More more on Normal Distributions

your name

2024-10-10

Math 2265 Sec 4.1 Normal Distribution Part III

- Work as a group!
- You will need to replace `"ans"` or `your_answer` in the source code
- Update your name in L3
- Add your group members' name below; students may lose one point if Question 0 is unanswered
- Make sure you save and `knit` your work (to html or pdf) before submitting it to Canvas

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Goal

- Learn the function that computes the percentile from given x or its Z-score.
- Learn the function that computes x or its Z-score or from given percentile.
- Master the functions `pnorm` and `qnorm`.

Question 0. Who are your group members? (List their first names should be sufficient)

Answer:

1. <name_1>
2. <name_2>

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If you need more time to get used to `Markdown`, use the `Visual` mode.

The icon is located in the upper-left corner next to `source`.

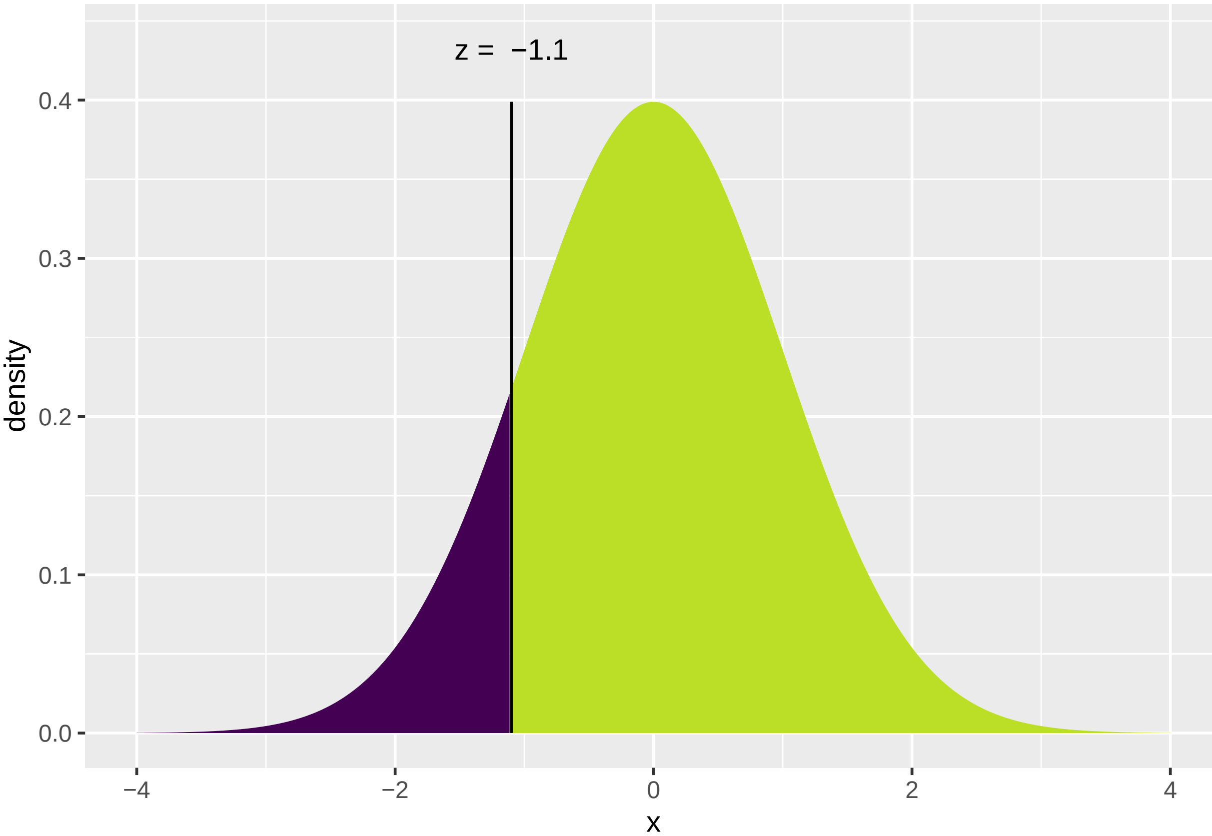
Example 1: Visualizing percentiles in the normal distribution with `xpnorm`

(a) P(Z < -1.1)

```
xpnorm(-1.1, mean = 0, sd = 1)
```

```
##
## If X ~ N(0, 1), then
##  P(X <= -1.1) = P(Z <= -1.1) = 0.1357
##  P(X >  -1.1) = P(Z >  -1.1) = 0.8643
```

```
##
```

