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Revenue Management: Flight Leg Revenue Optimization

16.75J/1.234J Airline Management

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Lecture Outline

1. Airline Revenue Maximization

- Pricing vs. Yield (Revenue) Management
- Computerized RM Systems

2. Single-leg Fare Class Seat Allocation Problem

- Serial Nesting of Booking Classes
- EMSRb Model for Seat Protection

3. Spiral Down in Fare Class YM Models

- EMSRb Sellup Model and Revenue Impacts

4. New Developments in RM Modeling

- Hybrid Forecasting of Price vs. Product Demand
- Marginal Revenue Optimization

1. Airline Revenue Maximization

- **Two components of airline revenue maximization:**

  Differential Pricing:

  - Various “fare products” offered at different prices for travel in the same O-D market

  Revenue Management (RM):

  - Determines the number of seats to be made available to each “fare class” on a flight, by setting booking limits on low fare seats

- **RM takes a set of differentiated prices/products and flight capacity as given:**

  - With high proportion of fixed operating costs for a committed flight schedule, revenue maximization to maximize profits

Computerized RM Systems

- **Size and complexity of a typical airline's seat inventory control problem requires a computerized RM system**
- **Consider a US Major airline with:**

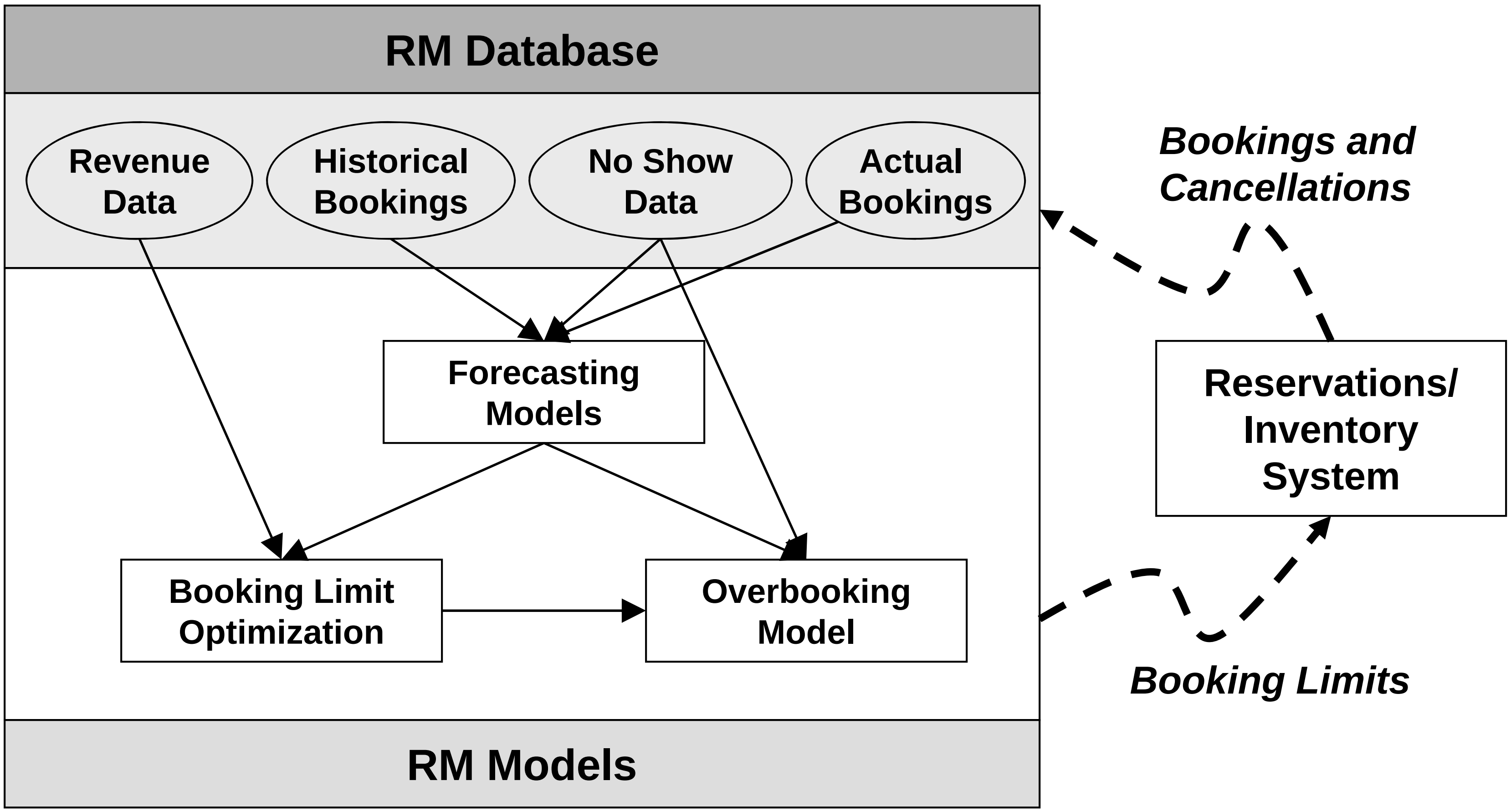
  2000 flight legs per day
  10 booking classes
  300 days of bookings before departure

- **At any point in time, this airline's seat inventory consists of 6 million booking limits:**
  - This inventory represents the airline's potential for profitable operation, depending on the revenues obtained
  - Far too large a problem for human analysts to monitor alone

Typical 3rd Generation RM System

- **Collects and maintains historical booking data by flight and fare class, for each past departure date.**
- **Forecasts future booking demand and no-show rates by flight departure date and fare class.**
- **Calculates limits to maximize total flight revenues:**
  - Overbooking levels to minimize costs of spoilage/denied boardings
  - Booking class limits on low-value classes to protect high-fare seats
- **Interactive decision support for RM analysts:**
  - Can review, accept or reject recommendations

Third Generation RM System



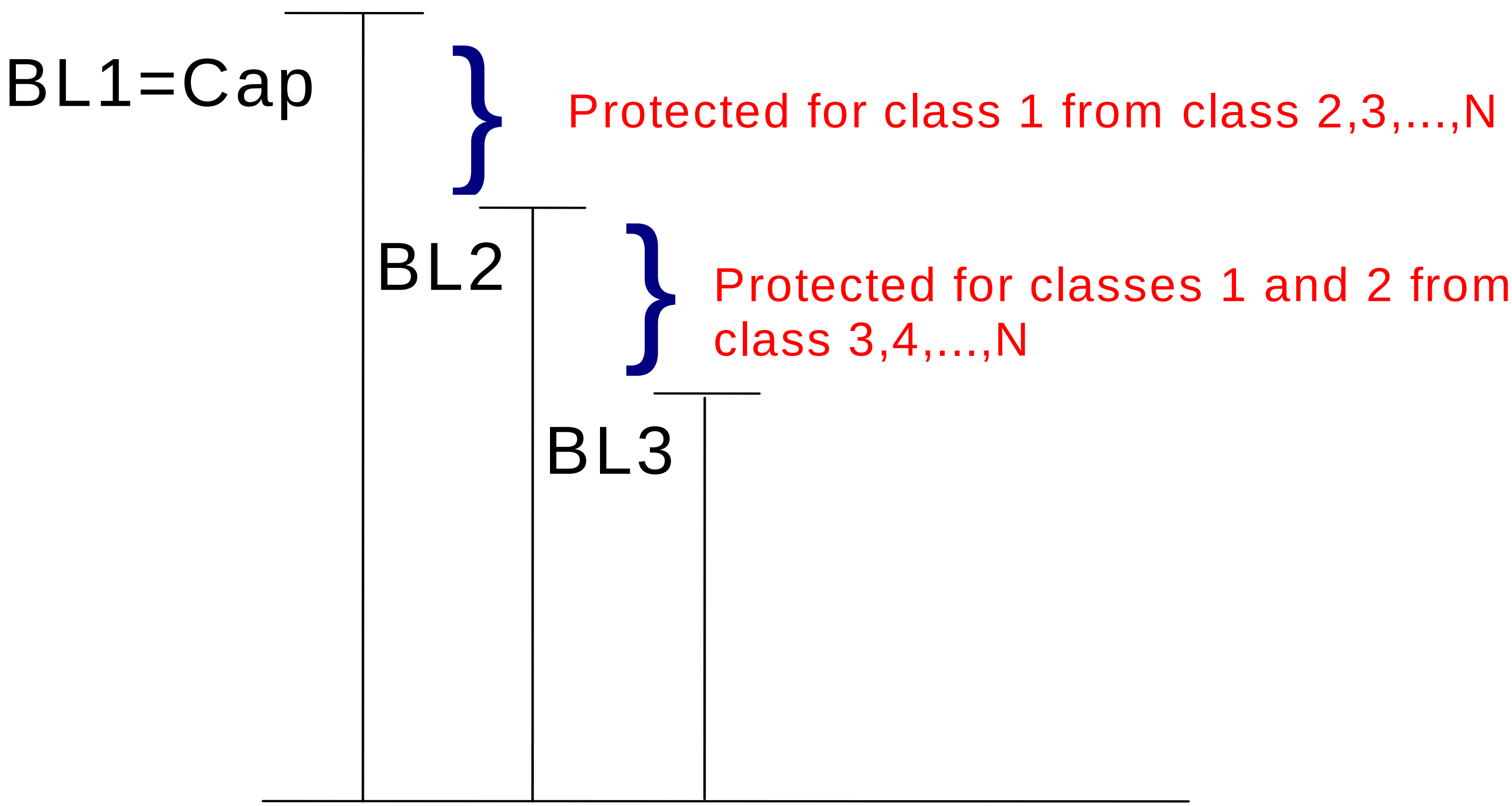
Revenue Management Techniques

- **Fare Class Mix (Flight Leg Optimization)**
  - Determine revenue-maximizing mix of seats available to each booking (fare) class on each flight departure
- **Overbooking**
  - Accept reservations in excess of aircraft capacity to overcome loss of revenues due to passenger “no-show” effects
- **Traffic Flow (O-D) Control (Network RM)**
  - Further distinguish between seats available to short-haul (one-leg) vs. long-haul (connecting) passengers, to maximize total network revenues
  - Currently implemented by most advanced and largest network airlines

