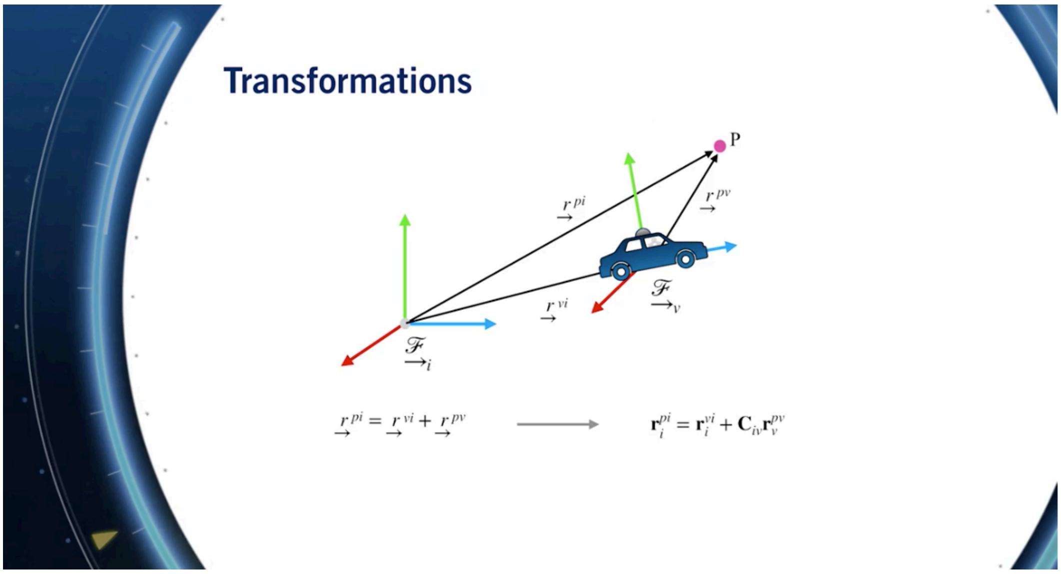
Module 3: GNSS/INS Sensing for Pose Estimation

INS: inertial navigation system

Lesson 1: 3D Geometry and Reference Frames

内容
- Understand how reference frames affect vector coordinates
- Compare and contrast different rotation representations
- Understand the importance of the ECEF, ECIF and Navigation reference frames

Coordinate Rotations
Vectors can be expressed in different coordinate frames:
空間内固定vector r
Frame Aから見ると、r_a
Frame Bから見ると、r_b
The coordinates of the vector are related through a rotation matrix: $r_b = C_{ba} r_a$
C_{ba}: take coordinates in frame A and rotates them into frame B.
subscript/superscriptの文法は常にright（start）→left（end）。



Transformations（上記Screenshotを参考）

How can we represent a rotation?

1. Rotation matrix: $C_{ba} = \begin{bmatrix} \vec{b}_1 \\ \vec{b}_2 \\ \vec{b}_3 \end{bmatrix} \begin{bmatrix} \vec{a}_1 & \vec{a}_2 & \vec{a}_3 \end{bmatrix}$
    1. This matrix defines the relationship between the basis vectors of two reference frames in terms of dot products.
    2. also called “direction cosine matrix (DCM)”

3. The Inverse of a rotation matrix is its Transpose.

2. Unit quaternions: $q = \begin{bmatrix} q_w \\ q_v \end{bmatrix} = \begin{bmatrix} cos\frac{\phi}{2} \\ \hat{u}sin\frac{\phi}{2} \end{bmatrix}$, $\|q\| = 1$
   1. In 3D space, according to Euler's rotation theorem, any rotation or sequence of rotations of a rigid body or coordinate system about a fixed point is equivalent to a single rotation by a give angle θ about a fixed axis (called the Euler axis) that runs through the fixed point. (https://en.wikipedia.org/wiki/Quaternions_and_spatial_rotation)
   2. つまり$\hat{u}$はthe Euler axisを表している。
   3. q_w: scalar part, q_v: vector part.
   4. unit quaternionsを使う理由：They don't suffer from singularities and they need only 4 parameters instead of 9.
   5. unit quaternionsとrotation matrixの変換： $r_b = C(q_{ba})r_a$, $C(q) = (q_w^2 - q_v^T q_v)1 + 2q_v q_v^T + 2q_w[q_v]_\times$

      where $[a]_\times = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$
   6. Quaternion Multiplication and Rotations
      1. Quaternion multiplication is a special operation that is associative but is not commutative in general (just like matrix multiplication):

         $$p \otimes q = \begin{bmatrix} p_w q_w - p_v^T q_v \\ p_w q_v + q_w p_v + [p_v]_\times q_v \end{bmatrix}$$
      2. Sequential rotation operations can also be performed by taking advantage of quaternion multiplication: $C(p \otimes q) = C(p)C(q)$