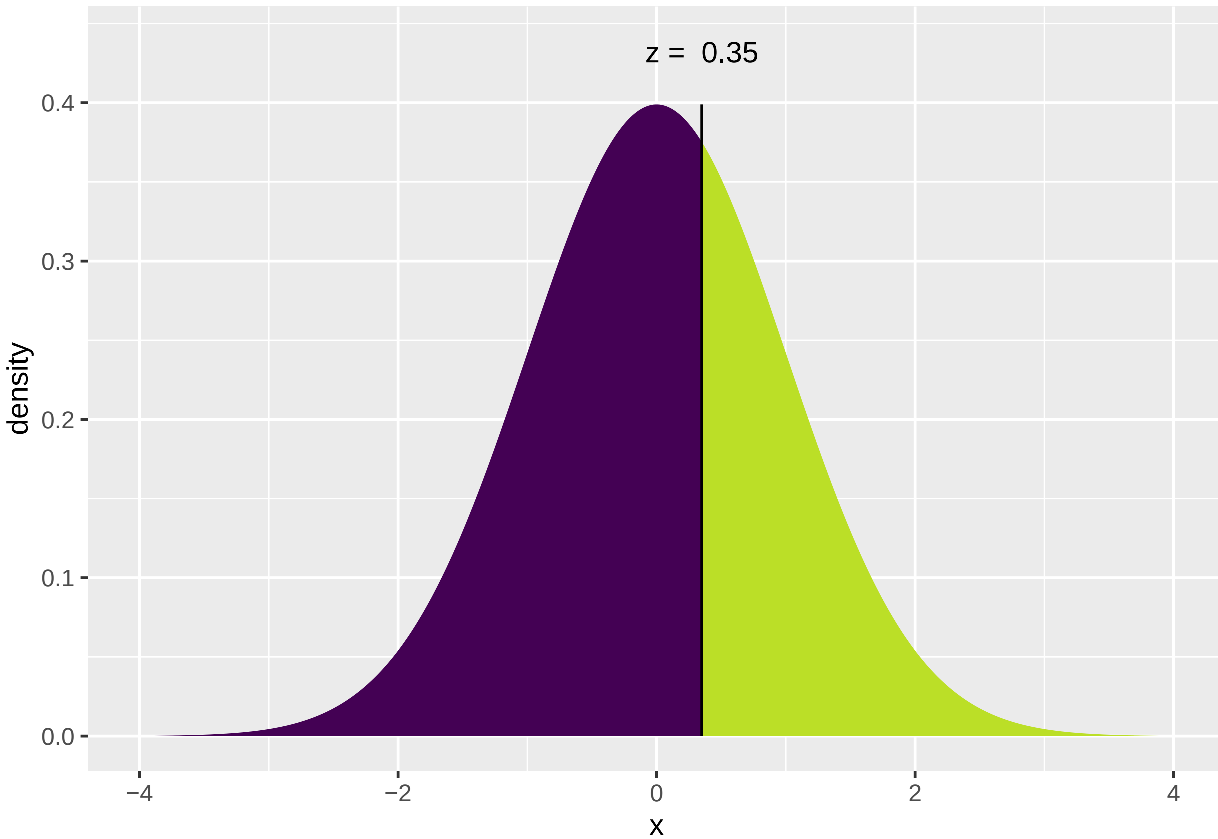



```
## [1] 0.1356661
```

(b) $P(Z < 0.35)$

```r
xpnorm(0.35, mean = 0, sd = 1)
```

```
##
## If X ~ N(0, 1), then
##  P(X <= 0.35) = P(Z <= 0.35) = 0.6368
##  P(X >  0.35) = P(Z >  0.35) = 0.3632
##
```

