Introduction to Numerical Sets and Primality

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1. Abstract

We present an introduction to higher-level mathematics by studying various subsets of integers, rational and real numbers. We define operations on these sets and study their properties. We introduce concepts of groups, rings, fields, and operations in this context. We introduce primality to study integers modulo given numbers and identify differences in these sets. The goal of his lesson is to improve students' comprehension of numbers beyond the traditional scope of the algebra curriculum.

2. Explanation

Many students face challenges when transitioning to University Mathematics, and have primarily worked within the field of the real numbers. Within this lesson, the subsets of the real numbers are thoroughly explained, and their properties and significance are analyzed. Within university-level mathematics, particularly within upper-division courses, students will study fields other than the real numbers. To fully grasp the concept of fields, one must understand rings. Similarly, a ring is meaningless without the knowledge of a group, which is incomprehensible without the knowledge of a set and a binary operation. Most of these topics are not introduced until Abstract Algebra, yet are essential to holistically understand lower-division subjects. Using these concepts, we will analyze the various subsets of the real numbers and the binary operations that can be computed within them. To ensure a thorough understanding, we describe the relationships between each subset of the real numbers, utilizing both set notation and visual diagrams. In mathematics, primaity holds unique properties and has many applications within ring and field theory, which necessitates an introduction at an early stage to ensure a thorough understanding. The second part of this lesson provides a brief introduction to primality, its applications, and methods to identify prime and composite numbers.

3. Methods

In this presentation, students are introduced to groups, rings, and fields briefly at a basic level and in a comprehensible manner. Then students are presented with the known field of the real numbers, and its various subsets are analyzed. First, the sets of the natural numbers, integers, and irrationals are described and displayed in mathematical set notation and then in a visually understandable manner using a drawn diagram.

Interactive activities within this lesson allow for the students to practice writing numbers within each set so they further understand each subset of the real numbers. This promotes the development of schemas, where students may correlate each subset with their own understanding of numbers. For instance, the numbers on dice are elements of the natural numbers, integers, and rationals, while the probability that they will roll a unique combination of numbers will always be displayed as a rational number.

Addition and multiplication within these sets are analyzed abstractly, such as how a rational number, when added or multiplied with another rational number, will always be rational, however, this will not always be the case with 2 irrational numbers. For example, $\frac{\pi}{2}$ and $\frac{2}{\pi}$ are both distinct irrational numbers, but their product is 1, which is a rational number.

Additional group activities surrounding numerical identification have been incorporated to promote collaborative learning and participation. Students will examine a given set of numbers and associate them with the presented subsets of the real numbers, having a table as a reference. This allows for new schemas to be formed, as well as a more thorough comprehension of numbers and their corresponding numerical sets.

Subsequently, students will be presented with additional numbers and asked to identify which numerical set they belong to, then briefly explain their reasoning in a manner that their classmates would understand. This not only strengthens their understanding of the correlations between numbers and numerical sets, but also promotes a holistic understanding of the material through verbal and written elaboration reasoning. Through this, students use critical thinking skills to communicate their understanding to their peers, which will further reinforce their understanding.

To test their comprehension, each student will complete an activity where two numbers are given, and either multiplied or added to each other. Then, they will identify the corresponding set to that number. For instance, $7 + 0.\bar{7} = 7.\bar{7}$, which is equivalent to $\frac{70}{9}$, a rational number. Through this, they combine all areas of the lesson into a single activity.

Within the second part of the lesson, the students will be introduced to the concept of primality, as well as prime numbers' properties and applications. They will first be provided with definitions of prime and composite numbers, and then shown examples of prime numbers and why they are prime.

The first activity within this lesson allows students to perform a brain dump by writing down all of the prime numbers they can think of between 1-100 in 2 minutes. Using a tangible number promotes student engagement and allows them to connect the concept of primality with their own understanding of numbers, which promotes schema formation. A chart will be given when the time is up to allow for each student to check their answers.

After the activity, students will be presented with a brief history of primality, which includes theorems devised by the mathematicians Euclid, Dusart, and Gauss, as well as theorems, propositions, and algorithms formulated by Pierre de Fermat. Then they will be introduced to three different methods of determining primality, including guess and check, evaluation of numbers, and primality tests. Through this, we will perform a live example of testing whether or not a number is divisible by 3 and 11 by evaluating each digit of the number. The lecture aspect of this lesson concludes with the presentation of the AKS Primality Test, where each step will be briefly explained.

