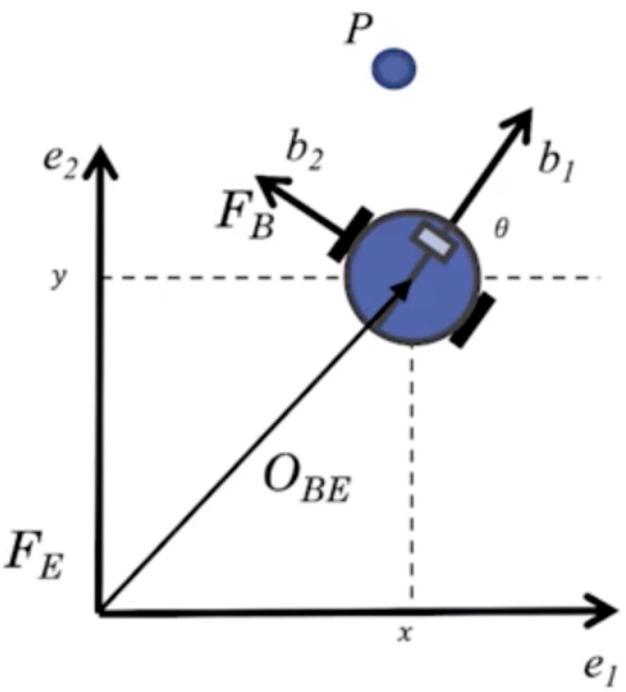
Module 4: Vehicle Dynamic Modeling
Lesson 1: Kinematic（運動学的な） Modeling in 2D

Kinematic Vs Dynamic Modeling
- At low speeds, it is often sufficient to look only at kinematic models of vehicles
  - Examples: two wheeled robot, bicycle model
- Dynamic modeling is more involved, but captures vehicle behavior more precisely over a wide operating range
  - Example: Dynamic vehicle model

Coordinate Frames
- Right handed
- Inertial frame
  - Fixed, usually relative to earth, East North Up, ENU, Earth-Centered Earth Fixed, ECEF (GNSS).
- Body frame
  - Attached to vehicle, origin at vehicle center of gravity, or center of rotation. (or the center point of the rear axle)
  - This frame is moving and rotating with respect to the fixed inertial frame as the vehicle moves about.
- Sensor frame
  - Attached to sensor, convenient for expressing sensor measurements.

Coordinate Transformation
Conversion between Inertial frame and Body coordinates is done with a translation vector and a rotation matrix



P OBSERVED BY 2-WHEELED ROBOT

Example: 2-wheeled robot
e→b: θ

$$C_{EB} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$$

b→e: -θ

$$C_{BE} = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$$

-Location of P in Body Frame B:

$$P_B = C_{EB}(\theta)P_E + O_{EB}$$

O_{EB}: translation term, expressed in body frame.

-Location of P in Inertial Frame E

$$P_E = C_{BE}(\theta)P_B + O_{BE}$$

O_{BE}: translation term, expressed in inertial frame.

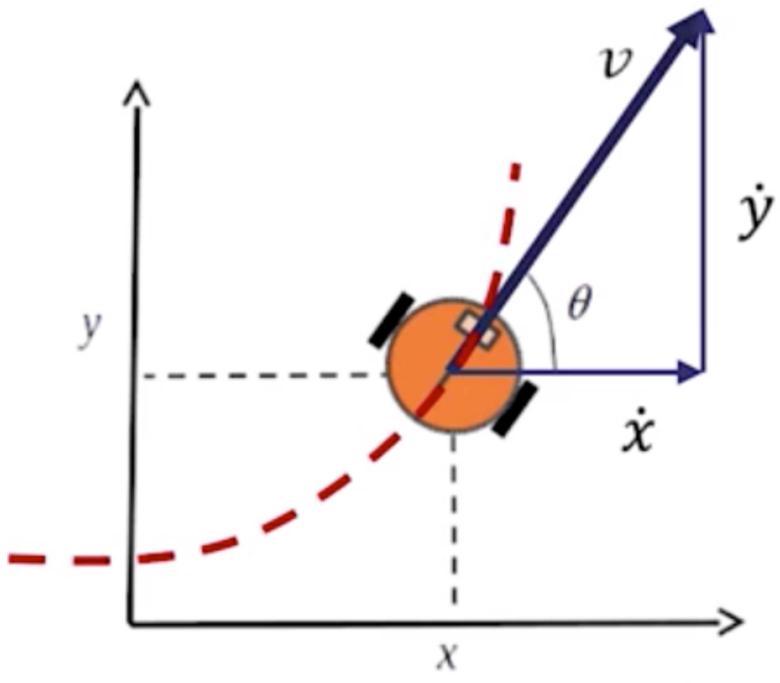
Homogeneous Coordinate Form
- A 2D vector in homogeneous form

$$P = \begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \bar{P} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

