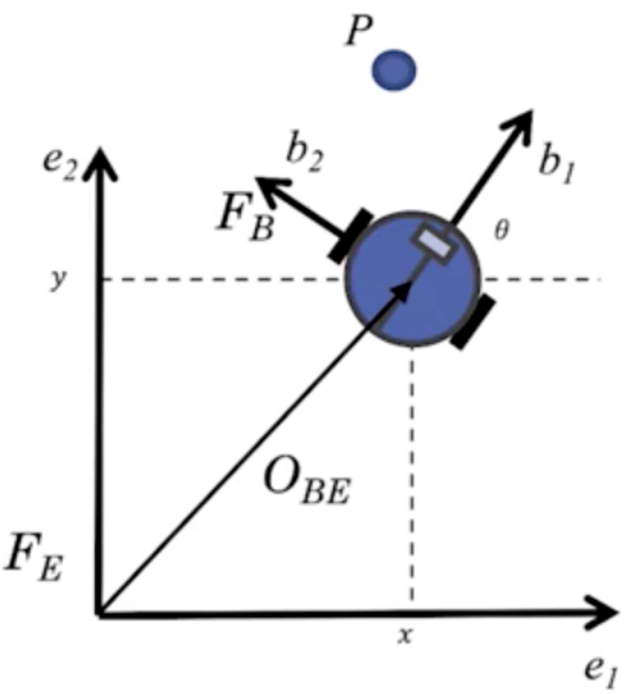
Lesson 1: Kinematic（運動学的な） Modeling in 2D

Generally, vehicle motion can be modeled either by considering the geometric constraint that defines its motion（Kinematic Modeling, especially at low speeds when the accelerations are not significant） or by considering all of the forces and moments acting on a vehicle（Dynamic Modeling）.

Kinematic Vs Dynamic Modeling
- At low speeds, it is often sufficient to look only at kinematic models of vehicles
  - Examples: two wheeled robot, bicycle model
- Dynamic modeling is more involved, but captures vehicle behavior more precisely over a wide operating range
  - Example: Dynamic vehicle model

Coordinate Frames
- Right handed
- Inertial frame（global world coordinate frame ）
  - Fixed, usually relative to earth, East North Up, ENU, Earth-Centered Earth Fixed, ECEF (GNSS).
- Body frame
  - Attached to vehicle, origin at vehicle center of gravity, or center of rotation. (or the center point of the rear axle)
  - This frame is moving and rotating with respect to the fixed inertial frame as the vehicle moves about.
- Sensor frame
  - Attached to sensor, convenient for expressing sensor measurements.

Coordinate Transformation
Conversion between Inertial frame and Body coordinates is done with a translation vector and a rotation matrix



Example: 2-wheeled robot

$$C_{EB} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$$

- C_{EB}: transform vector from the frame b to the frame e. clockwise

$$C_{BE} = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$$

- C_{BE}: project the frame e onto frame b. counter clockwise.

-Location of P in Body Frame B:

$$P_B = C_{EB}(\theta)P_E + O_{EB}$$

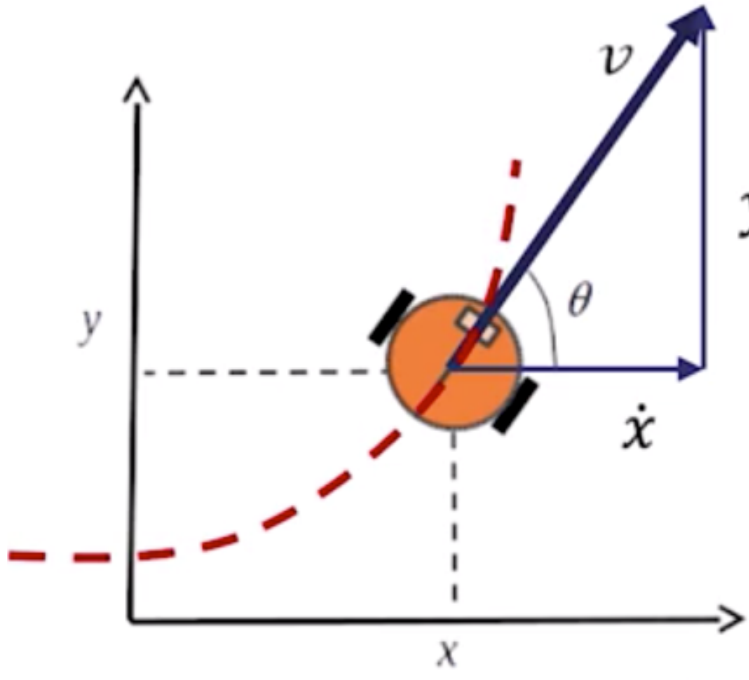
O_{EB}: translation term, expressed in body frame.

