CSC 418
Assignment 2

1a let the 2D affine transform be

$$T = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \qquad T \begin{bmatrix} a_1x & a_2x & a_3x \\ a_1y & a_2y & a_3y \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} b_1x & b_2x & b_3x \\ b_1y & b_2y & b_3y \\ 1 & 1 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} b_1x & b_2x & b_3x \\ b_1y & b_2y & b_3y \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a_1x & a_2x & a_3x \\ a_1y & a_2y & a_3y \\ 1 & 1 & 1 \end{bmatrix}^{-1}$$

the a matrix must be invertable, which means the colomns of a must be independent. which means a_1 a_2 a_3 must not all lie on the same line.

1b) for homography 2D, 3 linearly independent point mappings are required

let T be $\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix}$ centroid $p = \begin{bmatrix} \frac{x_1+x_2+x_3}{3} \\ \frac{y_1+y_2+y_3}{3} \\ 1 \end{bmatrix}$

point P_1 be $\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$ $P_2 = \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}$ $P_3 = \begin{bmatrix} x_3 \\ y_3 \\ 1 \end{bmatrix}$

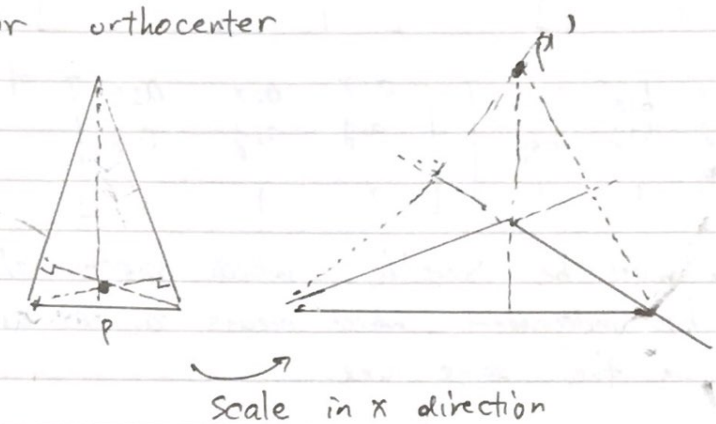
$$TP = \begin{bmatrix} a\frac{x_1+x_2+x_3}{3} + b\frac{y_1+y_2+y_3}{3} + c \\ d\frac{x_1+x_2+x_3}{3} + e\frac{y_1+y_2+y_3}{3} + f \\ 1 \end{bmatrix} \qquad TP_1 = \begin{bmatrix} ax_1 + by_1 + c \\ dx_1 + ey_2 + f \\ 1 \end{bmatrix}$$

$$TP_2 = \begin{bmatrix} ax_2 + by_2 + c \\ dx_2 + ey_2 + f \\ 1 \end{bmatrix} \qquad TP_3 = \begin{bmatrix} ax_3 + by_3 + c \\ dx_3 + ey_3 + f \\ 1 \end{bmatrix}$$

$$T\left(\frac{P_1+P_2+P_3}{3}\right) = \frac{TP_1 + TP_2 + TP_3}{3}$$

$\therefore$ centroid in invariant

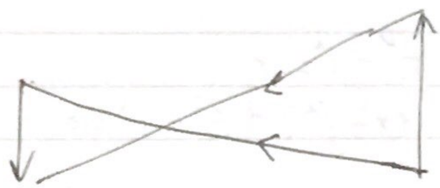
for orthocenter



$Tp \neq p'$ if T is affine

Otho center is not invariant

2a)



light travels in a straight line therefore the image is inverted

2b) let $w = c - p$, $v = (c-p) \times u$

$$T = \begin{bmatrix} v_x & u_x & w_x \\ v_y & u_y & w_y \\ v_z & u_z & w_z \\ 0 & 0 & 1 \end{bmatrix} \qquad P_{camera} = T^{-1} P_{world}.$$

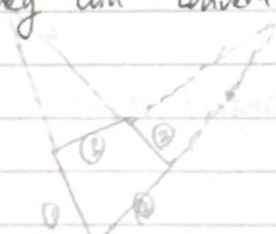
2c.)

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$$

then 2D point $p = \begin{bmatrix} \frac{xd}{z} \\ \frac{yd}{z} \end{bmatrix}$

2D) they must have the same z-value.

2e) if the vectors do not have the same z-value. then if they are parallel they will converge to one point if not they will convert to different points.