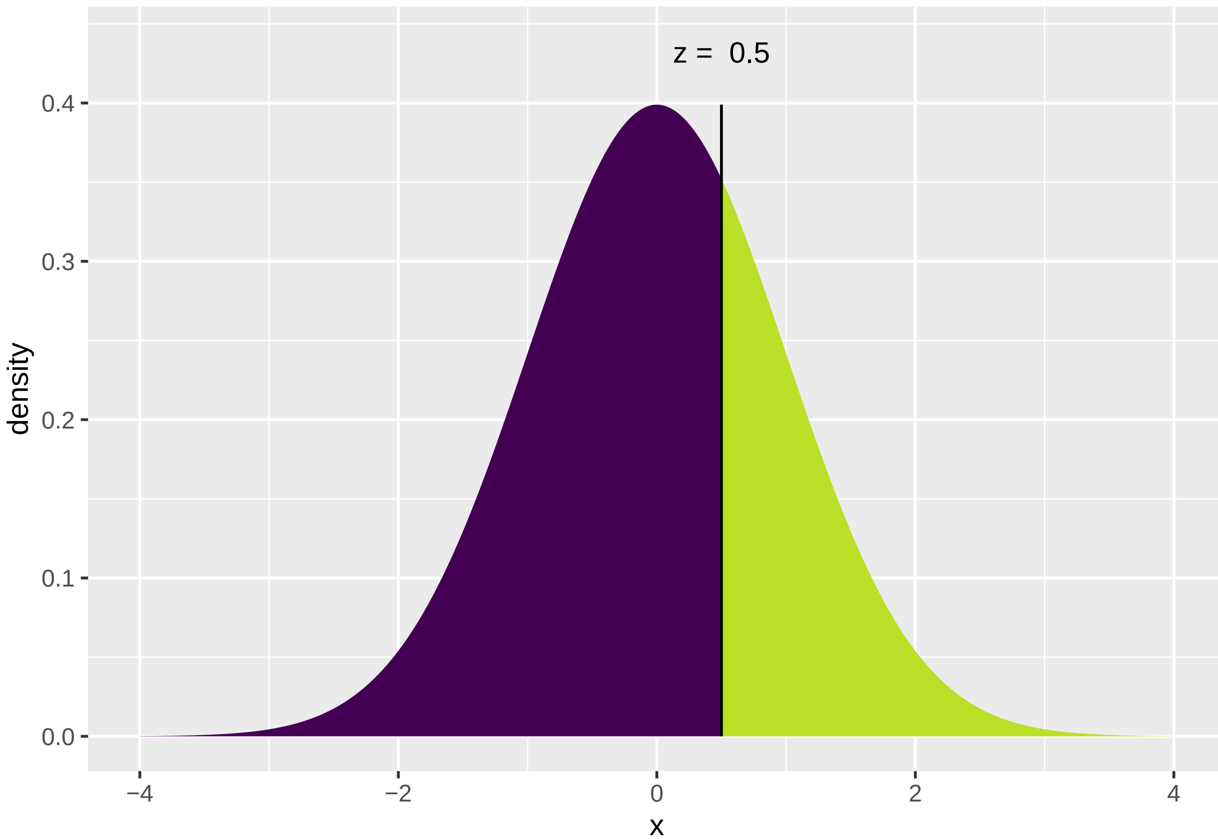



```
## [1] 0.6368307
```

(c) $P(Z > 0.5)$

```
xpnorm(0.5, mean = 0, sd = 1)
```

```
##
## If X ~ N(0, 1), then
##  P(X <= 0.5) = P(Z <= 0.5) = 0.6915
##  P(X >  0.5) = P(Z >  0.5) = 0.3085
##
```



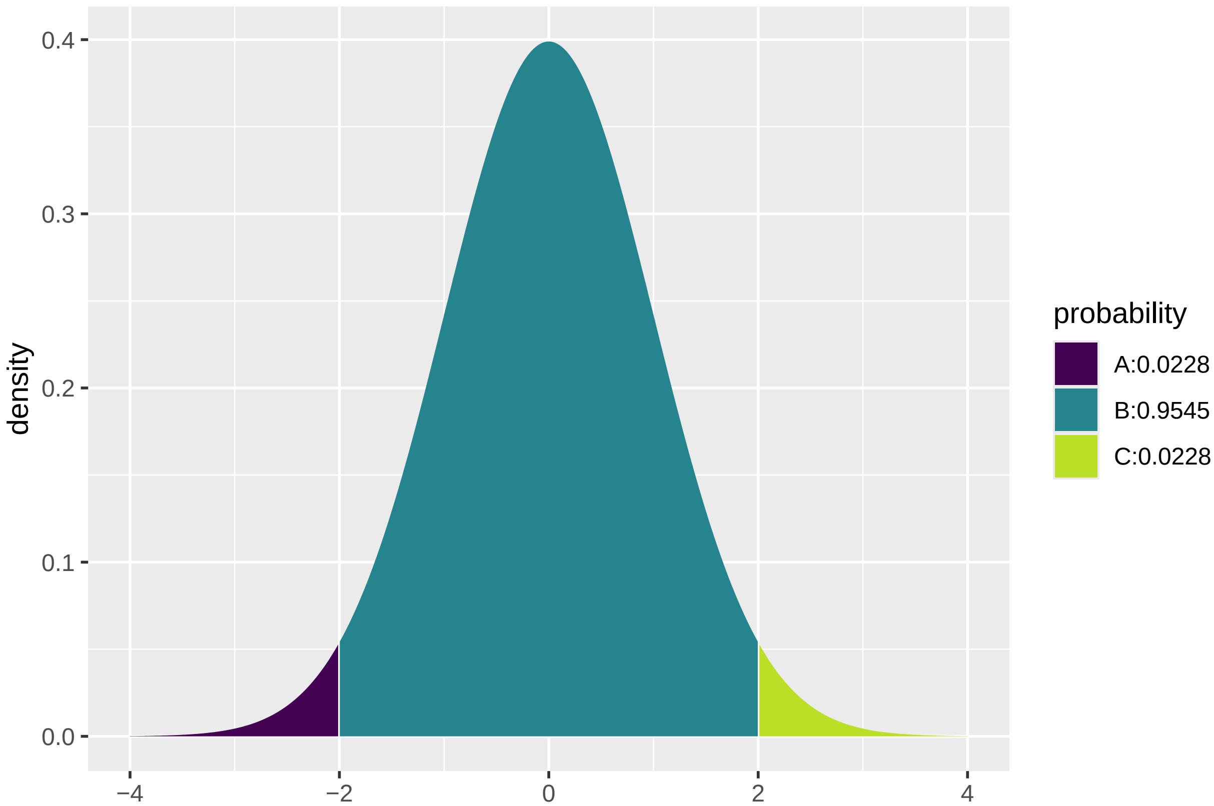

```
## [1] 0.6914625
```

Caveat: `xpnorm` computes both P(Z<0.5) and P(Z>0.5).

(d) P(|Z| < 2) = P (-2 < Z < 2)

```r
xpnorm(c(-2,2), mean = 0, sd = 1)
```

```
##
## If X ~ N(0, 1), then
##  P(X <= -2) = P(Z <= -2) = 0.02275   P(X <=  2) = P(Z <=  2) = 0.97725
##  P(X >  -2) = P(Z >  -2) = 0.97725   P(X >   2) = P(Z >   2) = 0.02275
##
```




```
## [1] 0.02275013 0.97724987
```

Caveat: This does not compute P(|Z|< 2), but P(Z<-2) and P(Z<2) for -2, 2. So, either we take the difference or use

```
pnorm(2, mean=0,sd=1) - pnorm(-2,mean=0,sd=1)
```

