Practice

$$\text{For } f(x) = \sin\left(\frac{\pi}{x}\right), \lim_{x \to 0} f(x) \text{ does not exist.}$$

Sol

Consider two sequences s_n and t_n satisfy

$$\frac{\pi}{s_n} = -\frac{\pi}{2} - 2n\pi, \frac{\pi}{t_n} = \frac{\pi}{2} + 2n\pi, \forall n \in \mathbb{N}$$

i.e.

$$s_n = \frac{\pi}{-\frac{\pi}{2} - 2n\pi} = \frac{2}{-1 - 4n},$$

$$t_n = \frac{\pi}{\frac{\pi}{2} + 2n\pi} = \frac{2}{1 + 4n}.$$

We have $f(s_n) = -1$ and $f(t_n) = 1$ for all $n \in \mathbb{N}$. Conclude that $\lim_{x \to 0} f(x)$ does not exist.

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