An Acturial outlook between United Kingdom and the Netherlands

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1 Developments since 1970

The United Kingdom (UK) has the 5th largest GDP in the world. This could mean that UK citizens are more able to afford and more willing to spend on insurance products. Therefore, we think that it could be a great opportunity to expand our operations there. This report summarises the methods we have used to estimate the mortality rates of UK and Dutch citizens, followed by the challenges posed by estimating future mortality rates and finally, the prediction results for certain actuarial policies.

Data: We examined the data obtained from the Human Mortality Database (HMDB) using period data for Deaths and for Exposure-to-risk. The observed time frame we used was between 1970 and 2008. The hazard rate μ_{xt} for now was estimated with the central death rate, i.e. $\hat{\mu}_{xt} = D_{xt}/E_{xt}$.

Results: We start by presenting some historical observations for the logarithm of the central death rate in the United Kingdom and the Netherlands. A plot of $\ln\hat{\mu}_{xt}$ against age between 25 and 100 can be seen in Figure 1. For both plots, we show the behavior of the logarithmic central death rate for both males and females in the years 1970 and 2008. Not much differences separate the two countries; both countries showed that mortality rates significantly improved over four decades. As well documented by Kalben(2000) , males appear to have a higher mortality rate than females. Assuming that hazard rates are constant in an age-period data cell, we estimated the probability of dying with $1-\exp(-\hat{\mu}_{xt})$ and used this to approximate the remaining complete period life expectancy of both males and females in the United Kingdom and the Netherlands for people who have just turned 65 years old and for people who have just been born, in both 1970 and 2008. The results show that for every age-group and cohort, the Dutch are expected to

live longer than the British except 65-year old males in 2008, whereby a British male of the same age in the same year is expected to live 0.1621 years more than his Dutch counterpart. A visualisation of the 16 results can be seen in Figure 2.

