# Module 5: Vehicle Longitudinal Control

## Lesson 1: Proportional-Integral-Derivative (PID) Control

## 単語:

- 1. transfer function: In engineering, a transfer function of an electronic or control system component is a mathematical function which theoretically models the device's output for each possible input.
- 2. Laplace transform: An integral transform. Transform a function of a real variable t (often time) to a function of a complex variable s (complex frequency). The Laplace transform is similar to the Fourier transform. While the Fourier transform of a function is a complex function of a real variable (frequency), the Laplace transform of a function is a complex function of a complex variable
- 3. regulate: To regulate an activity or process means to control it, especially by means of rules. 内容:
- review the basics of LTI (linear time-invariant) control
- explore the design of PID controllers
- design a PID control for a linear time-invariant system

前知識: classical control design including the use of transfer functions and the Laplace domain

## 各部分の機能(なぜControllerが必要?)

Dynamic and Kinematic Models for a vehicle based on the bicycle model

- These models aim to capture how the dynamic system reacts to input commands from the driver such as steering, gas and brake, and how it reacts to disturbances such as wind, road surfaces and different vehicle loads.
- The effects of the inputs and disturbances on the states such as velocity and rotation rate of the vehicle are defined by the kinematic and dynamic models we developed.

#### Controller

- Regulate some of these states of the vehicle by sensing the current state variables and then generating actuator signals to satisfy the commands provided.
- For longitudinal control, the controller senses the vehicle speed and adjust the throttle and brake commands to match the desired speed set by the autonomous motion planning system. つまりMotion Planningから欲しいSpeedが出て、Controllerはまず現在のSpeedをsenseして、 欲しいSpeedを出すようにThrottleやBrakeを計算して、commandを出す。

## A typical feedback control loop: Plant or Process model

- input: actuator signals
- output: state variables of the system
- outputs are measured by sensors, feedbackというのはsensor measurementsだろう。
- estimators are used to fuse measurements into accurate output estimates. sensor measurementsもインプットだろう。estimatorというのはcontrollerだ。例えばPID controller、もしくはfeedforward controller。
- The output estimates are compared to the desired or reference output variables, and the difference or error is passed to the controller.
- The controller can be seen as a mathematical algorithm that generates actuator signals so that the error signal is minimized and the plant state variables approach the desired state variables.

Plant System or Process (上記の続き)

## System Representation:

- The plant system could be linear or nonlinear
- Plant Representation: state-space form and transfer functions

- state-space form: track the evolution of an internal state to connect the input to the output
- transfer function: model the input to output relation directly.
  - the system must be linear and time-invariant
- Linear time-invariant systems can be expressed using transfer functions

## Transfer Function

- A transfer function G is a relation between input U and output Y
  - Y(s) = G(s)U(s)
  - $-s = \sigma + i\omega$
- Expressed in the Laplace domain, as a function of s, a complex variable

$$Y(s) = G(s)U(s) = \frac{N(s)}{D(s)}U(s)$$

- transfer function 02 form: polynomial or factorized (zero-pole-gain) form
- ここはzero-pole-gain (factorized) form is used.
- Zeros roots of numerator
- Poles roots of denominator
- Create Transform Function Model Using Zeros, Poles, and Gain
  - https://www.mathworks.com/help/control/ug/transfer-functions.html

$$G(s) = 5 \frac{s}{(s+1+i)(s+1-i)(s+2)}$$

- Z = [0]:
- -P = [-1 1i 1 + 1i 2];
- -K = 5:
- G = zpk(Z, P, K);
- Laplace transformを使う理由: allow easier analysis of an input-output relation and is useful in understanding control performance.
  - Having the controlled system components written in Laplace form makes it easier to determine the transfer function of the system.
  - 各部分がLaplace formを使えば、結合してシステム全体のLaplace formを出すのが簡単。下 にDynamic ModelとPID Controllerの結合の例から分かる。

#### Controller or Compensator

- Control algorithms can vary from simple to complex
  - simple: constant gain multiplication, lookup tables, linear equations
  - complex: non-linear functions and optimization over finite prediction horizons
- Some simple algorithms, widely used in industry:
  - Lead-lag controllers
  - PID controllers
- More complex algorithms
  - Particularly useful for non-linear system models, time-varying models, or models with constraints that limit output selection.
  - Nonlinear methods: Feedback linearization, Backstepping, Sliding mode
  - Optimization methods: Model predictive control
    - Optimization-based methods are heavily used in autonomous driving

