LIM assignment 3

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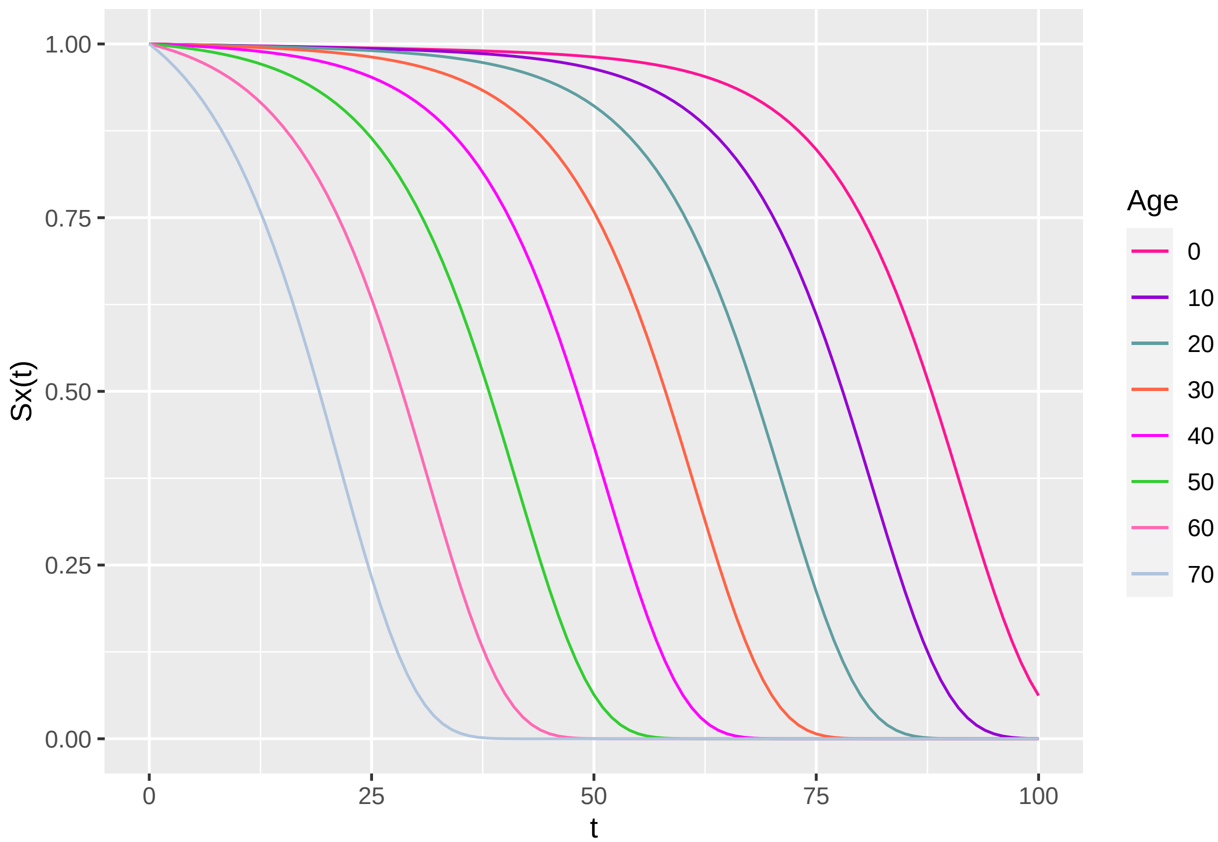
2.(a) We could not include the screenshot of our solution here, so we have submitted it separately. Apologies for the inconvenience.