-Location of P in Inertial Frame E

$$P_E = C_{BE}(\theta)P_B + O_{BE}$$

O_{BE}: translation term, expressed in inertial frame.

Homogeneous Coordinate Form
- A 2D vector in homogeneous form



2D KINEMATIC MODELING

$P = \begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \bar{P} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

-Transforming a point from body to inertial coordinates with homogeneous coordinates

$\bar{P}_E = [C_{EB}(\theta) \,|\, O_{EB}]\bar{P}_B$ （正しい? ）

2D Kinematic Modeling

-The kinematic constraint is nonholonomic (restrict the rate of change of the position of our robot, so robot can roll forward and turn while rolling, but cannot move sideways directly.)

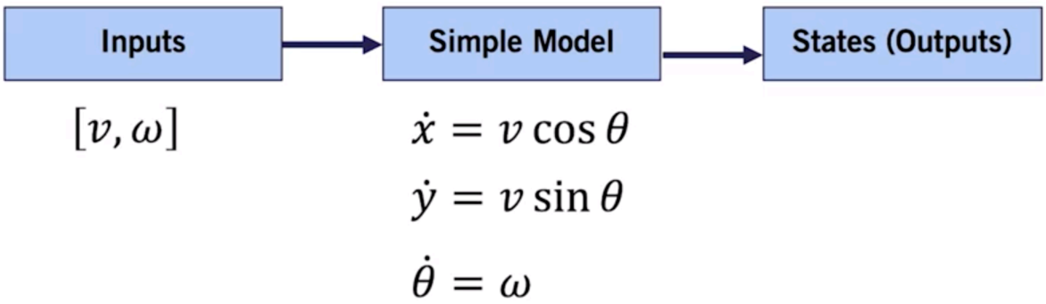
-A constraint on rate of change of degrees of freedom