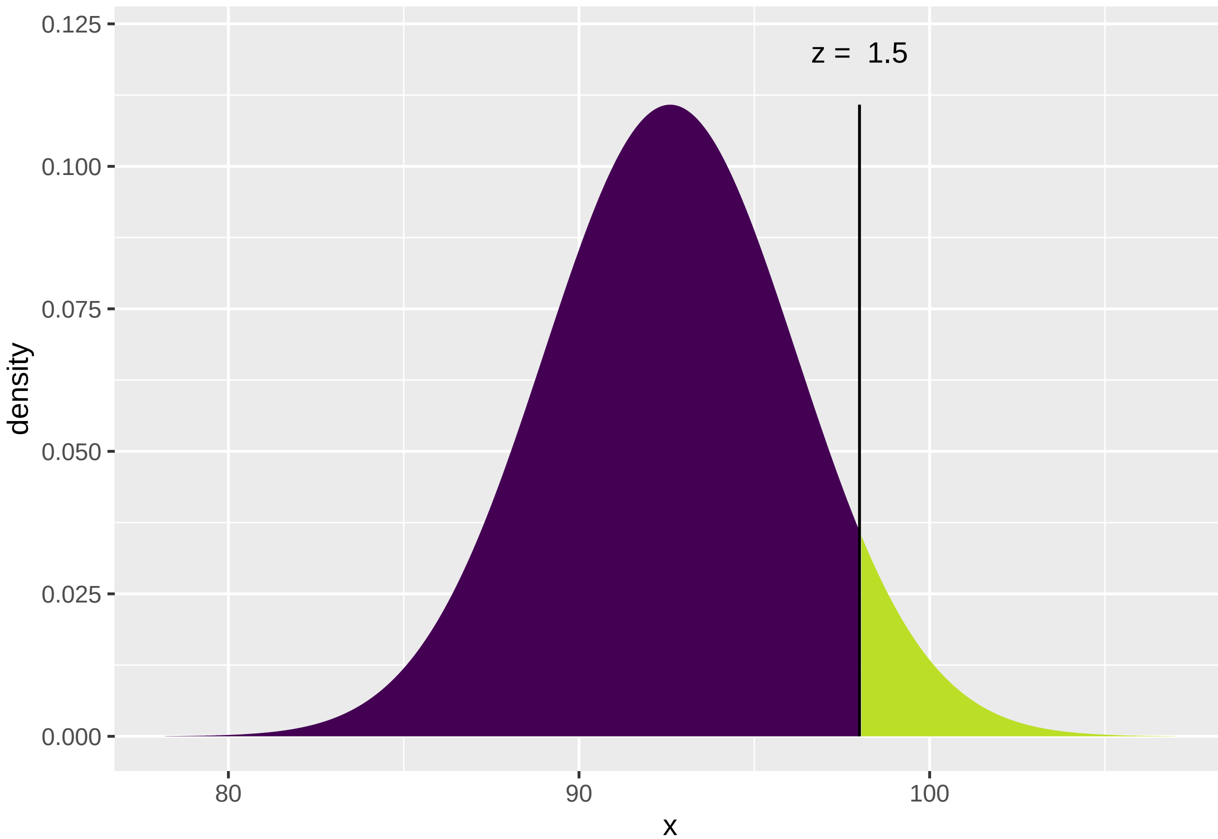
```
## [1] 0.9544997
```

Example 2: Visualizing percentiles in normal distributions with `xpnorm`

(a) x $= 98$, $\mu = 92.6$, $\sigma = 3.6$

```
xpnorm(98, mean=92.6, sd=3.6)
```

```
##
## If X ~ N(92.6, 3.6), then
##  P(X <= 98) = P(Z <= 1.5) = 0.9332
##  P(X >  98) = P(Z >  1.5) = 0.06681
##
```



