1.1- a)

- Floating-point numbers are represented as binary fractions in computers (IEEE 754), which are a whole number times a power of two. Because of it, most decimal fractions cannot be represented exactly as binary fractions.
- Therefore, the machine approximates the decimal floating-point numbers, entered by users, as the binary floating-point numbers, which are stored in it.
- It means that, for instance, 0.1 is not exactly 1/10 in the machine, but rounded from the true machine value.
- It is the same for 0.2 and 0.3. Thus, if you check '0.1 +0.2==0.3' on the computer, it'll return False, and you would realise that the result of '0.1 +0.2' is 0.30000000000000004 in the terminal.

- Hence, we cannot use comparisons like $x==y$ for floating-point numbers. Instead, we can check for $|x-y|<\tau$, where τ means a tolearance value. Otherwise, we can use a module in Python to check '0.1+0.2=0.3' as the following.

```
# Method 1. A tolerance value
print(abs((0.1 + 0.2) - 0.3) < 0.000001)


# Method 2. Using  the module
from decimal import *
getcontext().prec = 1 # Set a precision of the decimal number


# Method 2-1
print(Decimal(0.1) + Decimal(0.2) == Decimal('0.3'))
# Method 2-2
print(Decimal(0.1) + Decimal(0.2) == Decimal((0, (0, 3), -1)))
```

1.1 – b)

Augmented matrix
$$\left[\begin{array}{cccc|c} 1 & 2 & 5 & 3 & 4.5 \\ 0 & 1 & 2 & 4 & 1.5 \\ 2 & 3 & 5 & 3 & 4.5 \\ 4 & 2 & 1 & 0 & 2 \end{array}\right] \begin{array}{l} \\ \\ R_3 \to R_3 - 2R_1 \\ R_4 \to R_4 - 4R_1 \end{array}$$

$$\Leftrightarrow \begin{bmatrix} 1 & 2 & 5 & 3 & 4.5 \\ 0 & 1 & 2 & 4 & 1.5 \\ 0 & -1 & -5 & -3 & -4.5 \\ 0 & -6 & -19 & -12 & -16 \end{bmatrix} \begin{array}{l} \\ \\ R_3 \to R_3 + R_2 \\ R_4 \to R_4 + 6R_2 \end{array}$$

$$\Leftrightarrow \begin{bmatrix} 1 & 2 & 5 & 3 & 4.5 \\ 0 & 1 & 2 & 4 & 1.5 \\ 0 & 0 & -3 & 1 & -3 \\ 0 & 0 & -7 & 12 & -7 \end{bmatrix} \begin{array}{l} \\ \\ \\ R_4 \to R_4 - 7/3\, R_3 \end{array}$$

$$\Leftrightarrow \begin{bmatrix} 1 & 2 & 5 & 3 & 4.5 \\ 0 & 1 & 2 & 4 & 1.5 \\ 0 & 0 & -3 & 1 & -3 \\ 0 & 0 & 0 & 29/3 & 0 \end{bmatrix} \begin{array}{l} \\ \\ \\ R_4 \to \frac{3}{29} R_4 \end{array}$$

$$\Leftrightarrow \begin{bmatrix} 1 & 2 & 5 & 3 & 4.5 \\ 0 & 1 & 2 & 4 & 1.5 \\ 0 & 0 & -3 & 1 & -3 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{array}{l} R_1 \to R_1 - 3R_4 \\ R_2 \to R_2 - 4R_4 \\ R_3 \to (R_3 - R_4) \times (-1/3) \\ \end{array}$$

$$\Leftrightarrow \begin{bmatrix} 1 & 2 & 5 & 0 & 4.5 \\ 0 & 1 & 2 & 0 & 1.5 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{array}{l} R_1 \to R_1 - 5R_3 \\ R_2 \to R_2 - 2R_3 \\ \\ \\ \end{array}$$

$$\Leftrightarrow \begin{bmatrix} 1 & 2 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 0 & -0.5 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad R_1 \to R_1 - 2R_2$$

$$\Leftrightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 0.5 \\ 0 & 1 & 0 & 0 & -0.5 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\therefore \vec{x} = \begin{bmatrix} 0.5 \\ -0.5 \\ 1 \\ 0 \end{bmatrix}$$