-vehicle velocity always tangent to current path

-$\frac{dy}{dx} = tan\theta = \frac{sin\theta}{cos\theta}$

-Nonholonomic constraint

-$\dot{y}cos\theta - \dot{x}sin\theta = 0$



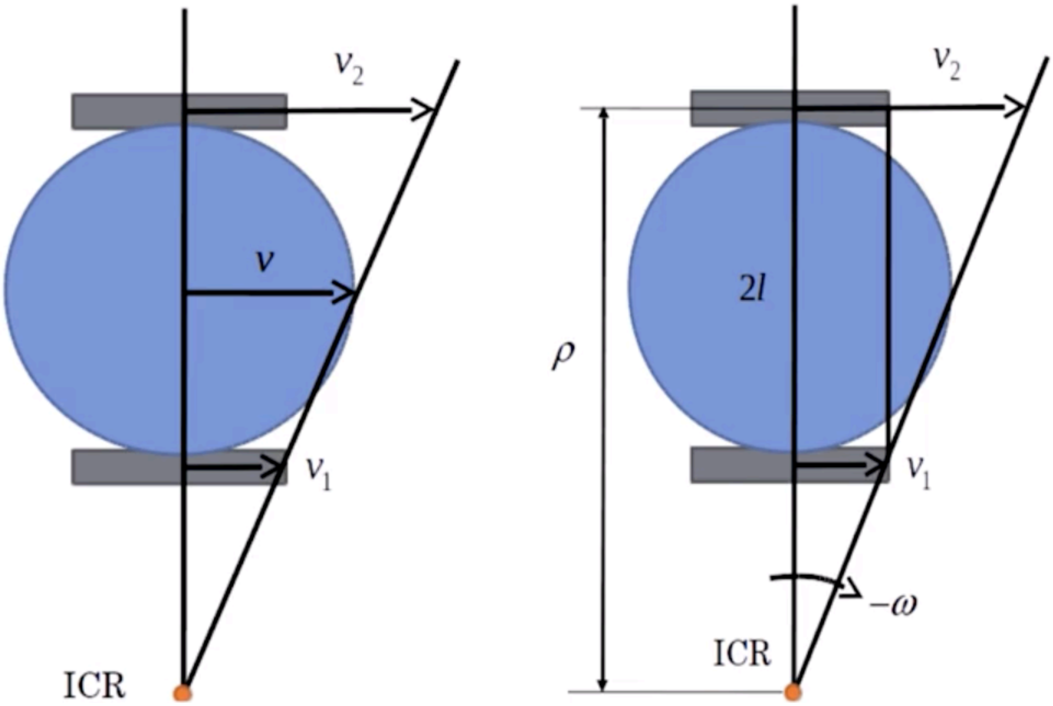
- Velocity components
  - $\dot{x} = vcos\theta$
  - $\dot{y} = vsin\theta$

Simple Robot Motion Kinematics

- inputs: the forward velocity, rotation rate
- represent the robot using a vector of three states: x, y heading

State: a set of variables often arranged in the form of a vector that fully describe the system at the current time.

Two-Wheeled Robot Kinematic Model
- Assume control inputs are wheel speeds
  - Center: p
  - Wheel to center: l



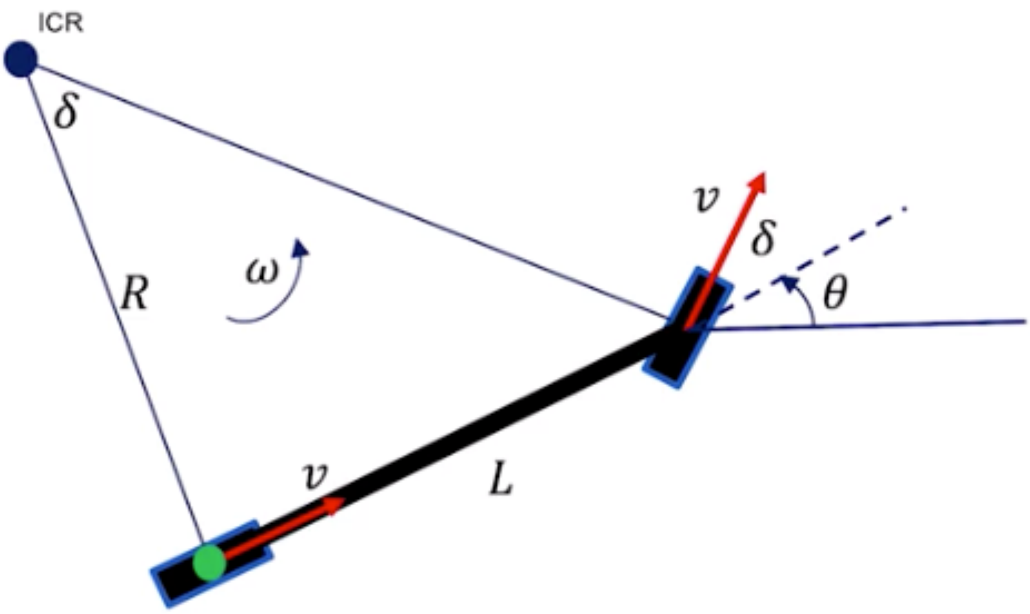
  - Wheel radius: r
  - Wheel rotation rates: w1, w2
- Velocity is the average of the two wheel velocities
  - $v = \dfrac{v_1 + v_2}{2} = \dfrac{rw_1 + rw_2}{2}$
- If the wheel velocities are different, the robot moves in a curved path about some instantaneous center of rotation or ICR.
- Use the instantaneous center of rotation (ICR)
- Equivalent triangles given the angular rate of rotation
  - $\omega = \dfrac{-v_2}{\rho} = \dfrac{-(v_2 - v_1)}{2l}$
  - $\omega = \dfrac{rw_1 - rw_2}{2l}$
- Continuous time model
  - $\dot{x} = [(\dfrac{rw_1 + rw_2}{2})cos\theta]$
  - $\dot{y} = [(\dfrac{rw_1 + rw_2}{2})sin\theta]$
  - $\dot{\theta} = (\dfrac{rw_1 - rw_2}{2l})$
- Discrete time model:
  - $x_{k+1} = x_k + \dot{x}_k \Delta t$
  - $y_{k+1} = y_k + \dot{y}_k \Delta t$
  - $\theta_{k+1} = \theta_k + \dot{\theta}_k \Delta t$
  - k: current time step

