Transportation models:

» The Transportation problems are one of the types of the LPP in which objective is to transport various quantities of a single homogenous commodity to different destinations in such a way that the total transportation cost is a min.

methods for initial basic feasible solutⁿ: (IBFS):

i) Northwest corner method (NWCM)
ii) lowest cost Entry method (LCEM)
iii) Vogdes approximation method (VAM)

i) NWCM → steps:

→ Construct an m×n matrix computed with rows & coloumns.
→ Indicate the row totals and coloumn totals at the end.
→ Starting with (1,1) cell at NWCM of the matrix
→ Adjust the supply and demand no.s in the respective rows and coloums allocations.
→ Continue the procedure until the total available quantity is fully allocated to the cells as required.

1) Solve the following Transporation problem by NWCM.

			Supply
7	6	9	20
5	7	3	28
4	5	8	17
Demand 21	25	19	

			Supply
(20) 7	8	9	20
5	(25) 7	(2) 3	28 27 (2)
4	5	(17) 8	17
21 1	25	19 17	65 55

IBFS condition: $m+n-1 = 5$ (allocations).

min Transportation cost by NWCM:

$= 20\times7 + 1\times5 + 25\times7 + 2\times3 + 17\times8$

$= \underline{462}$

(2)

				supply
19	30	50	10	7
70	30	40	60	9
40	8	70	20	18
demand 5	8	7	14	

(5) 19	(2) 30	~~50~~	~~10~~	~~7~~ 2
~~70~~	(6) 30	(3) 40	~~60~~	~~9~~ 3
~~40~~	~~8~~	(11) 70	14 20	~~18~~ 14
5	~~8~~ 6	~~7~~ 4	14	34 / 34

$m+n-1 = 6$

min TPC: $2\times30 + 5\times19 + 6\times30 + 3\times40 + 4\times70 + 14\times20$

$= 1,015$

ii) LCEM → steps: matrix method

→ Select the cell with lowest transportation cost among all the rows or coloumns of the TT.

→ If the min cost is not same then select any cell with the lowest cost

→ allocate as many units as possible to the cell in step 1 and Eliminate the rows in which either demand or supply Exhausted.

→ Adjust the demand and supply for the next allocation

→ repeat steps one & 4 for the reduced table until the entire demand & supply are exhausted to fill the demand or supply at diff destinations.

eg, Solve TP by LCM:

			supply
7	(20) 6	9	20
(4) 5	(5) 7	(19) 3	~~28~~ 9 5
(17) 4	5	8	17
demand 21 (4)	25 5	19	65 / 65

			supply
7	6	9	20
5	7	3	28
4	5	8	17
demand 21	25	19	

$m+n-1 = 5$

min TPC: $20\times6 + 4\times5 + 5\times7 + 19\times3 + 17\times4$
$= 380$.

2) Capacity

19	30	50	(7) 10	7
(2) 70	30	(7) 40	60	9 2
(3) 40	(8) 8	70	(7) 20	18 10 (3)
5 2	8	7	14 (7)	34 / 34

$m+n-1 = 6$.

min TPC: $7\times10 + 2\times70 + 7\times40 + 3\times40 + 8\times8 + 7\times20$
$= 814/-$

→ VAM → steps:

→ Construct the cost, requirement and availability matrix.

→ Compute a penalty for each row and column in the T.T.

→ The penality for a given row and coloumn is the diff between the smallest cost and next smallest cost element in that particular row or column.

→ Identify the row and coloumn with Largest penalty

→ Continue the procedure until the total available quantity is fully allocated to the cells as required.

1) Solve the TP by VAM:

			supply
7	6	9	20
5	7	3	28
4	5	8	17
dem 21	25	19	

7	(20) 6	9	20	1	1	1
(9) 5	7	(19) 3	~~28~~ (9)	2	2	4
(12) 4	(5) 5	8	17 (5)	1	1	1
~~21~~ 12	25	19				
1	1	5				
1	1					
3	1					

$m+n-1 = 5$

min TPC: $20\times6 + 9\times5 + 19\times3 + 12\times4 + 5\times5$

$= 295$

02/02/21:

Q) Solve the following prob by VAM.

					availability
3	4	6	8	8	20
2	10	0	5	8	30
7	11	20	40	3	15
1	0	9	14	16	13
Requirement 40	6	8	18	6	78/78

(20) 3	4	6	8	8	20	1	1	1	5	5
(4) 2	10	(8) 0	(18) 5	8	~~30~~ 22	2	3	3	3	3
(9) 7	11	20	40	(6) 3	15 9	4	4	4	3	3 4
(7) 1	(6) 0	9	14	16	~~13~~ 7	1	1	1	13	13
(9) ~~24~~ ~~31~~ 40	6	8	18	6	78/78					
1	4	6	3	5						
1	4		3	5						
1/1	4		3/2							

m+n-1 = 8

TPC : $20\times3 + 4\times2 + 8\times0 + 18\times5 + 9\times7 + 6\times3 + 7\times1 + 0$

$= 246$ ₹

⇒ Unbalanced Transportation problem:

If the sum of the supplies of all the sources is not equal to the sum of the demands of all the destinations then the problem is known as unbalanced Transportation problem.