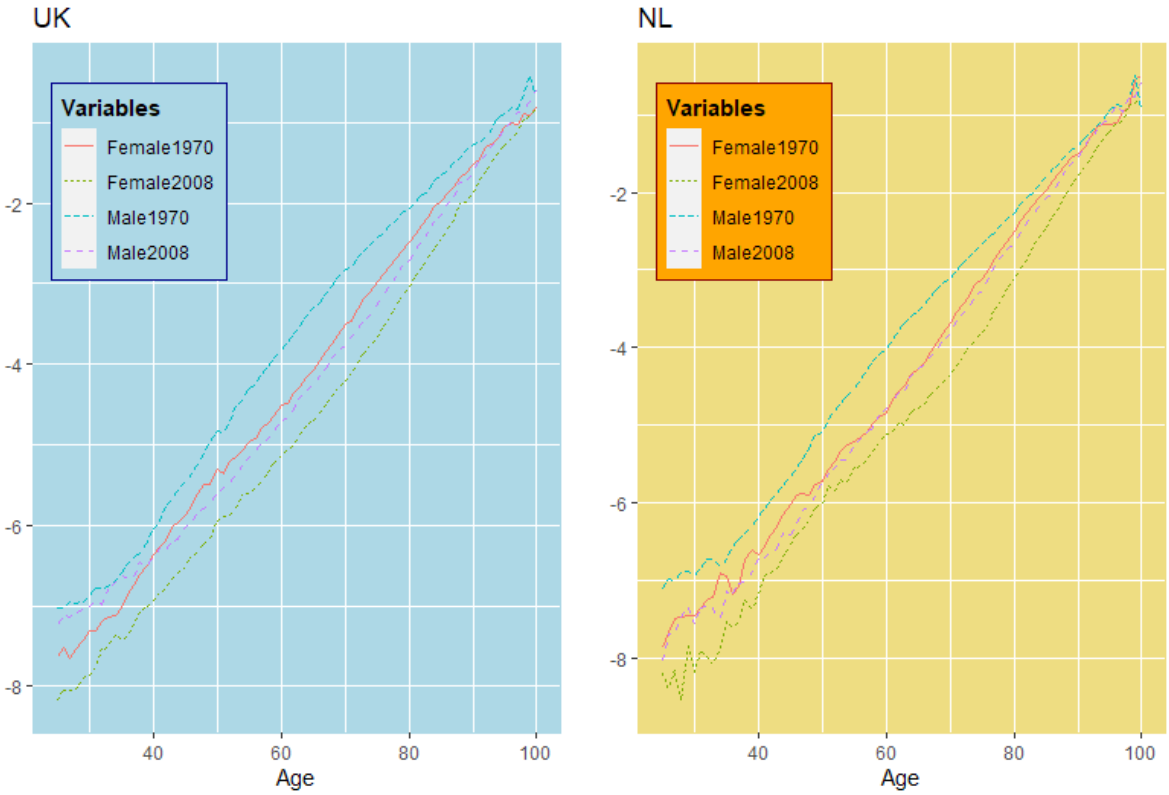

Figure 1: Log Central Death Rates

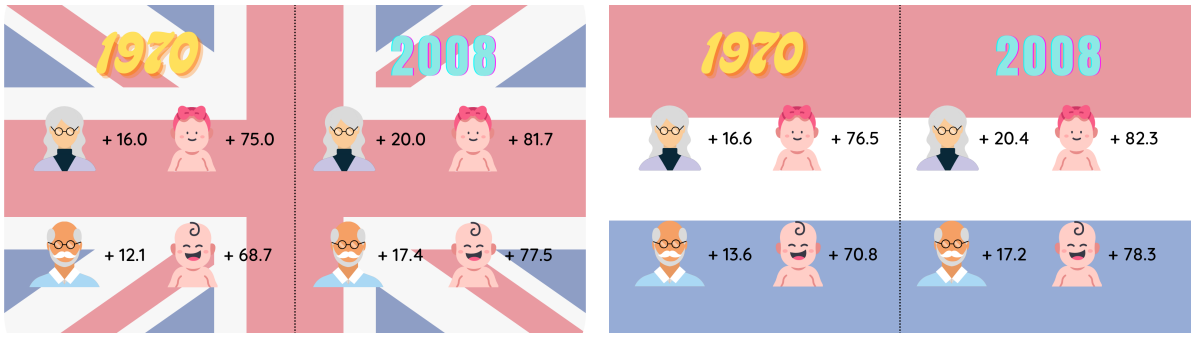

Figure 2: Remaining period life expectancy (years)

2 Current mortality per age

Model: Another estimator for the force of mortality is a scheme proposed by Gompertz (1825). The classical model of his theory assumes that for a given year, e.g. 2008, $\mu_{x,2008} = e^{a+bx}$ where x represents the age of a person while a and b are fixed parameters to be estimated. This model implicitly assumes that the death probabilities do not change over time which is quite unreasonable, nevertheless it serves as a great benchmark if we are able to estimate the parameters accurately.

Optimization and Estimation: This brings us to the subject of optimization. We implemented three methods using female data in the United Kingdom in the year 2008 and only taking ages 25 to 90 into account. The three different methods outlined with the results below involves solving for the arguments of various minimization problems:

1. Standard least square: $\min_{a,b} \sum_{x=25}^{90} \left[\frac{D_{x,2008}}{E_{x,2008}} - e^{a+bx}\right]^2$ $\hat{a} = -12.4$, $\hat{b} = 0.117$
2. Log transformation: $\min_{a,b} \sum_{x=25}^{90} \left[\ln \frac{D_{x,2008}}{E_{x,2008}} - (a+bx)\right]^2$ $\hat{a} = -10.8$, $\hat{b} = 0.096$
3. Weighted least square: $\min_{a,b} \sum_{x=25}^{90} D_{x,2008} \left[\frac{D_{x,2008}}{E_{x,2008}} - e^{a+bx}\right]^2$ $\hat{a} = -12.5$, $\hat{b} = 0.119$

Results: All three minimization problems yielded different estimates of a and b, with the standard least square and weighted least square yielding very similar estimates. However, the estimates obtained from log transformed least square were quite different from the other two, with $\hat{a}$ 1.4-1.5 more than the other two and $\hat{b}$ 0.021-0.023 lower than the other two. It might have been assumed that the log transformation method and standard least square would yield same results, however, it is not the same because it is not a monotonic transformation. If there was no square then the assumption would have been correct. Lastly, the weighted least square solution should yield similar results to the standard least square since they have the same first order condition, minimal differences observed would be due to the numerical accuracy of the algorithms used to solve it. Still weighted least square is a very useful modification especially in our case because it helps us handle very small values of the central death rates which may go below floating point limits of the software used to calculate. In other words, the weighted least square in our case is resistant to underflow. With these results we plotted both the logarithm of central death rate and central death rate, with each plot portraying the comparison between the different estimates and the observed, this are shown in Figure 3.