## **Proportional-Integral-Derivative Controller**

- In the time domain:
  - $u(t) = K_{P}e(t) + K_{I} \int_{0}^{t} e(t)dt + K_{D}\dot{e}(t)$
  - where  $K_P, K_I, K_D$  are the proportional, integral and derivative gains
  - e(t): error function

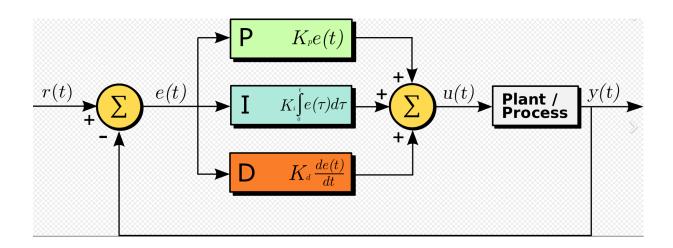
- u(t): input to the plant model。Plant Modelは実際に車の挙動、inputはsteering angle, throttle angle, brake angle、出力はこのinputを適用後の車の状態。
- In the Laplace domain:

$$U(s) = G_c(s)E(s) = (K_P + \frac{K_I}{s} + K_D s)E(s) = (\frac{K_D s^2 + K_P s + K_I}{s})E(s)$$

- この式から明らかにLaplace domainの式がtime domainの式より便利だのはすぐ分かれるだろう: a single transfer function for PID control.
- この式はtime domainの式と逆のようだね
  - Multiplying s in the Laplace domain is equivalent to taking a derivative in the time domain.
  - Dividing by s is equivalent to taking the integral.
- Not all gains need to be used for all systems.
  - If one or more of the PID gains are set to zero, the controller can be referred to as P, PD or PI.

$$G_c(s) = \frac{K_D s^2 + K_P s + K_I}{s}$$

- $K_D s^2 + K_P s + K_I$ : two zeros added. twoというのはsecond-order numeratorだから? そうだ、second-order numeratorなので、rootは2つあるのだ! つまり2つzerosだ。
- s: one pole added
- The pole is at the origin
- The zeros can be arbitrary places on the complex plane.
- PID controller design selects zero locations, by selecting P, I, and D gains
  - to achieve the desired output or performance based on the model for the plant
- There are several algorithms to tune PID gains (e.g. Zeigler-Nichols)



A BLOCK DIAGRAM OF A PID CONTROLLER IN A FEEDBACK LOOP. R(T) IS THE DESIRED PROCESS VALUE OR SETPOINT (SP), AND Y(T) IS THE MEASURED PROCESS VALUE (PV).

Closed Loop Response: Characteristics of P, I, and D Gains

## 定義

- Closed loop response: denotes the response of a system when the controller decides the inputs to apply to the plant model.
- Rise time: the time it takes to reach 90% of the reference value
- Overshoot: the maximum percentage the output exceeds this reference
- Settling time: the time to settle to within 5% of the reference(つまりreferenceの95%~105%の区間内?)

- Steady state error: the error between the output and the reference at steady-state。つまりシステムの固有誤差?
- reference (signal)はdesired valueだと思う。つまりr(t)です。

Closed Loop Response	Rise Time	Overshoot	Settling Time	Steady State Error
Increase $K_P$	Decrease	Increase	Small change	Decrease
Increase $K_I$	Decrease	Increase	Increase	Eliminate
Increase $K_D$	Small change	Decrease	Decrease	Small change

このtableについての解説: 下記のページは講師の解説より良い

- http://ctms.engin.umich.edu/CTMS/index.php?example=Introduction&section=ControlPID
- $K_I$ 
  - $K_I$  tends to help reduce steady-state error. It there is a persistent, steady error, the integrator builds and builds, thereby increasing the control signal and driving the error down.
  - A drawback of the integral term, however, is that it can make the system more sluggish (and oscillatory) since when the error signal changes sign, it may take a while for the integrator to "unwind".
- $K_D$ 
  - $K_D$  adds the ability of the controller to "anticipate" error.
  - With simple proportional control, if  $K_P$  is fixed, the only way that the control will increase is if the error increases. With derivative control, the control signal can become large if the error begins sloping upward, even while the magnitude of the error is still relatively small.
  - This anticipation tends to add damping to the system, thereby decreasing overshoot.
  - The addition of a derivative term, however, has no effect on the steady-state error.