Q) Solve the Transportation problem by NWCM.

Plant \ market	A	B	C	Availability
x	11	21	16	14
y	7	17	13	26
z	11	23	21	36
Requirement	18	28	25	76 / 71

It is an unbalanced problem so, it should be balanced by introducing dummies.

	(14) 11	21	16	0	~~14~~
	(4) 7	(22) 17	13	0	~~26~~ (22)
	11	(6) 23	(25) 21	(5) 0	36 (30) 5
	~~18~~ 4	28 (6)	25	5	76 / 76

TCS : 1219 ₹

Assignment problem is a special type of LPP. it deals in allocating the various items to various activities on one to one basis in such a way that the time or cost involved is minimized & sales or profits is maximised.

Hungarian method / RC method.

i) Subtract the min cost of each row of the cost matrix from all the elements of the respective row.

ii) modify the resulting matrix by subtracting min cost of each coloumn from all the elements of the respective coloumn obtaining the starting matrix.

Q)

120	100	80
80	90	110
110	140	120

$\xrightarrow{R}$

40	20	0
0	10	30
0	30	10

⇓ C

40	10	$\boxed{0}$
0	$\boxed{0}$	30
$\boxed{0}$	20	10

min assignment time : 110 + 90 + 80
= 280

Q, solve the following assignment prob

10	12	19	11
5	10	7	8
12	14	13	11
8	15	11	9

⇒

0	2	9	1
0	5	2	3
1	3	2	0
0	7	3	1

	1	2	3	4
A	$\cancel{0}$	$\boxed{0}$	7	1
B	$\cancel{0}$	3	$\boxed{0}$	3
C	1	1	$\cancel{0}$	$\boxed{0}$
D	$\boxed{0}$	5	1	1

min ass time : 8 + 12 + 7 + 11 = 38.

m = type of the problem = 4

n = no. of assignment = 4.

m = n.

Assignment schedule:

D → 1, A → 2, B → 3, C → 4

Q. Consider the problem of assigning 5 jobs to 5 persons. The assignment costs are given as follows:

	1	2	3	4	5
A	8	4	2	6	1
B	0	9	5	5	4.
C	3	8	9	2	6
D	4	3	1	0	3
E	9	5	8	9	5.

calculate min assignment of the given prob:

7	3	1	5	0
0	9	5	5	4
1	6	7	0	4
4	3	1	0	3
4	0	3	4	0

⇒

	1	2	3	4	5
A	7	3	$\cancel{0}$	5	$\boxed{0}$
B	$\boxed{0}$	9	4	5	4
C	1	6	6	$\boxed{0}$	4
D	4	3	$\boxed{0}$	$\cancel{0}$	3
E	4	$\boxed{0}$	2	4	$\boxed{\cancel{0}}$

ass time = 0 + 5 + 1 + 1 + 2 = 9

Schedule = 5 → A, 1 → B, 4 → C, 3 → D, 2 → E.

9)

$$\begin{bmatrix} 6 & 5 & 8 & 11 & 16 \\ 1 & 13 & 16 & 1 & 10 \\ 16 & 11 & 8 & 8 & 8 \\ 9 & 14 & 12 & 10 & 16 \\ 10 & 13 & 11 & 8 & 16 \end{bmatrix}$$

$\Downarrow$

$$\begin{bmatrix} 1 & 0 & 3 & 6 & 11 \\ 0 & 12 & 15 & 0 & 9 \\ 8 & 3 & 0 & 0 & 0 \\ 0 & 5 & 3 & 1 & 7 \\ 2 & 5 & 3 & 0 & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & \boxed{0} & 3 & 6 & 11 \\ \cancel{0} & 12 & 15 & \cancel{0} & 9 \\ 8 & 3 & \boxed{0} & \cancel{0} & \cancel{0} \\ \boxed{0} & 5 & 3 & 1 & 7 \\ 2 & 5 & 3 & \boxed{0} & 8 \end{bmatrix}$$

Horizontal - verticle method.

$$\begin{bmatrix} 1 & 0 & 3 & 6 & 11 \\ 0 & 12 & 15 & 0 & 9 \\ 8 & 3 & 0 & 0 & 0 \\ 0 & 5 & 3 & 1 & 7 \\ 2 & 5 & 3 & 0 & 8 \end{bmatrix}$$

5 - H, V
4 - H, V
3 - H, V
2 - H, V
1 - H, V

steps:

i. select min Element those lines are not covered.
  • min = 3

ii. Add at junctions.

iii. line → as it is

iv. not line → subtract

$$\begin{bmatrix} 3 & \boxed{0} & 3 & 9 & 11 \\ \boxed{0} & 9 & 12 & \cancel{0} & 6 \\ 11 & 3 & \cancel{0} & 3 & \boxed{0} \\ \cancel{0} & 2 & \boxed{0} & 1 & 4 \\ 2 & 2 & \cancel{0} & \boxed{0} & 5 \end{bmatrix}$$

Ass time: $5+1+12+8+8$
$= 34$.

Unbalanced assignment problem:

=> whenever the pay of matrix of assignment problem is not a square matrix i.e., no. of rows are not equal to no. of coloumns. The assignment prob is called unbalanced assignment prob. in such cases dummy rows or coloumns are added in the matrix to make it a square matrix.

