Linear Regression Basics

your name

2024-10-24

Math 2265 Chapter 8. Linear Regression

- Work as a group!
- You will need to replace `"ans"` or `your_answer` in the source code
- Update your name in L3
- Add your group members' name below; students may lose one point if Question 0 is unanswered
- Make sure you save and `knit` your work (to html or pdf) before submitting it to Canvas

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Goal

- Review the basic concepts in linear regression
  - Meaning of line fitting
    - Residuals
  - The *R*-value
- Next time we will learn/review
  - When not to use linear regression
    - Linearity
    - Nearly normal residuals
    - Constant variability
    - Independent observations
  - how to use an `R`-function to find the least squares line

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Question 0. Who are your group members? (List their first names)

Answer:

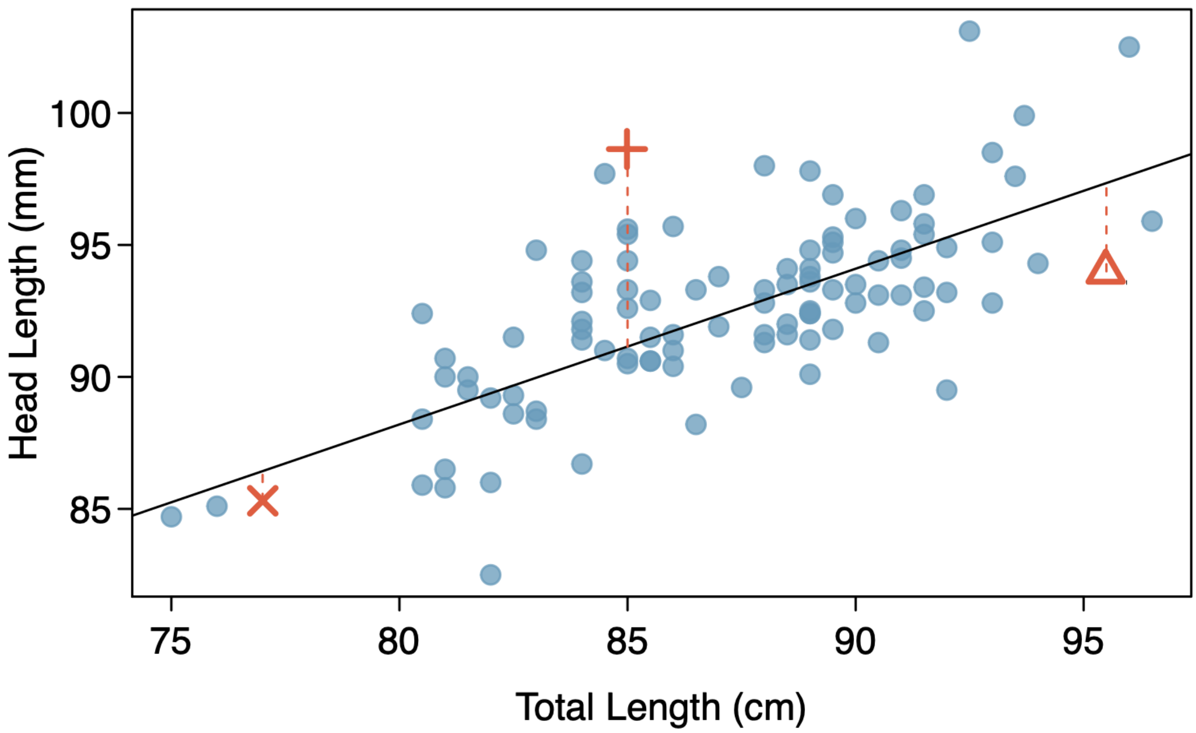
1. `<name_1>`
2. `<name_2>`

If you need more time to get used to `Markdown`, use the `Visual` mode.

The icon is located in the upper-left corner next to `source`.

Line fitting

We will use the possums data set:



- **data set**: `possum`
- **explanatory variable**: `total_l`
- **response variable**: `head_l`

Below, we define the line $y = 41 + 0.59x$ as a function first then make a scatter plot with the line and a residual.