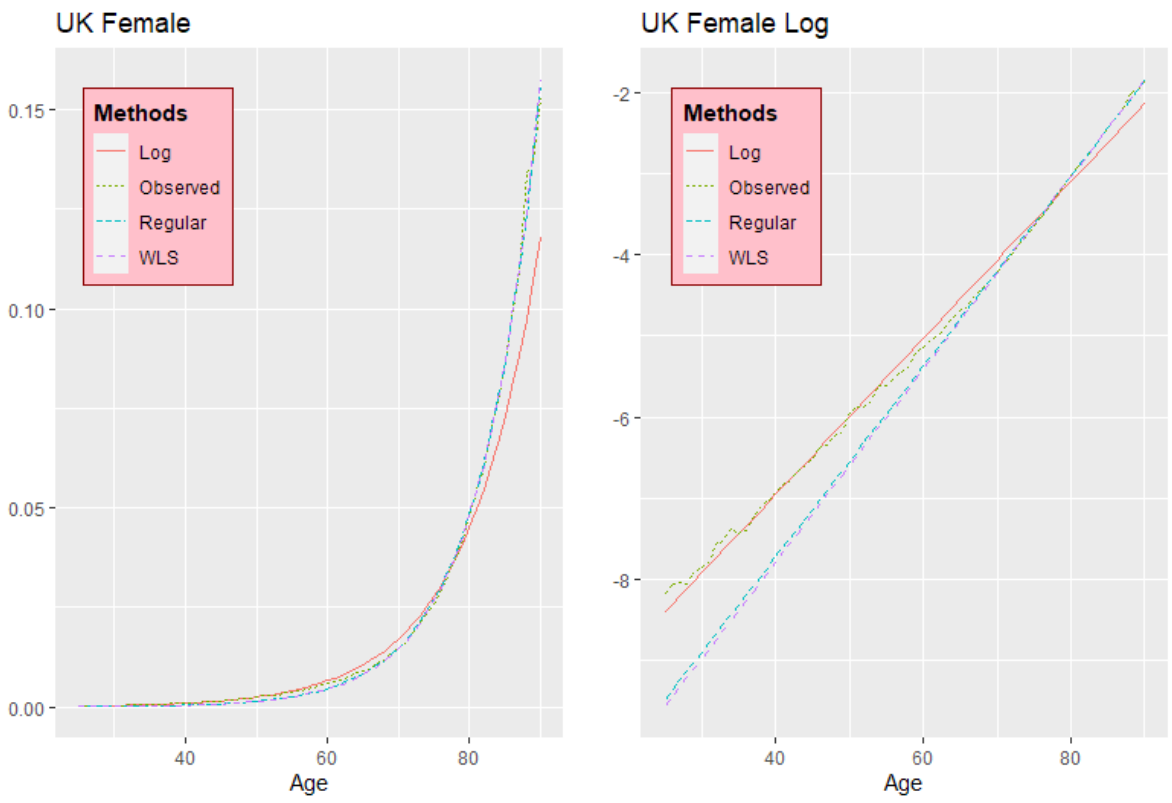

Figure 3: Central Death Rates

Plots: The logarithm of the estimated central death rates showed that the estimates obtained from the log transformed minimisation problem yielded a much more accurate result compared to the other two, however this advantage only lasts until age 75 where the other two estimates closely fit the observed central death rates better. We can also see that the estimated mortality rate increases ever so slowly from ages 25 until 60 before it skyrocketed. This could be attributed to old age groups having weaker immune systems hence becoming more prone to dying from diseases. By age 85, the estimated mortality rates are as high as 0.15.

3 Sensitivities in a dynamic model

Lee Carter: We then turn our attention to the sensitivity of the Lee-Carter Model, as this is believed to be a better estimation technique, which describes the force of mortality at a particular age and time by three parameters $\alpha_x, \beta_x, \kappa_t$. α_x which is invariant across time describes the average log level of mortality at a particular age, while κ_t which is invariant to age illustrates the overall speed of mortality improvement over the years. Lastly, β_x, like α_x invariant to time explains how much each age group changes when κ_t changes. The α_x estimate was calculated using the average over all time-points t of the logarithm of ratio between number of death and number of exposures $(\ln(D_{xt}/E_{xt}))$.

The other two estimates were obtained by the Singular Value Decomposition algorithm applied on the 101 by 39 matrix R, where each element represents an age (between 0 and 100) and a year (between 1970 and 2008) of $\ln(D_{xt}/E_{xt}) - \hat{\alpha}_x$. The resulting plots of each estimates for both female and male can be found in Figure 4, 5 and 6.

Results: Regardless of country or sex, Figure 4 shows three common features: a high infant mortality between ages 0 and 3, an accident hump around ages 15 and 20, and a linear ageing process from ages 50 onward. What is interesting to note is that for both female and male it shows that people from the UK have a higher average rate of mortality compared to the Netherlands. Furthermore, κ_t in Figure 6 for both female and male shows a steady decline trend, implying that mortality improvement goes down as time horizon increases with not much difference separating the two countries. Lastly, in Figure 5, for both female and male, the sensitivity to κ_t (β_x) generally decreases as one gets older except for ages between 25 and 60.

For the case of males in both countries, β_x peaks at around three-years old and peaks again at around 60-years old but not as high as the previous peak, UK peaking higher than the Netherlands. β_x values of Dutch and UK male trough at about age 30, UK male β_x values significantly lower than those of NL at that age. For the case of females in both countries, β_x peaks at around three-years old, and UK peaks again at age 50 while NL peaks 25 years later (at age 75). The biggest difference between β_x values for females in the two countries is at age 50, whereby the β_x value of UK females is more than twice the value of that of the Netherlands, otherwise both countries are relatively similar. For both sexes, β_x values of the Netherlands are always greater than those of the UK for the first phase of their lives, age 0 to 35 for females, and age 0 to 47 for males. After these ages, β_x values of the UK are always greater than those of the Netherlands.