2.(b)

```
library(ggplot2)
#(b)
A<-0.00022
B<-2.7*10^-6
c<-1.124
mx<-function(x){A+B*c^x}
s<-exp(-A)
g<-exp(-B/log(c))
tpx<-function(age,t){
  (s^t)*(g^(c^age*(c^t-1)))
}
age<-0:100
t<-seq(0, 100)
for(i in 10*c(0:8)) {
  nam <- paste("tp", i, sep = "")
  assign(nam, tpx(i,t))
}
D<-data.frame(t,tp0,tp10,tp20,tp30,tp40,tp50,
              tp60,tp70,tp80)

color = c("deeppink", "darkviolet",
          "cadetblue","tomato","magenta","limegreen",
          "hotpink", "lightsteelblue")
ggplot(D, aes(x=t)) +
  geom_line(aes(y=tp0, col="0"))+
  geom_line(aes(y=tp10, col="10"))+
  geom_line(aes(y=tp20, col="20"))+
  geom_line(aes(y=tp30, col="30"))+
  geom_line(aes(y=tp40, col="40"))+
  geom_line(aes(y=tp50, col="50"))+
  geom_line(aes(y=tp60, col="60"))+
  geom_line(aes(y=tp70, col="70"))+
  labs(x="t",y="Sx(t)")+
  scale_color_manual(name="Age", labels=c(as.character(10*c(0:7))),
                     values = c("0"=color[1],
                                "10"=color[2],
                                "20"=color[3],
```

```
                                "30"=color[4],
                                "40"=color[5],
                                "50"=color[6],
                                "60"=color[7],
                                "70"=color[8]))
```



2.(c)

```
px<-tpx(age,1)
qx<-1-tpx(age,1)
kpx<-c(1,cumprod(px[1:length(px)-1]))
lx_0<-1000000
lx<-lx_0 * kpx
dx<- lx*qx
ex<-sum(kpx)-cumsum(kpx[1:(length(kpx))])
life_table<-data.frame(age, lx, px,qx, dx, ex)
life_table
```

```
##     age         lx        px           qx       dx          ex
## 1     0 1000000.00 0.9997772 0.0002228393 222.8393 84.94436543
## 2     1  999777.16 0.9997768 0.0002231944 223.1446 83.94458827
## 3     2  999554.02 0.9997764 0.0002235935 223.4938 82.94503426
## 4     3  999330.52 0.9997760 0.0002240421 223.8921 81.94570373
## 5     4  999106.63 0.9997755 0.0002245463 224.3457 80.94659710
## 6     5  998882.28 0.9997749 0.0002251130 224.8614 79.94771482
```

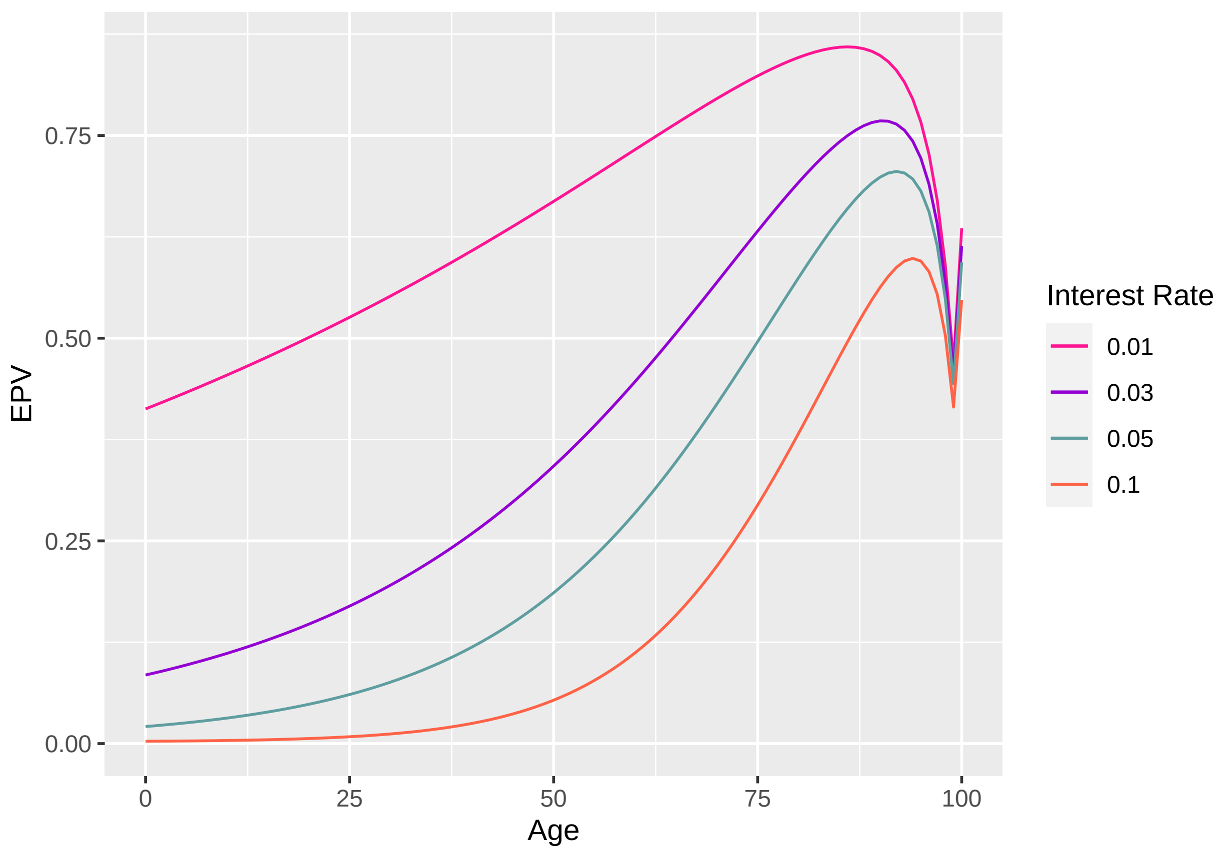
```
## 7     6  998657.42 0.9997742 0.0002257500   225.4470 78.94905740
## 8     7  998431.98 0.9997735 0.0002264661   226.1109 77.95062542
## 9     8  998205.87 0.9997727 0.0002272708   226.8631 76.95241955
## 10    9  997979.00 0.9997718 0.0002281754   227.7143 75.95444055
## 11   10  997751.29 0.9997708 0.0002291922   228.6768 74.95668926
## 12   11  997522.61 0.9997697 0.0002303350   229.7644 73.95916665
## 13   12  997292.85 0.9997684 0.0002316195   230.9925 72.96187381
## 14   13  997061.85 0.9997669 0.0002330633   232.3786 71.96481195
## 15   14  996829.48 0.9997653 0.0002346862   233.9421 70.96798248
## 16   15  996595.53 0.9997635 0.0002365102   235.7051 69.97138694
## 17   16  996359.83 0.9997614 0.0002385605   237.6921 68.97502711
## 18   17  996122.14 0.9997591 0.0002408650   239.9309 67.97890498
