

Practice

Solve $y' + p(x)y = q(x)$

Sol

Consider

$$H(x) = \int p(x)dx$$

Multiply $e^{H(x)}$ to both sides

$$e^{H(x)}y' + e^{H(x)}p(x)y = e^{H(x)}q(x)$$

This gives

$$\frac{d}{dx}[e^{H(x)}y] = e^{H(x)}q(x)$$

Integration gives

$$e^{H(x)}y = \int e^{H(x)}q(x)dx + C$$

and yields

$$y(x) = e^{-H(x)} \left[\int e^{H(x)}q(x)dx + C \right].$$

Note that $e^{H(x)}$ is called the *integrating factor*.

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