1 Mixture of Gaussian(Soft Clustering)

Give probabilities that an instance belongs to each cluster instead of assigning only one. Let $g(x; m, \sigma)$ be the probability of a point x based on a Gaussian Distribution with mean m and variance σ. Suppose there is a distribution generated by randomly selecting one of K Gaussians. We randomly draw a point from this distribution. Let p_k be the probability of choosing the k^{th} Gaussian. Then

$$p(x) = \sum_{k=1}^{K} p_k g(x; m_k, \sigma_k)$$

Now we want to find p_k, σ_k and m_k that maximize $p(x)$.
E step: compute the probability that point x_n is generated by distribution k, i stands for steps

$$p^{(i)}(k|x_n) = \frac{p_k^{(i)} g(x_n; m_k^{(i)}, \sigma_k^{(i)})}{\sum_{j=1}^{K} p_j^{(i)} g(x_n; m_j^{(i)}, \sigma_j^{(i)})}$$

M step: update p_k, σ_k and m_k.

$$\begin{aligned}
m_k^{(i+1)} &= \frac{\sum_{n=1}^{N} p^{(i)}(k|x_n) x_n}{\sum_{n=1}^{N} p^{(i)}(k|x_n)} \\
\sigma_k^{(i+1)} &= \sqrt{\frac{\sum_{n=1}^{N} p^{(i)}(k|x_n) \|x_n - n_k^{(i+1)}\|^2}{\sum_{n=1}^{N} p^{(i)}(k|x_n)}} \\
p_k^{(i+1)} &= \frac{1}{N} \sum_{n=1}^{N} p^{(i)}(k|x_n)
\end{aligned}$$