AREC422 Notes

Youpei Yan

February 14, 2019

Do $\hat{\beta}_0$ and $\hat{\beta}_1$ have variance? Yes. They are estimated, they have their own distribution. They have their expected values and they have their variances.

$$sd(\hat{\beta}_1) = \sqrt{var(\hat{\beta}_1)} = \frac{\sigma}{\sqrt{SST_x}}$$

where σ is in the GMA5, the error term's sd. However, in real life, we can only estimate this value using residuals:

$$\hat{\sigma}^2 = \frac{1}{n-2}\sum_{i=1}^{n} \hat{u_i}^2$$

$$se(\hat{\beta}_1) = \frac{\hat{\sigma}}{\sqrt{SST_x}}$$

It shows how precise the estimator $\hat{\beta}_1$ is.

1 Hypothesis Test

Examine how close this $\hat{\beta}_1$ is to **0**, we need some hypothesis testing techniques. (Also, we can change the 0 part to other numbers, but 0 is what we really care about in most of the econometric case.)

We often hear the phrase "significant impact", it actually has two meanings: (1) the magnitude of the impact is large, and (2) statistically significant at certain percentage level.

Based on the GMA1-5 + normality assumption, we have an error term u satisfying: $E(u) = 0$, $var(u|x) = \sigma^2$, and $u \sim Normal(0, \sigma^2)$. Then

$$\frac{\hat{\beta}_1 - \beta_1}{sd(\beta_1)} \sim Normal$$

Because $sd(\beta_1)$ is unknown, we use $se(\hat{\beta}_1)$ to estimate it. So

$$t_{\hat{\beta}_1} = \frac{\hat{\beta}_1 - \beta_1}{se(\hat{\beta}_1)} \sim t\ Distribution$$

What do we hypothesize about?

H_0: Null hypothesis, H_1: Alternative hypothesis.
Most of the time, we assume that: $H_0 : \beta_1 = 0$ vs. $H_1 : \beta_1 \neq 0$, or

$H_0 : \beta_1 = 0$ vs. $H_1 : \beta_1 > (<) 0$
(It is also possible to have that $H_0 : \beta_1 = 1$, in that case, we assume that

$$t_{\hat{\beta}_1} = \frac{\hat{\beta}_1 - 1}{se(\hat{\beta}_1)} \sim t\ Distribution$$

and perform the corresponding test.)

In econometrics, we want to reject H_0 because: If $\beta_1 = 0$ is true, the impact of x to y is 0. The relationship or the model we have is just wrong!

However, we have to be careful here. It is possible that we reject H_0 when H_0 is true. The corresponding probability is called α.

$$P(Reject\ H_0 | H_0\ is\ true) = \alpha$$

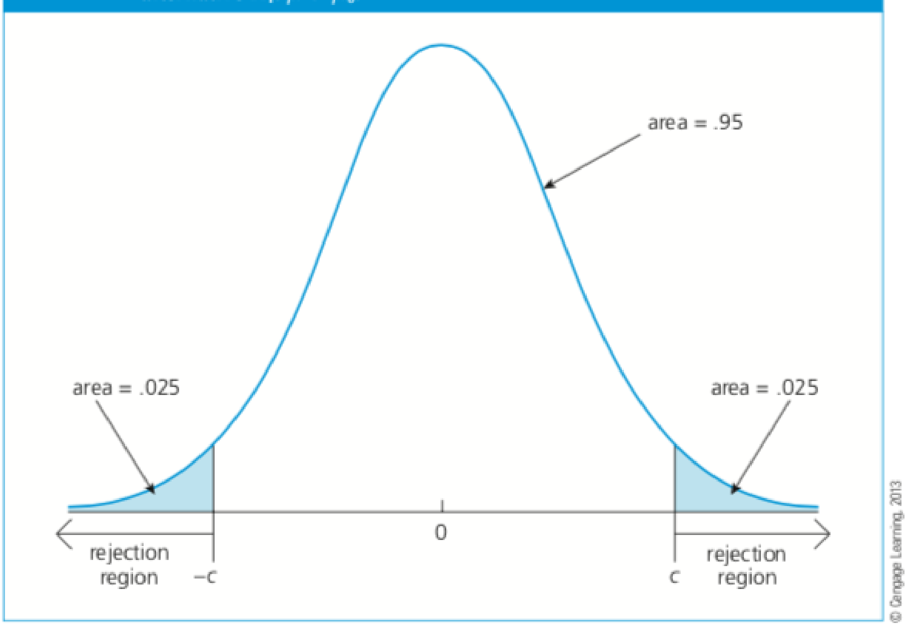
This is also called the probability of the Type I error.

When α is smaller, we have more confidence to reject H_0 without worrying that H_0 is actually true.

How do we know this α when we have a $\hat{\beta}_1$ estimated (given the t-distribution assumption)?

If $\hat{\beta}_1$ is far away enough from 0 (say, greater than a "value"), then $\hat{\beta}_1 >$ value is equivalent to $(\hat{\beta}_1 - 0)/se(\hat{\beta}_1) = t_{\hat{\beta}_1} >$ function of this value. This $t_{\hat{\beta}_1}$ is called the t-value because it has a t distribution.

FIGURE C.6 Rejection region for a 5% significance level test against the two-sided alternative H_1: $\mu \neq \mu_0$.



How far this β_1 is away from 0 (can we conclude that $\hat{\beta}_1$ is not 0)? When t-value > something on the x-axis, we say that $\hat{\beta}_1 >$ a value, and it is far enough. If $t_{\hat{\beta}_1}$ is located in the shaded regions, we can say that $\hat{\beta}_1$ is far away from 0. the corresponding α would be
$p(t_{\hat{\beta}_1}$ is in the shaded region $|\hat{\beta}_1$ is still 0)

Assume that we think $\alpha = 0.05$ is small enough to prevent this Type I error, then the area of each shaded region is 0.025.

Note that the shape of the t-distribution's bell is changing based on the size of the data (so does the degree of freedom (dof). Recall that using the sample removes 1 dof.) A t-distribution usually has a dof=n-k-1.

For instance, when $n \to \infty$, the thresholds for the t-values are ± 1.96. When $t_{\hat{\beta}_1} > 1.96$ or $t_{\hat{\beta}_1} < -1.96$, we can say that the impact is statistically different from zero (statistically significant) at the 5% level.

If we think that only when $\alpha = 0.01$ is small enough? The thresholds become to ± 2.576. In general, we will need the t-table to determine the "critical values" given a specific α and a specific dof.

Read the R examples and the t-table. Note that the t-value R calculated is for the following hypothesis: $H_0 : \beta_i = 0$ vs. $H_1 : \beta_i \neq 0$. This is also known as the 2-tailed test. Also note that the dof in the first example is 4, which is calculated as 6-1-1. The second example has a dof=5, which is calculated as 7-1-1. When, in the future, we have more regressors (say k regressors) on the RHS, we'll calculate the dof as (n-k-1).

In the first example, we are comparing the t-values to the critical value 4.604, and we can claim that (if we do not have the software's help) β_0 and β_1 are both statistically significant from zero at 0.01 significance level.

If we change H_1 to $\beta_i > 0$ or $\beta_i < 0$, the test becomes a 1-tailed test, and we will just need different critical values to compare with our t-values.