2. Single-Leg Seat Protection Problem

- **Given for a future flight leg departure:**
  - Total remaining booking capacity of (typically) the coach compartment
  - Several fare (booking) classes that share the same inventory of seats in the compartment
  - Forecasts of future booking demand by fare class between current DCP and departure
  - Revenue estimates for each fare (booking) class
- **Objective is to maximize total expected revenue:**
  - Protect seats for each fare class based on revenue value, taking into account forecast uncertainty and probability of realizing the forecasted demand

Serially Nested Buckets



EMSRb Model for Seat Protection: Assumptions

- **Modeling assumptions for serially nested classes:**
  - a) demand for each class is separate and independent of demand in other classes.
  - b) demand for each class is stochastic and can be represented by a probability distribution
  - c) lowest class books first, in its entirety, followed by the next lowest class, etc.
  - d) booking limits are only determined once (i.e., static optimization model)
- **Problem is to find <u>protection levels</u> for higher classes, and <u>booking limits</u> on lower classes**

EMSRb Model Calculations

- **To calculate the optimal protection levels:**

  Define $P_i(S_i) = $ probability that $X_i \geq S_i$,
  where S_i is the number of seats made available to class i, X_i is the random demand for class I

- **The expected marginal revenue of making the Sth seat available to class i is:**

  $EMSR_i(S_i) = R_i * P_i(S_i)$ where R_i is the average revenue (or fare) from class i

- **The optimal protection level, π_1 for class 1 from class 2 satisfies:**

  $EMSR_1(\pi_1) = R_1 * P_1(\pi_1) = R_2$

Example Calculation

Consider the following flight leg example:

Class	Mean Fcst.	Std. Dev.	Fare
Y	10	3	1000
B	15	5	700
M	20	7	500
Q	30	10	350

- **To find the protection for the Y fare class, we want to find the largest value of π_Y for which**

$$EMSR_Y(\pi_Y) = R_Y * P_Y(\pi_Y) \geq R_B$$

Example (cont'd)

$$EMSR_Y(\pi_Y) = 1000 * P_Y(\pi_Y) \geq 700$$

$$P_Y(\pi_Y) \geq 0.70$$

where $P_Y(\pi_Y)$ = probability that $X_Y \geq \pi_Y$.

- **Assume demand in Y class is *normally* distributed, then we can create a standardized normal random variable as $(X_Y - 10)/3$:**

  for $\pi_Y = 7$, Prob { $(X_Y - 10)/3 \geq (7 - 10)/3$ } = 0.841

  for $\pi_Y = 8$, Prob { $(X_Y - 10)/3 \geq (8 - 10)/3$ } = 0.747

  for $\pi_Y = 9$, Prob { $(X_Y - 10)/3 \geq (9 - 10)/3$ } = 0.63

- **$\pi_Y = 8$ is the largest integer value of π_Y that gives a probability $\geq$ 0.7 and we will protect 8 seats for Y class.**

General Case for Class n

- **Joint protection for classes 1 through n from class n+1**

$$\overline{X}_{1,n} = \sum_{i=1}^{n} \overline{X}_i$$

$$\hat{\sigma}_{1,n} = \sqrt{\sum_{i=1}^{n} \hat{\sigma}^2_i}$$

$$R_{1,n} = \frac{\sum_{i=1}^{n} R_i * \overline{X}_i}{\overline{X}_{1,n}}$$

- **We then find the value of π_n that makes**

$$\mathbf{EMSR_{1,n}(\pi_n) = R_{1,n} * P_{1,n}(\pi_n) = R_{n+1}}$$

- **Once π_n is found, set BL_{n+1}= Capacity - π_n**

EMSRb Seat Protection Model

CABIN CAPACITY =		**135**				
AVAILABLE SEATS =		**135**				
BOOKING	**AVERAGE**	**SEATS**	**FORECAST DEMAND**		**JOINT**	**BOOKING**
CLASS	**FARE**	**BOOKED**	**MEAN**	**SIGMA**	**PROTECT**	**LIMIT**
Y	$ 670	0	12	7	6	**135**
M	$ 550	0	17	8	23	**129**
B	$ 420	0	10	6	37	**112**
V	$ 310	0	22	9	62	**98**
Q	$ 220	0	27	10	95	**73**
L	$ 140	0	47	14		**40**
	SUM	**0**	**135**			

Dynamic Revision and Intervention

- **RM systems revise forecasts and re-optimize booking limits at numerous "checkpoints":**
  - Monitor actual bookings vs. previously forecasted demand
  - Re-forecast demand and re-optimize at fixed checkpoints or when unexpected booking activity occurs
  - Can mean substantial changes in fare class availability from one day to the next, even for the same flight departure
- **Substantial proportion of fare mix revenue gain comes from dynamic revision of booking limits:**
  - Human intervention is important in unusual circumstances, such as "unexplained" surges in demand due to special events

Revision of Forecasts and Limits as Bookings Accepted

CABIN CAPACITY =		**135**				
AVAILABLE SEATS =		**63**				
BOOKING	**AVERAGE**	**SEATS**	**FORECAST DEMAND**		**JOINT**	**BOOKING**
CLASS	**FARE**	**BOOKED**	**MEAN**	**SIGMA**	**PROTECT**	**LIMIT**
Y	$ 670	2	10	5	5	**63**
M	$ 550	4	13	7	19	**58**
B	$ 420	5	5	2	27	**44**
V	$ 310	12	10	5	40	**36**
Q	$ 220	17	20	6	63	**23**
L	$ 140	32	15	4		**0**
	SUM	**72**	**73**			

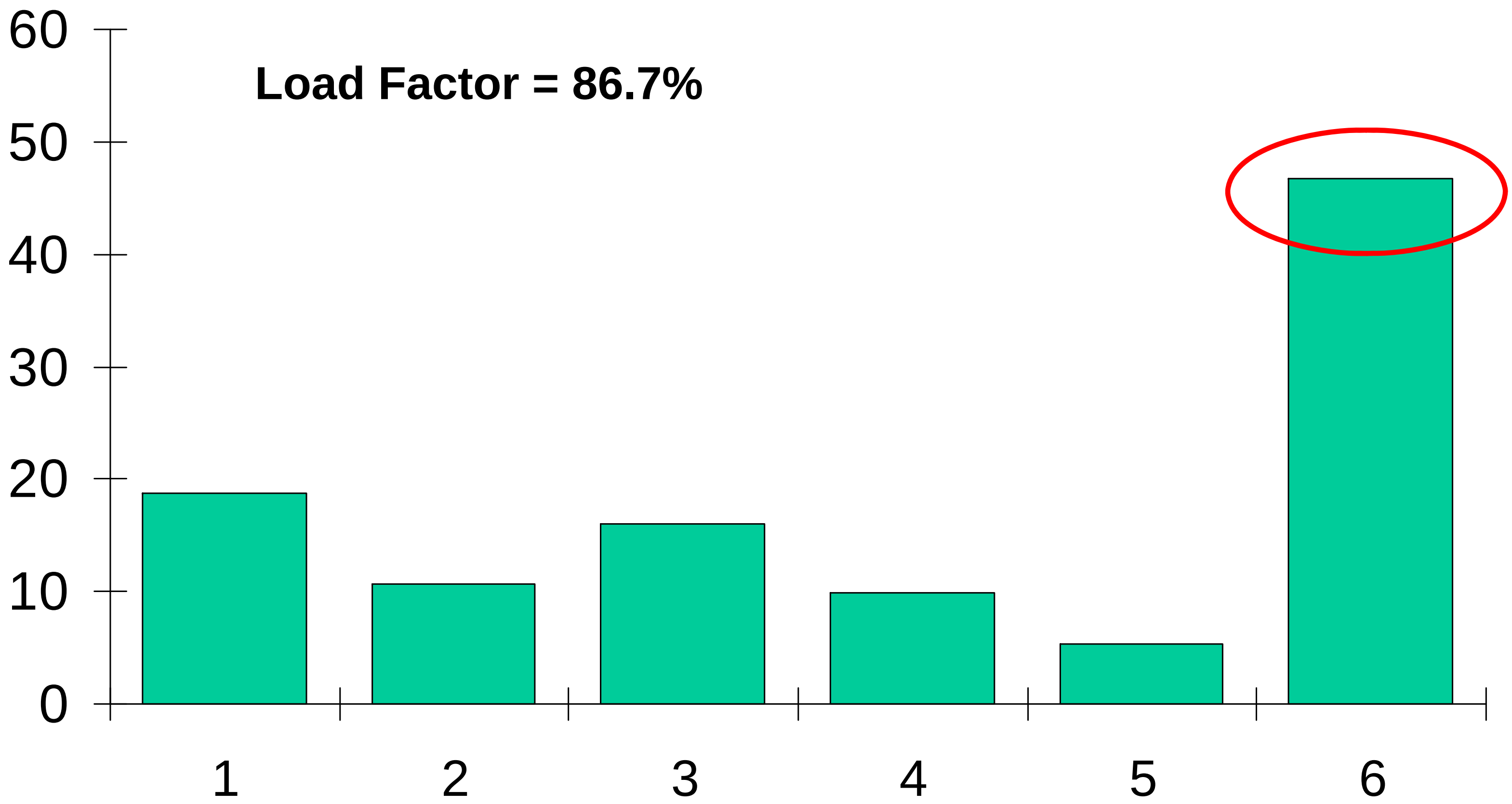
Higher than expected Q bookings close L class