Ultimately, the P, I and D gains must be selected with knowledge of the interaction of their effects to adjust the system response to get the right closed loop performance.

## Example: Second Order System: Mass-Spring-Damper System

- Dynamic Equation of the System
  - $-m\ddot{x} + b\dot{x} + kx = F, x(0) = 0$
  - input: F, output: x
- Laplace Transform (s-domain)
  - $-ms^2X(s) + bsX(s) + kX(s) = F(s)$
- Transfer Function of Output to Input

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + k}$$

 a second-order system with two poles defined by the mass, spring constant and damping coefficient.

## Open-Loop Step Response

- To evaluate the system characteristics, we excite the system by using a unit step input.
  - This is normally the first step to evaluate the dynamic characteristics of a plant.
  - Let m=1, b=10, k=20, F=1.
- This response is called the open-loop response since there is no controller applied to the system at this point.

## Closed-loop Response

- 定義: If a controller is added to the plant and the output of the model is measured and compared with the desired output or reference signal
- The poles of the open-loop system define the characteristics of the closed-loop response.

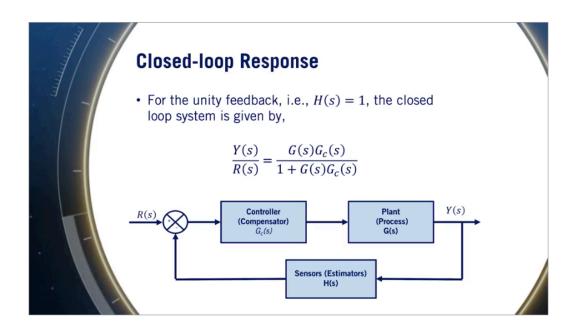
Step Response (Spring-Mass Damper System)

- P Controller: a fast rise time, significant overshoot and prolonged oscillation leading to a long settling time.
- PD Controller: improves the step response in terms of overshoot and settling time but slows down the rise time.
- PI Controller: maintains a short rise time and is able to reduce oscillations and overshoot leading to a fast settling time as well.
- The simple PI control is an excellent design for the spring-mass damper system.

## Step Response for PID Control

- PID controller

$$G_{PID}(s) = (K_P + K_D s + \frac{K_I}{s})$$



Glosed loop system
$$\frac{Y(s)}{R(s)} = \frac{G(s)G_C(s)}{1 + G(s)G_C(S)}$$

$$= \frac{\frac{1}{ms^2 + bs + k}(K_P + K_Ds + \frac{K_I}{s})}{1 + \frac{1}{ms^2 + bs + k}(K_P + K_Ds + \frac{K_I}{s})}$$

$$= \frac{K_P + K_Ds + \frac{K_I}{s}}{ms^2 + bs + k + K_P + K_Ds + \frac{K_I}{s}}$$

$$= \frac{K_Ps + K_Ds^2 + K_I}{ms^3 + bs^2 + ks + K_Ps + K_Ds^2 + K_I}$$

$$= \frac{K_Ps + K_Ds^2 + K_I}{ms^3 + (b + K_D)s^2 + (k + K_P)s + K_I}$$

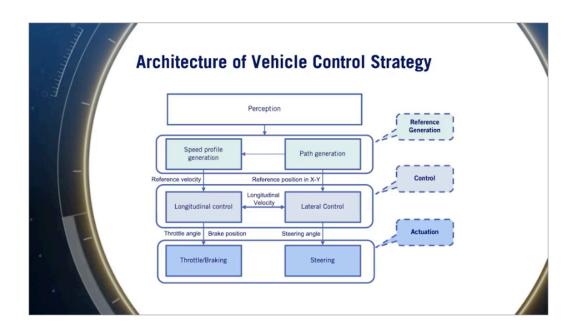
- By carefully tuning the controller gains, we can use the benefits of all three to eliminate overshoot and still maintain very short rise and settling times.