Q, A company is faced the problem of assigning 6 different machines to 5 different jobs. The costs are estimated and given below Jobs.

machines	J_1	J_2	J_3	J_4	J_5
m_1	6	2	5	2	6
m_2	2	5	8	7	7
m_3	7	8	6	9	8
m_4	6	2	3	4	5
m_5	9	3	8	9	7
m_6	4	7	4	6	8

J_1	J_2	J_3	J_4		
6	2	5	2	6	0
2	5	8	7	7	0
7	8	6	9	8	0
6	2	3	4	5	0
9	3	8	9	7	0
4	7	4	6	8	0

=>

4	$\boxed{0}$	2	$\cancel{0}$	1	$\cancel{0}$
$\boxed{0}$	3	5	5	2	$\cancel{0}$
5	6	3	7	3	$\boxed{0}$
4	$\cancel{0}$	$\boxed{0}$	2	$\cancel{0}$	$\cancel{0}$
7	1	5	7	2	0
2	5	1	4	3	0

output
13
23
30
39
47

Horizontal vertical method

4	0	2	0	1	0
5	3	5	5	2	0
5	6	3	7	3	0
4	0	0	2	0	0
7	1	5	7	2	0
2	5	1	4	3	0

6 → 4

5 -

min = 1

5	~~0~~	2	$\boxed{0}$	1	1
$\boxed{0}$	2	4	4	1	~~0~~
5	5	2	6	2	$\boxed{0}$
5	~~0~~	~~0~~	2	$\boxed{0}$	1
7	$\boxed{0}$	4	6	1	~~0~~
2	4	$\boxed{0}$	3	2	~~0~~

assg time =

$= 2+2+0+5+3+4$

$= 16$.

maximization case in assignment problem:

In some cases, the pay-of Elements of the assignment problem may represent profits instead of costs so that the objective will be maximize total revenue or profit. The problem of maximization can be converted into minimized case by selecting the largest Element among all Elements of the profit matrix and then subtracting it from all other Elements in the matrix.

Q) 5 jobs are to be processed and 5 machines available. Any machine can process any job with resulting profit (in rupees) as follows

	A	B	C	D	E
1	32	38	40	28	40
2	40	24	28	21	36
3	41	27	33	30	37
4	22	38	41	36	36
5	29	33	40	35	39

what is the max profit that may be expected if an optimum assignment is made.

max element = 41

9	3	1	13	1
1	17	13	20	5
0	14	8	11	4
19	3	0	5	5
12	8	1	6	2

min element = 1

$\xRightarrow{R}$

8	2	0	12	0
0	16	12	19	4
0	14	8	11	4
19	3	0	5	5
11	7	0	5	1

$\Downarrow$ C

8	~~0~~	~~0~~	7	$\boxed{0}$
$\boxed{0}$	14	12	14	4
~~0~~	12	8	6	4
19	1	~~0~~	$\boxed{0}$	5
11	5	$\boxed{0}$	~~0~~	1

9	0	1

⇒ min → introduce dummies in questioned box
max → introduce dummies in the box we get.

Travelling salesman problem: TSP is similar to assignment problem except that in the formal there is an additional ~~restr~~ destruction that a salesman who starts from his home city visit each city only once and that returns to his home city

1. Solve the following TSP so as to minimize the cost / cycle

$$
\text{from}\quad
\begin{array}{c|ccccc|}
 & A & B & C & D & E \\
\hline
A & - & 3 & 6 & 2 & 3 \\
B & 3 & - & 5 & 2 & 3 \\
C & 6 & 5 & - & 6 & 4 \\
D & 2 & 2 & 6 & - & 6 \\
E & 3 & 3 & 4 & 6 & - \\
\hline
\end{array}
$$

(TO over columns A–E)

$$
\begin{bmatrix}
\infty & 3 & 6 & 2 & 3 \\
3 & \infty & 5 & 2 & 3 \\
6 & 5 & \infty & 6 & 4 \\
2 & 2 & 6 & \infty & 6 \\
3 & 3 & 4 & 6 & \infty
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
\infty & 1 & 4 & 0 & 1 \\
1 & \infty & 3 & 0 & 1 \\
2 & 1 & \infty & 2 & 0 \\
0 & 0 & 4 & \infty & 4 \\
0 & 0 & 1 & 3 & \infty
\end{bmatrix}
$$

$$\Downarrow$$

$$
\begin{bmatrix}
\infty & 1 & 3 & \boxed{0} & 1 \\
1 & \infty & 2 & \cancel{0} & 1 \\
2 & 1 & \infty & 2 & \boxed{0} \\
\boxed{0} & \cancel{0} & 3 & \infty & 4 \\
\cancel{0} & \cancel{0} & \boxed{0} & 3 & \cancel{\infty}
\end{bmatrix}
$$

$m = 5$; $n = 4$

$m \neq n$. min Elem = 1

$m = 5$; $n = 5$.