3. “Spiral-Down” in Fare Class RM

- **Much evidence of “spiral-down” when fare classes do not correspond to independent demand segments**
  - This fundamental assumption of fare class YM has been violated since day one, but becomes even more critical with decreasing fare product differentiation

- **Unadjusted fare class forecasts based on historical bookings are guaranteed to under-estimate demand for higher fare levels**
  - Previous “buy-down” is recorded as lower fare demand
  - Exacerbated by inadequate detruncation methods
  - EMSRb under-protects based on under-forecasts of high-fare demands
  - Allowing more buy-down to occur, and the cycle continues

Standard RM Allows Spiral Down in Less Restricted Fare Structures



Passenger Diversion and "Sell-up"

- In practice, passengers unable to obtain a low fare seat might accept a higher fare
- We define "sell-up" as the diversion of passenger demand to a higher fare
- Correctly incorporating the likelihood of sell-up into seat protection calculations can increase total revenues
- In all cases, sell-up is assumed to occur from a lower class to the adjacent higher class

Incorporating Sell-up Rate from Class 2

- **Define the sell-up rate from Class 2 to Class 1:**

  SU_2 = probability random class 2 passenger will sell up to class1

- **The optimal protection level, π_1 for class 1 from class 2 must now satisfy:**

  $$EMSR_1(\pi_1) = R_1\, P_1(\pi_1) + R_1\, [1-P_1(\pi_1)]\, SU_2 = R_2$$

- **Logic: If protected Class 1 seat is not taken by Class 1 demand, then with remaining probability $[1-P_1(\pi_1)]$ that seat is available for sell-up from class 2**

Sellup Model Changes Fare Ratios

- **The optimal protection level, π_1 for class 1 from class 2 must now satisfy:**

$$EMSR_1(\pi_1) = R_1 * P_1(\pi_1) + R_1 * [1-P_1(\pi_1)]* SU_2 = R_2$$

$$R_1 * P_1(\pi_1) + R_1 * SU_2 - R_1 P_1(\pi_1)* SU_2 = R_2$$

$$P_1(\pi_1) [R_1 - R_1 SU_2] = R_2 - R_1 * SU_2$$

$$P_1(\pi_1) = \frac{R_2 - R_1 SU_2}{[R_1 - R_1 SU_2]} = \frac{R_2 - R_1 SU_2}{R_1 [1- SU_2]}$$

Belobaba/Weatherford EMSRb Sell-up Model (*Decision Sciences*, 1996)

- **EMSRb model modified for expected sell-up (extended to multiple classes below Class 2):**

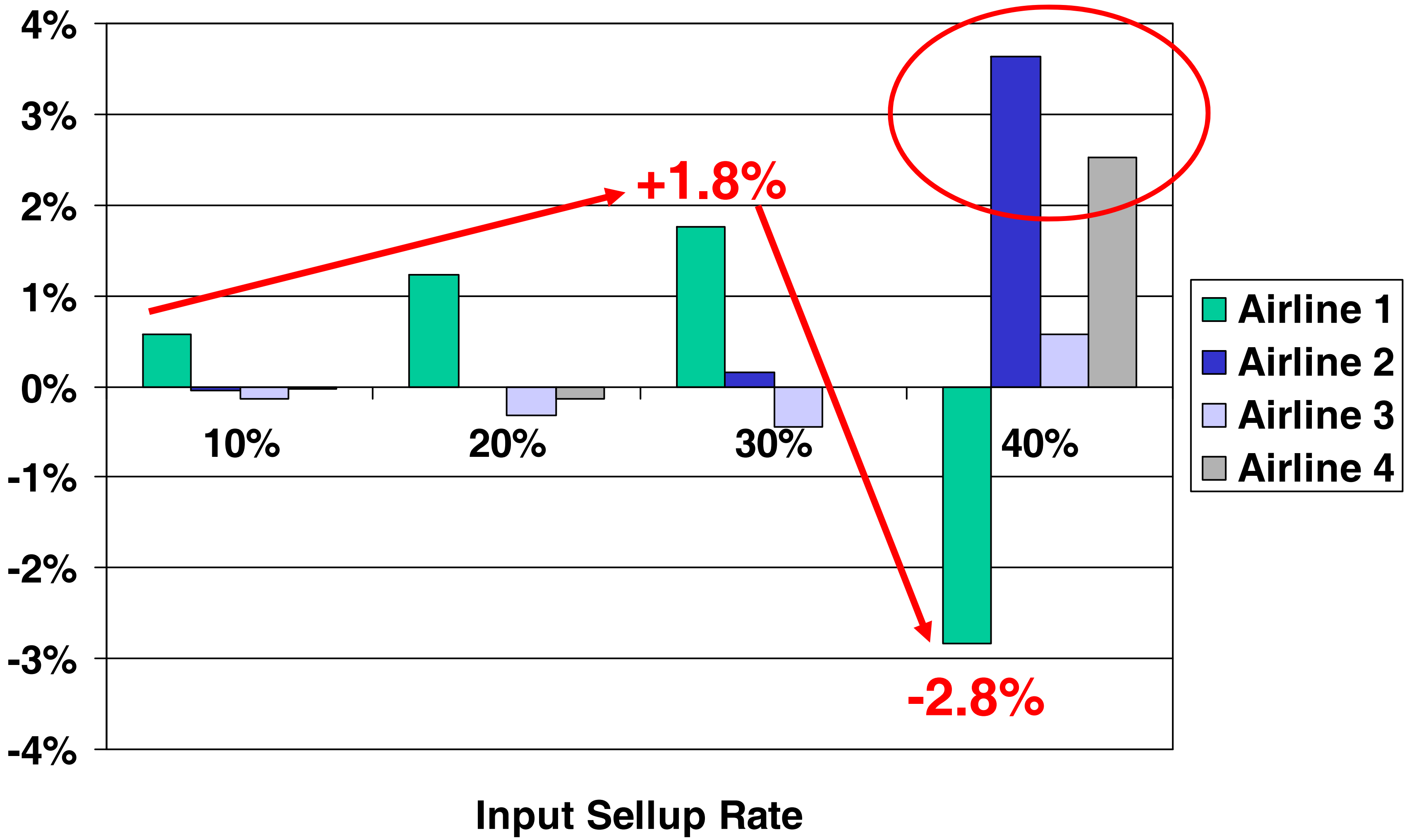
$$P(\pi_n) = \frac{R_{n+1} - R_{1,n} \times SU_{n+1,n}}{R_{1,n}(1 - SU_{n+1,n})}$$

$P(\pi_n)$ is the probability of selling the π_nth seat in class n or higher
π_n is the protection level for class 1 to n
$R_{1,n}$ is the weighted average revenue for classes 1 to n
R_{n+1} is the revenue from the class below class n
$SU_{n+1,n}$ is the probability of sell-up from class n+1 to n