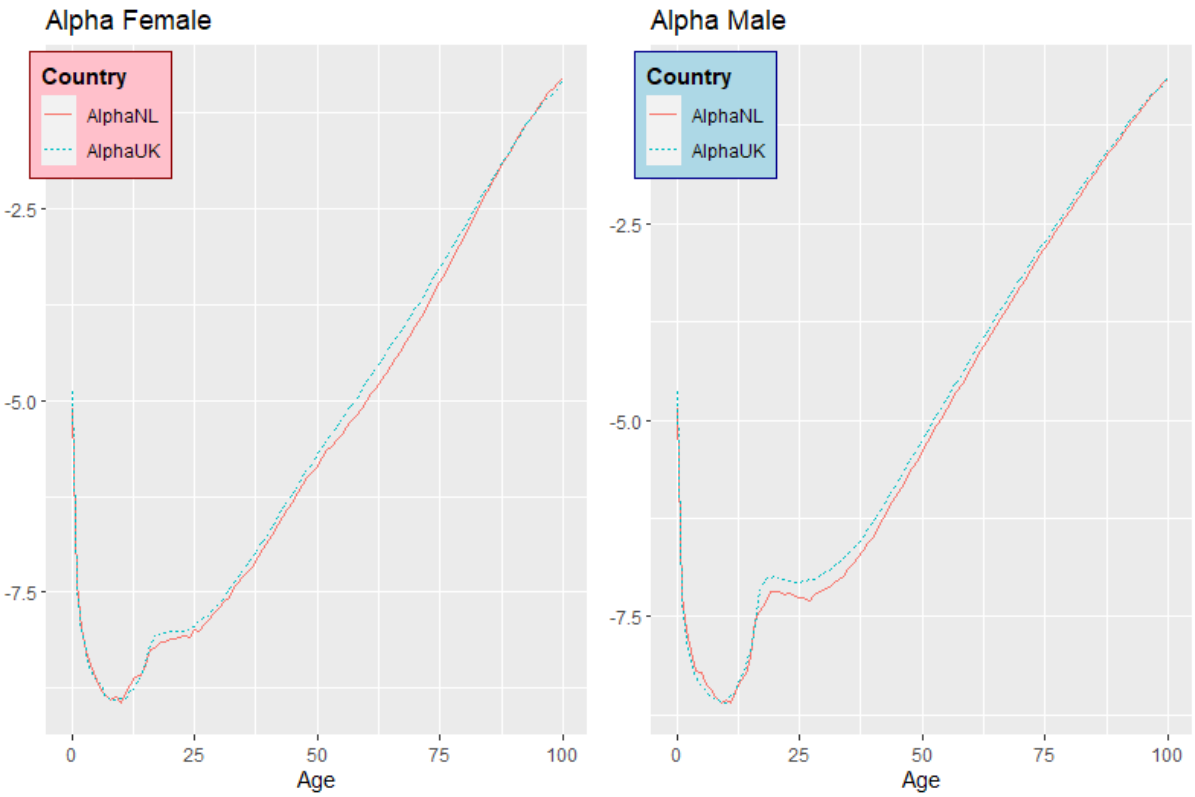

Figure 4: Alpha

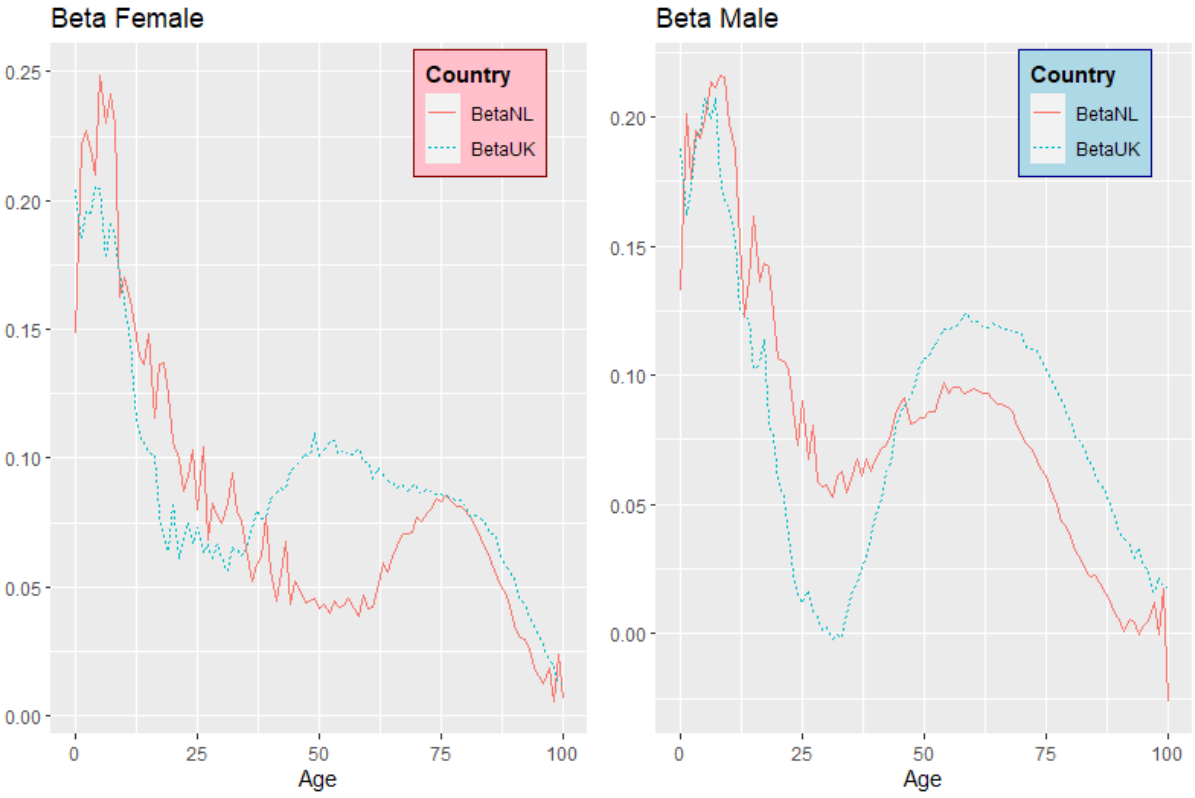

Figure 5: Beta

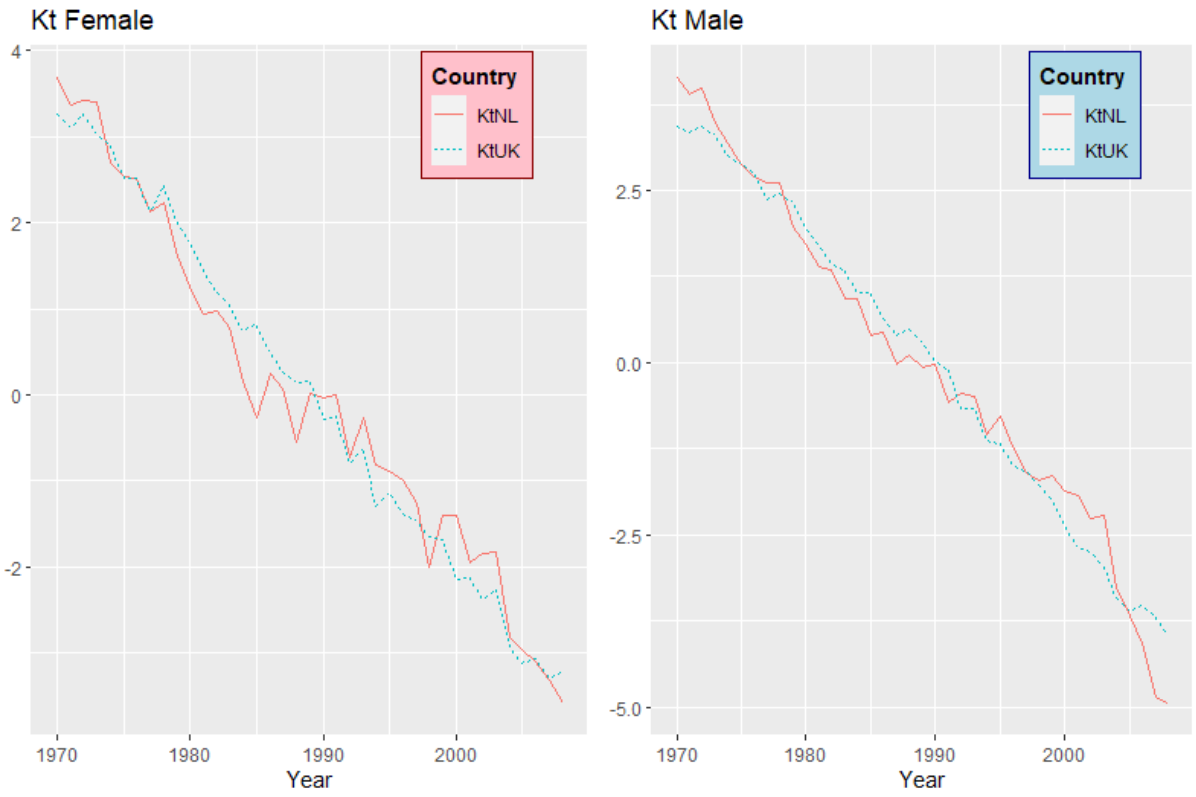

Figure 6: Mortality Trend Kt

4 Simulation of a dynamic model

Method: A good way to understand stochastic nature of the projected one year probability of death using the Lee-Carter model would be to do some simulations. We set a 50-year horizon and generated 25 random trajectories for a 65-year old male from the United Kingdom and the Netherlands. We estimated the one-year death probabilities q_{65t} using $1-\exp(-\exp(\hat{\alpha}_{65} + \hat{\beta}_{65}\kappa_t))$ whereby a random walk with drift model $\kappa_{t+1} = \kappa_t + \theta + \sigma\delta_t$ was used recursively to calculate the future κ_t with starting values -3.97 and -4.95 for UK and the Netherlands respectively. δ_t is assumed to be standard Gaussian i.i.d. and with this assumption, the estimates for parameters θ and σ were computed by matching the first two moments of $\Delta\kappa_t$ which yielded -0.195 & 0.161 for British male and -0.239 & 0.283 for Dutch male.