3. Euler Angles: $C(\theta_3, \theta_2, \theta_1) = C_3(\theta_3)C_2(\theta_2)C_1(\theta_1)$
   1. These angles represent an arbitrary rotation as the composition of 3 separate rotations about different principle axes.例えば、z, y, x軸。
   2. $C_3 = \begin{bmatrix} cos\theta_3 & -sin\theta_3 & 0 \\ sin\theta_3 & cos\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $C_2 = \begin{bmatrix} cos\theta_2 & 0 & sin\theta_2 \\ 0 & 1 & 0 \\ -sin\theta_2 & 0 & cos\theta_2 \end{bmatrix}$,

      $C_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta_1 & -sin\theta_1 \\ 0 & sin\theta_1 & cos\theta_1 \end{bmatrix}$
   3. Unfortunately, Euler angle representations are subject to what are called singularities.
      1. Singularities complicate state estimation because they represent particular rotations from which two Euler angles are indistinguishable. （singular alignment）
      2. (https://en.wikipedia.org/wiki/Rotation_matrix)によると、次の3つ例がある：
         1. multiples of 360°: $(90^\circ, 45^\circ, -105^\circ) = (-270^\circ, -315^\circ, 255^\circ)$
         2. singular alignment: $(72^\circ, 0^\circ, 0^\circ) = (40^\circ, 0^\circ, 32^\circ)$
         3. bistable flip: $(45^\circ, 60^\circ, -30^\circ) = (-135^\circ, -60^\circ, 150^\circ)$
      3. Neither quaternions nor rotation matrices suffer from this problem at the expense of using more parameters.

Matrix Singularity (https://documentation.statsoft.com/STATISTICAHelp.aspx?path=Glossary/GlossaryTwo/M/MatrixSingularity)
A matrix is singular if the elements in a column (or row) of the matrix are linearly dependent on the elements in one or more other columns (or rows) of the matrix.

For example, if the elements in one column of a matrix are 1, -1, 0, and the elements in another column of the matrix are 2, -2, 0.
- A unique, regular matrix inverse cannot be computed for singular matrices, but generalized inverses can be computed for any singular matrix.

Which rotation representation should I use?

	Rotation Matrix	Unit quaternion	Euler angles
Expression	C	$q = \begin{bmatrix} cos\frac{\phi}{2} \\ \hat{u}sin\frac{\phi}{2} \end{bmatrix}$	$\{\theta_3, \theta_2, \theta_1\}$
Parameters	9	4	3
Constraints	$CC^T = 1$	$\|\|q\|\| = 1$	None*
Singularities?	No	No	Yes!

- To use a unit quaternion to actually rotate a vector, also require some additional algebra beyond simple matrix multiplication.
- Euler angles are intuitive to visualize.

Reference Frames | ECIF (Earth-Centered Inertial Frame)
ECIF coordinate frame is fixed, Earth rotates about the z-axis.
$\mathscr{F}_{ECIF}$

x	fixed w.r.t. stars
y	fixed w.r.t. stars
z	true north

fixed w.r.t. stars, means that although the earth rotates about the z-axis, the x and y axes do not move.

Reference Frames | ECEF (Earth-Centered Earth-Fixed Frame)
$\mathscr{F}_{ECEF}$

x	prime meridian (on equator)
y	RHR (right-hand rule)
z	true north

単語:
- prime meridian: The prime meridian is the line of longitude, corresponding to zero degrees and passing through Greenwich, England, from which all the other lines of longitude are calculated.
- meridian: A meridian is an imaginary line from the North Pole to the South Pole. Meridians are drawn on maps to help you describe the position of a place.

Reference Frames | Navigation
Although the ECEF and ECIF are useful when discussing satellites and inertial sensing onboard aircraft, for a practical car applications, usually want to use a frame that is fixed w.r.t. the ground.
NED Frame

x	true north
y	true east
z	down (along gravity)

ENU Frame

x	east
y	north
z	up

Reference Frames | Sensor & Vehicle
For localization, often ignore the distinction between the vehicle and sensor frame and assume that if can track the sensor, can track any point on the vehicle, given proper calibration.