-Transforming a point from body to inertial coordinates with homogeneous coordinates

$$\bar{P}_E = [C_{EB}(\theta) \,|\, O_{EB}]\bar{P}_B$$ （正しい？）

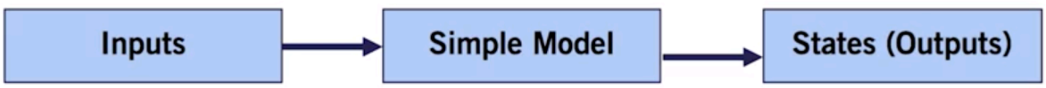
2D Kinematic Modeling



-The kinematic constraint is nonholonomic (restrict the rate of change of the position of our robot, so robot can roll forward and turn while rolling, but cannot move sideways directly.)
-A constraint on rate of change of degrees of freedom
-vehicle velocity always tangent to current path
-$\frac{dy}{dx} = tan\theta = \frac{sin\theta}{cos\theta}$
-Nonholonomic constraint
-$\dot{y}cos\theta - \dot{x}sin\theta = 0$
-Velocity components
-$\dot{x} = vcos\theta$
-$\dot{y} = vsin\theta$

Simple Robot Motion Kinematics

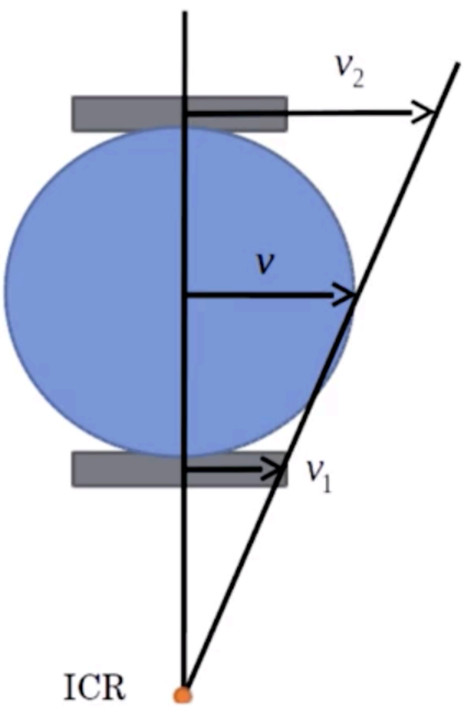
2D KINEMATIC MODELING



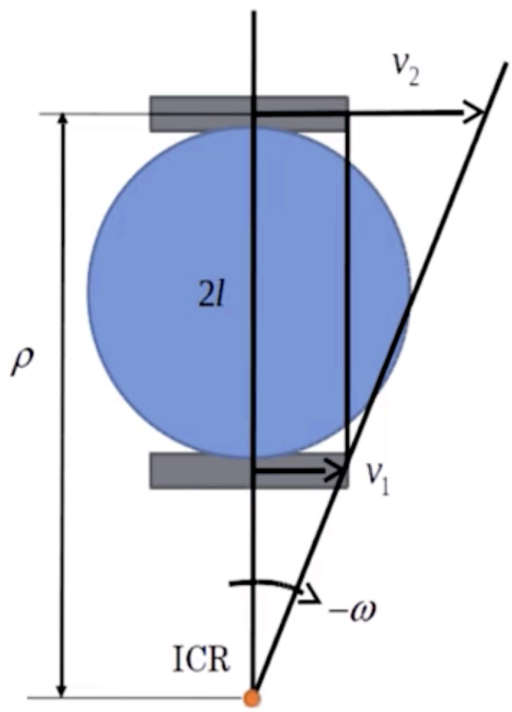
$[v, \omega]$

$$\dot{x} = v\cos\theta$$
$$\dot{y} = v\sin\theta$$
$$\dot{\theta} = \omega$$

SIMPLE ROBOT MOTION KINEMATICS



VELOCITY EQUALS AVERAGE OF TWO WHEEL VELOCITIES



INSTANTANEOUS CENTER OF ROTATION (ICR)

- inputs: the forward velocity, rotation rate
- represent the robot using a vector of three states: x, y heading

State: a set of variables often arranged in the form of a vector that fully describe the system at the current time.