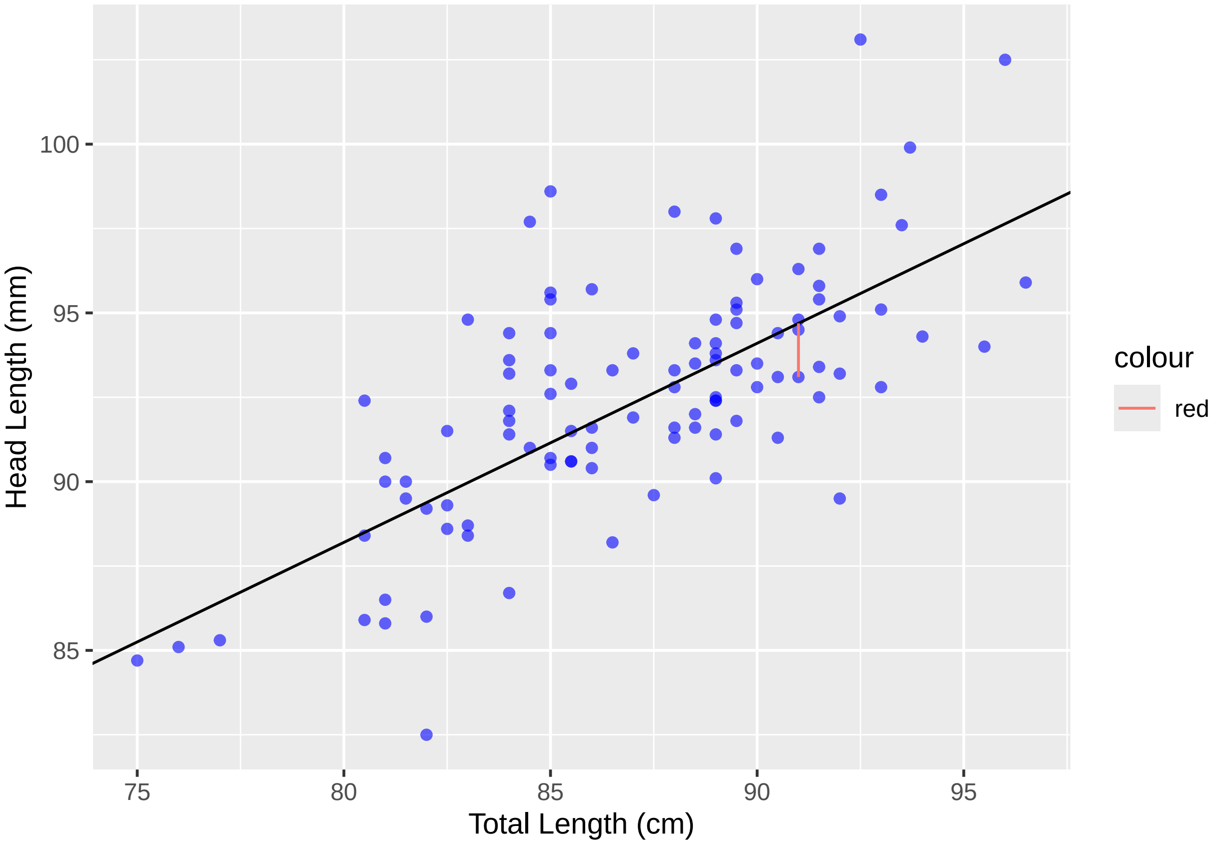
```r
# We define the function of the line and use `_hat` since it is an estimate
y_hat <- function(x) {
  41 + 0.59 * x
}

# We choose one sample point to demonstrate the residual
some_number = 53 # try different numbers between 1 and 104
possum$total_l[some_number]
```

```
## [1] 91
```

```r
sample_x <- possum$total_l[some_number]
sample_y <- possum$head_l[some_number]

ggplot(data = possum, mapping = aes(x=total_l, y=head_l)) +
  geom_point(color='blue', alpha = 0.6) +
  geom_abline(slope = 0.59, intercept = 41) +
  # the following line draws the residual in red
  geom_line(data = data.frame(x = c(sample_x, sample_x), y = c(sample_y, y_hat(sample_x))), aes(x = x, 
  xlab("Total Length (cm)") +
  ylab("Head Length (mm)")
```