Results: Figure 7 shows the 25 random walks for the two countries. The key takeaway from the plots is that the uncertainty regarding the one year probability of death increases as the projection horizon gets longer with the Netherlands simulation showing a higher level of uncertainty. Nevertheless the simulation did give us insightful knowledge that from 2008 until 2058 the rate of mortality gradually decreases, and by 2058 it is up to four times lower than half a century ago. If we consider how health technology has advanced

exponentially and how poverty rates have improved over the past decades, our findings may even be an underestimation of the improvements in mortality rates.

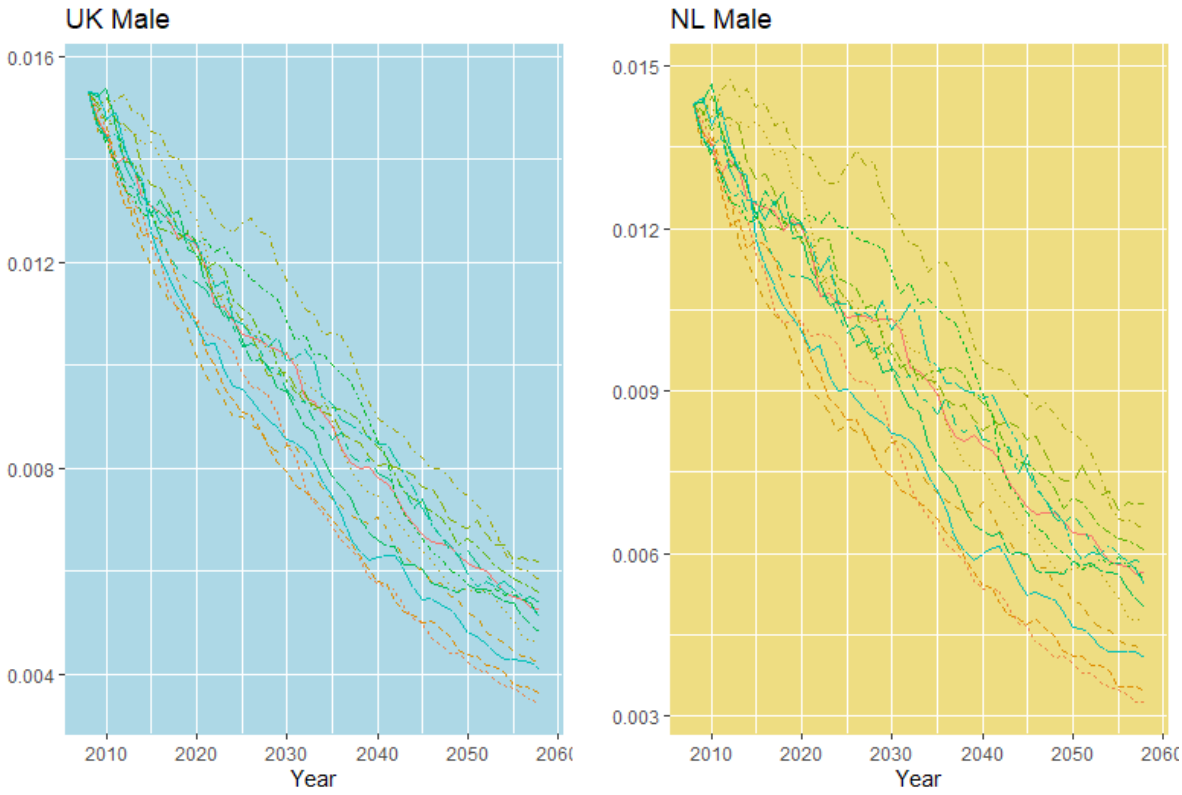

Figure 7: 25 Random Walks death probability

5 Uncertainty in predictions

Prediction: As our ambition is to expand our business to the United Kingdom, our goal is to forecast the remaining cohort life expectancy, the value of an annuity policy and the value of a death benefit. These three values require the one-year cohort death probabilities; and to be able to estimate a less bias approximation due to the huge uncertainty as mention in the previous section of the report, we generated 1000 instead of 25 simulations for our calculations. Due to extreme noises for higher ages, the one-year death probabilities will only be simulated until age 90, whereas following ages will be extrapolated. We used a scheme proposed by Kannistö, which uses hazard rates for ages 80 to 90, to extrapolate hazard rates for ages above 90. The hazard rates enabled us to extrapolate one-year death probabilities for those aged 91 and higher. We also assumed that the maximum years a person can live is 120 years.

Results: We did the aforementioned forecasting for 65-year old females in 2008 in both the Netherlands and the United Kingdom. In Figure 8, for UK females of the description mentioned, the expected remaining life expectancy is 20.90 years (sd = 0.40), which is

half a year shorter than their counterpart from the Netherlands who is expected to live for another 21.49 years (sd = 0.51). It is not hard to see why we should charge a 65-year old female in the Netherlands more than a female in the UK of the same age, after all the first result suggested less annuity payment would be made on average in the UK. In Figure 9, the annuity value for a policy paying 1000 euros yearly at a fix discount rate of 3% is 15407.02€(sd = 205.91) and 15811.66€(sd = 262.87) in the UK and the Netherlands respectively. Similarly because we expect a 65-year old Dutch female to live longer on average than her British counterpart, the latter should expect to be charged more for health insurance. In Figure 10, the death benefit value for a policy paying 1000 euros at the end of the year of death at a fix discount rate of 3% is 551.25€(sd = 6.00) and 539.47€(sd = 7.66) in the UK and the Netherlands respectively.