Lesson 2: The Kinematic Bicycle Model

kinematic bicycle model: a classic model that does surprisingly well at capturing vehicle motion in normal driving conditions.

Bicycle Kinematic Model
- 2D bicycle model (simplified car model)
- Front wheel steering
  - the front wheel orientation can be controlled relative to the heading of the vehicle
  - the front wheel represents the front right and left wheels of the car
  - the rear wheel represents the rear right and left wheels of the car
- reference point x, y: the center of the rear axle, the center of the front axle, or the center of gravity (cg).

Rear Wheel Reference Point
- Robot modelの上、更にsteering angle for the front wheelを定義する（δ）
  - measured relative to the forward direction of the bicycle
- velocity points in the same direction as each wheel前輪、後輪の速度大きさが一緒、方向違う
  - no slip condition: wheel cannot move laterally or slip longitudinally

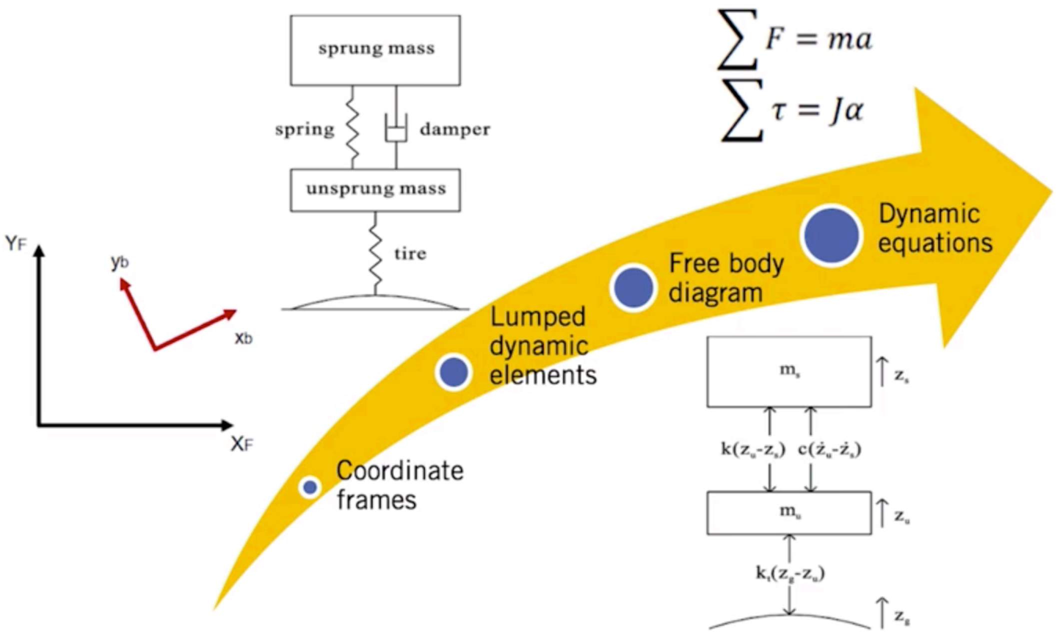


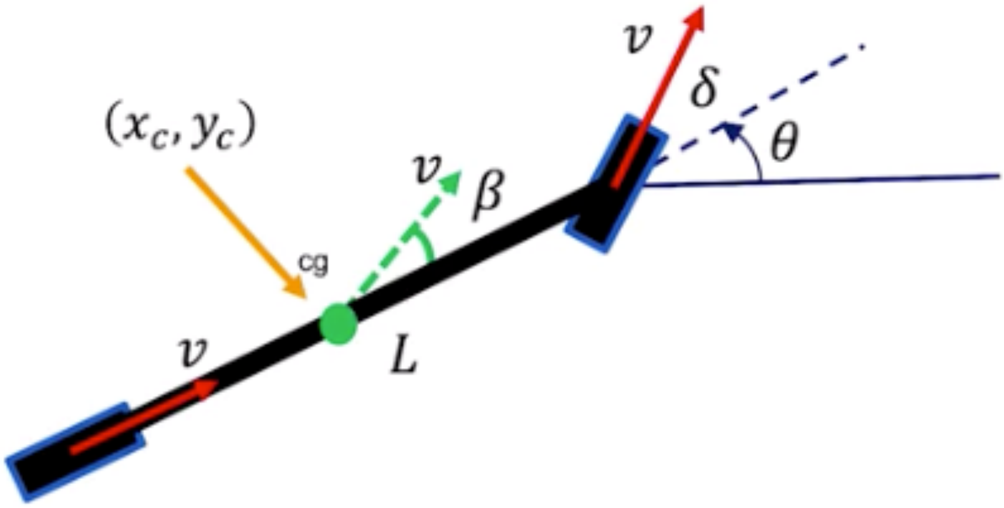
- Apply Instantaneous Center of Rotation (ICR)
- Similar triangles
  - $tan\delta = \dfrac{L}{R}$
- Rotation rate equation
  - $\dot{\theta} = \omega = \dfrac{v}{R} = \dfrac{vtan\delta}{L}$

- If the desired point is at the center of the rear axle（x方向とy方向は全部inertial coordinate systemの軸だから、速度は地球座標系に分解するから）
  - $\dot{x}_r = vcos\theta$
  - $\dot{y}_r = vsin\theta$
  - $\dot{\theta} = \dfrac{vtan\delta}{L}$