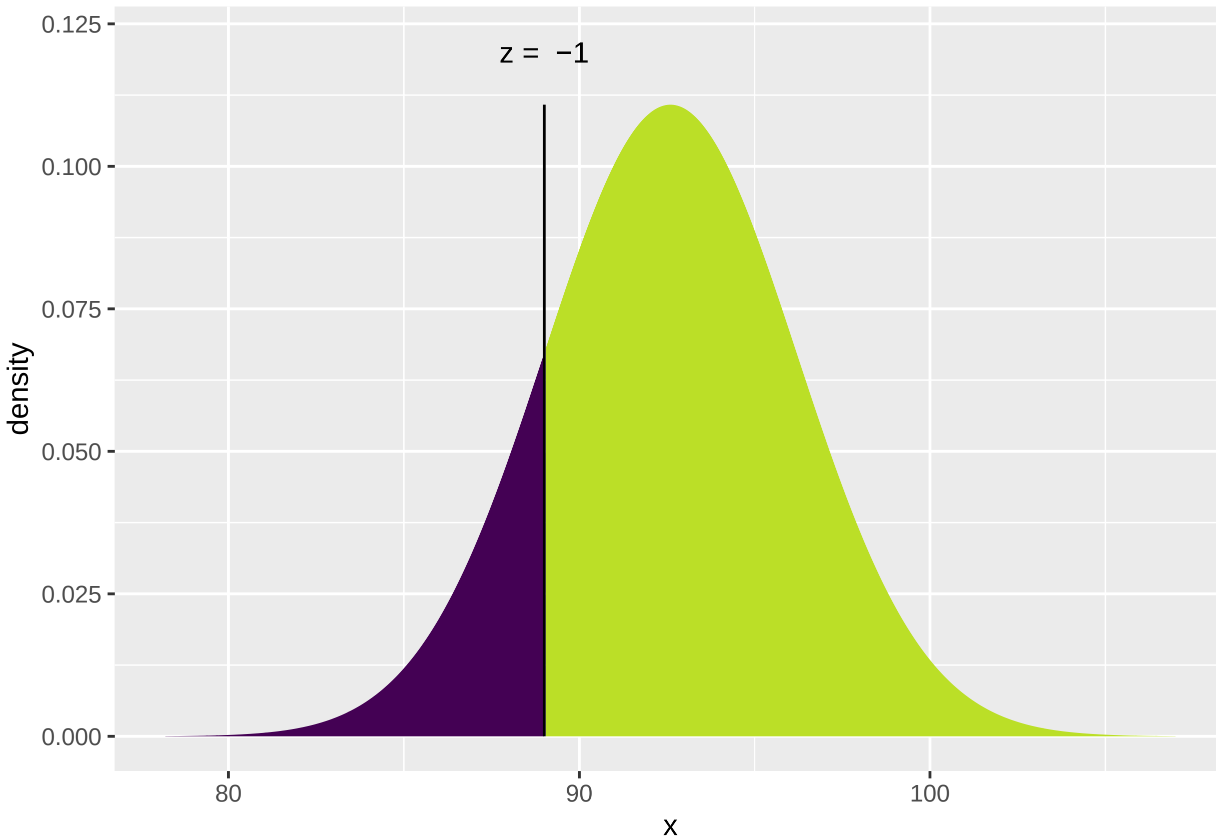

```
## [1] 0.9331928
```

Note that the Z-score, 1.5, is computed.

(b) x = 89, $\mu = 92.6$, $\sigma = 3.6$

```
xpnorm(89, mean=92.6, sd=3.6)
```

```
##
## If X ~ N(92.6, 3.6), then
##  P(X <= 89) = P(Z <= -1) = 0.1587
##  P(X >  89) = P(Z >  -1) = 0.8413
##
```




```
## [1] 0.1586553
```

Example 3: Visualizing Z-scores from Percentiles

What is the Z-score for the 80th percentile with mean = 0 and standard deviation = 1?

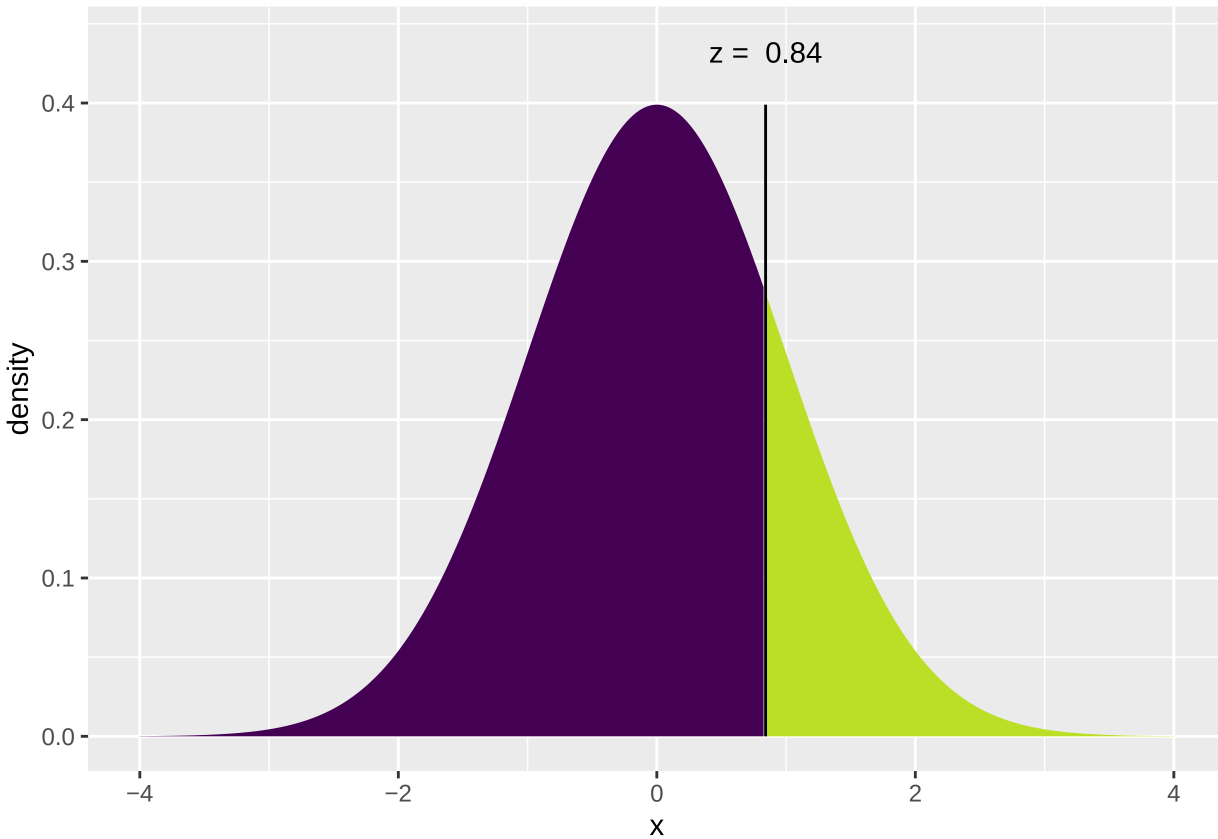
```r
qnorm(0.8, mean = 0, sd = 1)
```

```
## [1] 0.8416212
```

Now check out the output of `xqnorm`.

```r
xqnorm(0.8, mean = 0, sd = 1)
```

```
##
## If X ~ N(0, 1), then
##  P(X <= 0.8416212) = 0.8
##  P(X >  0.8416212) = 0.2
##
```




```
## [1] 0.8416212
```

HW Problem

Speeding on the I-5 Freeway

The distribution of passenger vehicle speeds traveling on the Interstate 5 Freeway (I-5) in California is nearly normal with:

- Mean speed (μ): 72.6 miles/hour
- Standard deviation (σ): 4.78 miles/hour

(a) What percent of passenger vehicles travel slower than 80 miles/hour?