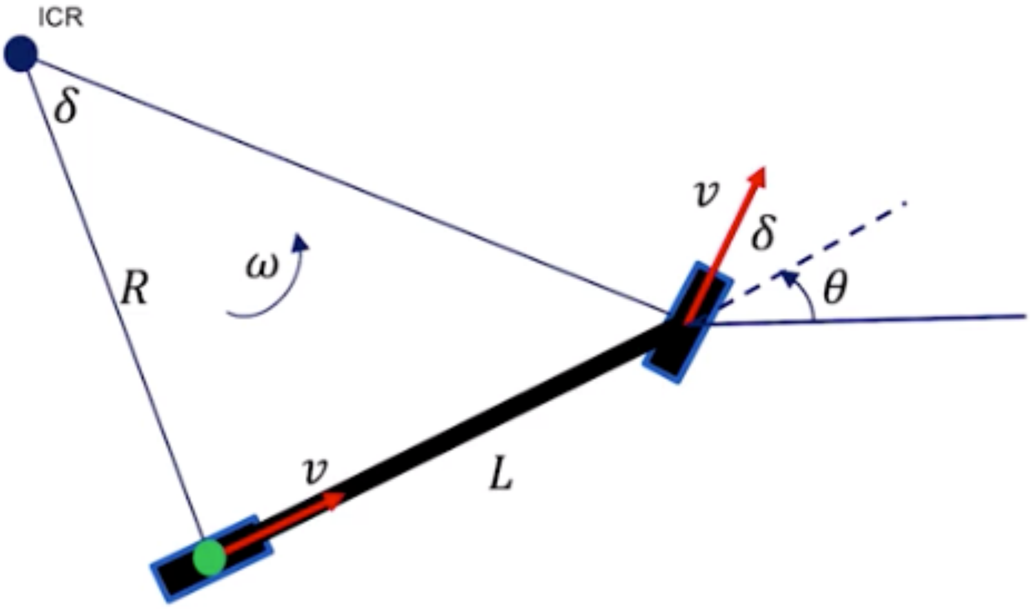
Two-Wheeled Robot Kinematic Model
- Assume control inputs are wheel speeds
  - Center: p
  - Wheel to center: l
  - Wheel radius: r
  - Wheel rotation rates: w1, w2
- Velocity is the average of the two wheel velocities
  - $v = \frac{v_1 + v_2}{2} = \frac{rw_1 + rw_2}{2}$
- If the wheel velocities are different, the robot moves in a curved path about some instantaneous center of rotation or ICR.
- Use the instantaneous center of rotation (ICR)
- Equivalent triangles given the angular rate of rotation
  - $\omega = \frac{-v_2}{\rho} = \frac{-(v_2 - v_1)}{2l}$
  - $\omega = \frac{rw_1 - rw_2}{2l}$
- Continuous time model
  - $\dot{x} = [(\frac{rw_1 + rw_2}{2})cos\theta]$
  - $\dot{y} = [(\frac{rw_1 + rw_2}{2})sin\theta]$
  - $\dot{\theta} = (\frac{rw_1 - rw_2}{2l})$
- Discrete time model:
  - $x_{k+1} = x_k + \dot{x}_k \Delta t$
  - $y_{k+1} = y_k + \dot{y}_k \Delta t$
  - $\theta_{k+1} = \theta_k + \dot{\theta}_k \Delta t$
  - k: current time step

Lesson 2: The Kinematic Bicycle Model

kinematic bicycle model: a classic model that does surprisingly well at capturing vehicle motion in normal driving conditions.