- If the desired point is at the center of the front axle（上記と同じ、vを地球座標系に分解する）
  - $\dot{x}_f = vcos(\theta + \delta)$
  - $\dot{y}_f = vsin(\theta + \delta)$
  - $\dot{\theta} = \frac{vsin\delta}{L}$
  - この場合、回転半径は$\frac{L}{sin\delta}$だから。
- If the desired point is at the center of the gravity (cg)
  - $\dot{x}_c = vcos(\theta + \beta)$
  - $\dot{y}_c = vsin(\theta + \beta)$
  - $\dot{\theta} = \frac{vcos\beta tan\delta}{L}$
  - この場合の回転半径は、$\frac{\frac{L}{tan\delta}}{cos\beta}$。$\frac{L}{tan\delta}$はちょうど後輪中心時の回転半径。
  - β: slip angle
  - use as the basis of modeling of the dynamics of vehicles
  - $\beta = tan^{-1}(\frac{l_r tan\delta}{L})$
  - Lr: distance from the rear wheel to the cg

State-space Representation
- Modify CG kinematic bicycle model to use steering rate input
  - State: $[x, y, \theta, \delta]^T$ Inputs: $[v, \varphi]^T$
  - φ: the rate of change of the steering angle
  - $\dot{x}_c = vcos(\theta + \beta)$
  - $\dot{y}_c = vsin(\theta + \beta)$
  - $\dot{\theta} = \frac{vcos\beta tan\delta}{L}$
  - $\dot{\delta} = \varphi$





GRAVITY REFERENCE POINT MODEL

φが必要である理由： It is not usually possible to instantaneously change the steering angle of a vehicle from one extreme of its range to another, as is currently possible with our kinematic model. Since δ is an input that would be selected by a controller, there is no restriction on how quickly it can change which is somewhat unrealistic.