- Vector quantities can be expressed in different reference frames
- Rotations can be parametrized by rotation matrices, quaternions or Euler angles
- ECEF, ECIF and Navigation frames are important in localization

Lesson 2: The Inertial Measurement Unit (IMU)

単語
- inertial space (or inertial frame of reference): (http://www.cleonis.nl/physics/phys256/inertial_space.php)によると、the background reference that is provided by the phenomenon of inertia. (https://en.wikipedia.org/wiki/Inertial_frame_of_reference)によると、in this frame of reference, a body with zero net force acting upon it does not accelerate; that is, such a body is at rest or moving at a constant velocity. Conceptually, the physics of a system in an inertial frame have no causes external to the system.
- tuning fork: A tuning fork is a small steel instrument which is used to tune instruments by striking it against something to produce a note of fixed musical pitch.

内容
- Describe the individual components (accelerometers and gyroscopes) of an inertial measurement unit and their basic operating principles.
- Define the measurement models used for accelerometers and gyroscopes.

Inertial Measurement Unit
- In space there are few landmarks to rely on for guidance.
- In modern self-driving cars, IMUs play a very similar role.
  - Filling in during periods when navigation information from other sensors is either unavailable or unreliable.
- An IMU is typically composed of 3 gyroscopes and 3 accelerometers.
- More expensive IMUs use more complex components and have more accurate calibration models that can remove the effects of temperature fluctuations, for example.

The Gyroscope
- Historically: a spinning disc that maintains a specific orientation relative to inertial space, providing an orientation reference.
  - due to angular momentum, resist changes in orientation.
- Microelectromechanical systems (MEMS) are much smaller and cheaper
  - MEMS consists of a small silicon tuning fork that changes its resonance properties based on an applied rotation or orientation change.
  - measure rotational rates instead of orientation directly
  - measurements are noisy and drift over time

The Accelerometer
- An accelerometer measures acceleration along a single axis.
- Accelerometers measure acceleration relative to free-fall - this is also called the proper acceleration or specific force: $a_{meas} = f = \dfrac{F_{non-gravity}}{m}$

- Sitting still at your desk, your proper acceleration is g upwards. (think of the 'normal' force holding you up)
- In localization, typically require the acceleration relative to a fixed reference frame
  - 'coordinate' acceleration
  - computed using fundamental equation for accelerometers in a gravity field: $f + g = \ddot{r}_i$

The Accelerometer | Examples
- An accelerometer in a stationary car measures: $f = \ddot{r}_i - g \approx 0 - g = -g$, ($g$ 'up')
- An accelerometer on the International Space Station measures: $f = \ddot{r}_i - g \approx g - g = 0$, (zero-g)

Measurement Model: Gyroscope

$\omega(t) = \omega_s(t) + b_{gyro}(t) + n_{gyro}(t)$

$\omega_s(t)$: angular velocity of the sensor expressed in the sensor frame.

$b_{gyro}(t)$: slowly evolving bias

$n_{gyro}(t)$: noise term
- Although gyroscopes do measure the rotation of the Earth, it's often safe to ignore this for applications where we care only about motion over a short duration.

Measurement Model: Accelerometer

$a(t) = C_{sn}(t)(\ddot{r}_n^{sn}(t) - g_n) + b_{accel}(t) + n_{accel}(t)$

$C_{sn}(t)$: orientation of the sensor (computed by integrating the rotational rates from the gyroscope)
- Since the accelerometers measure acceleration in the IMU body frame, need to keep track of the orientation at all times in order to be able to perform the necessary subtraction.
  - An accurate orientation estimate is critical for accurate position estimates.
    - When we convert the measured specific force into an acceleration, have to make sure that the direction of gravity is correct.
    - Otherwise even a small error in orientation can cause us to think that we're accelerating when we're not.

g_n: gravity in the navigation frame.

Inertial Navigation: Important Notes
1. If inaccurately keep track of $C_{sn}(t)$, we incorporate components of g_n into $\ddot{r}_n^{sn}(t)$.
   1. This will ultimately lead to terrible estimates of position ($r_n^{sn}(t)$)
2. Both measurement models ignore the effect of Earth's rotation.
3. Only consider strapdown IMUs - where the individual sensors are rigidly attached to the vehicle and are not gimballed.

- A 6-DOF IMU is composed of three gyroscopes and three accelerometers, mounted orthogonally.
- A strapdown gyroscope measures a rotational rate in the sensor frame.
- A strapdown accelerometer measures a specific force (or acceleration relative to free-fall) in the sensor frame.

Since strapdown IMUs are tricky to calibrate and drift over time, need another sensor to periodically correct the pose estimates.

Lesson 3: The Global Navigation Satellite Systems (GNSS)

単語
- ionosphere: 電離層。a region of the earth's atmosphere, extending from about 60 kilometers to 1000km above the earth's surface, in which there is a high concentration of free electrons formed as a result of ionizing radiation entering the atmosphere from space.

- ephemeris: 天体暦。a table giving the future positions of a planet, comet, or satellite.