First, compute the Z-score for $x = 80$:

```
z_80 <- (80 - 72.6) / 4.78
z_80
```

```
## [1] 1.548117
```

Now, compute the percentile using the standard normal distribution:

```
p_80 <- pnorm(z_80, mean = 0, sd = 1)
p_80
```

```
## [1] 0.939203
```

Alternatively, you can compute the percentile directly without calculating the Z-score:

```r
p_80_direct <- pnorm(80, mean = 72.6, sd = 4.78)
p_80_direct
```

```
## [1] 0.939203
```

(b) What percent of passenger vehicles travel between 60 and 80 miles/hour?

First, compute the Z-score for $x = 80$:

```r
z_60 <- (80 - 72.6) / 4.78
z_60
```

```
## [1] 1.548117
```

Now, compute the percentile using the standard normal distribution:

```r
p_60 <- pnorm(z_60, mean = 0, sd = 1)
p_60
```

```
## [1] 0.939203
```

Alternatively, compute directly without Z-scores:

```r
p_60_direct <- pnorm(60, mean = 72.6, sd = 4.78)
p_60_direct
```

```
## [1] 0.004194693
```

Compute the percentile (we already computed `p_80_direct` above).

```r
p_80_direct - p_60_direct
```

```
## [1] 0.9350083
```

(c) How fast do the fastest 5% of passenger vehicles travel?

We need to find the speed corresponding to the 95th percentile (since the fastest 5% are above this speed).

Compute the speed using the inverse normal distribution function `qnorm`:

```r
speed_95th <- qnorm(0.95, mean = 72.6, sd = 4.78)
speed_95th
```

```
## [1] 80.4624
```

(d) Approximate what percentage of the passenger vehicles travel above the speed limit of 70 miles/hour.

Compute the Z-score for $x = 70$:

```r
z_70 <- (70 - 72.6) / 4.78
z_70
```

```
## [1] -0.5439331
```

Compute the percentile up to 70 miles/hour:

```r
p_70 <- pnorm(z_70, mean = 0, sd = 1)
p_70
```

```
## [1] 0.2932438
```

Alternatively, compute directly:

```
p_70_direct <- pnorm(70, mean = 72.6, sd = 4.78)
p_70_direct
```

```
## [1] 0.2932438
```

Compute the percentage of vehicles traveling above 70 miles/hour:

```
percentile_above_70 <- 1 - p_70
```

Answers:

Question	Answer
(a) Percent traveling slower than 80 mph	93.94%
(b) Percent traveling between 60 and 80 mph	93.48%
(c) Speed of the fastest 5% of vehicles	Above 80.46 mph
(d) Percent traveling above the speed limit of 70 mph	70.68%

Here is visualization with `xpnorm` and `xqnorm`:

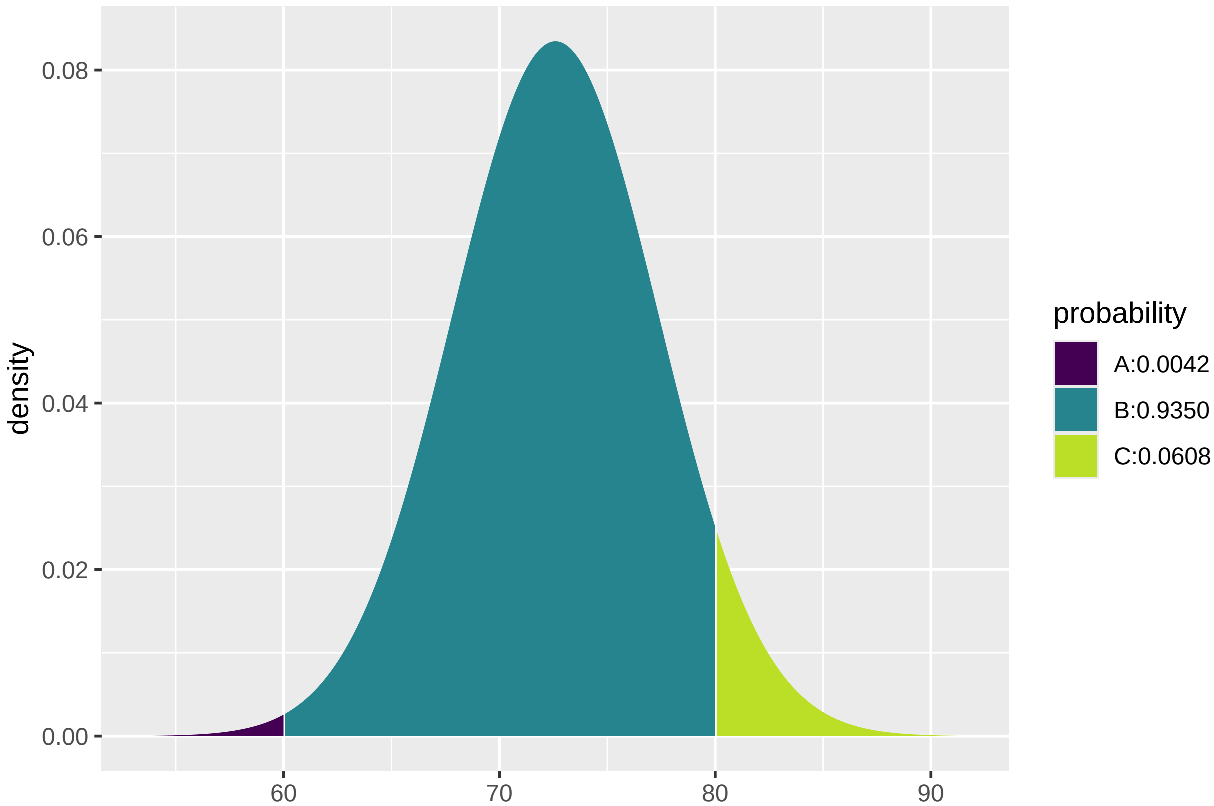
```
xpnorm(80, mean = 72.6, sd = 4.78)
```

```
##
## If X ~ N(72.6, 4.78), then
##  P(X <= 80) = P(Z <= 1.548) = 0.9392
##  P(X >  80) = P(Z >  1.548) = 0.0608
##
```


```
## [1] 0.939203
```

```r
xpnorm(c(60,80), mean = 72.6, sd = 4.78)
```