Lesson 3: Dynamic Modeling in 2D

単語：
1. throttle: The throttle of a motor vehicle or aircraft is the device, lever, or pedal that controls the quantity of fuel entering the engine and is used to control the vehicle's speed.
2. shaft: In a machine, a shaft is a rod that turns around continually in order to transfer movement in the machine.
3. gear: The gears on a machine or vehicle are a device for changing the rate at which energy is changed into motion.
4. hub: The hub of a wheel is the part at the center.

Start taking into account the forces and moments acting on the vehicle
強み： can lead to higher fidelity predictions, than are possible with kinematic models

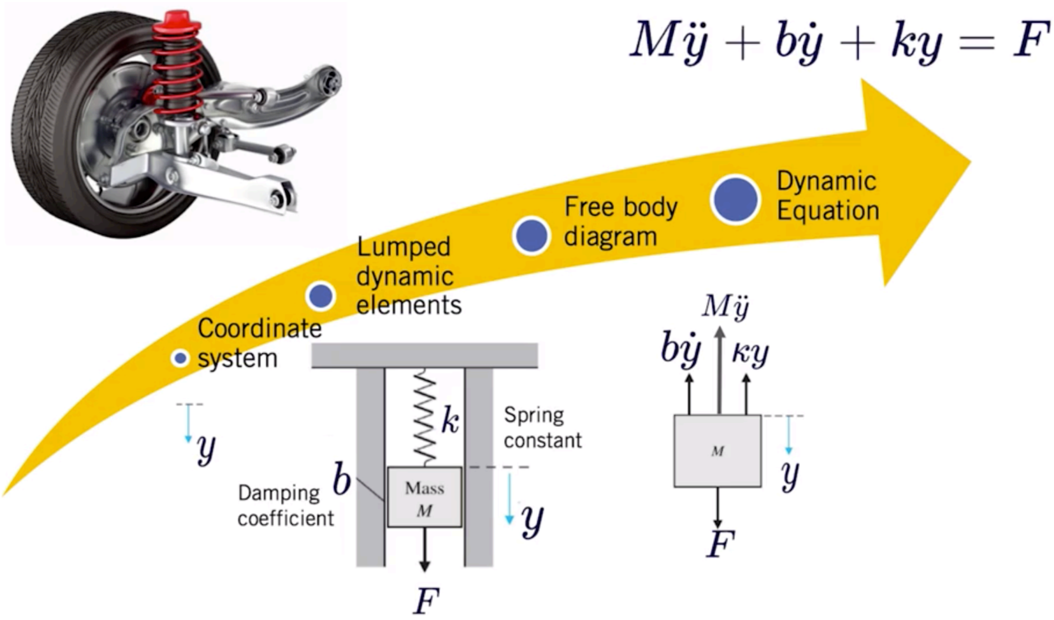
Why Dynamic Modeling is Important?
- At higher speed and slippery roads, vehicles do not satisfy no slip condition
- Many sub systemsにもkinematic constraintが適用できない。例えばdrive train: the balance of torques is needed to capture the connection from throttle position to wheel torque through the engine and transmission systems.