## 19   18  995882.21 0.9997565 0.0002434552   242.4527 66.98302277
## 20   19  995639.75 0.9997536 0.0002463666   245.2924 65.98738302
## 21   20  995394.46 0.9997504 0.0002496390   248.4893 64.99198856
## 22   21  995145.97 0.9997467 0.0002533172   252.0876 63.99684259
## 23   22  994893.88 0.9997425 0.0002574515   256.1369 63.00194871
## 24   23  994637.75 0.9997379 0.0002620983   260.6929 62.00731096
## 25   24  994377.05 0.9997327 0.0002673214   265.8183 61.01293391
## 26   25  994111.24 0.9997268 0.0002731921   271.5834 60.01882267
## 27   26  993839.65 0.9997202 0.0002797907   278.0671 59.02498302
## 28   27  993561.58 0.9997128 0.0002872076   285.3584 58.03142143
## 29   28  993276.23 0.9997045 0.0002955440   293.5568 57.03814521
## 30   29  992982.67 0.9996951 0.0003049140   302.7743 56.04516254
## 31   30  992679.90 0.9996846 0.0003154459   313.1368 55.05248264
## 32   31  992366.76 0.9996727 0.0003272835   324.7853 54.06011588
## 33   32  992041.97 0.9996594 0.0003405889   337.8785 53.06807391
## 34   33  991704.09 0.9996445 0.0003555439   352.5943 52.07636981
## 35   34  991351.50 0.9996276 0.0003723530   369.1327 51.08501831
## 36   35  990982.37 0.9996088 0.0003912462   387.7181 50.09403595
## 37   36  990594.65 0.9995875 0.0004124817   408.6021 49.10344130
## 38   37  990186.05 0.9995637 0.0004363498   432.0675 48.11325525
## 39   38  989753.98 0.9995368 0.0004631769   458.4312 47.12350127
## 40   39  989295.55 0.9995067 0.0004933298   488.0489 46.13420572
## 41   40  988807.50 0.9994728 0.0005272204   521.3195 45.14539822
## 42   41  988286.18 0.9994347 0.0005653122   558.6902 44.15711204
## 43   42  987727.49 0.9993919 0.0006081256   600.6624 43.16938455
## 44   43  987126.83 0.9993438 0.0006562457   647.7977 42.18225772
## 45   44  986479.03 0.9992897 0.0007103299   700.7255 41.19577869
## 46   45  985778.30 0.9992289 0.0007711170   760.1504 40.21000039
## 47   46  985018.15 0.9991606 0.0008394373   826.8610 39.22498223
## 48   47  984191.29 0.9990838 0.0009162238   901.7395 38.24079094
## 49   48  983289.55 0.9989975 0.0010025248   985.7721 37.25750139
## 50   49  982303.78 0.9989005 0.0010995182  1080.0608 36.27519761
## 51   50  981223.72 0.9987915 0.0012085275  1185.8358 35.29397389
## 52   51  980037.88 0.9986690 0.0013310397  1304.4694 34.31393600
## 53   52  978733.42 0.9985313 0.0014687256  1437.4908 33.33520259
## 54   53  977295.92 0.9983765 0.0016234618  1586.6026 32.35790666
## 55   54  975709.32 0.9982026 0.0017973567  1753.6977 31.38219734
## 56   55  973955.62 0.9980072 0.0019927785  1940.8778 30.40824171
## 57   56  972014.75 0.9977876 0.0022123868  2150.4726 29.43622697
## 58   57  969864.27 0.9975408 0.0024591689  2385.0601 28.46636269
## 59   58  967479.21 0.9972635 0.0027364792  2647.4867 27.49888348
## 60   59  964831.73 0.9969519 0.0030480838  2940.8880 26.53405175
```