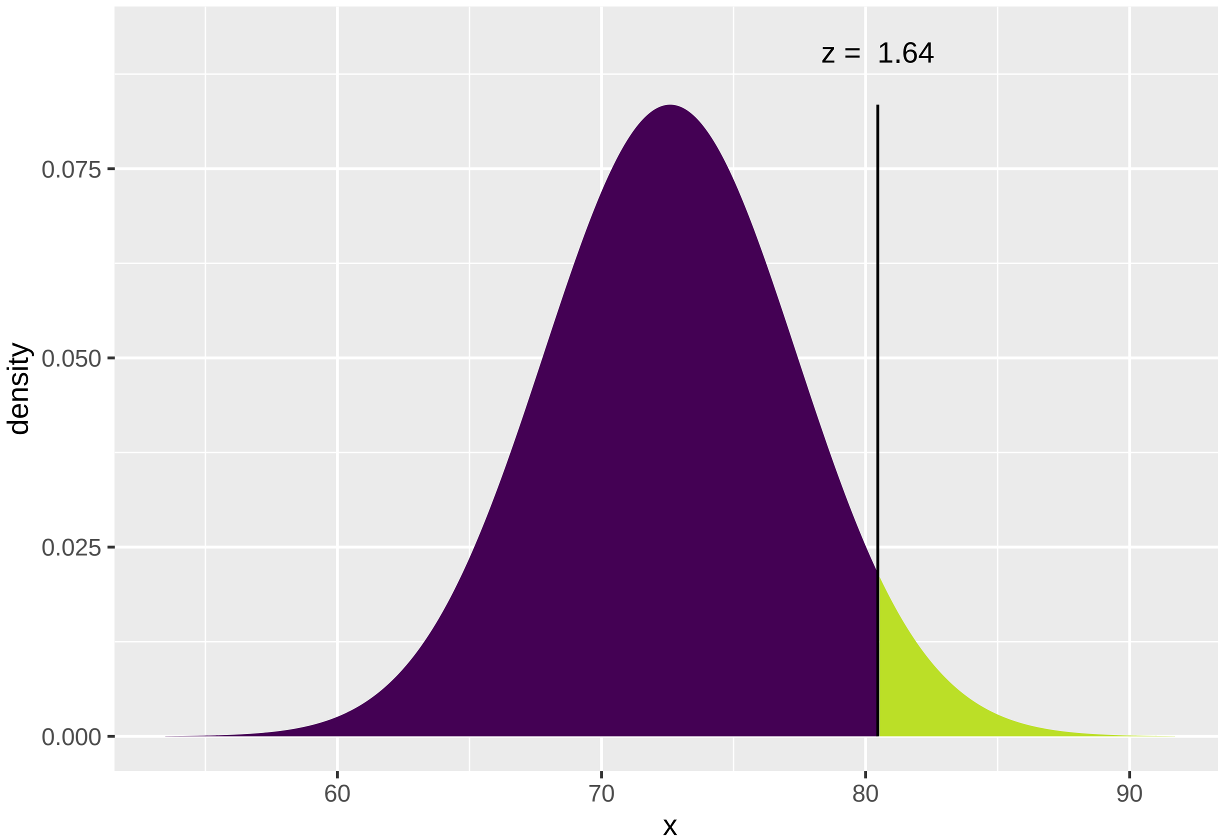
```
##
## If X ~ N(72.6, 4.78), then
##  P(X <= 60) = P(Z <= -2.636) = 0.004195  P(X <= 80) = P(Z <=  1.548) = 0.939203
##  P(X >  60) = P(Z >  -2.636) = 0.9958    P(X >  80) = P(Z >   1.548) = 0.0608
##
```




```
## [1] 0.004194693 0.939202954
```

```
xqnorm(0.95, mean = 72.6, sd = 4.78)
```