① and ② are parrellel in 3D
③ and ④ are parrallel in 3D

3a) $f(x,y,z) = (R - (x^2+y^2)^{\frac{1}{2}})^2 + z^2 + r^2 = 0$

$= R^2 - 2R(x^2+y^2)^{\frac{1}{2}} + (x^2+y^2) + z^2 + r^2 = 0$

$\vec{n} = \nabla f = (-R(\tfrac{1}{2})(x^2+y^2)^{-\frac{1}{2}}(2x) + 2x,\ -2R(\tfrac{1}{2})(x^2+y^2)(2y) + 2y,\ 2z)$

$= \left(\frac{-2Rx}{\sqrt{x^2+y^2}} + 2x,\ \frac{-Ry}{\sqrt{x^2+y^2}} + 2y,\ 2z \right)$

3b) $f(x', y', z') = \vec{n}(P - P_0)$

$= \left(\frac{2Rx}{\sqrt{x^2+y^2}} + 2x\right)(x'-x) + \left(\frac{2Ry}{\sqrt{x^2+y^2}} + 2y\right)(y'-y)$

$+ 2z \cdot (z' - z)$

3c) let $x = R\cos\lambda$, $y = R\sin\lambda$, $z = r$

$f(x,y,z) = (R - \sqrt{R\cos^2(\lambda) + R\sin^2(\lambda)})^2 + r^2 - r^2 = 0$

$\therefore$ point lies on plane

3d) $\vec{t} = q'(\lambda) = (-R\sin\theta,\ R\cos\theta,\ 0)$

3e) need to show $\vec{t} = q'(\lambda)$ and $\vec{n}$ of plane are orthogonal

$\cancel{\vec{t} \cdot \vec{n} = \frac{2R^2\sin\theta}{R} - 2R\sin\theta + \frac{2R^2\cos\theta}{R} \pm 2R\cos\theta}$

$\vec{t} \cdot \vec{n} = \left(\frac{-2R \cdot R\cos\lambda}{R} + 2R\cos\lambda\right)(-R\sin\lambda) + \left(\frac{-R \cdot R\sin\lambda}{R} + 2R\sin\lambda\right)(R\cos\lambda) + zero$

$= 0 + 0 + 0 = 0$

$\therefore$ $\vec{t} = q(\lambda)$ lies on the tangent plane

4a No.

4b. on next page

4 c)
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walk (node)
    if ( node-is-leaf)
        if ( box_frustum_intersection )
            add-to-list-of-oaks
            return.
    else
        walk ( node -> left )
        walk ( node -> right )
```

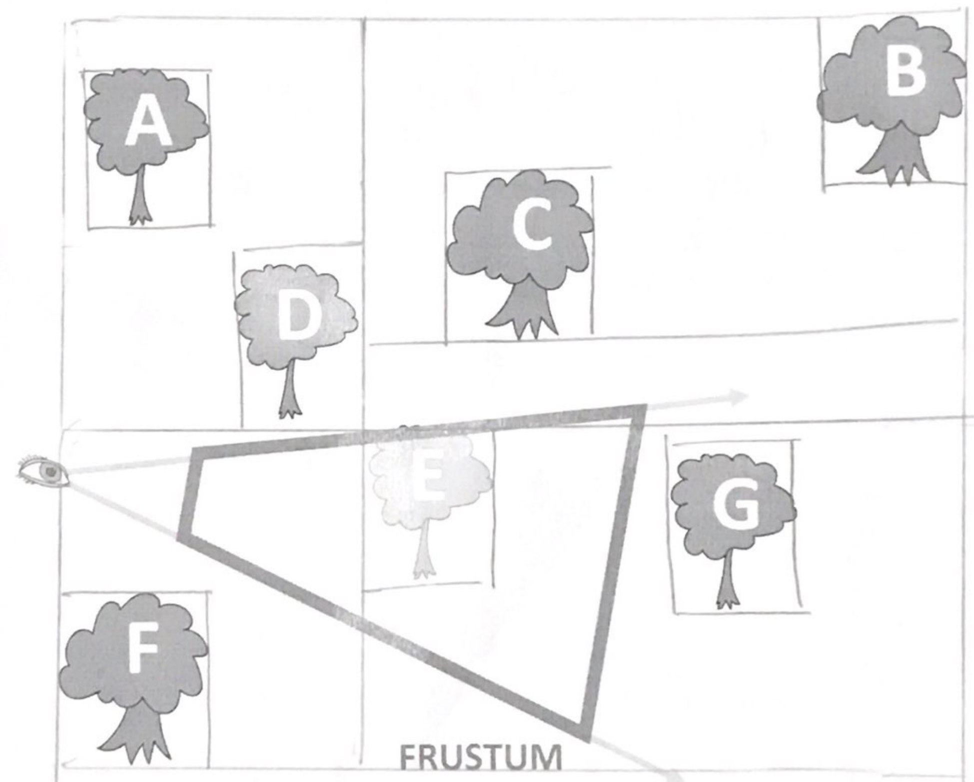

Figure 2: A 2D scene composed of seven oak trees viewed within a frustum (blue parallelogram).