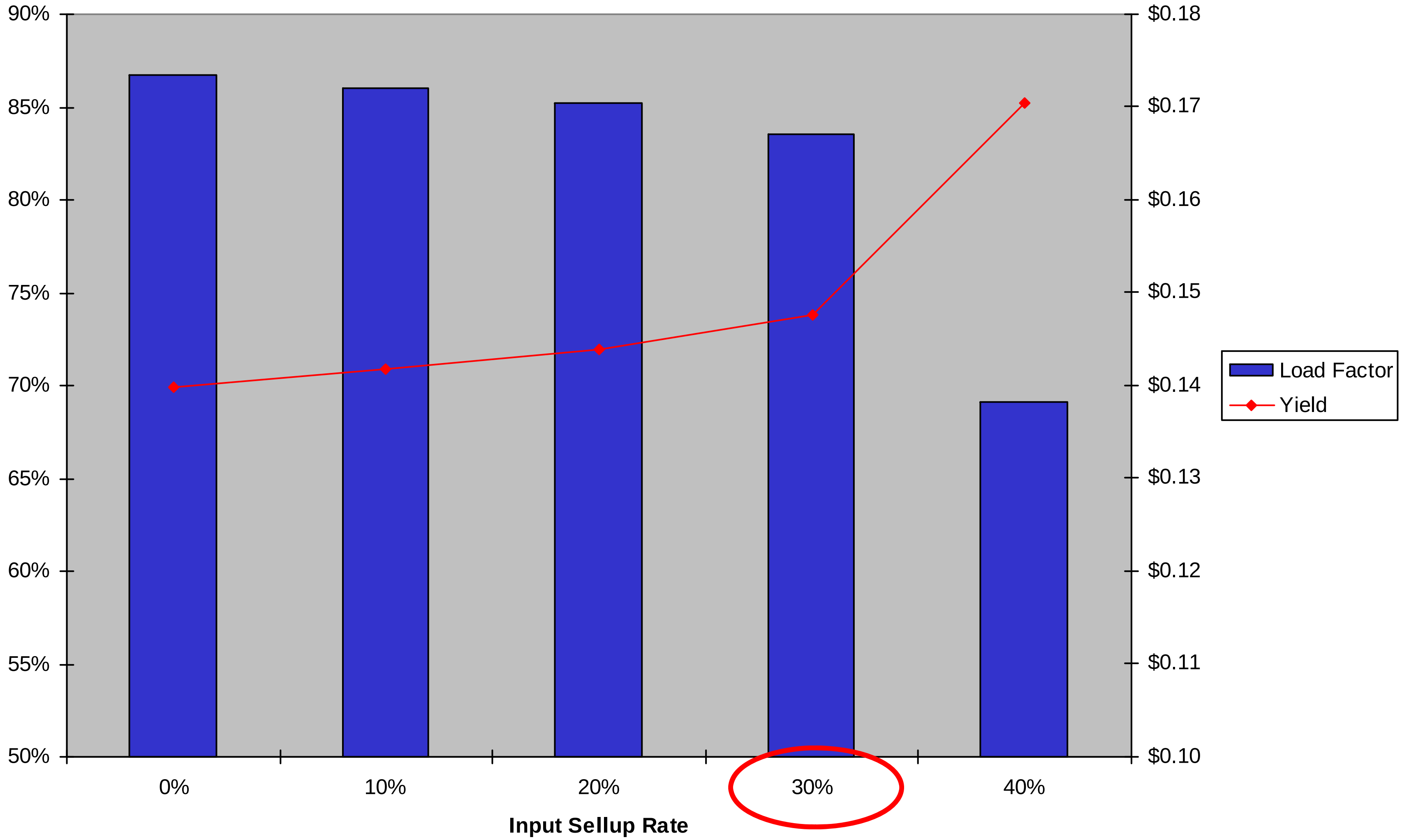
EMSR Sellup Model Revenue Impacts

- **EMSRb Sell-up Model available in vendor RM systems (e.g., PROS) – what are revenue impacts?**
  - Designed to adjust EMSRb fare ratios and increase higher class protections to account for sellup potential in Leg RM systems
  - Previous studies showed benefits even in restricted fare structures
- **Use of EMSRb Sell-up model by Airline 1 in PODS Network S increases revenues by 1.8%**
  - But requires inputs by airline of expected sellup rates
  - And carries risk of over-protecting and lower revenues
  - Not used by many airlines, including most PROS customers
- **Practical reality: Sell-up model with moderate sell-up inputs increases revenues**

Revenue Gains w/ EMSRb Sellup Model (Used by Airline 1)



EMSRb Sellup Model Decreases LF, Increases Yield and Revenue (to a point)



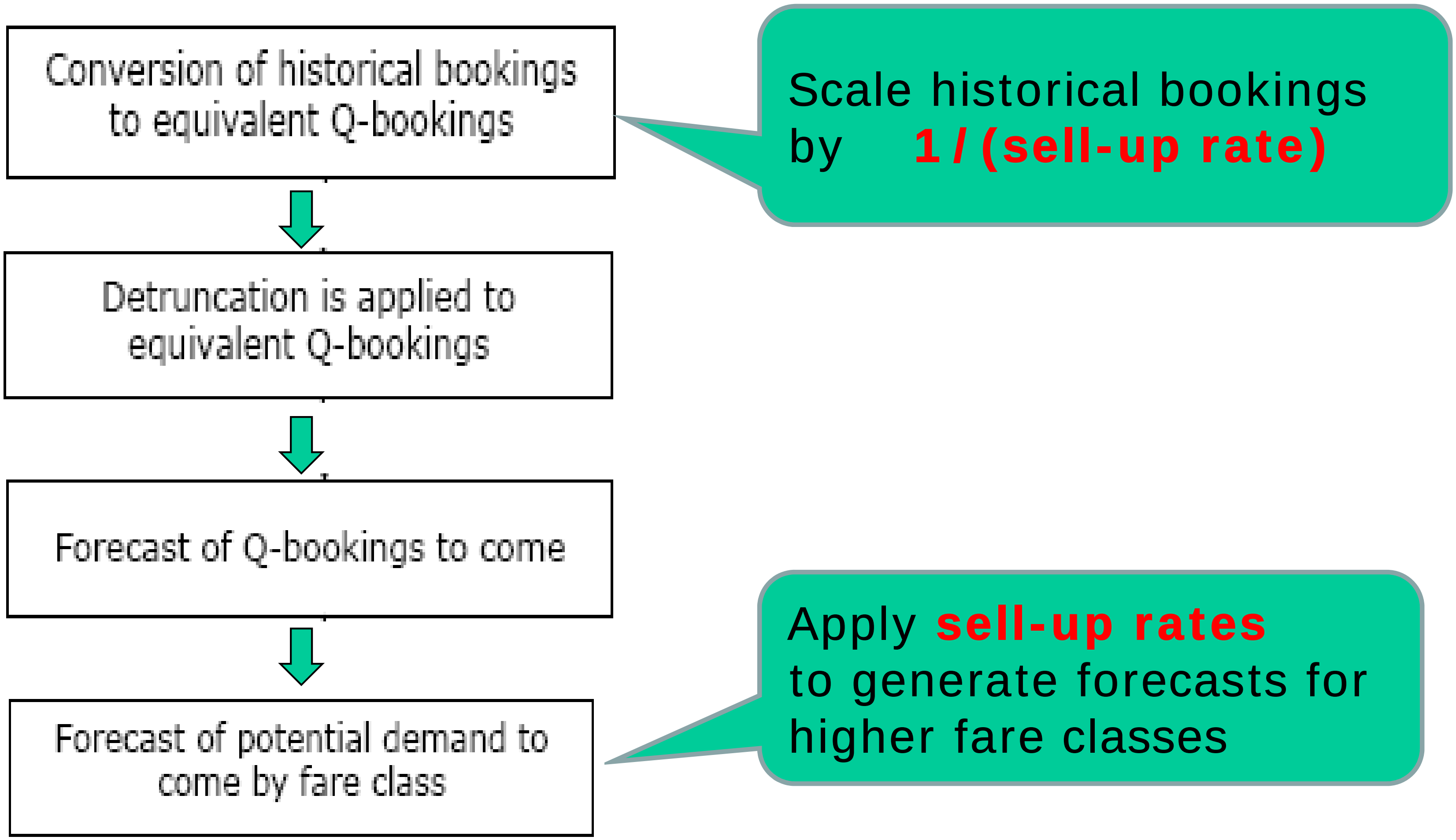
4. New Developments in RM Modeling

- **Methods developed and/or tested in MIT PODS Revenue Management research consortium**
  - Funded by nine large international airlines
  - Passenger Origin Destination Simulator used to evaluate revenue impacts of RM models in competitive markets