$m = n$:

$$
\begin{bmatrix}
\infty & \boxed{0} & 2 & \boxed{0} & \cancel{0} \\
\boxed{0} & \infty & 1 & \cancel{0} & \cancel{0} \\
2 & 1 & \infty & 3 & \boxed{0} \\
\cancel{0} & \cancel{0} & 3 & \infty & 4 \\
\cancel{0} & \cancel{0} & \boxed{0} & 4 & \infty
\end{bmatrix}
$$

assignment time: $3 + 3 + 2 + 4 + 4$

$\therefore$ The optimum assignment is $A \to B \to D \to A$ and not feasible

	A	B	C	D	E
A	∞	$\cancel{25}$	2	$\boxed{0}$	$\cancel{0}$
B	$\cancel{\infty}$	∞	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$
C	2	1	∞	3	$\boxed{0}$
D	$\cancel{\infty}$	$\boxed{0}$	3	∞	4
E	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	4	∞

select min no. from the problem & select it & then repeat the same process.

∴ The optimum assignment is $A \to D \to B \to C \to E \to A$

∴ Assignment value $= 2+5+4+2+3 = 16$

Imp.
Q) Solve the following salesman problem given by the following data.
$C_{12} = 20$, $C_{13} = 4$, $C_{14} = 10$, $C_{23} = 5$.
$C_{34} = 6$, $C_{25} = 10$, $C_{35} = 6$, $C_{45} = 20$, where $C_{ij} = C_{ji}$ and there is no route between cities i and j. If a value of C_{ij} is not known.

	1	2	3	4	5
1	∞	20	4	10	∞
2	20	∞	5	∞	10
3	4	5	∞	6	6
4	10	∞	6	∞	20
5	∞	10	6	20	∞

$\Rightarrow$

∞	16	0	6	∞
15	∞	0	∞	5
0	1	∞	2	2
4	∞	0	∞	14
∞	4	0	14	∞

$\Downarrow$

∞	15	$\boxed{0}$	4	∞
15	∞	$\cancel{0}$	∞	3
$\boxed{0}$	$\cancel{0}$	∞	$\cancel{0}$	$\cancel{0}$
4	∞	0	∞	12
∞	3	0	12	∞

min Element = 3

∞	12	$\boxed{0}$	1	∞
12	$\cancel{0}$	$\cancel{0}$	∞	$\boxed{0}$
$\boxed{0}$	$\cancel{0}$	∞	$\cancel{0}$	$\cancel{0}$
1	∞	$\cancel{0}$	∞	9
∞	$\boxed{0}$	$\cancel{0}$	9	∞

Replacement problem: The problem of replacement is concerned with situations that arise when some items such as machines, Electric light bulbs Etc need replacement due to their low efficiency, failure or breakdown.

Following are the situations when the replacement of certain items needs to be done:

i) The old item has failed and doesn't work at all.

ii) The old item is Expected to fail shortly

iii) A better design of equipment has been developed due to improvement in technology

Q, The maintainence cost and resell value/yr of machine whose purchase price is 1000 is given below

year	1	2	3	4	5	6	7	8
operating cost	9000	1200	1600	2100	2800	3700	4700	5900
resell value	4000	2000	1200	600	500	400	400	400

when should the machine be replaced

year ①	operating cost ②	cumulative cost ③	Resell (5) Value ④	Depriciation ⑥	Total Cost ⑦ ⑤+⑥	Avg cost ⑥/①
1	900	900	4000	3000	3900	3900
2	1200	2100	2000	5000	7100	3550
3	1600	3700	1200	5800	9500	3166.6
4	2100	5800	600	6400	12200	3050
$\boxed{5}$	2500	8600	500	6500	15100	$\boxed{3020}$ *
6	3700	12300	400	6600	18900	3150
7	4700	17000	400	6600	23600	3371.4
8	5900	22900	400	6600	29500	3687.5

Note: In the 5th year the min avg cost is 3020 shown by star (*) in the above table so replacement should take place in the end of 5th year.

9) The cost have increased as they continue in service due to increase direct operating cost and increased maintainence. The initial cost is 3500 and the trade in value drops as time passes until it reaches a constant value of rupees 500.
Given the cost of operating, maintaining and the trade in value, determine the proper length of service before cost should be replaced

Year of service	: 1	2	3	4	5
year end trade in-value	: 1900	1050	60	500	500
Annual operating Cost	: 1500	1800	2100	2400	2700
Annual maintaining Cost	: 300	400	600	800	1000

Year	operating cost (2)	cum cost (3)	Scrape (5)	Depreciation (5)	Total cost	Avg cost
1	1800	1800	1900	1600	3400	3400
2	2200	4000	1050	2450	6450	3225
3	2700	6700	60	3440	10140	3380
4	3200	9900	500	3000	12900	$\boxed{3225}$
5	3700	13600	500	3000	16600	3320

Game theory: It is a body of knowledge which is concerned to the study of decision making in situation where 2 ~~one~~ or more rational opponents are involved under condition of competition.

2 persons zero sum game: It is the situation which involves 2 persons or players and gains made by one person is equal to the loss encure by other.

Classification:

The games can be classified according to:

i) No. of players.

ii) Nature of stratergies

iii) Nature of final outcome.

Straterggy: It is a pre determined rule by which each player decides his course of action from his list available to him.