```
## 61   60  961890.84 0.9966018 0.0033982113  3268.7083 25.57216092
## 62   61  958622.13 0.9962084 0.0037916077  3634.7191 24.61353878
## 63   62  954987.41 0.9957664 0.0042336000  4043.0347 23.65855137
## 64   63  950944.38 0.9952698 0.0047301652  4498.1240 22.70760700
## 65   64  946446.25 0.9947120 0.0052880089  5004.8162 21.76116074
## 66   65  941441.44 0.9940853 0.0059146520  5568.2985 20.81971931
## 67   66  935873.14 0.9933815 0.0066185277  6194.1023 19.88384617
## 68   67  929679.04 0.9925909 0.0074090890  6888.0747 18.95416713
## 69   68  922790.96 0.9917031 0.0082969290  7656.3310 18.03137617
## 70   69  915134.63 0.9907061 0.0092939131  8505.1818 17.11624154
## 71   70  906629.45 0.9895867 0.0104133270  9441.0289 16.20961209
## 72   71  897188.42 0.9883300 0.0116700384 10470.2233 15.31242367
## 73   72  886718.20 0.9869193 0.0130806770 11598.8743 14.42570548
## 74   73  875119.32 0.9853362 0.0146638316 12832.6024 13.55058615
## 75   74  862286.72 0.9835597 0.0164402661 14176.2231 12.68829943
## 76   75  848110.50 0.9815668 0.0184331558 15633.3529 11.84018894
## 77   76  832477.14 0.9793317 0.0206683441 17205.9240 11.00771179
## 78   77  815271.22 0.9768254 0.0231746205 18893.6011 10.19244057
## 79   78  796377.62 0.9740160 0.0259840198 20693.0918  9.39606296
## 80   79  775684.53 0.9708679 0.0291321407 22597.3508  8.62037843
## 81   80  753087.18 0.9673415 0.0326584844 24594.6858  7.86729125
## 82   81  728492.49 0.9633932 0.0366068080 26667.7847  7.13879876
## 83   82  701824.71 0.9589745 0.0410254900 28792.7024  6.43697406
## 84   83  673032.00 0.9540321 0.0459679011 30937.8686  5.76394206
## 85   84  642094.13 0.9485072 0.0514927715 33063.2066  5.12184792
## 86   85  609030.93 0.9423355 0.0576645431 35119.4902  4.51281699
## 87   86  573911.44 0.9354463 0.0645536903 37048.1012  3.93890556
## 88   87  536863.34 0.9277630 0.0722369904 38781.3917  3.40204222
## 89   88  498081.94 0.9192023 0.0807977153 40243.8832  2.90396027
## 90   89  457838.06 0.9096743 0.0903257142 41354.5499  2.44612221
## 91   90  416483.51 0.8990827 0.1009173439 42030.4098  2.02963870
## 92   91  374453.10 0.8873248 0.1126751990 42191.5778  1.65518560
## 93   92  332261.52 0.8742924 0.1257075806 41767.7923  1.32292407
## 94   93  290493.73 0.8598724 0.1401276327 40706.1990  1.03243034
## 95   94  249787.53 0.8439479 0.1560520640 38979.8600  0.78264281
## 96   95  210807.67 0.8264006 0.1735993612 36596.0773  0.57183514
## 97   96  174211.60 0.8071126 0.1928873917 33603.2203  0.39762354
## 98   97  140608.38 0.7859697 0.2140302860 30094.4508  0.25701517
## 99   98  110513.92 0.7628655 0.2371344912 26206.6632  0.14650124
## 100  99   84307.26 0.7377061 0.2622938963 22113.2800  0.06219398
## 101 100   62193.98 0.7104160 0.2895839526 18010.3789  0.00000000
```

3.