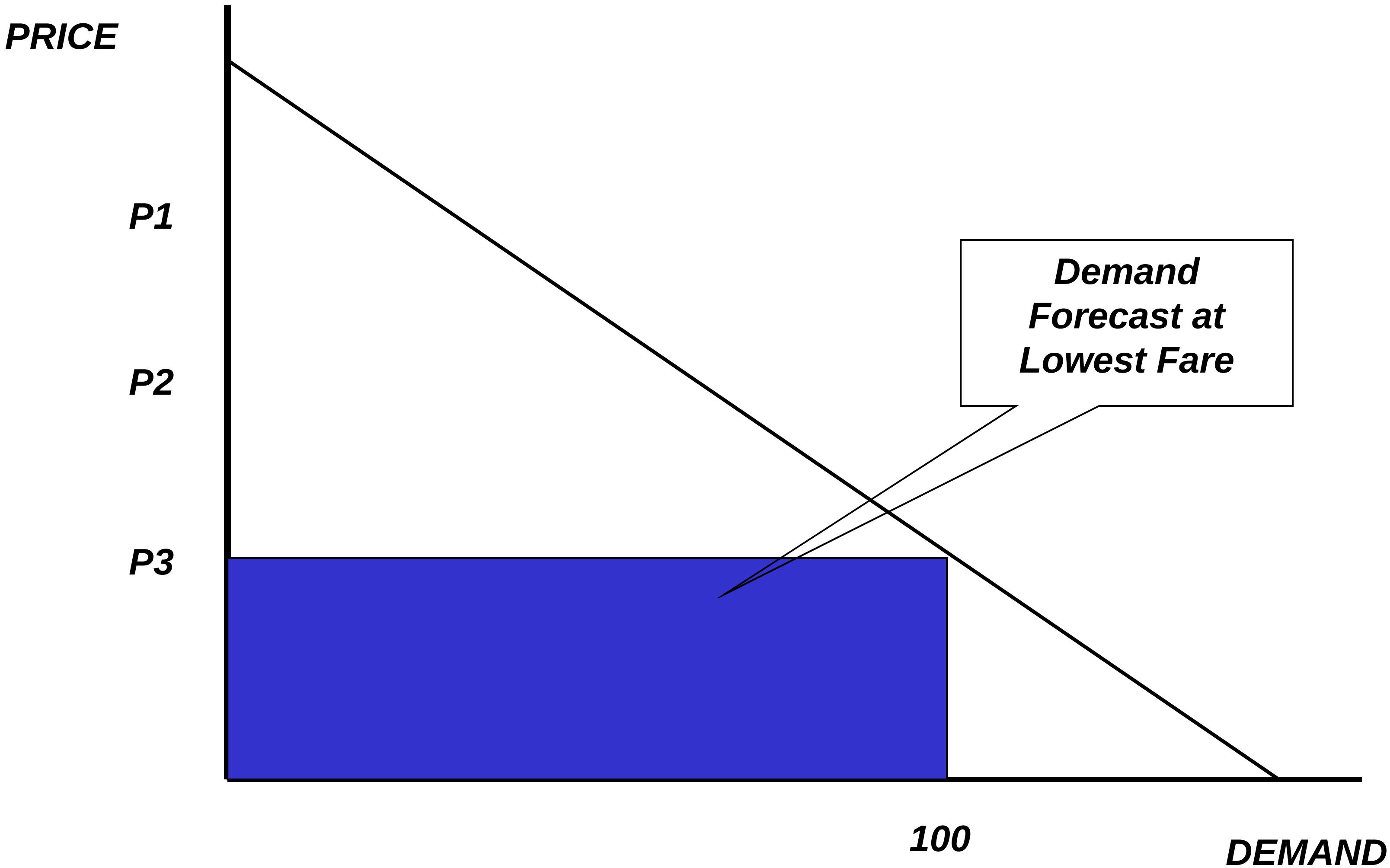
- **New RM models to account for sell-up and risk of buy-down in less restricted fare structures**
  - Hybrid Forecasting by demand segment
  - Marginal Revenue Optimization to account for buy-down
  - Estimation of WTP from historical booking data
  - Forecast Adjustment to account for competitor's availability

Q-Forecasting of Price-Oriented Demand

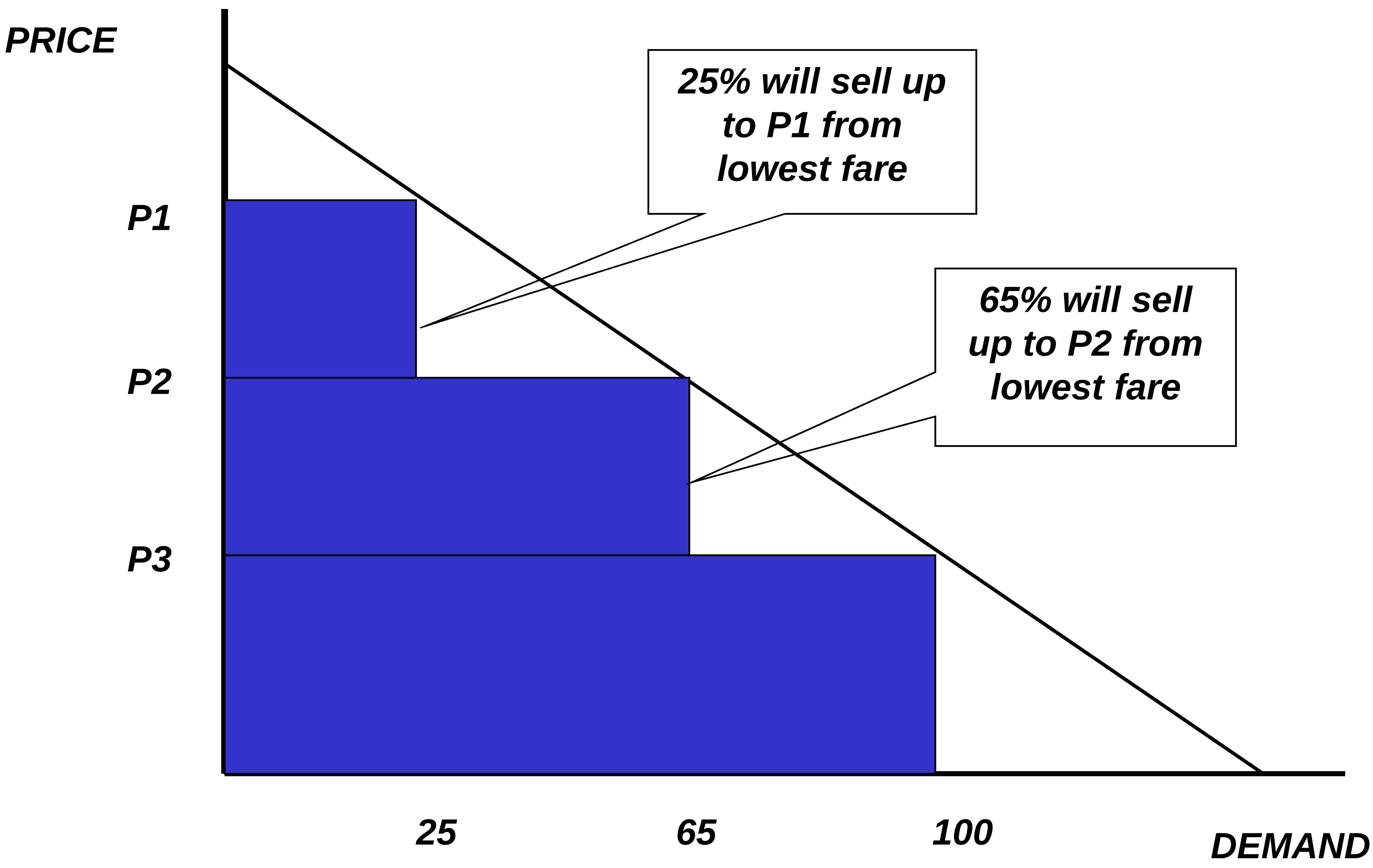
- **Q forecasting assumes fully undifferentiated fares**