There are 2 types of game theory.

i) Graphical method

ii) Non-graphical method (dominance method).

steps for graphical method:

→ The game matrices of $2 \times n$ or $m \times 2$ is divided into 2×2 sub matrices.

→ Taking the probabilities of the 2 alternatives of the first player say 'A' as P_1 and $(1-P_1)$.

→ The boundaries of the 2 alternatives strategies of the first player are shown by 2 $\parallel^{el}$ lines shown on the graph.

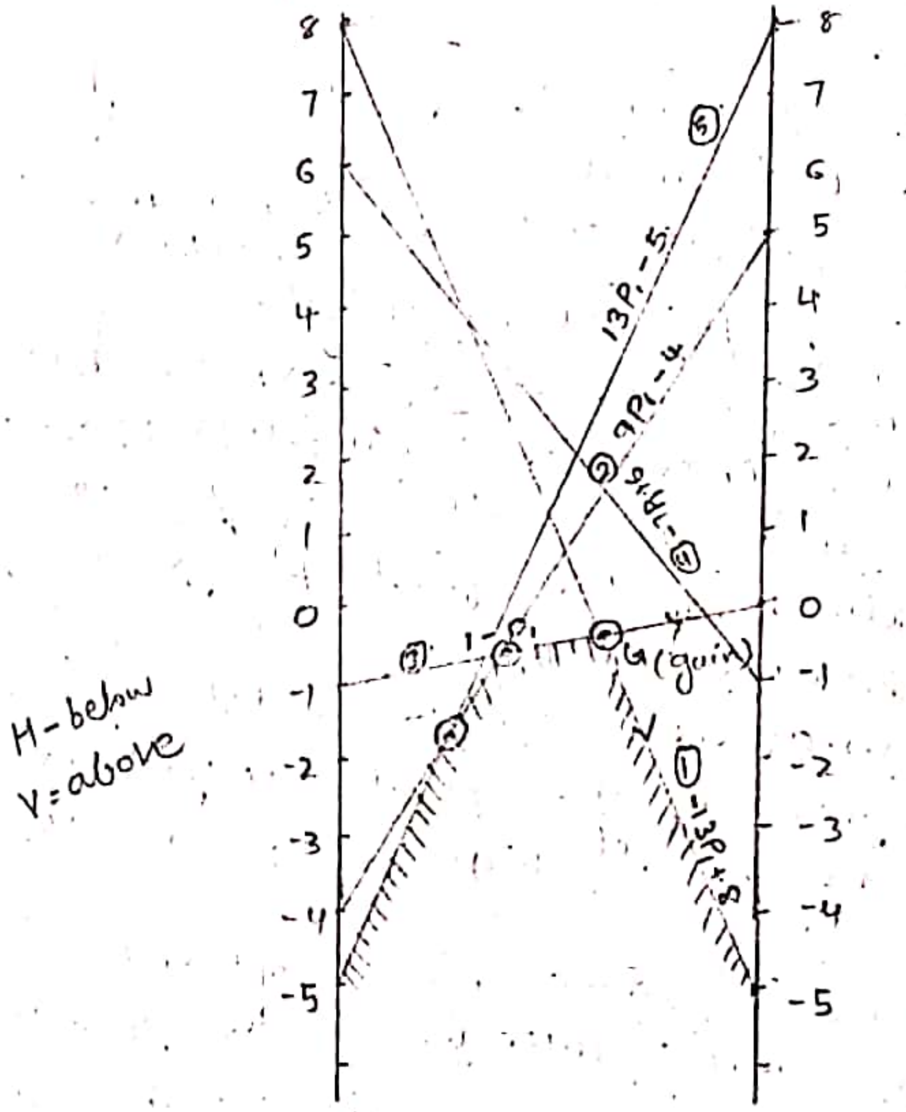
⇒ The gain equation of different sub-gains are plotted on the graph.

Q) Solve the following gain graphically.

Player B

		1	2	3	4	5	
Player A	1	-5	5	0	-1	8	P_1 — R
	2	8	-4	-1	6	-5	$(1-P_1)$ — L

If B select strategy	Expected payoff (A)
1	$-5P_1 + 8(1-P_1)$ $= -13P_1 + 8$
2	$5P_1 - 4 + 4P_1$ $= 9P_1 - 4$
3	$0P_1 - 1 + P_1 = P_1 - 1$
4	$-P_1 + 6 - 6P_1$ $= 6 - 7P_1$
5	$8P_1 - 5 + 5P_1 = 13P_1 - 5$



Horizontal - below the line
Vertical - Above the line

① ③ [No (-) values]