```r
whole_life_insurance<-function(age, i, life_table){
  qx<-life_table$qx
  px<-1-qx
  kpx<-c(1, cumprod(px[(age+1):(length(px)-1)]))
  kqx<-kpx*qx[(age+1):length(qx)]
  discount_factors<-(1+i)^-(1:length(kqx))
  sum(discount_factors*kqx)
}
plot_by_age<-function(){
  ages<-0:100
```

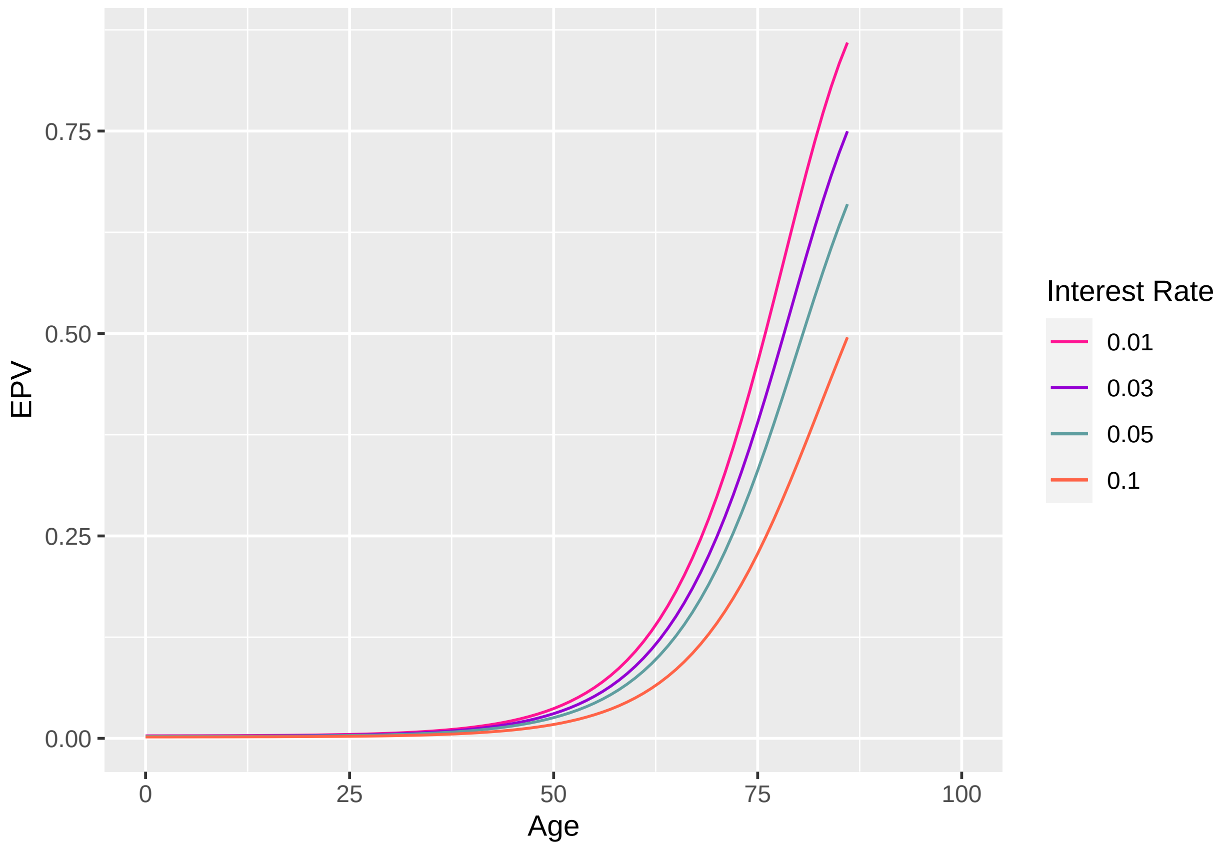
```
  EPV1<-sapply(ages, whole_life_insurance, i=0.01, life_table=life_table)
  EPV3<-sapply(ages, whole_life_insurance, i=0.03, life_table=life_table)
  EPV5<-sapply(ages, whole_life_insurance, i=0.05, life_table=life_table)
  EPV10<-sapply(ages, whole_life_insurance, i=0.1, life_table=life_table)
  a<-data.frame(ages,EPV1,EPV3,EPV5,EPV10)
  ggplot(a,aes(x=ages))+geom_line(aes(x=ages,y=EPV1, col="0.01"))+
    geom_line(aes(y=EPV3, col="0.03"))+
    geom_line(aes(y=EPV5, col="0.05"))+
    geom_line(aes(y=EPV10, col="0.1"))+
    labs(main="Whole Life Insurance",x="Age",y="EPV")+
  scale_color_manual(name="Interest Rate",
                     labels=c(as.character(c(0.01,0.03,0.05,0.1))),
                     values = c("0.01"=color[1],
                                "0.03"=color[2],
                                "0.05"=color[3],
                                "0.1"=color[4]))
}
plot_by_age()
```



Here, we can see how the EPV increases with age because it becomes more and more likely that the death benefit is paid sooner as the policyholder ages. Also, higher interest rates lead to lower EPVs since the insurer can set aside less money today to accrue the needed future amount if the interest rate is higher.