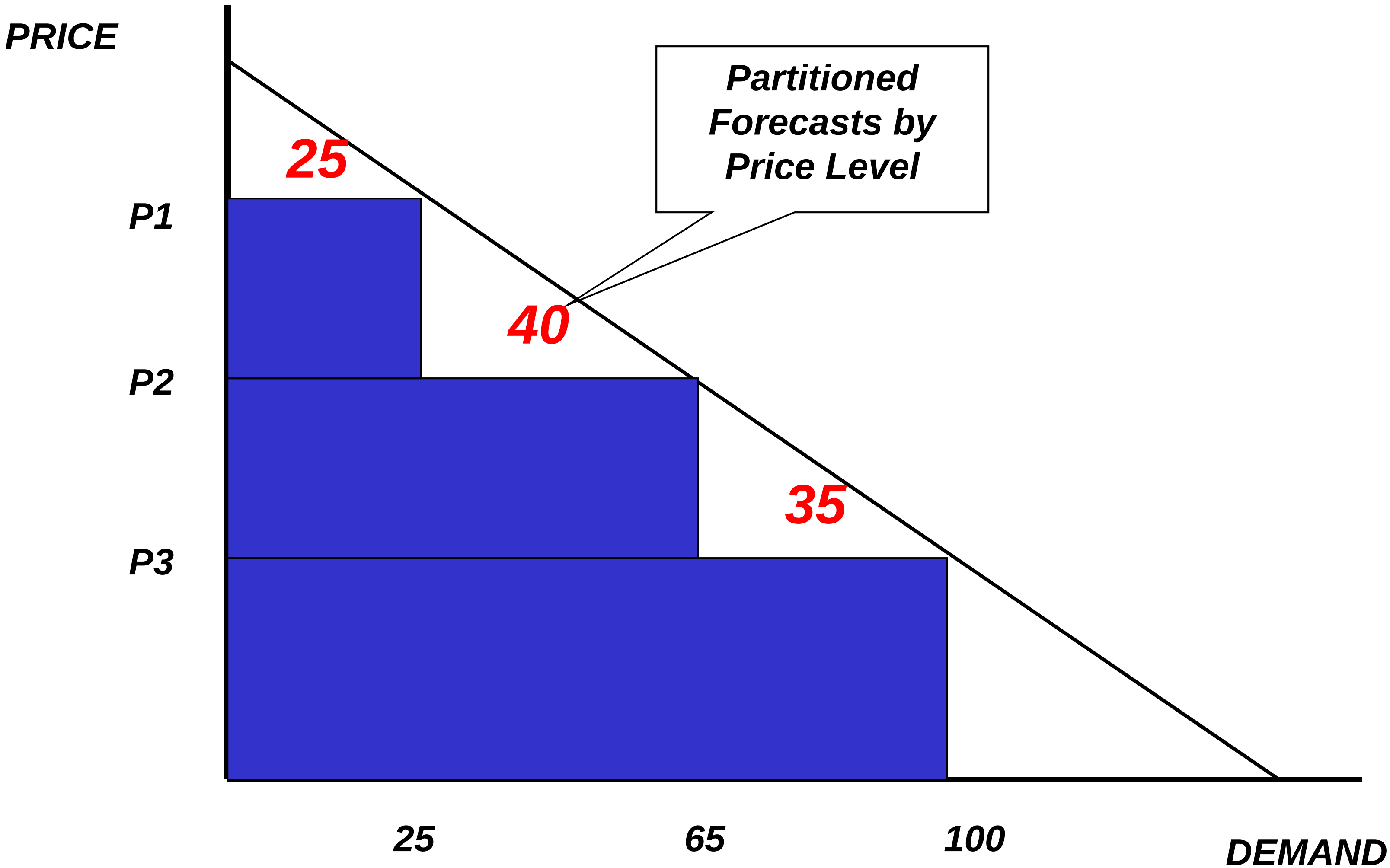
Generate Flight-Specific Forecast of Potential Demand at Lowest Fare



Apply Sell-up Estimates



Create "Partitioned" Forecasts by WTP



Hybrid Forecasting by Demand Segment

- ***Hybrid Forecasting*** **generates separate forecasts for price and product oriented demand:**

Price-Oriented:

- **Passengers will only purchase lowest available class**
- **Generate conditional forecasts for each class, given lower class closed**
- **Forecast demand by WTP**

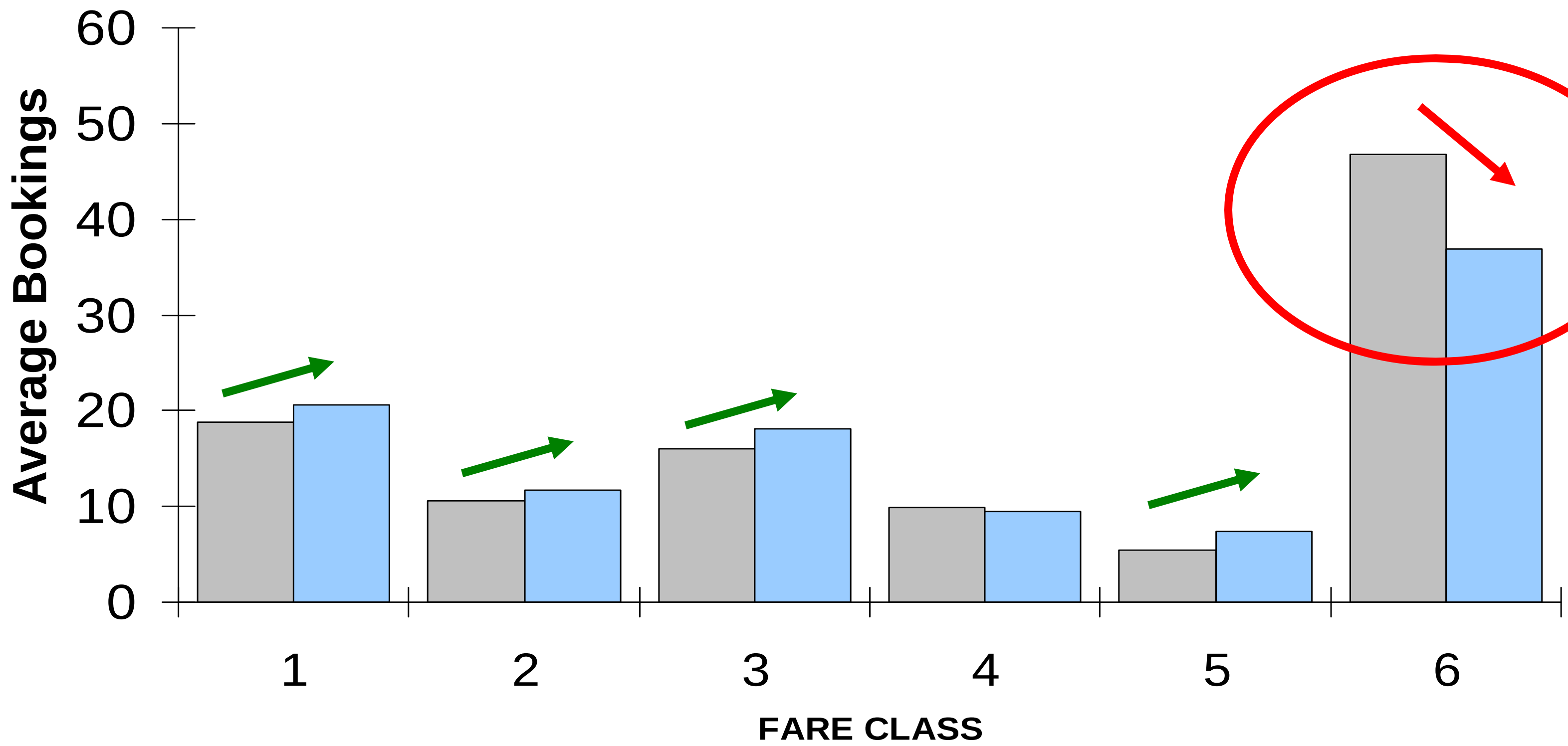
Product-Oriented:

- **Passengers will book in their desired class, based on product characteristics**
- **Use Traditional RM Forecasting by fare class**

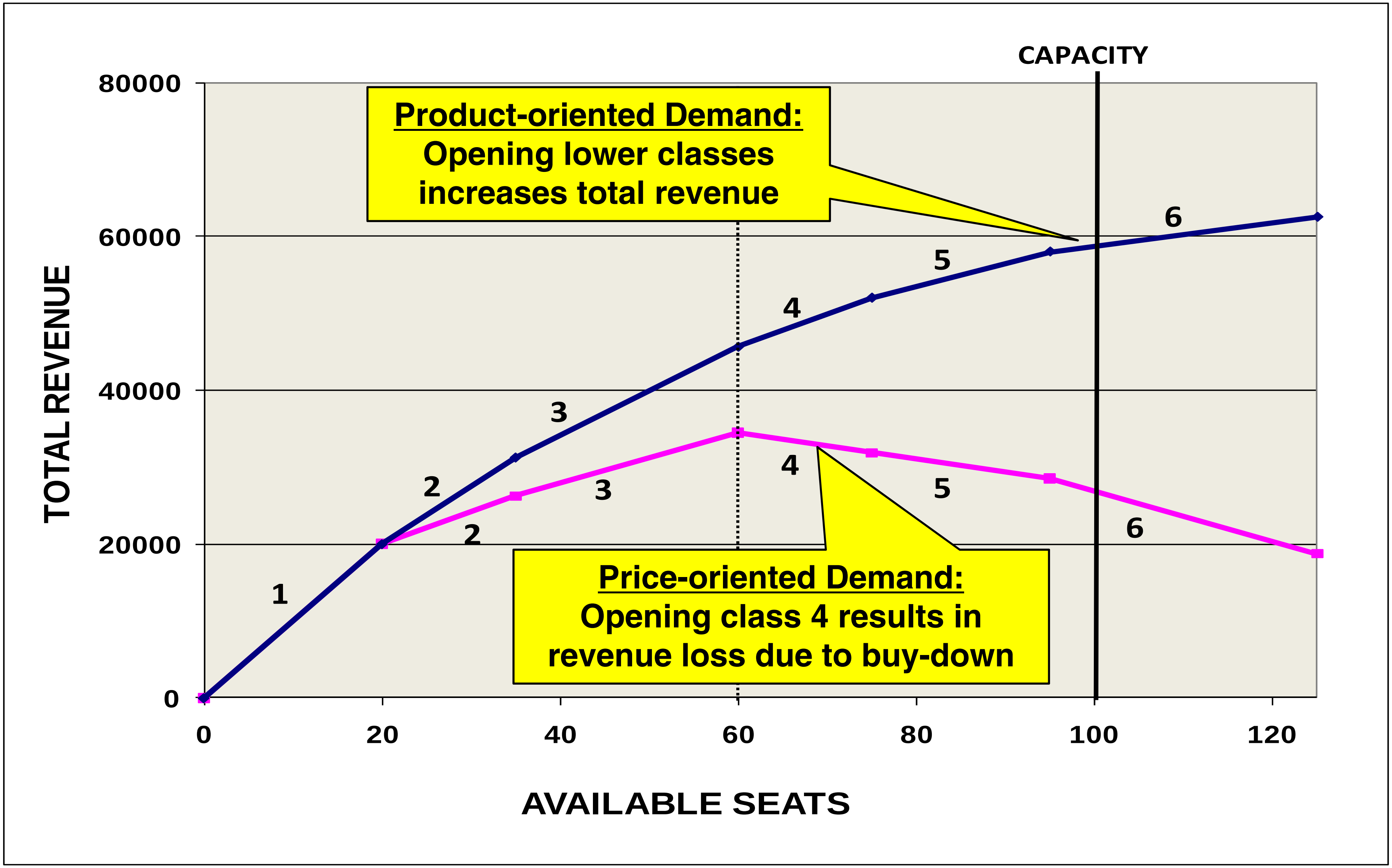
Forecast of total demand for itinerary/class