This lesson will conclude with a final activity that builds off the first, having each student write down as many prime numbers between 100-1000 as they can within 2 minutes. Another key will be provided so that each student may verify their answers. As a post-lesson activity, each student will complete an assignment involving the tests to determine whether a number is divisible by 4, 8, and 9.

4. Examples

What kind of number systems are there?

$\mathbb{N}$ = Natural Numbers = $\{1, 2, 3 \ldots\}$

$\mathbb{Z}$ = Integers = $\{-3, -2, -1, 0, 1, 2, 3\}$

$\mathbb{Q}$ = Rationals = $\left\{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{N}\right\}$

$\mathbb{I}$ = Irrationals = $\{r \in \mathbb{R} : r \notin \mathbb{Q}\} = \mathbb{Q}^c$

$\mathbb{R}$ = Real Numbers = $\mathbb{Q} \cup \mathbb{I}$

ADDITION & MULTIPLICATION

$\mathbb{N} * \mathbb{Z} = \mathbb{Z}$

$\mathbb{Z} * \mathbb{Q} = \mathbb{Q}$

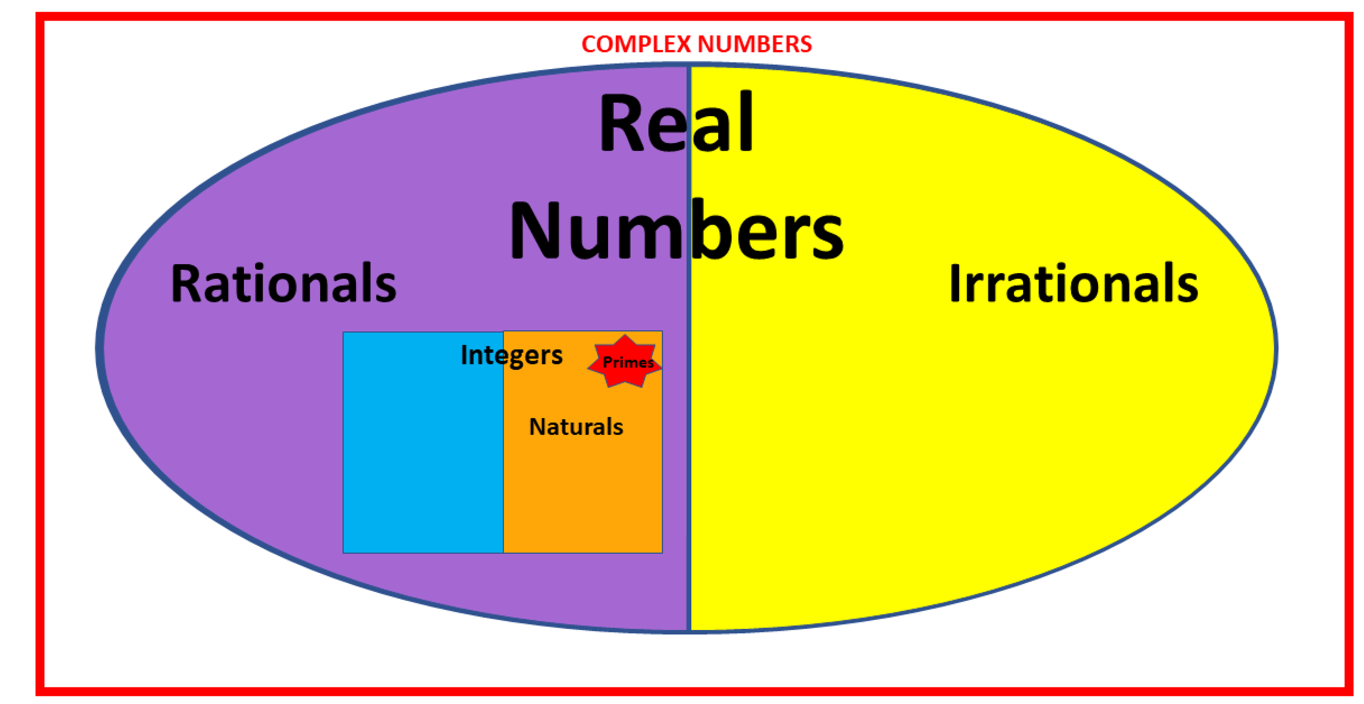
$\mathbb{Z} * \mathbb{I} = \mathbb{I}$

$\mathbb{Q} * \mathbb{Q} = \mathbb{Q}$

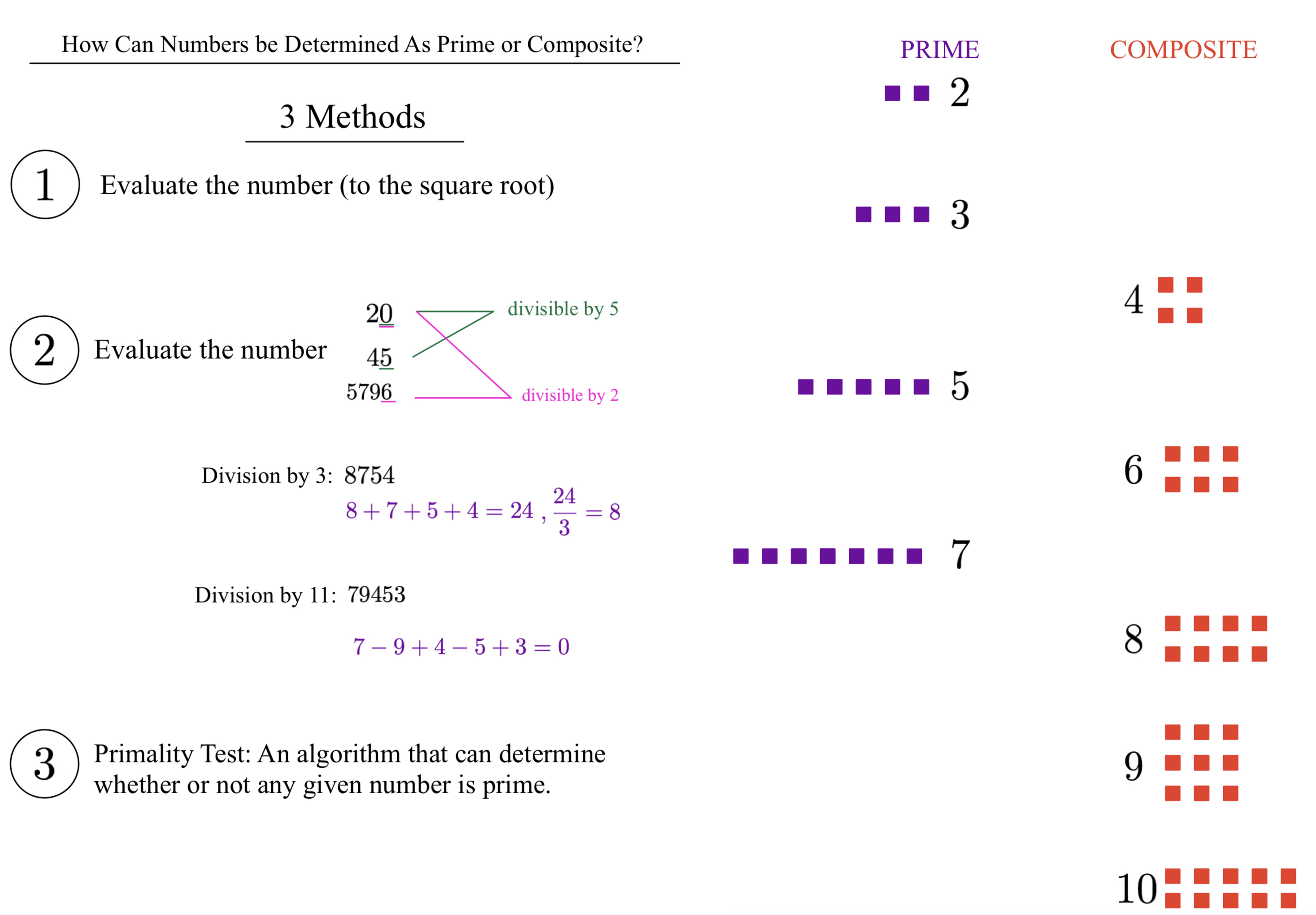
$\mathbb{I} * \mathbb{I} = \mathbb{I}$ or $\mathbb{Q}$

$\mathbb{Q} * \mathbb{I} = \mathbb{I}$

5. Utilized Diagrams



6. Additional Examples



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