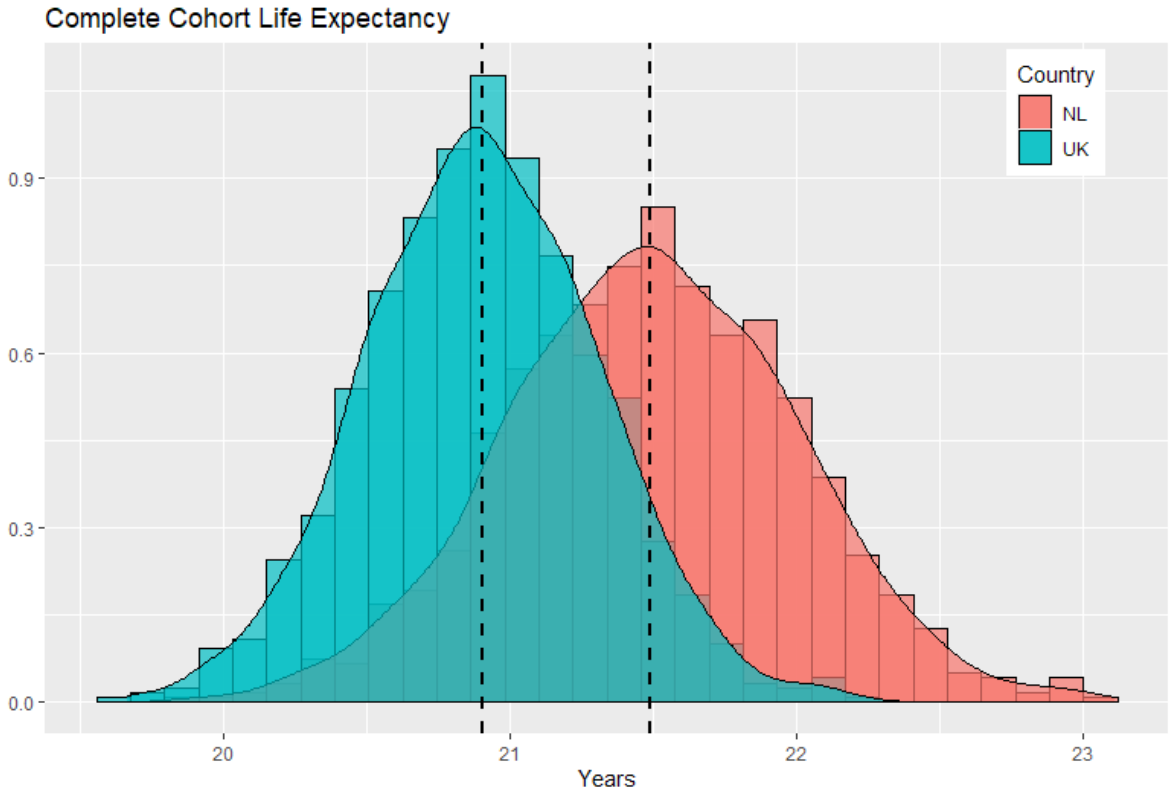

Figure 8: Histogram density of the complete cohort life expectancy

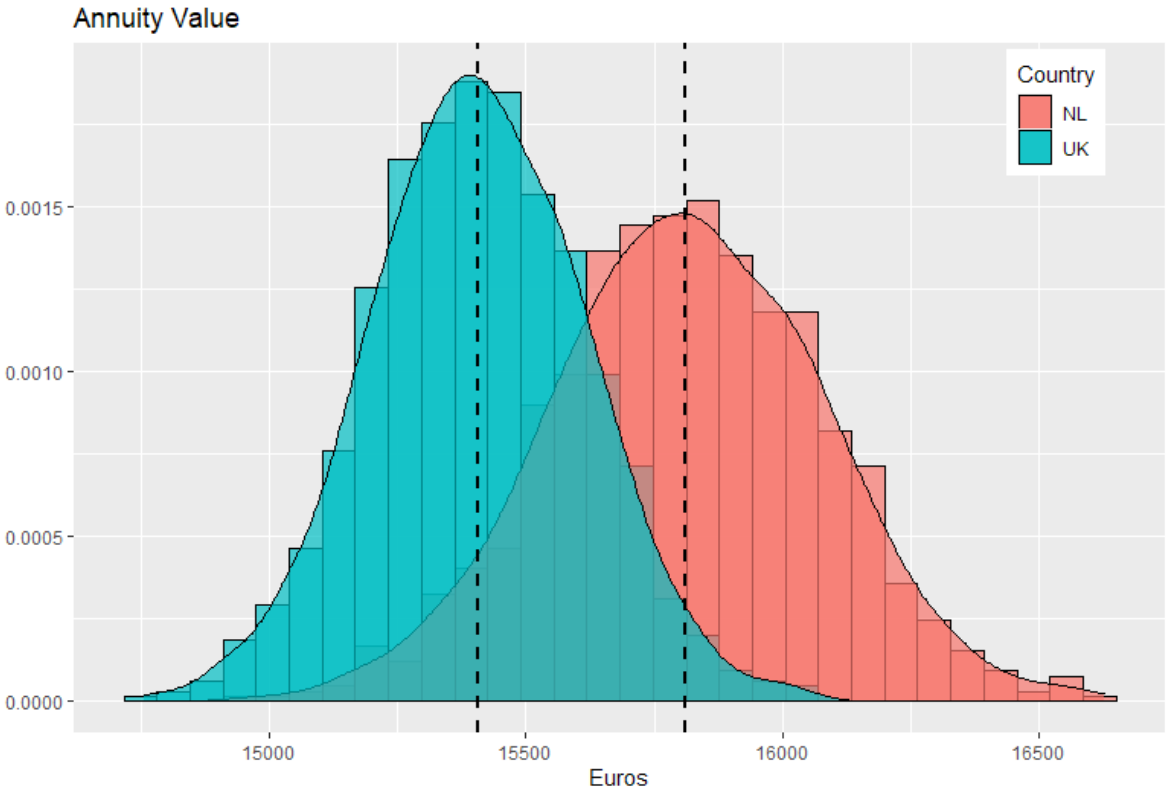

Figure 9: Histogram density of the annuity value

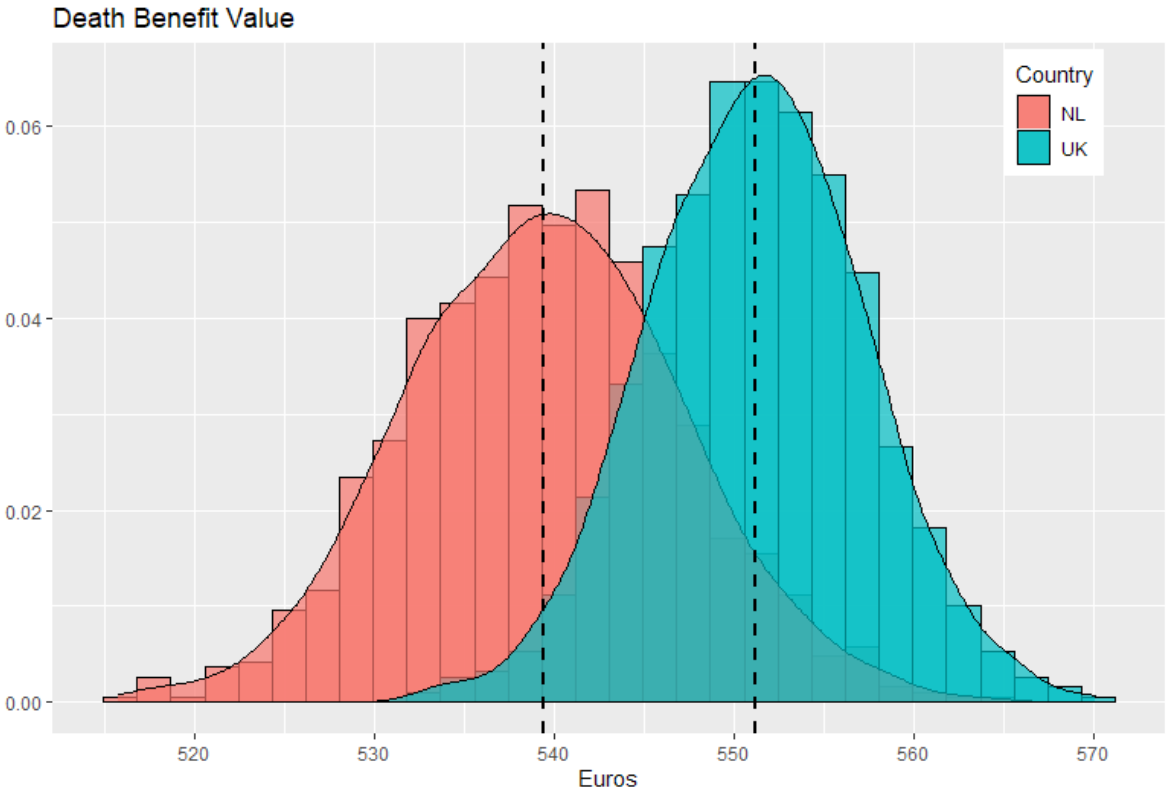

Figure 10: Histogram density of death benefit value

6 References

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Gompertz, B. (1825). XXIV. On the nature of the function expressive of the law of human mortality, and on a new mode of determining the value of life contingencies. In a letter to Francis Baily, Esq. FRS c. *Philosophical transactions of the Royal Society of London*, (115), 513-583.