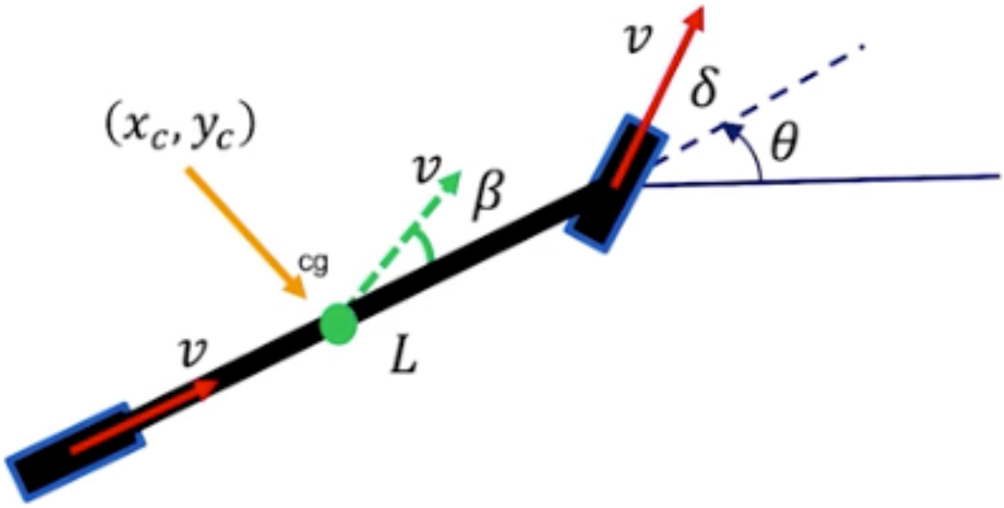
Bicycle Kinematic Model
- 2D bicycle model (simplified car model)
- Front wheel steering
  - the front wheel orientation can be controlled relative to the heading of the vehicle
  - the front wheel represents the front right and left wheels of the car
  - the rear wheel represents the rear right and left wheels of the car
- reference point x, y: the center of the rear axle, the center of the front axle, or the center of gravity (cg).

Rear Wheel Reference Point
- Robot modelの上、更にsteering angle for the front wheelを定義する（δ）
  - measured relative to the forward direction of the bicycle
- velocity points in the same direction as each wheel前輪、後輪の速度大きさが一緒、方向違う
  - no slip condition: wheel cannot move laterally or slip longitudinally