Computing residuals with respect to this line $y = 41 + 0.59x$ is fairly simple. Recall that the residual is the true value of y minus the estimation $\hat{y}$. The explanatory variable `total_l` plays the role of x, and the response variable `head_l` plays the role of y. Let's check these two variables first.

```
library(dplyr)
```

```
##
## Attaching package: 'dplyr'

## The following objects are masked from 'package:stats':
##
##     filter, lag

## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

```
possum %>% select(total_l, head_l)
```

```
## # A tibble: 104 x 2
##    total_l head_l
##      <dbl>  <dbl>
##  1    89     94.1
##  2    91.5   92.5
##  3    95.5   94
##  4    92     93.2
##  5    85.5   91.5
##  6    90.5   93.1
##  7    89.5   95.3
##  8    91     94.8
##  9    91.5   93.4
## 10    89.5   91.8
## # i 94 more rows
```

We will add another column `head_l_hat` containing the estimations by using the line.

```
# We compute the estimates and add it to as a column
possum$head_l_hat <- y_hat(possum$total_l)
head(possum$head_l_hat)
```

```
## [1] 93.510 94.985 97.345 95.280 91.445 94.395
```

Now we display the three columns.

```
# if you see an error, run library(dplyr) first
# library(dplyr)
possum %>% select(total_l, head_l, head_l_hat)
```

```
## # A tibble: 104 x 3
##    total_l head_l head_l_hat
##      <dbl>  <dbl>      <dbl>
##  1    89     94.1       93.5
##  2    91.5   92.5       95.0
##  3    95.5   94         97.3
##  4    92     93.2       95.3
##  5    85.5   91.5       91.4
##  6    90.5   93.1       94.4
##  7    89.5   95.3       93.8
##  8    91     94.8       94.7
##  9    91.5   93.4       95.0
## 10    89.5   91.8       93.8
## # i 94 more rows
```

The residual for each point is the value of head_l minus head_l_hat. The vector calculation comes in handy. We will compute it, attach it to the data frame, and display the four columns.

```
# if you see an error, run the previous cell first
possum$residuals <- possum$head_l - possum$head_l_hat
possum %>% select(total_l, head_l, head_l_hat, residuals)
```

```
## # A tibble: 104 x 4
##    total_l head_l head_l_hat residuals
##      <dbl>  <dbl>      <dbl>     <dbl>
##  1    89     94.1       93.5    0.590
##  2    91.5   92.5       95.0   -2.48
##  3    95.5   94         97.3   -3.34
##  4    92     93.2       95.3   -2.08
##  5    85.5   91.5       91.4    0.0550
##  6    90.5   93.1       94.4   -1.30
##  7    89.5   95.3       93.8    1.49
##  8    91     94.8       94.7    0.110
##  9    91.5   93.4       95.0   -1.58
## 10    89.5   91.8       93.8   -2.01
## # i 94 more rows
```

The positive residuals mean that the points are lying above the line, and the negative residuals mean the points are below the line. The `least squares` line means that the sum of squares of the residuals is the least among all possible line fittings. In this example, the value is

```
sum_squares <- sum( (possum$residuals)^2 )
sum_squares
```

```
## [1] 692.6659
```

Note: The line we computed above is not the actual least squares line. We'll verify this by computing the sum of squares of the residuals and comparing it to the least squares regression model.

Lastly, we can compute the R-value (the correlation factor) using the function `cor`.

```
r <- cor(possum$total_l, possum$head_l, method = "pearson")
r; r^2 # R and R^2
```

```
## [1] 0.6910937
```

```
## [1] 0.4776105
```

Next time

```
model <- lm(head_l ~ total_l, data = possum)
print(sum( residuals(model)^2))
```

```
## [1] 687.041
```

```
summary(model)
```

```
##
## Call:
## lm(formula = head_l ~ total_l, data = possum)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -7.1877 -1.5340 -0.3345  1.2788  7.3968
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 42.70979    5.17281   8.257 5.66e-13 ***
## total_l      0.57290    0.05933   9.657 4.68e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.595 on 102 degrees of freedom
## Multiple R-squared:  0.4776, Adjusted R-squared:  0.4725
## F-statistic: 93.26 on 1 and 102 DF,  p-value: 4.681e-16
```

Question:

Add the line from the summary the the possum scatter plot.