$a_1 = 5$	$a_2 = 3$ (−)	9
(−) $b_1 = 8$	(−) $b_2 = 1$	5
1	13	14 / 14
c_1	c_2	

∴ Gain value =

$$G_i = \frac{a_1 c_1 + a_2 c_2}{c_1 + c_2}$$

$$= \frac{-5 + 13 \cdot 4 \cdot 5}{14} = \frac{\cancel{-99}}{14}$$

$$= \frac{-5 + 0}{14} = \frac{-5}{14}$$

$$= -0.35$$

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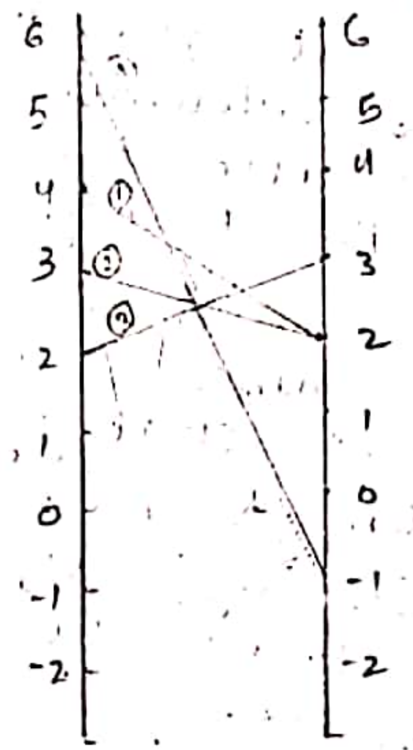
Q) Solve the following 2×4 gain graphi-
-cally.

		B				
		1	2	3	4	
A	1	2	2	3	-1	P_1
	2	4	3	2	6	$(1-P_1)$



Strategy B	expected pay off (A)
1	$4-2P_1$
2	$3-P_1$
3	$2+P_1$
4	$6-7P_1$

Types of strategy:
There are 2 types of strategy:
i) Pure strategy
ii) Mixed strategy.

Pure strategy: It is a pre-determined course of action to be employed by the player. The player known it in advance. It is usually represented by a number with which the course of action is associated.

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Mixed strategy: In mixed strategy the player decides his course of action in accordance with same fixed prob-ability distribution. Probability are associated with each course of action and the selection is done as per these probabilities.

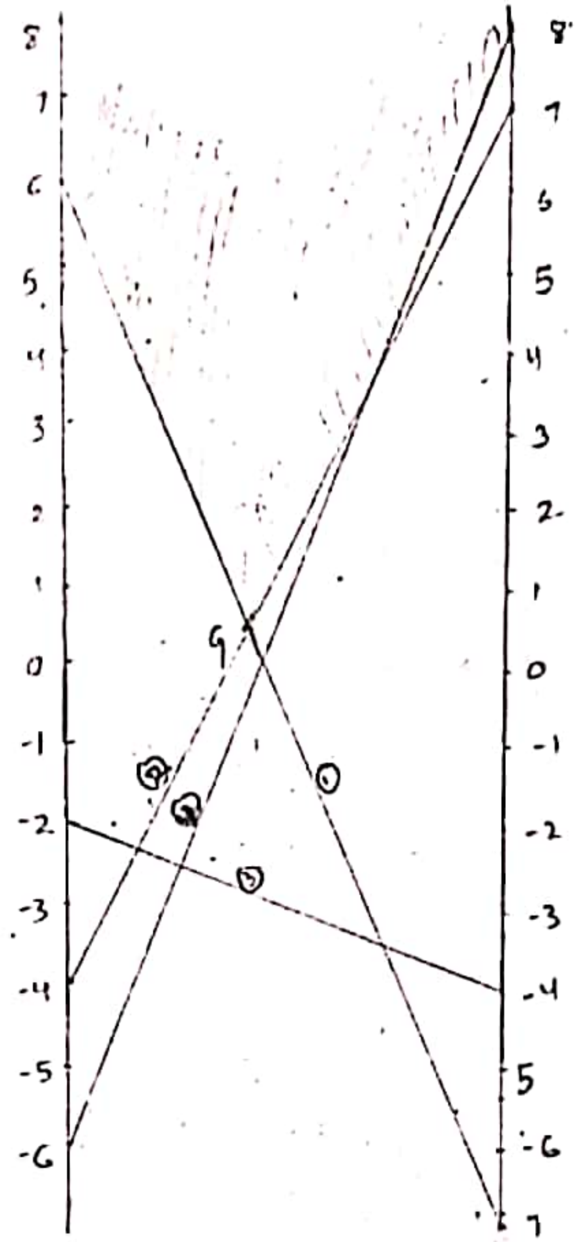
Sadding point: The sadding point in a pay-off matrix is 1 which is the smallest value in its row and largest value in its coloumn. The saddle point is also known as Equilibrium point in the theory of games.

Dominance method: Dominance method is also applicable to pure strategy and mixed strategy problem. In pure strategy the solution is obtained by itself while in the mixed strategy it can be used for simplifying the problem.

Q. Solve the following game graphically.

	b_1 (q_1)	b_2 ($1-q_1$)
a_1	-7	6
a_2	7	-4
a_3	-4	-2
a_4	8	-6
	R	L

selected strategy (A)	Expected pay off.
a_1	$-13q_1+6$
a_2	$11q_1-4$
a_3	$-2q_1-2$
a_4	$14q_1-6$



	①	②	
	-7 (c_1)	6 (a_2)	11
	7 (b_1)	-4 (b_2)	13
	10	14	24
	c_1	c_2	

$$\text{Game value } (g) = \frac{a_1c_1 + a_2c_2}{c_1+c_2}$$

$$= \frac{-70+84}{24}$$

$$= \frac{14}{24}$$

$$= 0.58$$

Q) Explain the principle of dominance in game theory and solve the following game

		B			
		B_1	B_2	B_3	B_4	row min
A	A_1	8	10	9	14	8
	A_2	10	11	8	12	8
	A_3	13	$\boxed{12}$	14	13	12
column max		13	12	14	14	

∴ Saddle point = 12.