## Lesson 2: Longitudinal Speed Control with PID

単語:

1. cruise: If a car, ship, or aircraft cruises somewhere, it moves there at a steady comfortable speed.

内容:



- define the full vehicle planning and control architecture
- design a PID controller for cruise control/speed regulation

## Architecture of Vehicle Control Strategy

- 4つ部分のStack: Perception, Reference Generation, Control, Actuation
- Reference Generation: The path and the speed profiles are the reference inputs needed by our controllers.
- Control: For both the lateral and longitudinal control of an autonomous vehicle, the only task that needs to be performed is to follow the plan as precisely as possible, and thereby minimize the error between the actual and reference path and speed.

## 例: Cruise Control operating at highway speeds

定義: A cruise control system performs the function of maintaining a fixed reference speed using throttle commands, and accelerating or decelerating to a new reference speed as requested by the driver. When the vehicle is subjected to different loads and resistances, the throttle angle will be changed by the cruise controller accordingly.

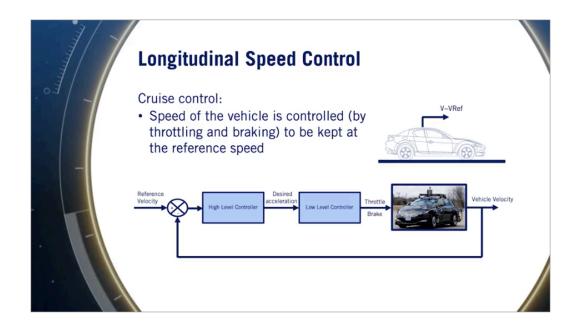
High and low-level control structure based on PID and engine maps.

### バリアント:

- adaptive cruise control: can vary the reference point based on measurements of a lead vehicle
- semi autonomous system, like traffic jam assist: can operate throughout the vehicle speed range and create spacing gaps for merging vehicles.

Block diagram of cruise controller and plant vehicle model:

- The low level controller is not essential to the control task.
- The high level controller takes the difference between the set point velocity and the vehicle actual velocity, and generates the desired vehicle acceleration to close the gap.



- The low level controller gets the vehicle acceleration and generates a throttle or braking actuation to track the reference acceleration.
- two-stage approachの効果: allows us to go beyond just PID control and impose limits or profiles directly on the accelerations that are requested of the vehicle in order to maintain speed.
- high level controllerは欲しい速度と実際速度の差から欲しい加速度を出す。low level controller は欲しい加速度を実現するために必要なThrottle及びBrakeを出す。そうだ! あくまでThrottle及びBrakeは直接に加速度に影響するんだ! そこからまた速度に影響するのだ。

## Upper Level Controller

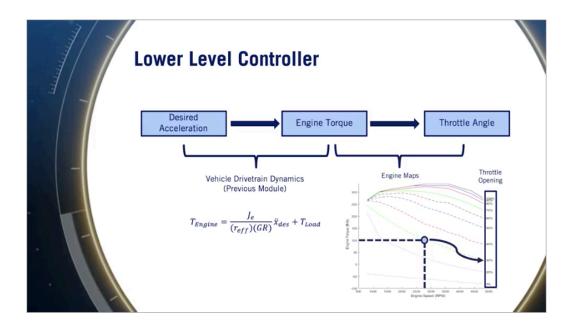
- The upper level or high level controller determines how much acceleration is needed at each time step based on the velocity error. 出力は加速度だけ!
- Use PID controller

$$\ddot{x}_{des} = K_P(\dot{x}_{ref} - \dot{x}) + K_I \int_0^t (\dot{x}_{ref} - \dot{x}) dt + K_D \frac{(\dot{x}_{ref} - \dot{x})}{dt}$$

