

Mixture of Gaussian(Soft Clustering)

Give probabilities that an instance belongs to each cluster instead of assigning only one. Let $g(x; m, \sigma)$ be the probability of a point x based on a Gaussian Distribution with mean m and variance σ . Suppose there is a distribution generated by randomly selecting one of K Gaussians. We randomly draw a point from this distribution. Let p_k be the probability of choosing the k^{th} Gaussian. Then

$$p(x) = \sum_{k=1}^K p_k g(x; m_k, \sigma_k)$$

Now we want to find p_k , σ_k and m_k that maximize $p(x)$.

E step: compute the probability that point x_n is generated by distribution k , i stands for steps

$$p^{(i)}(k | x_n) = \frac{p_k^{(i)} g(x_n; m_k^{(i)}, \sigma_k^{(i)})}{\sum_{j=1}^K p_j^{(i)} g(x_n; m_j^{(i)}, \sigma_j^{(i)})}$$

M step: update p_k , σ_k and m_k .

$$m_k^{(i+1)} = \frac{\sum_{n=1}^N p^{(i)}(k | x_n) x_n}{\sum_{n=1}^N p^{(i)}(k | x_n)}$$
$$\sigma_k^{(i+1)} = \sqrt{\frac{\sum_{n=1}^N p^{(i)}(k | x_n) \|x_n - m_k^{(i+1)}\|^2}{\sum_{n=1}^N p^{(i)}(k | x_n)}}$$
$$p_k^{(i+1)} = \frac{1}{N} \sum_{n=1}^N p^{(i)}(k | x_n)$$