```
# We define the function of the line and use `_hat` since it is an estimate
y_hat <- function(x) {
  41 + 0.59 * x
}

# We choose one sample point to demonstrate the residual
some_number = 53 # try different numbers between 1 and 104
possum$total_l[some_number]
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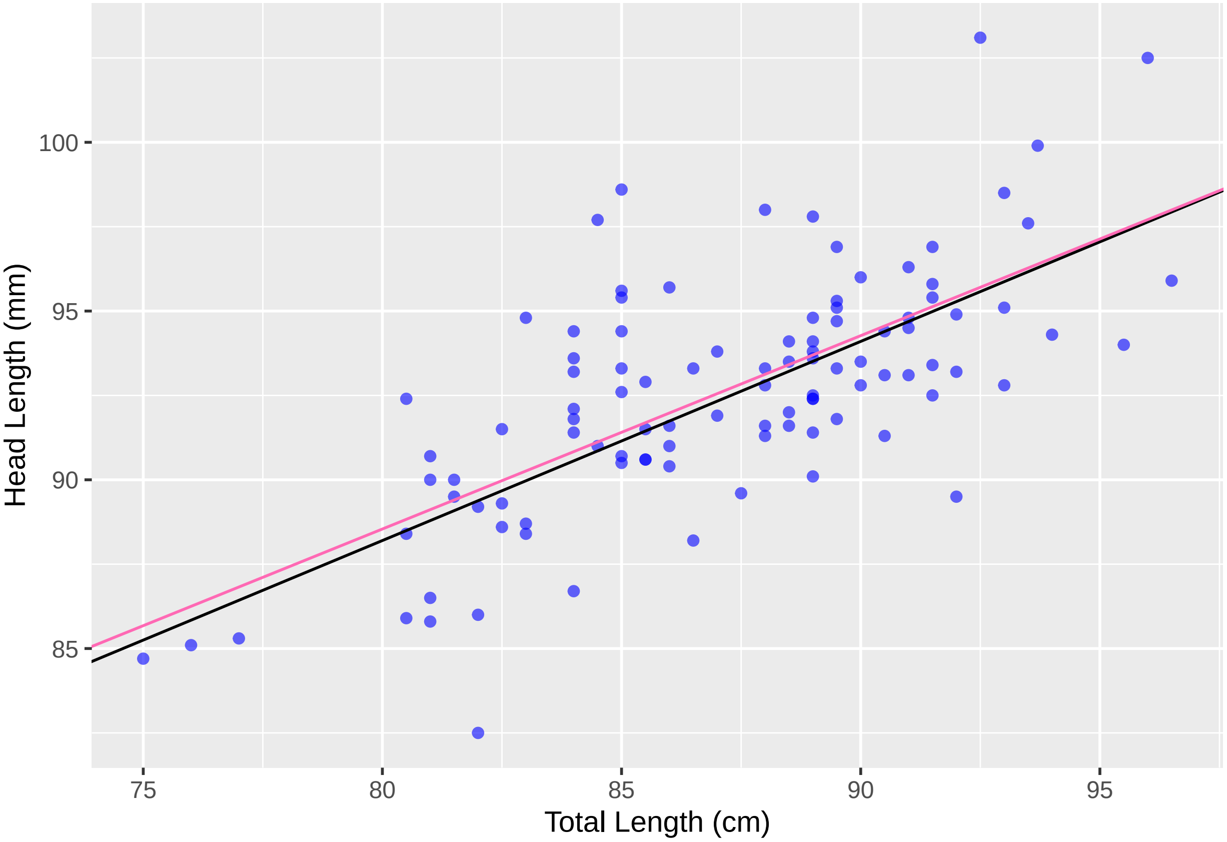
```
sample_x <- possum$total_l[some_number]
sample_y <- possum$head_l[some_number]

ggplot(data = possum, mapping = aes(x=total_l, y=head_l)) +
  geom_point(color='blue', alpha = 0.6) +
  geom_abline(slope = 0.59, intercept = 41) +
  #
  geom_abline(slope = 0.5729, intercept = 42.7098, color = 'hotpink') +
  xlab("Total Length (cm)") +
  ylab("Head Length (mm)")
```



Share your work and help your group members before uploading your work to Canvas