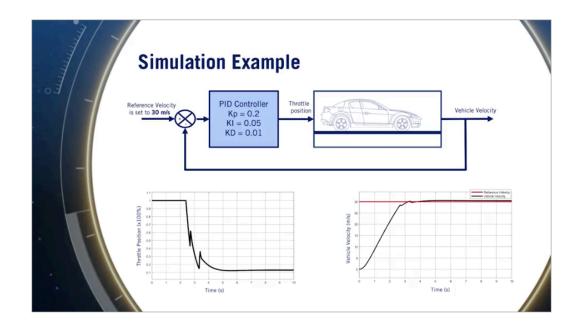
- $\ddot{x}_{des}$ : desired acceleration
- $\dot{x}_{ref}$ : reference velocity
- $\dot{x}$ : vehicle velocity
- 実装: To implement such a controller in software, we discretize the controller, changing the integral to a summation over a fixed length time steps.
  - The derivative term can be approximated with the finite difference over a fixed time step if either the reference acceleration or the estimated vehicle acceleration is not available.

### Lower Level Controller

- The low-level controller generates the throttle and braking signals to follow the desired acceleration calculated by the high-level controller.
- Assumptions:
  - Only throttle actuations is considered (no braking)
    - only throttle is needed to manage the speed of the vehicle during cruise control, and that the driver will take over if braking is required to avoid an incident.
  - The torque converter is locked (gear 3+)
    - operate in gear 3 or higher such that the torque converter is locked, meaning that torque from the engine passes directly to the transmission without loss.



- The tire slip is small (gentle longitudinal maneuvers)
  - as cruise control motions are typically gentle.
- The low-level controller seeks to generate the desired acceleration from the high level controller by increasing or decreasing the torque produced by the engine.
  - The desired acceleration is translated to a torque demand, and the torque demand is then converted to a throttle angle command.
  - The steady-state engine map: generated in testing the engine at different operating points
    - In these standard maps, the desired engine torque and the current engine RPM define the required throttle position, and can be interpolated if needed.
    - This approach is a data-driven approximation, but it works quite well in practice.

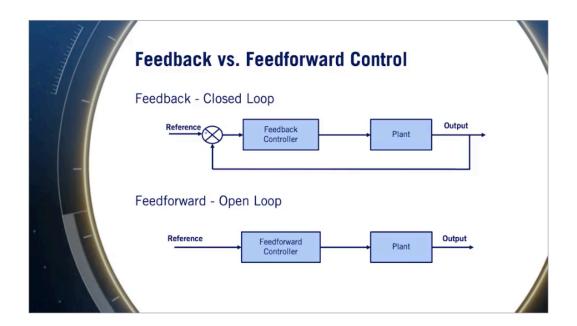


## Simulation Example

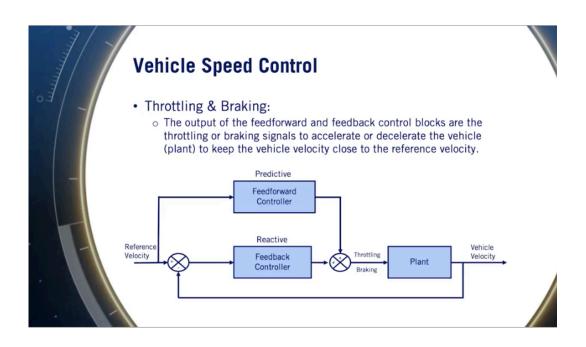
- Because of the engine map non-linearity, we see some interesting artifacts in the vehicle response as it closes in on the reference speed.
- You'll see even more interesting effects in the simulated vehicles in Carla during the final assessment for this course, with gear changes causing big challenges for pure PID control.

## Lesson 3: Feedforward Speed Control

内容:



- Integrate both feedforward and feedback control into a combined control architecture
- Apply this architecture to longitudinal vehicle control