```
temporary_life_insurance <- function(age, n, i, life_table) {
  qx <- life_table$qx
  px <- 1 - qx
```

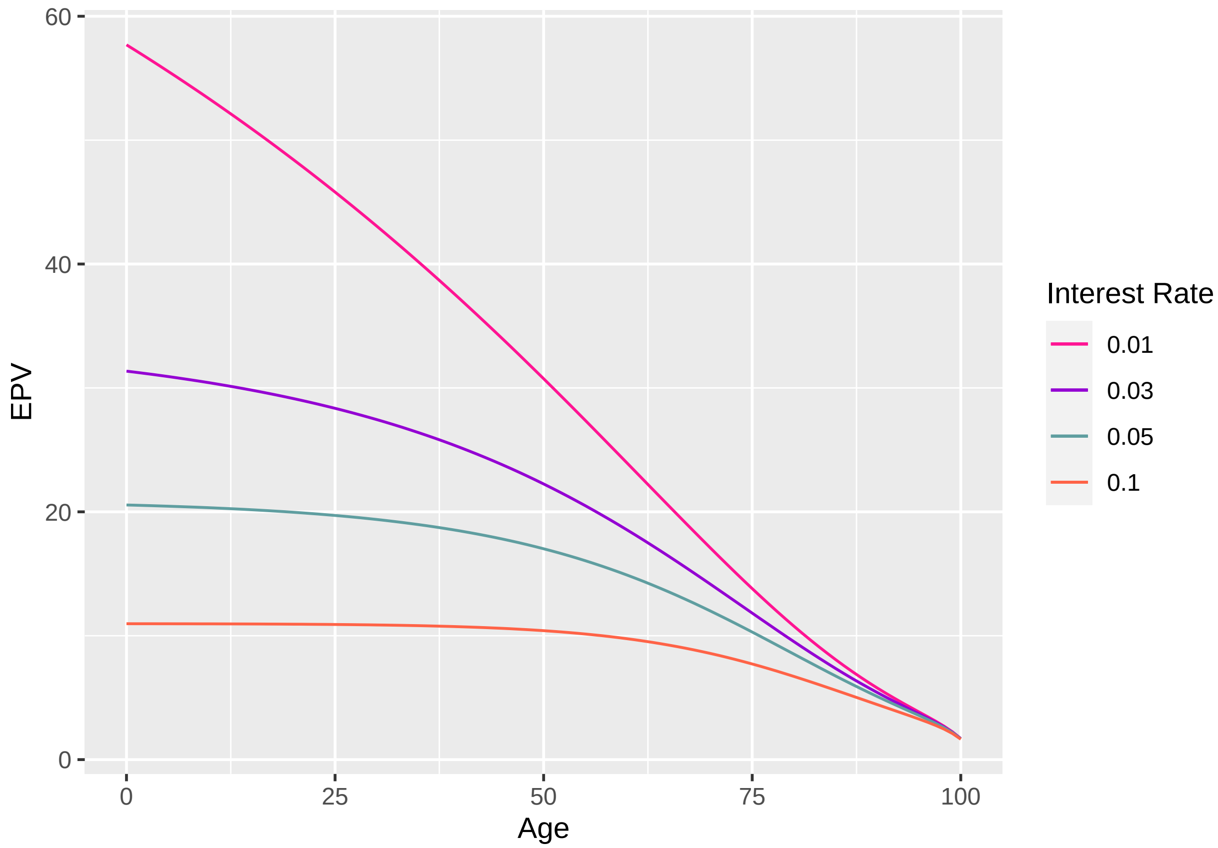
```r
  kpx <- c(1, cumprod(px[(age+1):(age+n-1)]))
  kqx <- kpx * qx[(age+1):(age+n)]
  discount_factors <- (1 + i) ^ - (1:length(kqx))
  sum(discount_factors * kqx)
}
plot_by_age<-function(){
  ages<-0:100
  EPV1<-sapply(ages, temporary_life_insurance, n=15, i=0.01, life_table=life_table)
  EPV3<-sapply(ages, temporary_life_insurance, n=15, i=0.03, life_table=life_table)
  EPV5<-sapply(ages, temporary_life_insurance, n=15, i=0.05, life_table=life_table)
  EPV10<-sapply(ages, temporary_life_insurance, n=15, i=0.1, life_table=life_table)
  a<-data.frame(ages,EPV1,EPV3,EPV5,EPV10)
  ggplot(a,aes(x=ages))+geom_line(aes(y=EPV1, col="0.01"))+
    geom_line(aes(y=EPV3, col="0.03"))+
    geom_line(aes(y=EPV5, col="0.05"))+
    geom_line(aes(y=EPV10, col="0.1"))+
    labs(x="Age",y="EPV", main="Temporary Life Insurance, 15 years")+
  scale_color_manual(name="Interest Rate",
                     labels=c(as.character(c(0.01,0.03,0.05,0.1))),
                     values = c("0.01"=color[1],
                                "0.03"=color[2],
                                "0.05"=color[3],
                                "0.1"=color[4]))
}
plot_by_age()
```



The same can be seen here, except the graph lines stop short due to the delay period.

4.