Solve the following 2 person 0 sum game

	B_1	B_2	B_3	
A_1	5	7	11	5
A_2	2	-1	8	-1
A_3	18	-6	10	-6
	18	7	11	

$\boxed{\begin{array}{l} R = \min \\ C = \max \end{array}}$

∵ No saddle point go to dominance methods

Dominance method (R-C method)

	C_1	C_2	C_3
R_1	5	7	11
R_2	2	-1	8
R_3	18	-6	10

∴ R_1 is dominated to R_2

Eliminate R_2

5	7	11
18	-6	10

∴ C_3 is dominated to C_2, Eliminate C_3.

5	7	24
18	-6	2
13	13	26/26

$$\text{game value} = \frac{a_1c_1 + a_2c_2}{c_1 + c_2}$$

$$= \frac{65 + 91}{26}$$

$$= \frac{156}{26} = 6$$

8, In an Election campaign, the strategies adopted by the ruling and opposition party with pay-off

	A_1	A_2	A_3	
B_1	55	40	35	35
B_2	70	70	55	55
B_3	75	55	65	55
	75	70	65	

assume a zero-sum game, find the optimum strategies for both parties and expected pay-off for the ruling party.

∴ No saddle point, go to dominance method.

	C_1	C_2	C_3
R_1	55	40	35
R_2	70	70	55
R_3	75	55	65

=> R_3 is dominating to R_1, Eliminate R_1.

C_1	C_2	C_3
70	70	55
75	55	65

=> C_1 dominating C_3, Eliminate C_1

	B_2	B_3	
A_2	70	55	10
A_3	55	65	15
	10	15	25 / 25

$$\text{game value} = \frac{700 + 825}{25}$$

$$= 61$$

Strategies of $A \rightarrow A_2, A_3$

strategies of $B \rightarrow B_2, B_3$

9, Reduce the following game dominance method and find the value of the game.

$$
\begin{array}{c|cccc|c}
 & I & II & III & IV & \\
\hline
I & 3 & 2 & 4 & 0 & 0 \\
II & 3 & 4 & 2 & 4 & 2 \\
III & 4 & 2 & 4 & 0 & 0 \\
IV & 0 & 4 & 0 & 8 & 0 \\
\hline
 & 4 & 4 & 4 & 4 &
\end{array}
$$

Row - min
Col - max

R_3 dominating R_1, Eliminate R_1

$$
\begin{array}{c|cccc|}
 & I & II & III & IV \\
\hline
II & 3 & 4 & 2 & 4 \\
III & 4 & 2 & 4 & 0 \\
IV & 0 & 4 & 0 & 8 \\
\hline
\end{array}
$$

C_1 dominating C_3, Eliminate C_1

$$
\begin{array}{c|ccc|}
 & II & III & IV \\
\hline
II & 4 & 2 & 4 \\
III & 2 & 4 & 0 \\
IV & 4 & 0 & 8 \\
\hline
\end{array}
\qquad
\begin{bmatrix} 2 & 4 & 0 \\ 4 & 0 & 8 \end{bmatrix}
$$

Average method:

$$\frac{R_3+R_4}{2} : \begin{bmatrix} 4 & 2 & 4 \\ 3 & 2 & 4 \end{bmatrix}$$

Eliminate R_4:

$$
\begin{array}{c|ccc|}
 & C_1 & C_3 & C_4 \\
\hline
R_2 & 4 & 2 & 4 \\
R_3 & 2 & 4 & 0 \\
\hline
\end{array}
$$

C_2 is dominating C_4, Eliminate C_2.

$$
\begin{array}{|cc|c}
\hline
2 & 4 & 4 \\
4 & 0 & 2 \\
\hline
\end{array}
$$

$$\text{game value} = \frac{a_1c_1 + a_2c_2}{c_1+c_2}$$

Q) Using the dominance property obtain the optimal strategies for both the players determine the value of games.

2	4	3	8	4	2
5	6	3	7	8	3
6	7	9	8	7	6
4	2	8	4	3	2
5	7	9	8	8	

saddle point = 6

Sequencing models: The selection of an appropriate order for a series of jobs to be done on a finite no. of service facilities is called sequencing.

A sequencing problem could involve:

i) Jobs in a manufacturing plant.

ii) ~~Aircrafate~~ Aircraft scheduling in a factory.

iii) Programs to be run on a computer.

Jobs: The jobs or items or costumers or orders are the primary for sequencing. There should be a no. of jobs say 'n' to be processed

Processing time: Every operation requires certain time at each of machine. If the time is certain, Then the determination of schedule is easy when processing times are uncertain, then the schedule is complex.

Total Elapsed time: It is the time between starting of first job & completing the last one.

Idle time: It is a time the machine remains idle during the total Elapsed time.

Q, we have 5 jobs each of which must go through the 2 machines A and B in order AB. Processing time in hours are given below.

Job:	1	2	3	4	5
machine A:	5	1	9	3	10
machine B:	2	6	7	8	4.