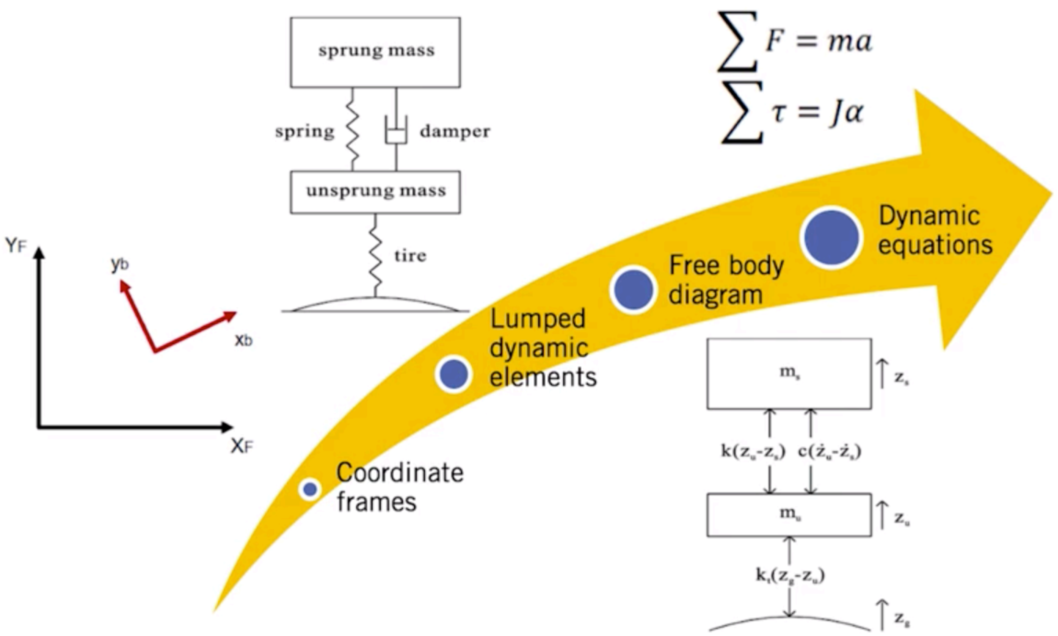


REAR WHEEL REFERENCE POINT MODEL

- Apply Instantaneous Center of Rotation (ICR)
- Similar triangles
  - $tan\delta = \dfrac{L}{R}$
- Rotation rate equation
  - $\dot{\theta} = \omega = \dfrac{v}{R} = \dfrac{vtan\delta}{L}$
- If the desired point is at the center of the rear axle（x方向とy方向は全部inertial coordinate systemの軸だから、速度は地球座標系に分解するから）
  - $\dot{x}_r = vcos\theta$
  - $\dot{y}_r = vsin\theta$
  - $\dot{\theta} = \dfrac{vtan\delta}{L}$
- If the desired point is at the center of the front axle（上記と同じ、vを地球座標系に分解する）
  - $\dot{x}_f = vcos(\theta + \delta)$
  - $\dot{y}_f = vsin(\theta + \delta)$
  - $\dot{\theta} = \dfrac{vsin\delta}{L}$
- If the desired point is at the center of the gravity (cg)
  - $\dot{x}_c = vcos(\theta + \beta)$
  - $\dot{y}_c = vsin(\theta + \beta)$
  - $\dot{\theta} = \dfrac{vcos\beta tan\delta}{L}$



GRAVITY REFERENCE POINT MODEL



DYNAMIC MODELING STEPS

- β: slip angle
- use as the basis of modeling of the dynamics of vehicles
- $\beta = tan^{-1}(\frac{l_r tan\delta}{L})$
- Lr: distance from the rear wheel to the cg

State-space Representation

- Modify CG kinematic bicycle model to use steering rate input
  - State: $[x, y, \theta, \delta]^T$ Inputs: $[v, \varphi]^T$
  - φ: the rate of change of the steering angle
  - $\dot{x}_c = vcos(\theta + \beta)$
  - $\dot{y}_c = vsin(\theta + \beta)$

- $\dot{\theta} = \dfrac{v cos\beta tan\delta}{L}$
- $\dot{\delta} = \varphi$

Lesson 3: Dynamic Modeling in 2D

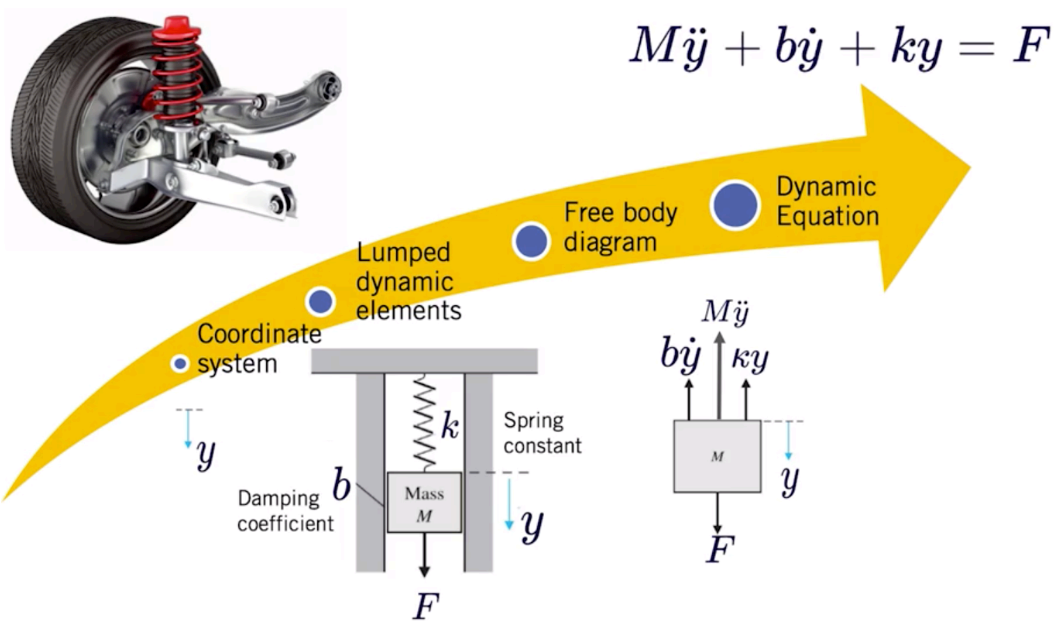
Start taking into account the forces and moments acting on the vehicle
強み: can lead to higher fidelity predictions, than are possible with kinematic models

Why Dynamic Modeling is Important?
- At higher speed and slippery roads, vehicles do not satisfy no slip condition
- Many sub systemsにもkinematic constraintが適用できない。例えばdrive train: the balance of torques is needed to capture the connection from throttle position to wheel torque through the engine and transmission systems.