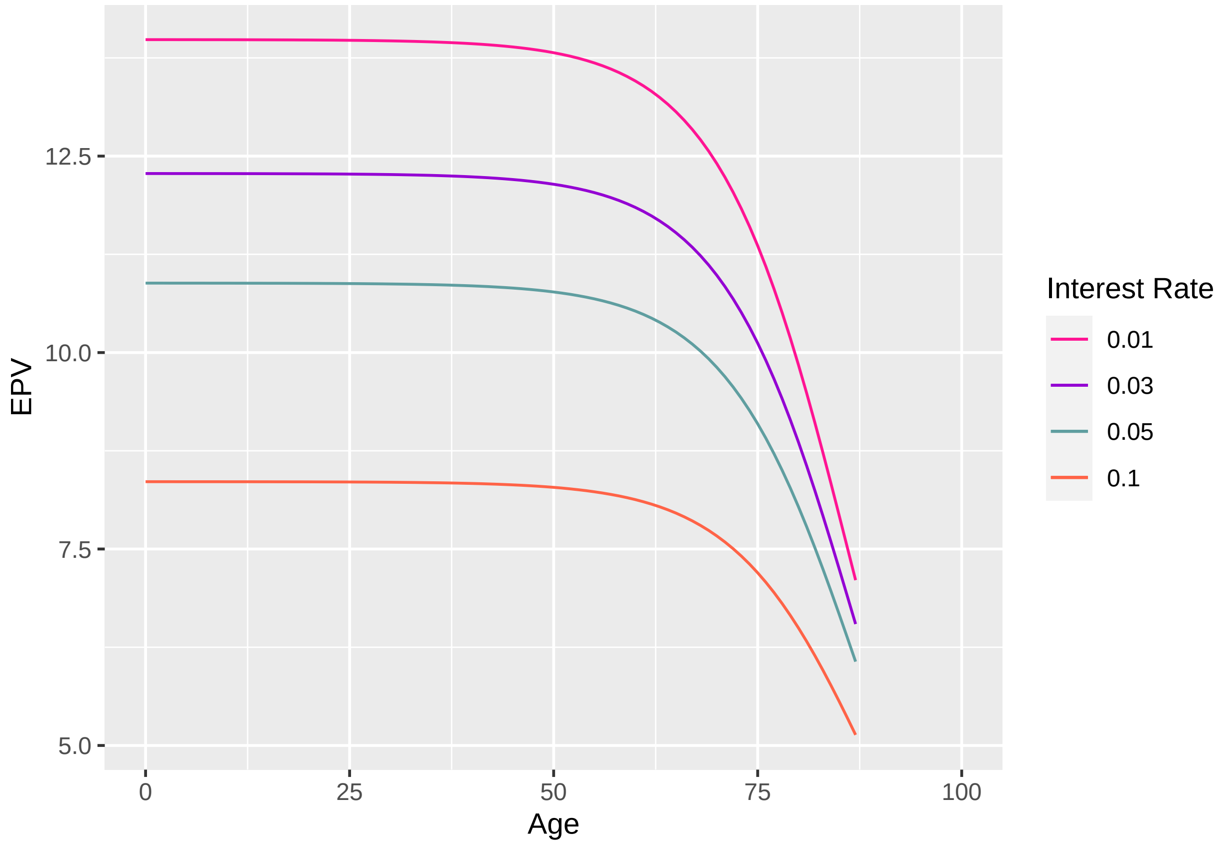
```
life_annuity_due <- function(age, i, life_table) {
  px <- 1-life_table$qx
  kpx <- c(1, cumprod(px[(age+1):length(px)]))
  discount_factors <- (1+i) ^ - (0:(length(kpx)-1))
  sum(discount_factors * kpx)
}
plot_by_age<-function(){
  ages<-0:100
  EPV1<-sapply(ages, life_annuity_due, i=0.01, life_table=life_table)
  EPV3<-sapply(ages, life_annuity_due, i=0.03, life_table=life_table)
  EPV5<-sapply(ages, life_annuity_due, i=0.05, life_table=life_table)
  EPV10<-sapply(ages, life_annuity_due, i=0.1, life_table=life_table)
  a<-data.frame(ages,EPV1,EPV3,EPV5,EPV10)
  ggplot(a,aes(x=ages))+geom_line(aes(y=EPV1, col="0.01"))+
    geom_line(aes(y=EPV3, col="0.03"))+
    geom_line(aes(y=EPV5, col="0.05"))+
    geom_line(aes(y=EPV10, col="0.1"))+
    labs(x="Age",y="EPV",main="Life Annuity due")+
  scale_color_manual(name="Interest Rate",
                     labels=c(as.character(c(0.01,0.03,0.05,0.1))),
                     values = c("0.01"=color[1],
                                "0.03"=color[2],
                                "0.05"=color[3],
                                "0.1"=color[4]))
}
plot_by_age()
```



For the annuity, it becomes less and less likely that the policyholder will receive the annuity payments as they age, therefore the EPV of the annuity decreases with age. The annuity EPV also decreases with interest rate for the same reasons as mentioned in the insurance section.

