1 GAN

Min-Max Problem

$$\min_G \max_D E_{x\sim P_{true}}[\ln D(x)] + E_{z\sim P(z)}[1 - \ln D(G(z))]$$

$x \sim P_{true}$ and $z \sim P(z)$ implies Monte-Carlo

$$E[\ln D(X)] = \frac{1}{N}\sum_{i=1}^{N} \ln D(x_i)$$
$$E[1 - \ln D(G(Z))] = \frac{1}{N}\sum_{i=1}^{M} \ln D(G(z_i))$$

2 WGAN

For G fixed, the optimal D is

$$D_G(x) = \frac{P_{true}(x)}{P_{true}(x) + P_G(x)}$$

KL divergence

$$KL(p\|q) = \sum_i p_i \frac{\ln p_i}{q_i} = E_p[\ln \frac{p}{q}]$$

Jensen–Shannon divergence

$$JSD(p,q) = \frac{1}{2}KL(p\|m) + \frac{1}{2}KL(q\|m),$$
$$m = \frac{1}{2}(p+q)$$

Let p, q be P_{true}, P_G,

$$\min_G JSD(P_{true}, P_G)$$

Earth-Mover distance / Wasserstein-1 distance

$$WS(P_{true}, P_G) = \sup_{\|D\|_L \le 1} \{E_{x\sim P_{true}}[D(x)] - E_{y\sim P_G}[D(y)]\}$$

where D is 1-Lipschitz continuous, i.e.

$$\|D(x) - D(y)\| \le \|x - y\| \tag{1}$$