2.1 – a) $A = LU$

$$\begin{bmatrix} 5 & 6 & 9 \\ 3 & 6 & 2 \\ 1 & 2 & 5 \end{bmatrix} \quad R_2 \to R_2 - \tfrac{3}{5} R_1$$

$$\Leftrightarrow \begin{bmatrix} 5 & 6 & 9 \\ 0 & 12/5 & -17/5 \\ 1 & 2 & 5 \end{bmatrix} R_3 \to R_3 - \tfrac{1}{5} R_1$$

$$6 - \tfrac{3}{5} \times 6 = \frac{30-18}{5} = \frac{12}{5}$$

$$2 - \tfrac{3}{5} \times 9 = \frac{10-27}{5} = -\frac{17}{5}$$

$$\Leftrightarrow \begin{bmatrix} 5 & 6 & 9 \\ 0 & 12/5 & -17/5 \\ 0 & 4/5 & 16/5 \end{bmatrix} R_3 \to R_3 - \tfrac{1}{3} R_2$$

$$2 - \frac{6}{5} = \frac{10-6}{5} = 4/5 \qquad 5 - 9/5 = \frac{25-9}{5} = 16/5$$

$$\Leftrightarrow \begin{bmatrix} 5 & 6 & 9 \\ 0 & 12/5 & -17/5 \\ 0 & 0 & 13/3 \end{bmatrix} = U$$

$$\frac{16}{5} + \frac{17}{15} = \frac{48+17}{15} = \frac{65}{15}\,\frac{13}{3} = \frac{13}{3}$$

$$\begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 6 \\ 2 \end{bmatrix} \quad \begin{bmatrix} 5 \end{bmatrix}$$

$\div 5 \quad \div 6 \quad \div 5$

$$\begin{bmatrix} 1 & & \\ 3/5 & 1 & \\ 1/5 & 1/3 & 1 \end{bmatrix}$$

$$\therefore L = \begin{bmatrix} 1 & 0 & 0 \\ 3/5 & 1 & 0 \\ 1/5 & 1/3 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 5 & 6 & 9 \\ 0 & 12/5 & -17/5 \\ 0 & 0 & 13/3 \end{bmatrix}$$

2.1 – b)

$$A = \begin{bmatrix} 2 & 5 & -1 & 2 \\ 0 & 2 & 4 & -4 \\ 1 & 0 & 3 & 0 \\ 0 & 2 & -3 & 1 \end{bmatrix} \quad R_3 \to R_3 - \tfrac{1}{2} R_1$$

$$\Leftrightarrow \begin{bmatrix} 2 & 5 & -1 & 2 \\ 0 & 2 & 4 & -4 \\ 0 & -5/2 & 7/2 & -1 \\ 0 & 2 & -3 & 1 \end{bmatrix} \quad \begin{matrix} R_3 \to R_3 + 5/4 R_2 \\ R_4 \to R_4 - R_2 \end{matrix}$$

$$\Leftrightarrow \begin{bmatrix} 2 & 5 & -1 & 2 \\ 0 & 2 & 4 & -4 \\ 0 & 0 & 17/2 & -6 \\ 0 & 0 & -7 & 5 \end{bmatrix} \quad R_4 \to R_4 + 14/17 R_3$$

$\frac{7}{2} + \cancel{4} \cdot \frac{5}{\cancel{4}} = \frac{7+10}{2}$ $\quad -1 - \cancel{4} \cdot \frac{5}{\cancel{4}} = -6$

$$\Leftrightarrow \begin{bmatrix} 2 & 5 & -1 & 2 \\ 0 & 2 & 4 & -4 \\ 0 & 0 & 17/2 & -6 \\ 0 & 0 & 0 & 1/17 \end{bmatrix}$$

$5 - 6 \cdot \frac{14}{17} = \frac{85 - 84}{17} = \frac{1}{17}$

$\det(A) = 2 \times \cancel{2} \times \cancel{17/2} \times \cancel{1/17} = 2$

$\therefore 2$