Students & the Struggle with Math Quantifiers

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Abstract

The purpose of this work is to study student understanding of quantifiers. We compare three groups: freshmen that have no background with quantifiers, mathematical logic students who have learned about quantifiers and seniors who have studied and used quantifiers in their previous courses. Most of the previous studies on this topic were based on questionnaires with both formal symbolic quantifiers (the universal quantifier "∀" and the existential quantifier "∃") and informal language-based quantifiers. Most questions asked the participants to decide if a statement given was True or False and to justify their reasoning. Often participants were asked to rewrite a formal statement informally or vice versa. The analysis of data showed that students at every level struggle with quantifiers, and some methods were proposed to aid the learning of these concepts.

Our research began with a pilot study analyzing answers given by college freshmen in a mathematical logic course. Each question had a formal (or informal) statement, and the students were asked to rewrite it informally (or formally). The results depended strongly on the number of quantifiers included and the majority of students had problems with the proper order of two or more quantifiers. Next, we prepared a more in-depth survey that was administered to three groups of students at different levels of their academic developments. We also compared two different ways of teaching quantifiers at the introductory level (active learning vs. traditional style). We analyzed data using statistical T-test applied to different samples and presented the results. Our overall goal is to study the obstacles for comprehending mathematical quantifiers and improve the methodology about teaching and assessing the retention of this knowledge a few years after the initial learning happened.

Introduction

I was motivated to do research with mathematical quantifiers because I myself struggled trying to understand these concepts when I was an undergraduate student at CSU Northridge. Now that I have a better grasp of these ideas, I hope to find methods that can aid students in their efforts with the comprehension of mathematical quantifiers. Our overall goal is to study the obstacles for comprehending mathematical quantifiers and improve the methodology about teaching and assessing the retention of this knowledge a few years after the initial learning happened. Dr. Katrina Piatek-Jimenez conducted a study titled "Students' Interpretations of Mathematical State-ments Involving Quantification" who found in her results that statements of the form "There exists ... for all ..." (which can be referred to as EA statements) evoked a larger variety of interpretations than statements of the form "For all ... there exists ..." (AE statements). [1] Dr. Piatek-Jimenez discovered that the most effective method for her students in regards to comprehending mathematical quantifiers was being able to conceptualize both the AE and EA interpretations of a statement and seeing how these two statements have different meanings. [1] On the other hand, similar studies were conducted by Dr. Ed Dubinsky, who I consider to be at the forefront of student understanding of mathematical quantification. I say this bold statement because Dr. Dubinsky is referenced in numerous articles regarding student comprehension of quantifiers and he himself has written a plethora of articles about the matter in question. The earliest one I could find was from 1987 where Dr. Dubinsky attempted to understand the psychological process of comprehending quantification and use it to design instruction, especially using computers, that can foster the development of this concept by students. [2] Dr. Dubinsky's more recent works have suggested that we should stop using normal language and examples that students can relate to in everyday situations as a means to supporting students in understanding statements in mathematics that involve existential and universal quantifiers. [3] His work proposes that students struggle tremendously trying to accept many of the rules that mathematics follows for the reason that mathematics obeys certain rigid rules that our students do not necessarily pick up on their own [3] from everyday language. It is for this reason that we need to teach our students that the language of mathematics works differently from natural language. [3] Apart from this, my own research will be in contrast with some of the work that Dr. Dubinsky published since I used everyday normal language analogies to help the students from our study comprehend mathematical quantifiers with questions that are relatable to their life's. For example I made questions that said "All dogs are poodles" or "Some cats are mammals". These questions are relatable because many people have dogs or cats, so they can relate from a personal experience to the very material they are learning. Which in turn make the transition to mathematical statements with quantifiers a little bit easier, because they already have background and relatable experiences, so the schemas have been built into their brains.

Hypothesis/Problem Description

We compared two different ways of teaching quantifiers at the introductory level (active learning vs. traditional style), but before this we administered a pre-test to see how much background students had with mathematical quantifiers. After both teaching styles were used, a post test was given to see if any information was useful in helping students better comprehend mathematical quantifiers. From the results of the post test we would use a statistical T-test to compare them to the pre-test to see if learning did indeed happen. After this, we would compare the results of both classes with the seniors who also took the pre and post tests to see how much retention was kept of the material. We hope in the end that learning is achieved, but our overall goal is to study the obstacles for comprehending mathematical quantifiers and improve the methodology about teaching and assessing the retention of this knowledge a few years after the initial learning happened.

