More on Normal Distributions

your name

2024-10-08

Math 2265 Sec 4.1 Normal Distribution Part II

- Work as a group!
- You will need to replace `"ans"` or `your_answer` in the source code or answer questions
- Update your name in L3
- Add your group members' name below; students may lose one point if Q0 is unanswered
- Make sure you save and `knit` your work (to html or pdf) before submitting it to Canvas

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Goal

- Learn the function that computes the percentile from given x or its Z-score.
- Learn the function that computes x or its Z-score or from given percentile.

Remark: - These processes are inverses to each other and understanding graphs (regions) is beneficial. - The Z-score is a way to transform any normal distribution to the standard normal distribution $\mu = 0$ and $\sigma = 1$.

Question 0. Who are your group members? (List their first names should be sufficient)

Answer:

1. `<name_1>`
2. `<name_2>`

Load Packages

If you need more time to get used to `Markdown`, use the `Visual` mode.

The icon is located in the upper-left corner next to `source`.

Example 1: Percentage from Z-scores

When the Z-score is zero, then it splits the graph equally into two regions.

```
pnorm(0, mean = 0, sd = 1)
```

```
## [1] 0.5
```

In terms of the formula,

$$P(Z < 0) = 0.5$$

Task 1: Percentages from Z-scores

(a) Z = -1.1

```
pnorm(-1.1, mean = 0, sd = 1)
```

```
## [1] 0.1356661
```

In terms of the formula,

$$P(Z < -1.1) = 0.1356661$$

(b) Z = 0.35

```
pnorm(0.35, mean = 0, sd = 1)
```

```
## [1] 0.6368307
```

In terms of the formula,

$$P(Z < 0.35) = 0.6368307$$

(c) Do you remember how to compute $P(Z > 0.5)$?

```
1 - pnorm(0.5, mean = 0, sd = 1)
```

```
## [1] 0.3085375
```

In terms of the formula,

$$P(Z > 0.5) = 0.3085375$$

Example 2: Z-scores from Percentages

What is the Z-score for the 80th percentile with mean = 0 and standard deviation = 1?

We need to find the Z-score z such that $P(Z < z) = 0.8$, where $0.8 = 80/100$. We can use the `qnorm` function. The syntax is the same as `pnorm` but the first

argument (parameter) is the percentile instead of the Z-score.

```
qnorm(0.8, mean = 0, sd = 1)
```

```
## [1] 0.8416212
```

That is (with marginal error),

$$P(Z < 0.8416212) = 0.8$$

We can check it with `pnorm`.

```
pnorm(0.8416212, mean = 0, sd = 1)
```

```
## [1] 0.8
```

Task 2:

(a) Fine the Z-score for 13.57 percentile.

```
qnorm(0.1357, mean = 0, sd = 1)
```

```
## [1] -1.099844
```

That is (with marginal error),

$$P(Z < -1.099844) = 0.1357$$

(b) Fine the Z-score for 63.68 percentile.

```
qnorm(0.6368, mean = 0, sd = 1)
```

```
## [1] 0.3499183
```

That is (with marginal error),

$$P(Z < 0.3499183) = 0.6368$$

Example 3: arbitrary mean and standard deviation

- As some of you may have guessed, both `pnorm` and `qnorm` can be used with arbitrary mean and standard deviation.
- In this case, they internally compute the Z-score from given x and vice versa.
- For the purpose of Math 2265, students are expected to know how conversion works both ways (x to Z-score and Z-score to x); the sec 4.1 video (second part) explains how to do this.

Revisiting the possums example.

The possums example in the video follows the normal distribution with mean 92.6 and sd 3.6.

When $x = 98$, the Z-score is 1.5. With `pnorm`, we computed the percentile (approximately) 0.9331

```
z_98 <- (98-92.6)/3.6
p_98 <- pnorm(z_98, mean = 0, sd = 1)
p_98
```

```
## [1] 0.9331928
```

This process can be combined as

```
pnorm(98, mean = 92.6, sd = 3.6)
```

```
## [1] 0.9331928
```

When $x = 89$, the Z-score is -1, the percentile was (approximately) 0.16.

```
z_89 <- (89-92.6)/3.6
p_89 <- pnorm(z_89, mean = 0, sd = 1)
p_89
```

```
## [1] 0.1586553
```

Task 3

Complete the code to compute the percentile for $x = 89$.

```
pnorm(89, mean = 92.6, sd = 3.6)
```

```
## [1] 0.1586553
```

- We will cover exercises with `qnorm` next time, but it should be enough for up to Q6 in HW4.
- Also, we will use `xpnorm` to visualize today's code.

Example 68-95-99.7% Rule

- The percentage within one standard deviation is 68%,
- The percentage within two standard deviations is 95%
- The percentage within three standard deviations is 99.7%

The percentage within one standard deviation is 68% means

$$P(|Z| < 1) = P(-1 < Z < 1) = 0.68$$

So,

$$P(|Z| < 2) = 0.95$$
$$P(|Z| < 3) = 0.997$$

Task 4: Compute $P(|Z| < 2)$ using `pnorm`.

```
# There are a few different ways to compute it
p_sd2 <- pnorm(2)-pnorm(-2)
p_sd2
```

```
## [1] 0.9544997
```

This output should be close to 0.95.

Task 6. Knit your code and check your outcomes.

You are only allowed to upload pdf or html

Task 7. Check your answer

for Task 4

```
if(p_sd2-0.9544997<0.00005) {
  print("Your answer is correct")
} else {
  print("Check your answer")
}
```

```
## [1] "Your answer is correct"
```

Share your work and help your group members before uploading your work to Canvas