Ryan Louie

Sig Sys Problem Set 10

① $\dot{y} + y = x$

which can be simply transformed

$sY + Y = X$

$(s+1)\,Y = X$

$H(s) = \frac{Y}{X} = \frac{1}{s+1}$

??? ~~why not~~ take the inverse laplace transform!

because it is a step response

All zero

$\int_0^\infty \frac{1}{s+1} e^{-st}\, dt = \int_0^\infty H(s) e^{-st} dt = \int_0^\infty h(s) e^{-st} dt$

$\Rightarrow \frac{1}{(s+1)(s)}$

We can rearrange w/ fractions by parts

$\frac{1}{(s+1)(s)} = \frac{A}{s} + \frac{B}{s+1}$

$1 = A(s+1) + Bs$

when $s = -1$, $B = -1$

$s = 0$, $A = 1$

Thus,

$\frac{1}{(s+1)s} = \boxed{\frac{1}{s} - \frac{1}{s+1}}$

We want to apply the inverse laplace

Luckily we have some properties than can save from integrating by hand.

$u(t) \overset{\mathcal{L}}{\Longleftrightarrow} \frac{1}{s},\ Re\{s\} > 0, \quad e^{-at}u(t) \overset{\mathcal{L}}{\Longleftrightarrow} \frac{1}{s+a},\ Re\{s\} > -a$

$\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = u(t)$

$\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} = e^{-(1)t}u(t)$

$\mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{1}{s+1}\right\} = (1 - e^{-t})u(t) \overset{?}{=} y(t)$

Why is DC gain defined like this

(2) A) DC Gain $= \cancel{\lim_{s\to 0} H(s)} = \lim_{s\to 0} \frac{Y(s)}{Y_{sp}(s)}$

If $K(s) = K_1/s$, for any general $H(s)$

$$\frac{Y(s)}{Y_{sp}(s)} = \frac{K_1/s \cdot H(s)}{1 + K_1/s \cdot H(s)}$$

$$\lim_{s\to 0} \frac{Y(s)}{Y_{sp}(s)} = \lim_{s\to 0} \frac{\overbrace{K_1/s\ H(s)}^{\to\infty}}{1 + K_1/s\ H(s)}$$

$$= \frac{\infty}{1+\infty}$$

$$= \frac{\infty}{\infty}$$

$$= \boxed{1}$$

This is independent of the value of K_1

B) Assume $H(s) = \frac{1/\tau}{s + 1/\tau}$

$$\frac{Y(s)}{Y_{sp}(s)} = \frac{KH}{1+KH} = \frac{K \frac{1/\tau}{s+1/\tau}}{1 + K\frac{1/\tau}{s+1/\tau}} = \boxed{\frac{K/\tau}{s + (K+1)/\tau}} = \boxed{\frac{K_1/s\tau}{s + (K_1/s + 1)/\tau}}$$

still in integral control space $K = K_1/s$

O zeros transfer = 0

x poles transfer ∞

If $K \gg 1/\tau$

$\frac{Y(s)}{Y_{sp}(s)} = \infty$ when $K_1/s\tau = \infty$ or $s + (K_1/s + 1)/\tau = 0$

~~$s = 0$~~

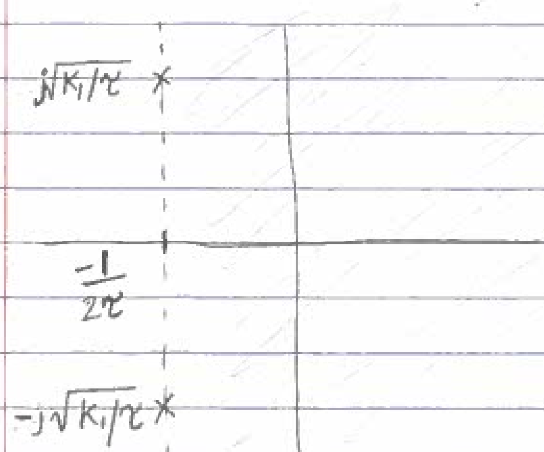
$$\cancel{s^2 + \frac{K_1}{\tau} + \frac{s}{\tau} = 0}$$

$$s^2 + \frac{1}{\tau}s + \frac{K_1}{\tau} = 0$$

$$s = \frac{-1/\tau \pm \sqrt{1/\tau^2 - 4K_1/\tau}}{2}$$

Since $1/\tau^2$ is probs small,

$$s = -\frac{1}{2\tau} \pm \frac{\cancel{2}j\sqrt{K_1/\tau}}{\cancel{2}}$$



$$☺ = \frac{-0.01 \pm \sqrt{(.01)^2 - 4}}{2}$$

step response $= \mathcal{L}^{-1}\{\frac{1}{s} H(s)\}$

(4) A. $\frac{1}{s} H(s) = \frac{1}{(s)} \frac{1}{(s^2 - 0.01s + 1)} = \frac{A}{s} + \frac{B}{(s^2 - 0.01s + 1)}$

$1 = A(s^2 + 0.01s + 1) + Bs$

when $s=0$, $A = 1$;

I used MATLAB step(sys) & pzplot(sys) to create step response & pole zero maps.

B. $\frac{Y}{Y_{sp}} = \frac{KH}{1+KH} = \frac{K \cdot 1/(s^2 - 0.01s + 1)}{1 + K \cdot 1/(s^2 - 0.01s + 1)}$

~~let $R = s^2 - 0.01s + 1$. Then $K/R/(1 + K/R) =$~~

$\frac{K}{K+R}$ wolfram alpha $\boxed{\frac{K}{K + s^2 - 0.01s + 1}}$

~~see plot digitally in ipynb~~

C. Integral Control

where $K = K_i/s$

$$\frac{Y}{Y_{sp}} = \frac{KH}{1+KH} \qquad \text{let } R = s^2 - 0.01s + 1$$

$$= \frac{K/R}{1+K/R}$$

$$= \frac{K_i/Rs}{1+K_i/Rs} \qquad \text{Thanks Wolfram}$$

$$= \frac{K_i}{K_i + Rs}$$

$$= \boxed{\frac{K_i}{K_i + s^3 - 0.01s^2 + s}}$$

D. Derivative Control

where $K = s \cdot K_d$ let $R = s^2 - 0.01s + 1$

$$\frac{Y}{Y_{sp}} = \frac{K/R}{1+K/R}$$

$$= \frac{K_i \cdot s/R}{1+K_i \cdot s/R} \qquad \text{Thank Wolfram}$$

$$= \frac{K_s}{K_s + R}$$

$$= \frac{K \cdot s}{K \cdot s + s^2 - 0.01s + 1}$$