```
temporary_life_annuity_due <- function(age, n, i, life_table) {
  px <- 1 - life_table$qx
  kpx <- c(1, cumprod(px[(age+1):(age+n-1)]))
  discount_factors <- (1 + i) ^ - (0:(length(kpx)-1))
  sum(discount_factors*kpx)
}
plot_by_age<-function(){
  ages<-0:100
  EPV1<-sapply(ages, temporary_life_annuity_due, n=15, i=0.01, life_table=life_table)
  EPV3<-sapply(ages, temporary_life_annuity_due, n=15, i=0.03, life_table=life_table)
  EPV5<-sapply(ages, temporary_life_annuity_due, n=15, i=0.05, life_table=life_table)
  EPV10<-sapply(ages, temporary_life_annuity_due, n=15, i=0.1, life_table=life_table)
  a<-data.frame(ages,EPV1,EPV3,EPV5,EPV10)
  ggplot(a,aes(x=ages))+geom_line(aes(y=EPV1, col="0.01"))+
    geom_line(aes(y=EPV3, col="0.03"))+
    geom_line(aes(y=EPV5, col="0.05"))+
    geom_line(aes(y=EPV10, col="0.1"))+
    labs(x="Age",y="EPV",main="Temporary Life Annuity due, 15 years")+
  scale_color_manual(name="Interest Rate",
                     labels=c(as.character(c(0.01,0.03,0.05,0.1))),
                     values = c("0.01"=color[1],
                                "0.03"=color[2],
```

```
                                        "0.05"=color[3],
                                        "0.1"=color[4]))
}
plot_by_age()
```



Here, the EPVs follow the same decreasing trend but only once a certain age is reached, due to the fixed number of payments.