Steps to build a typical dynamic model:
1. set up coordinate frames
2. break down dynamic system into lumped dynamic elements
    1. 例えばspring mass damper: lumped elementsはthe spring, the mass, the damper

3. define a model for each lumped element
- 例えば、the linear spring

4. sketch the free body diagram for each rigid body in the list of elements
5. Dynamic equations: Newton's second law



DYNAMIC MODELING - VEHICLE SHOCK ABSORBER (SUSPENSION)

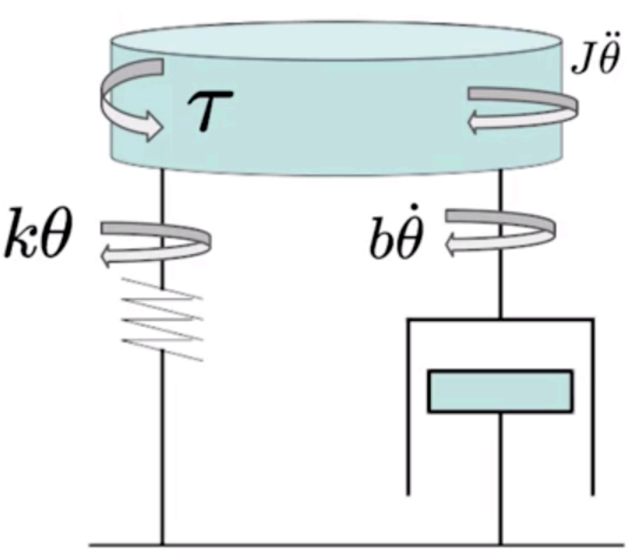
Dynamic Modeling - Translational System

（例、rolling cart）
- Deal with forces and torques

- Roughly, need to equate all forces
- Governed by Newton's second law

Example - Vehicle Shock Absorber (Suspension)
- The shock absorber relies on its spring and hydraulics（油圧） cylinder with flow restriction to absorb shocks.
- use a linear spring and damper model: spring resist displacement in y and damper resist the y velocity.
- no variation in this process to handle rotational or torsional systems

Dynamic Modeling - Rotational Systems



- Inertial, J
  - -Torsional force, τ
  - -Forces resisting that torsional force
  - -Spring force
  - -Damping force
  - -Inertia force

Example - Tire Model

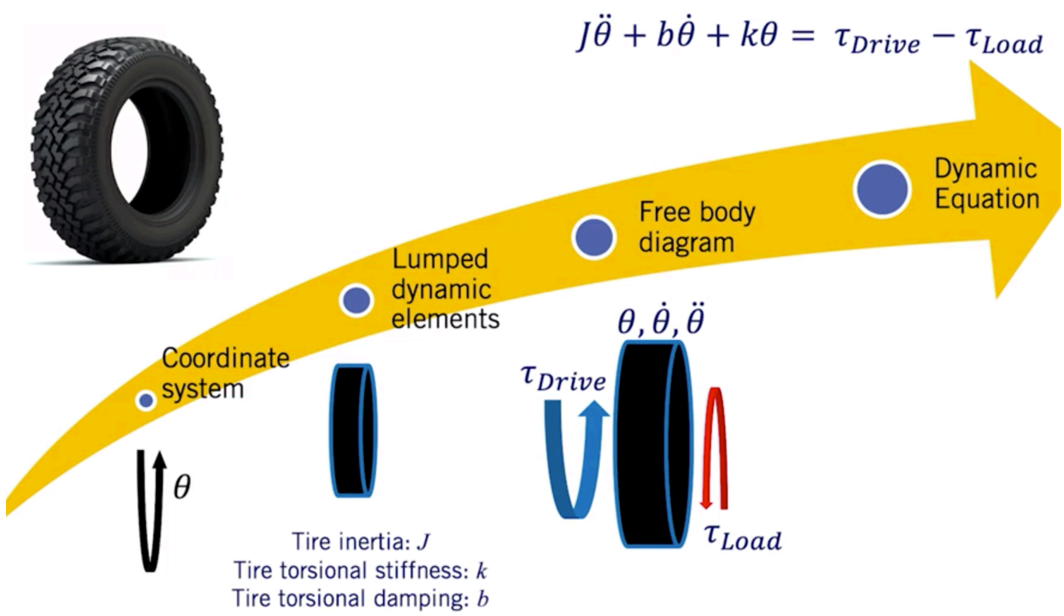
-We drive the wheel with a drive torque（駆動トルク） from the vehicle's drive shaft and resist this with a load torque（負荷トルク） coming from the tires interaction with the road's surface