Methodology

This research was conducted at California State University Channel Islands where there was a mixture of students at different levels of their academic developments but the majority were mathematics majors. For our research, we firstly analyzed one of the questions from the final exam from a mathematical logic course of the previous semester (Fall 2021), making it our pilot study where the majority of the students were sophomores. This pilot study consisted of 1-question from their final exam with 4-parts in this question. Each part was a mathematical quantifier statement that asked the students to either rewrite the statement in English (if it was already given in symbolic logic) or symbolic logic (if it was already given in English). After this, we waited until the following semester for the new mathematical logic courses (Spring 2022) where we began our main research with a pre-test to see how much background our new participants had with mathematical quantifiers (some did have some background already, but very little). Some of the questions in the pre-test can be found in the pictures below (top two-pictures) as well as some pictures of the lecture slides (bottom three-pictures). The questions in the pre-test consisted of either English statements that asked the students to rewrite the statement in symbolic logic form or in English if the question was already in symbolic logic form (the post-test had very similar questions with a few words changed). Then 2-different teaching styles were used for the two-mathematical logic courses that were being taught in the Spring of 2022. The morning logic course, which we will refer to Logic 1 and the afternoon logic course, which we will refer to Logic 2, were taught with the same examples, ideas and even the same lecture slides. The only difference would be that Logic 1 was going to learn about quantifiers first from a relatable English perspective, then transition to symbolic logic and finally learn about mathematical quantifier statements (traditional style). On the other hand, Logic 2 was going to first learn about symbolic logic then transition into mathematical quantifier statements and finally learn quantifiers from a relatable English perspective (active style). After the teaching styles were used, we administered a post-test to see if any information was useful in helping students better comprehend mathematical quantifiers. We then used a statistical T-test to compare both Logic 1 and Logic 2 courses to see if learning was indeed achieved and if one teaching style was better than the other. We also gave the same pre and post-tests to the seniors who had prior knowledge about mathematical quantifiers from a previous course. We compared the seniors results with the 2-logic courses to see how much material was retained and to see if one-style is better than the other. We hope in the end that learning was indeed acquired, but the our overall goal is to study the obstacles for comprehending mathematical quantifiers and improve the methodology about teaching and assessing the retention of this knowledge a few years after the initial learning happened.

1.) There exists a unique social security number for every person born in the U.S.A.
Which one of the following logical sentences best represents the above statement? (Use x for people and y for numbers)

- a.) ∃ y in natural's ∀ x in U.S.A.
- b.) ∀ y in U.S.A. ∃ x in natural's
- c.) ∃ y in natural's ∃ x in U.S.A.
- d.) ∀ x in U.S.A. ∃ y in natural's
- e.) ∀ x in U.S.A. ∀ y in natural's
- f.) ∀ x in U.S.A. ∃ y in natural's
- g.) ∀ y in natural's, x in U.S.A.
- h.) ∀ x in U.S.A. ∀ y in natural's
- i.) None of the above

4.) *

Write the following in your own words without symbols:

$$\exists\, m\; \forall\, x \text{ in reals } \exists\, y \text{ in integers } |x - y^m| < \frac{1}{2}$$

Methodology

English to Symbols Examples (2 Predicates)

- **Consider the statement: All tigers are cats.**
  - **What are the predicates and then transform statement into symbols**
    - Let T(x) denote "is a tiger".
    - Let C(x) denote "is a cat".
    - $\forall x\ (T(x) \longrightarrow C(x))$
- **Consider the statement: Some quadrilaterals are squares.**
  - **What are the predicates and then transform statement into symbols**
    - Let Q(x) denote "is a quadrilaterals".
    - Let S(x) denote "is a squares".
    - $\exists x\ (Q(x) \wedge S(x))$

T or F: Math 2-Order (Nested) Quantifier Statements

- $\forall x \in \mathbf{R}, \exists y \in \mathbf{R} : x + y = 4$
  - Ask the students to rewrite the statement in English and determine if true or false
  - We can set x to be ***ANY*** real number then we can find ***AT LEAST ONE*** y based on x such that x+y=4 is true.
  - x=10, y =-6 (True)
- $\exists y \in \mathbf{R}, \forall x \in \mathbf{R} : x + y = 4$
  - Ask the students to rewrite the statement in English and determine if true or false
  - ***AT LEAST ONE*** y can be found ***BEFORE*** any other variable is set. After this y is set, we can put **ANY** x and it will make the statement x+y=4 to be true.
  - y=-6, x=11 (False)

T or F: Relatable Examples

1. **All dogs are poodles.**
   - **Not every dog is a poodle.**
   - **There exist other types of dogs. (FALSE)**
2. **Some books have hard covers.**
   - **It's true that some books do have hard covers. (TRUE)**
3. **Every U.S. citizen pays sales tax.**
   - **People that live in Oregon do not pay sales tax. (FALSE)**

Future Conclusions

My future work is to compare the three groups with a statistical T-test to see how much material was retained and to see if one-style of teaching is better than the other.

Bibliography

[1] K. Piatek-Jimenez, "Students' interpretations of mathematical statements involving quantification," Mathematics Education Research Journal, vol. 22, pp. 41–56, November 2010.
[2] E. Dubinsky, "On helping students construct the concept of quantification," in Proceedings of the Annual Conference of the International Group for the Psychology of Mathematics Education (A. Borbas, ed.), vol. 1, pp. 255–262, OOK Printing House Veszprem, July 1988.
[3] E. Dubinsky and O. Yiparaki, "On student understanding of ae and ea quantification," pp. 1–64, November 2000.

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