Hybrid Forecasting Increases Revenues by 2.2% by Changing Fare Class Mix

- Load Factor drops from 86.7% to 83.7%, but yield increases as fewer bookings are taken the lowest fare class.



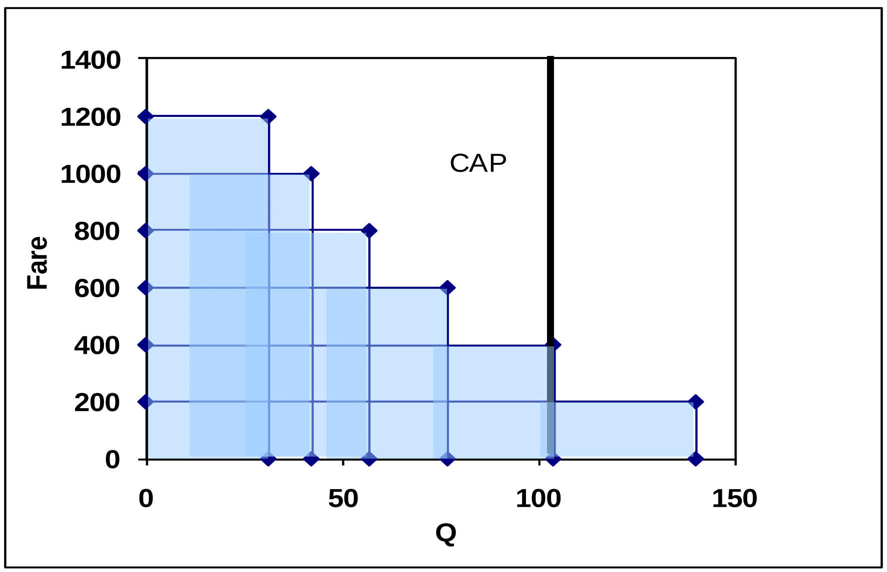
Marginal Revenue Optimization for Price-Oriented Demand

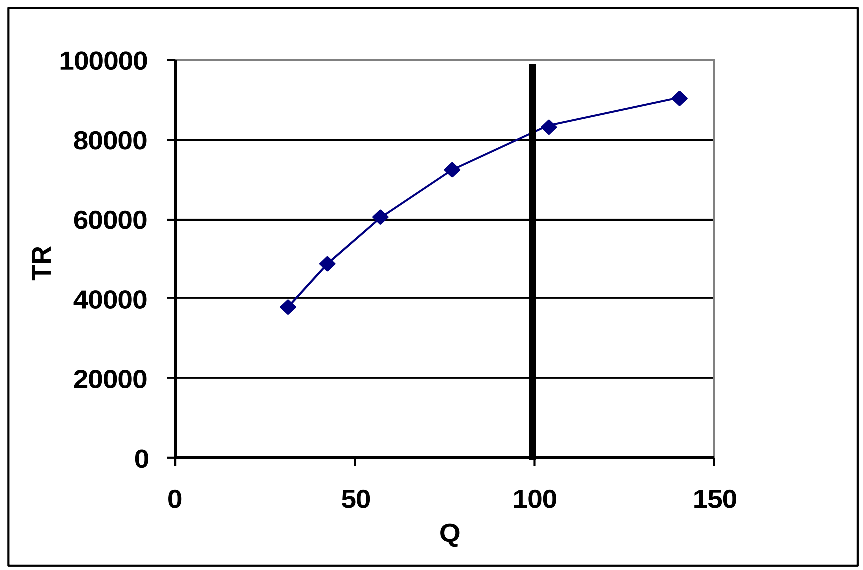


Fully Differentiated Fare Structure

Deterministic Demand , Single Leg

Fare Product	Fare fi	Demand di	Qi	Fully Differentiated TRi	MRi
1	$ 1,200	31.2	31.2	$ 37,486	$ 1,200
2	$ 1,000	10.9	42.1	$ 48,415	$ 1,000
3	$ 800	14.8	56.9	$ 60,217	$ 800
4	$ 600	19.9	76.8	$ 72,165	$ 600
5	$ 400	26.9	103.7	$ 82,918	$ 400
6	$ 200	36.3	140.0	$ 90,175	$ 200





Fully Undifferentiated Fare Structure

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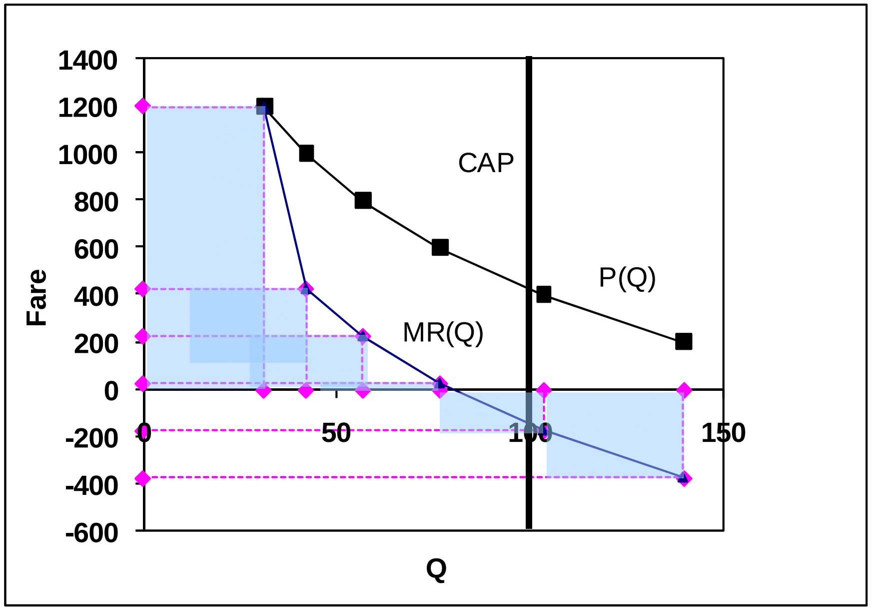
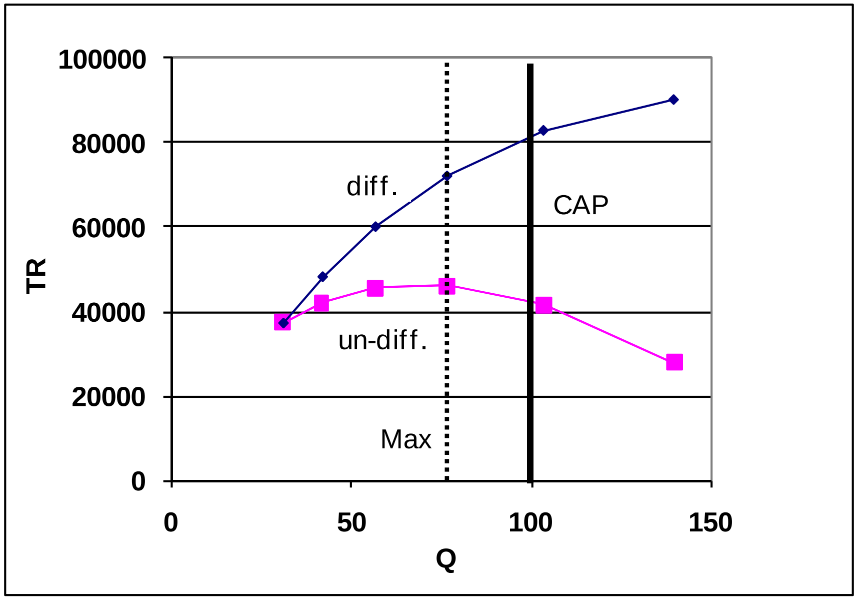
Fare Product	Fare fi	Demand di	Qi	Fully Undifferentiated TRi	MRi
1	$ 1,200	31.2	31.2	$ 37,486	$ 1,200
2	$ 1,000	10.9	42.1	$ 42,167	$ 428
3	$ 800	14.8	56.9	$ 45,536	$ 228
4	$ 600	19.9	76.8	$ 46,100	$ 28
5	$ 400	26.9	103.7	$ 41,486	$ (172)
6	$ 200	36.3	140.0	$ 28,000	$ (372)