Dynamic modelの応用

-used to improve state estimation methods when fusing sensor data to track motion

- used to aid in controller design to track a

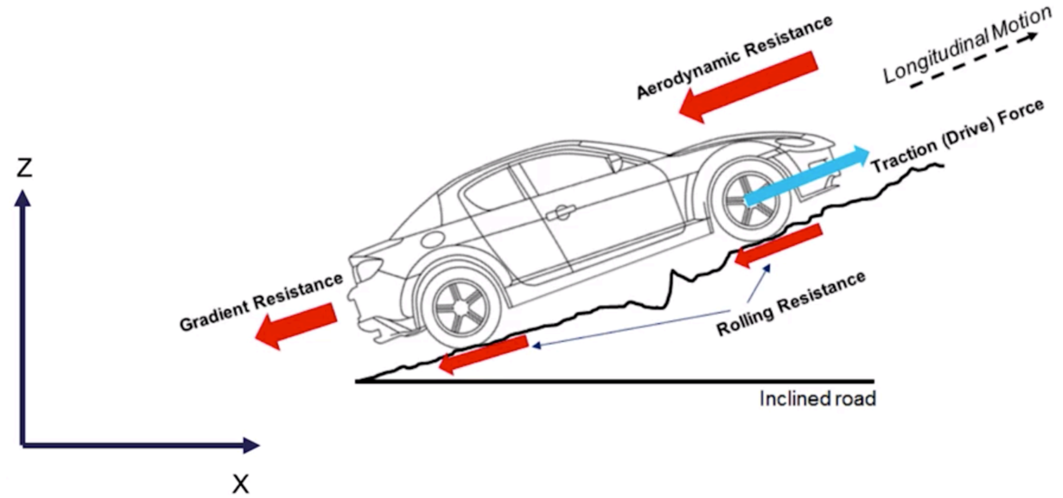
ROTATIONAL SYSTEMS



TIRE MODEL

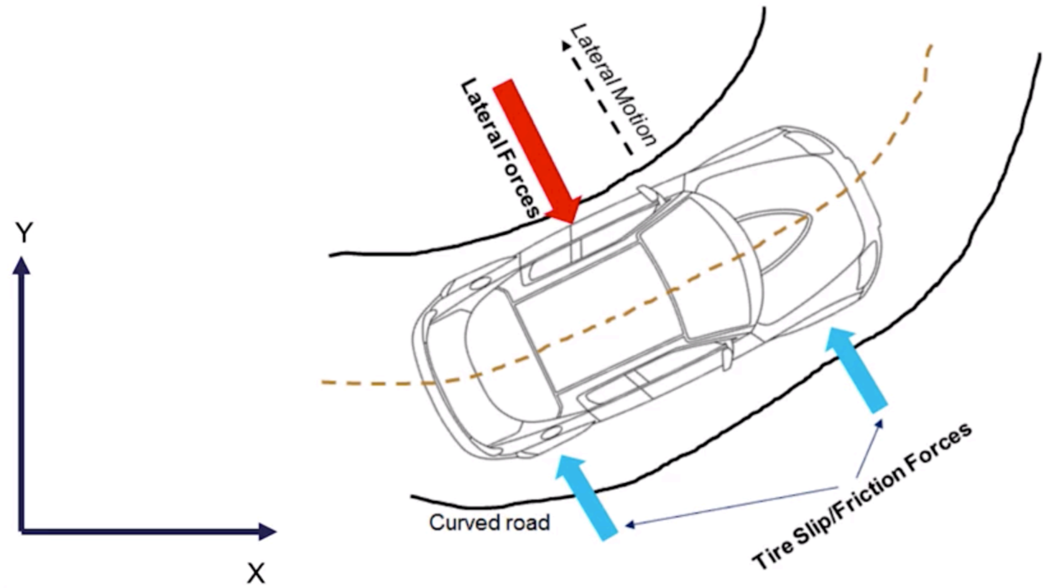
desired trajectory or path
- help self-driving engineers define the limits of vehicle performance to avoid from planning unsafe trajectories that a car cannot track.

2D Dynamics - Vehicle Longitudinal Motion (x, z plane)



2D DYNAMICS - VEHICLE LONGITUDINAL MOTION

2D Dynamics - Vehicle Lateral Motion (x, y plane)



2D DYNAMICS - VEHICLE LATERAL MOTION

- centrifugal force（遠心力）