Steps to build a typical dynamic model
1. set up coordinate frames
2. break down dynamic system into lumped dynamic elements
    1. 例えばspring mass damper: lumped elementsはthe spring, the mass, the damper

3. define a model for each lumped element
- 例えば、the linear spring

4. sketch the free body diagram for each rigid body in the list of elements
5. Dynamic equations: Newton's second law

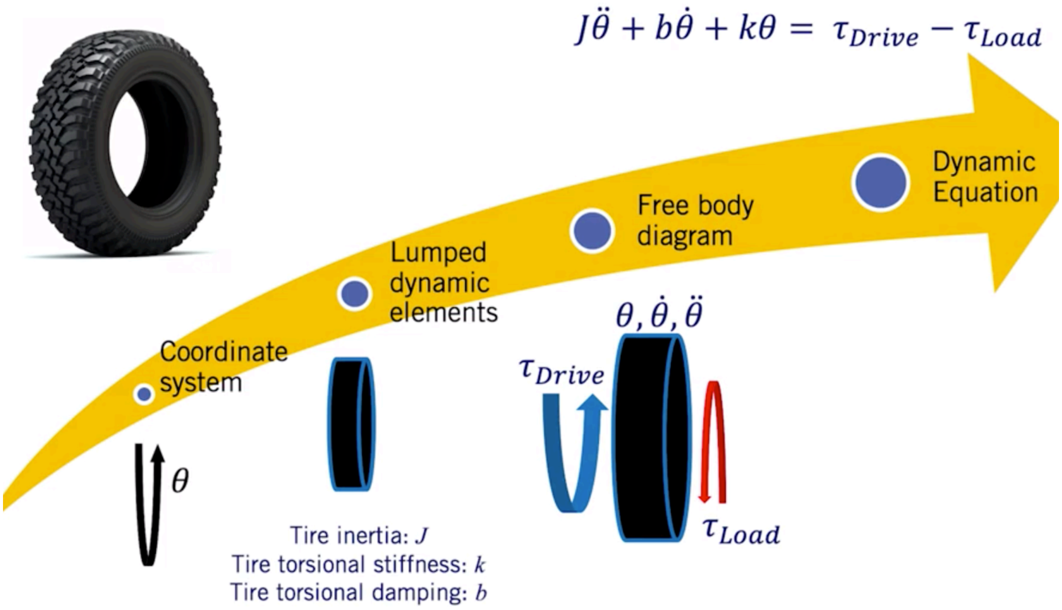
Dynamic Modeling - Translational System
（例、rolling cart）
- Deal with forces and torques
- Roughly, need to equate all forces



DYNAMIC MODELING - VEHICLE SHOCK ABSORBER (SUSPENSION)

- Governed by Newton's second law

Example - Vehicle Shock Absorber (Suspension)
- The shock absorber relies on its spring and hydraulics（油压） cylinder with flow restriction to absorb shocks.
- use a linear spring and damper model: spring resist displacement in y and damper resist the y velocity.



TIRE MODEL

- no variation in this process to handle rotational or torsional systems. つまりrotationalやtorsionalシステムは同じ。

Dynamic Modeling - Rotational Systems

使われる場合：combustion engine shafts, gear boxes, torque converters and tires.

- Inertial, J
- Torsional force, τ
- Forces resisting that torsional force
  - Spring force
  - Damping force
  - Inertia force