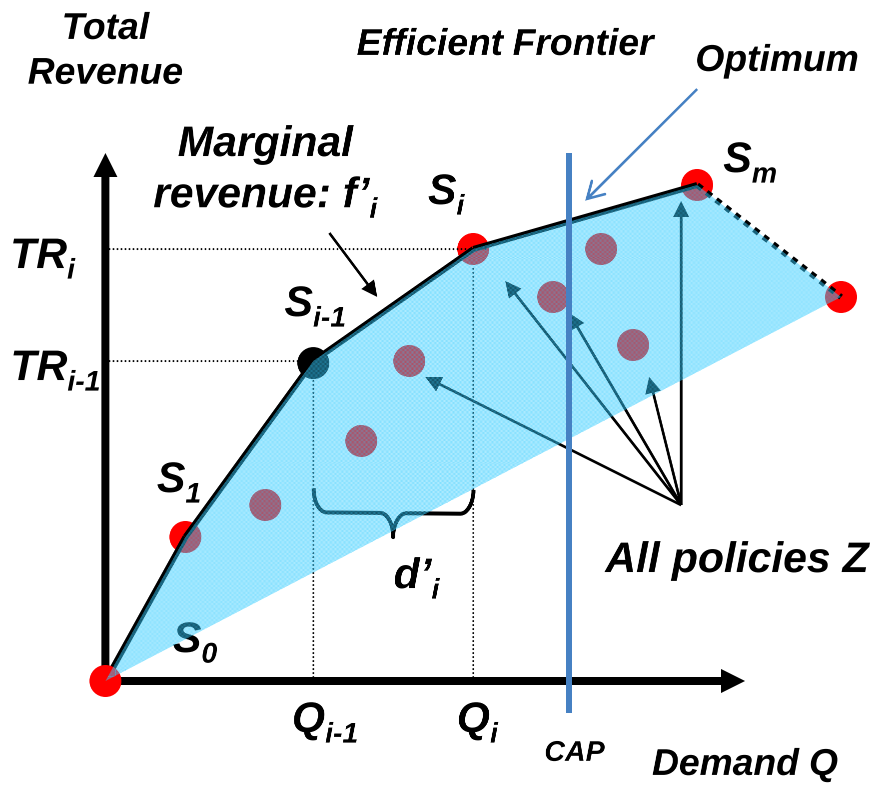




Optimization: General Formulation

Arbitrary Fare Structure, Deterministic Demand, Single Leg

Fare products	$f_j,\ j=1,\ldots,n$
Policy $Z \subseteq N$ (any set of open classes)	$\{\},\{1\},\{1,3\},\ldots$
Demand	$d_j(Z)$
Accumulated Dem.	$Q(Z)=\sum_{j\in Z} d_j(Z)$
Total Revenue	$TR(Z)=\sum_{j\in Z} d_j(Z)\, f_j$
Objective	$\max TR(Z)$ $s.t.\quad Q(Z)\le cap$



Marginal Revenue Transformation

Policies on the efficient frontier

Policy	Dem.	TR
	Q_1	TR_1
S_2	Q_2	TR_2
…	…	
S_m	Q_m	TR_m

→

Independent demand

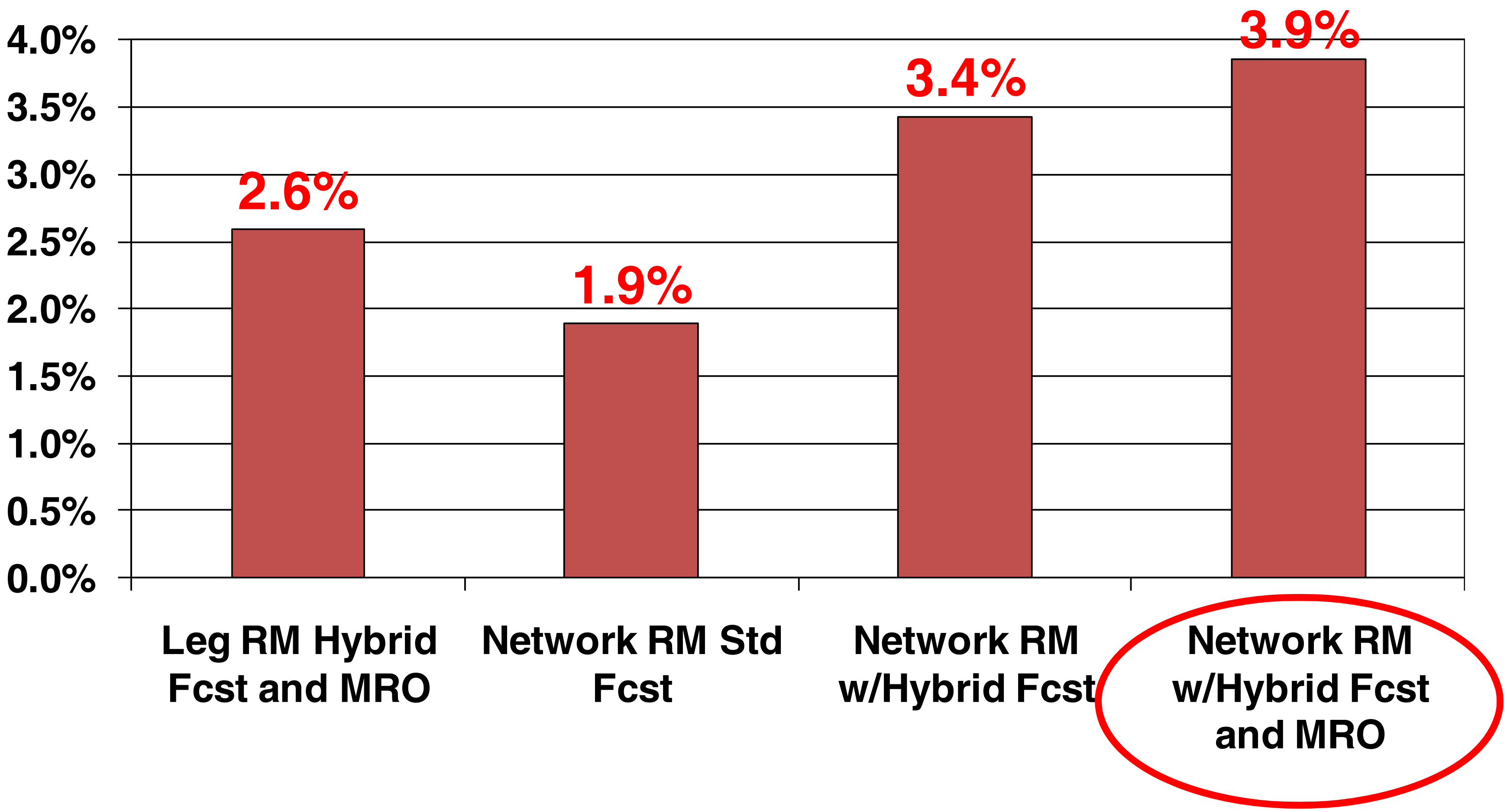
Partition Dem.	Adj. Fare
$d_1^{'} = Q_1$	$f_1^{'} = f_1$
$d_2^{'} = Q_2 - Q_1$	$f_2^{'} = (TR_2 - TR_1)/d_2^{'}$
…	…
$d_m^{'} = Q_m - Q_{m-1}$	$f_m^{'} = (TR_m - TR_{m-1})/d_m^{'}$

Marginal Revenue Transformation Theorem

✈ The transformed policies are independent.

✈ Optimization using the original fare structure and the marginal revenue transformed in policy space gives identical results.

Revenue Gains of Hybrid Forecasting and Marginal Revenue Optimization



Changing Fare Structures Caught Airline RM Systems by Surprise

- **Major shifts in airline pricing strategies since 2000**
  - Growth of low-fare airlines with relatively unrestricted fares
  - Matching by legacy carriers to protect market share
  - Increased consumer use of internet search engines
  - Greater consumer resistance to complex fare structures

- **With less restricted fares, traditional RM systems were no longer able to maximize revenues**
  - Spiral down contributed to dramatically lower yields and historical record load factors
  - Airline RM science was once again been replaced by human judgment and “rule-based” decision making