内容
- Develop a model of GNSS positioning based on pseudoranging and trilateration.
- Become familiar with the sources of GNSS positioning error.
- Describe two ways to improve GNSS.

Korean Air Lines Flight 007
- Korean Air Flight 007 was shot down in 1983 after deviating into Soviet airspace due to improper use of their Inertial Navigation System.
- This prompted the U.S. to open GPS for worldwide use.

GNSS | Accurate Global Positioning
- Global Navigation Satellite System (GNSS) is a catch-all term for a satellite system(s) that can be used to pinpoint a receiver's position anywhere in the world.
- The two that are fully operational as of 2018 are GPS and GLONASS, the Russian equivalent.
- Several other systems are nearing completion, including the European Galileo constellation.

GPS | Global Positioning System
- Composed of 24 to 32 satellites in 6 orbital planes
  - Altitude of ~20,200 km (12,550 miles)
  - Orbital period of ~12 hours
- Satellites are decommissioned and replaced periodically.
- The constellation is designed such that at least 4 satellites are visible at any surface point on earth at all times.
- Each satellite broadcasts on two frequencies
  - L1 (1575.42 MHz, civilian + military)
  - L2 (1227.6 MHz, military)

GPS | Computing Position
- Each GPS satellite transmits a signal that encodes
  1. its position (via accurate ephemeris information)
  2. time of signal transmission (via onboard atomic clock)
- Each broadcast signal contains a pseudo-random code that identifies the satellite position and the time of transmission of the signal.
  - signalを送信するタイミング。送信側から受信側までのかかる時間ではない。
- The basic principle behind GPS is time of arrival ranging.
- The receiver computes a distance to each visible satellite by comparing its own internal clock with that of the time of transmission.
- The time difference is converted to a distance using knowledge that electromagnetic signals propagate at the speed of light.
- At least four satellites are required to solve for 3D position, three if only 2D is required (e.g., if altitude is known).

Trilateration
- The process of recovering position from several distances to known landmarks is called trilateration.
  - Triangulationとは違う。triangulation: compute positions based on angle measurements.
- For each satellite, measure the pseudo range as follows:

$$\rho^{(i)} = c(t_r - t_s) = \sqrt{(p^{(i)} - r)^T(p^{(i)} - r)} + c\Delta t_r + c\Delta t_a^{(i)} + \eta^{(i)}$$

  - r: receiver (3D) position.
  - $p^{(i)}$: position of satellite i.
  - Δt_r: receiver clock error.
  - $\Delta t_a^{(i)}$: atmospheric propagation delay.
  - η: measurement noise.
  - c: speed of light.

- t_s, t_r: time sent, time received.
- The term pseudo range refers to the fact that the range information is corrupted by the error sources above.
- Each pseudo range measurement defines a circle in 2D or a sphere in 3D.
- By using at least 4 satellites, can solve for:
  - r: receiver (3D) position.
  - Δt_r: receiver clock error.
- If more than 4, can use method of least squares to find the maximum likelihood position assuming Gaussian noise.

GPS | Error Sources (I)
- Ionospheric delay (unknown amount)
  - Charged ions in the atmosphere affect signal propagation.
- Multipath effects
  - Surrounding terrain, buildings can cause unwanted reflections.
  - Increase the distance traveled by the signal before reaching the receiver.
- Ephemeris & clock errors
  - Any small error in clock synchronization or satellite position information can have catastrophic consequences.
  - A clock error of $1 \times 10^{-6}s$ gives a 300m position error!
  - Both a Ephemeris data and satellite clocks are updated and recalibrated often, but the calibration can be out of date.
- Geometric Dilution of Precision (GDOP)
  - The configuration of the visible satellites affects position precision.
    - Poor config - high GDOP
    - Good config - low GDOP
  - For higher accuracy, a configuration with satellites spread across the sky is preferable.

GPS | Improvements
- Differential GPS can correct receiver positioning estimates by making use of the more accurately know positions of one or more fixed base stations.
  - Corrections are broadcast on separate frequencies to the GNSS receiver in the moving vehicle.
- Real-Time Kinematic, or RTK GPS makes use of carrier phase information to improve positioning accuracy.
- Typically quite costly to implement.

Basic GPS	**Differential GPS (DGPS)**	**Real-Time Kinematic (RTK) GPS**
mobile receiver	mobile receiver + fixed base station(s)	mobile receiver + fixed base station
no error correction	estimate error caused by atmospheric effects	estimate relative position using phase of carrier signal
~10m accuracy	~10cm accuracy	~2cm accuracy

- A GNSS works through trilateration via pseudoranging from at least 4 satellites (for a 3D position fix).