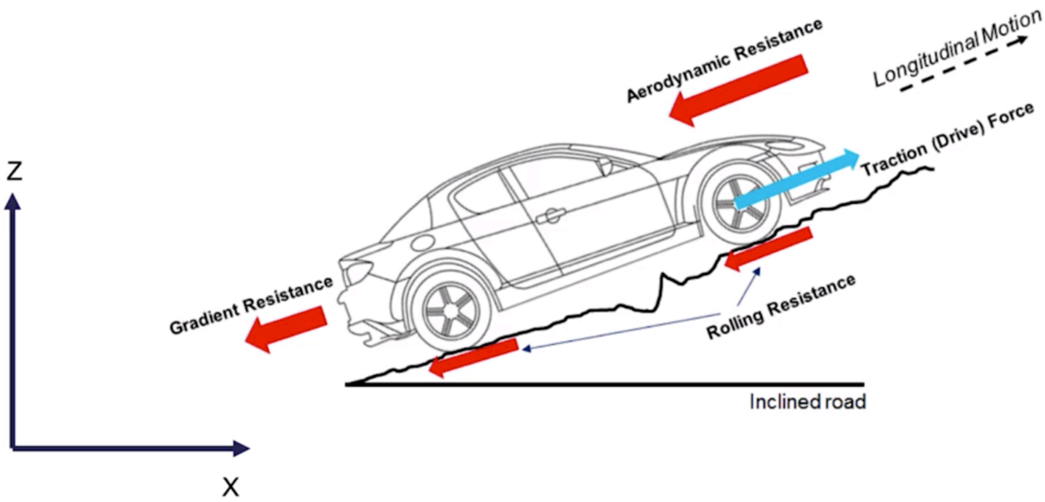
Example - Tire Model

- The tire model has rotational inertia J due to its rotating mass.
- Torsional stiffness k and damping b, defined by the material properties of the tire in the wheel hub.
- We drive the wheel with a drive torque（駆動トルク） from the vehicle's drive shaft and resist this with a load torque（負荷トルク） coming from the tires interaction with the road's surface

Dynamic modelの応用

- used to improve state estimation methods when fusing sensor data to track motion
- used to aid in controller design to track a desired trajectory or path
- help self-driving engineers define the limits of vehicle performance to avoid from planning unsafe trajectories that a car cannot track.

2D Dynamics - Vehicle Longitudinal Motion (x, z plane)

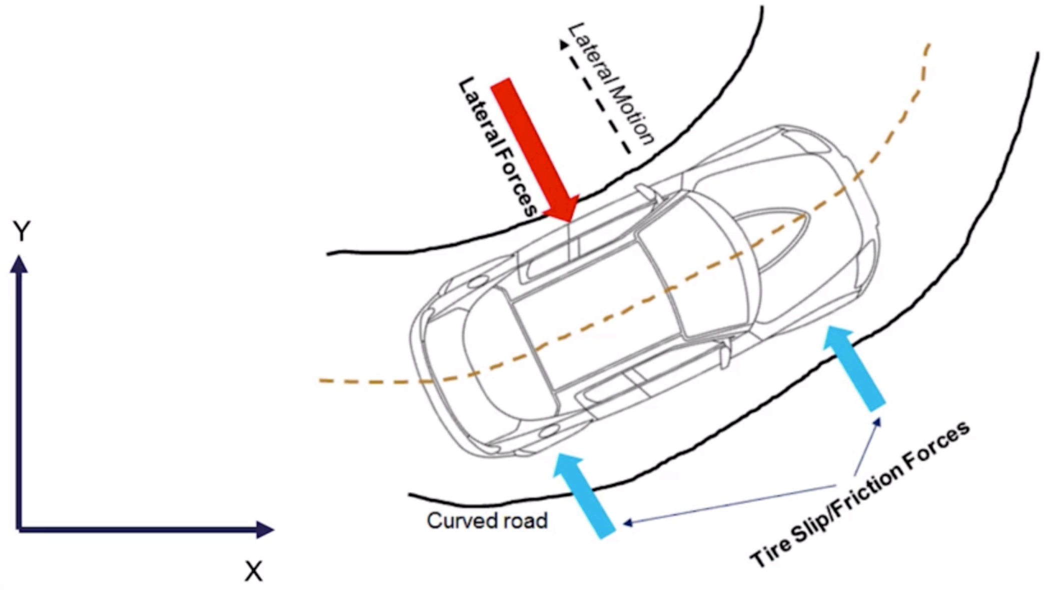


2D DYNAMICS - VEHICLE LONGITUDINAL MOTION

2D Dynamics - Vehicle Lateral Motion (x, y plane)

- centrifugal force（遠心力）

Vehicle Modeling: Vehicle Dynamics (2016)
http://publications.lib.chalmers.se/records/fulltext/244369/244369.pdf



2D DYNAMICS - VEHICLE LATERAL MOTION