#### Combined Feedforward and Feedback Control

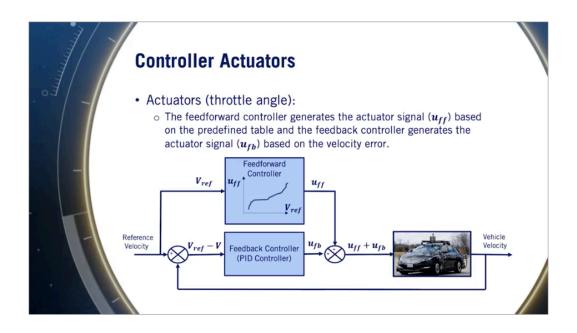
- You can think of feedforward control as providing the necessary inputs (a predictive response)
  expected to keep the plant tracking its reference signal, and the feedback controller correcting
  for errors that result from either disturbances or inaccuracies in the plant model used by the
  feedforward controller.
- The input to the plant is simply the addition of the feedforward and feedback inputs.
- Feedforward controllers provide a predictive response as they produce a reference output to achieve a particular tracking response, particularly when the required inputs are non-zero.
- Feedback controllers provide a reactive response, which eliminates control errors due to the disturbances as they occur.
- The combination of feedback and feedforward control is widely used because of this complementary relationship.

## non-zero offsetの意味

 Because autonomous vehicles require non-zero steering commands to maintain a constant radius turn and a constant throttle or brake command to maintain constant speed or deceleration rates, feedforward commands are extremely beneficial in improving tracking performance in automated driving.

## Feedforward and Feedback controller in Longitudinal Speed Control

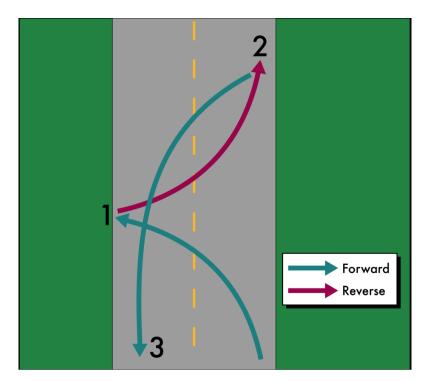
- The reference speed or drive cycle is defined by a higher level planner.
- The reference velocity is the input to the feedforward block, and the velocity error is the input to the feedback or PID control block.
- Both controllers produce two vehicle actuation signals, the throttle and brake commands.
- The role of the low-level controller achieving the desired acceleration through the use of a mapping from accelerations to engine commands is now going to be handled by the feedforward block.
- Feedforward block:
  - The feedforward block gets only the reference signal as input, and its primary objective is to accurately set the inputs of the plan.
  - やり方: Convert the entire longitudinal dynamics model into a fixed lookup table or reference map, that maps the reference velocity to the corresponding actuators signals assuming the vehicle is at steady state.
  - This feedforward approach works well at steady state, but ignores the internal dynamics of the vehicle powertrain.



#### Feedforward Table

Reference Velocityから $heta_{throttle}$ までの流れ

- 1. Reference Velocity 
  2. Wheel Angular Speed:  $\omega_{w} = \frac{V_{ref}}{r_{eff}}$
- 3. Engine Angular Speed:  $\omega_e = \frac{\omega_w^3}{GP}$



**THREE-POINT TURN** 

- 4. Engine Torque:  $T_{Engine} = T_{Load}$ 
  - 1. Assuming steady state operation, the dynamics of the powertrain says that the engine torque must be equal to the total load torque acting on the vehicle.
  - 2. We can compute the combined load torque using the current state of the vehicle, including its current speed, and the road slope.
- 5. Throttle Angle (through Typical Engine Map):  $\omega_e$  ,  $T_{Eng}$ ກາຣ $\theta_{throttle}$ 
  - 1. The engine map is defined for discrete steady state values of engine torque and RPM.
  - 2. And is interpolated as needed, based on the current vehicle operating point.

## PIDとPID+FeedforwardのSimulation比較

- 1. Because the PID controller needs errors to exist before it can correct them, its response lag the feedforward approach, which immediately applies the relevant input reference values.
- 2. The feedforward tracking is still not perfect, however, as the vehicle response is ultimately governed by its inertia, and the feedforward approach we've presented relies on steady state modeling of the car.

Zoox's Vehicle is bidirectional, that means they won't have to do U-turns or a three-point turns. Have all wheels steering.

## テストメモ:

- 1. If you need to increase the overshoot of a control loop system? "Decrease  $K_D$ " should not be selected. Increasing  $K_D$ はdecreases overshootだが、Decreasing  $K_D$ はincreasing overshootと関係ない?
- 2. What types of inaccuracies are corrected by a feedback controller? "High level controller simplification: changing the integral to a summation over fixed length time steps in the Integral term" should not be selected.