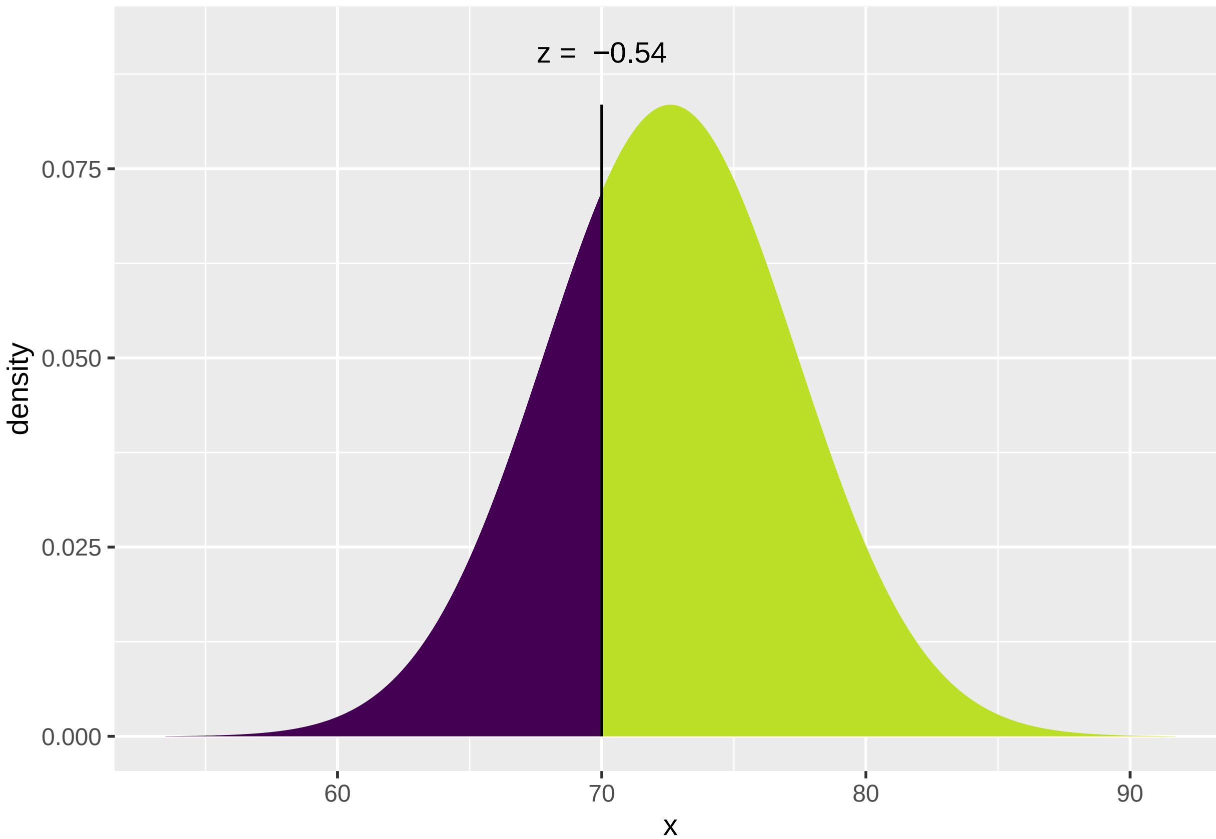
```
##
## If X ~ N(72.6, 4.78), then
##  P(X <= 80.4624) = 0.95
##  P(X >  80.4624) = 0.05
##
```




```
## [1] 80.4624
```

```r
xpnorm(70, mean = 72.6, sd = 4.78)
```

```
##
## If X ~ N(72.6, 4.78), then
##  P(X <= 70) = P(Z <= -0.5439) = 0.2932
##  P(X >  70) = P(Z >  -0.5439) = 0.7068
##
```




```
## [1] 0.2932438
```

Task 6. Knit your code and check your outcomes.

You are only allowed to upload pdf or html

Task 7. Check your answer

```r
# Correct answers for comparison
correct_answers <- list(
  a = 0.9394,  # Percent traveling slower than 80 mph
  b = 0.9348,  # Percent traveling between 60 and 80 mph
  c = 80.46,   # Speed of the fastest 5% of vehicles
  d = 0.7068   # Percent traveling above the speed limit of 70 mph
)

# Part (a): Check percentile for slower than 80 mph
if (abs(p_80_direct - correct_answers$a) < 0.0005) {
  print("Part (a): Your answer is correct")
} else {
  print("Part (a): Check your answer")
}
```

```
## [1] "Part (a): Your answer is correct"
```

```r
# Part (b): Check percentile for between 60 and 80 mph
if (abs(p_80_direct - p_60_direct - correct_answers$b) < 0.0005) {
  print("Part (b): Your answer is correct")
```

```r
} else {
  print("Part (b): Check your answer")
}
```

```
## [1] "Part (b): Your answer is correct"
```

```r
# Part (c): Check speed of the fastest 5%
if (abs(speed_95th - correct_answers$c) < 0.05) {
  print("Part (c): Your answer is correct")
} else {
  print("Part (c): Check your answer")
}
```

```
## [1] "Part (c): Your answer is correct"
```

```r
# Part (d): Check percentage traveling above 70 mph
if (abs(percentile_above_70 - correct_answers$d) < 0.0005) {
  print("Part (d): Your answer is correct")
} else {
  print("Part (d): Check your answer")
}
```

```
## [1] "Part (d): Your answer is correct"
```

Share your work and help your group members before uploading your work to Canvas