Determine a sequence for the 5 jobs.

Sol,

Job:	1	2	3	4	5
mac. A:	5	1	9	3	10
mac B:	2	6	7	8	4

Find the min among MA and MB round up, if it is then write job value.

Sequencing Table:

MA → ← MB

2	4	3	5	1

Table II: Elapsed table.

Job	machine A		machine B	
	Input	Output	Input	output
2	0+1	1	1+6	7
4	1+3	4	7+8	15
3	4+9	13	15+7	22
5	13+10	23	23+4	27
1	23+5	28	28+2	$\boxed{30}$

Total elapsed time = 30 hrs.

Idle time on machine A = 30 − 28 = 2 hrs. ⇒ machine A in total

" " " B = 30 − 27 = 3hrs. ⇒ machine B in total

Types of sequencing problem:

1. Problems with n jobs through 1 machine
2. Problems with n jobs through 2 machines
3. " " through 3 machines
4. " " through m machines.

Q> Find the sequence that minimizes the total elapsed time required to complete in performing the following jobs on 3 machines in the order

Processing time / jobs	1	2	3	4	5	6
machine A	8	3	7	2	5	1
machine B	3	4	5	2	1	6
machine C	8	7	6	9	10	9

Jobs :	1	2	3	4	5	6
mx(A+B) :	11	7	12	4	6	7
my(B+C) :	11	11	11	11	11	15

19

22

→mx my←

Sequencing Table :

4	5	2	6	1	3

elapsed Table

Job	machine A Input	machine A Output	machine B Input	machine B Output	machine C input	machine C output
4	0+2	2	2+2	4	4+9	13
5	2+5	7	7+1	8	13+10	23
2	7+3	10	10+4	14	23+7	30
6	10+1	11	14+6	20	30+9	39
			20+3	23	39+8	47
[illegible]	11+8		[illegible]	31	47+6	$\boxed{53}$

Total Elapsed time = 53 hrs.

Idle time M-A = 53 - 26 = 27 hrs

" M-B = 53 - 21 = 32 hrs

" M-C = 53 - 49 = 4 hrs

Determine the optimal sequence of jobs that minimise the total elapsed time based on the following information

Jobs:	A	B	C	D	E	F	G
machine M_1	3	8	7	4	9	8	7
machine M_2	4	3	2	5	1	4	3
machine M_3	6	7	5	11	5	6	12

Sol,

Jobs:	A	B	C	D	E	F	G
M_x :	7	11	9	9	10	12	10
M_y :	10	10	7	16	6	10	15

Sequencing table: $M_x \rightarrow$ $M_y \leftarrow$

A	D	G	F	B	C	E

Elapsed table:

Job	M_1 Input	M_1 output	M_2 Input	M_2 output	M_3 Input	M_3 output
A	0+3	3	3+4	7	7+6	13
D	3+4	7	7+5	12	13+11	24
G	7+7	14	14+3	17	24+12	36
F	14+8	22	22+4	26	36+6	42
B	22+8	30	30+3	33	42+7	49
C	30+7	37	37+2	39	49+5	54
E	37+9	46	46+1	47	54+5	59.

Total Elapsed time = 59 hrs

Idle time, $m_1 = 59 - 46 = 13$ hrs.

$m_2 = 59 - 22 = 37$ hrs.

$m_3 = 59 - 52 = 7$ hrs.

Q) Find an optimal sequence for the following sequencing problem of 4 jobs and 5 machines when passing is not allowed of which processing time (in hrs) is given below.

Job :	1	2	3	4
m_1 :	6	5	4	7
m_2 :	4	5	3	4
m_3 :	1	3	4	2
m_4 :	2	4	5	2
m_5 :	8	9	7	1

Job :	1	2	3	4
m_x (2+3+4+5) :	15	21	19	9
m_y (1+2+3+4) :	13	17	16	15

Sequencing table :

4	2	3	1

Elapsed table:

Job	m_1 Input	m_1 output	m_2 i/p	m_2 o/p	m_3 i/p	m_3 o/p	m_4 i/p	m_4 o/p	m_5 i/p	m_5 o/p
4	0+7= =7	7	7+4	11	11+2	13	13+2	15	15+1	16
2	7+5	12	12+5	17	17+3	20	20+4	24	24+9	33
3	12+4	16	17+3	20	20+4	24	24+5	29	33+7	40
1	16+6	22	22+4	26	26+1	27	29+2	31	40+8	48

Elapsed time : 48 hrs,

Idle time for m_1 : 48 - 22 = 26 hrs

m_2 : 48 - 16 = 32 hrs

m_3 : 48 - 10 = 38 hrs

m_4 : 48 - 13 = 35 hrs

m_5 : 48 - 25 = 23 hrs.

Scanned with CamScanner

graphical method

Q) 2 jobs: let there be 2 jobs A and B of which is to be processed on n machines say $m_1, m_2 \ldots m_m$ is a 2 different orders. The technological ordering of each of the 2 jobs through m machines is known in advance. The processing times on the given machines is also known. Each machine can perform only 1 job at a time. The objective is to determine an optimal sequence of processing the jobs so as to minimize the total elapsed time.

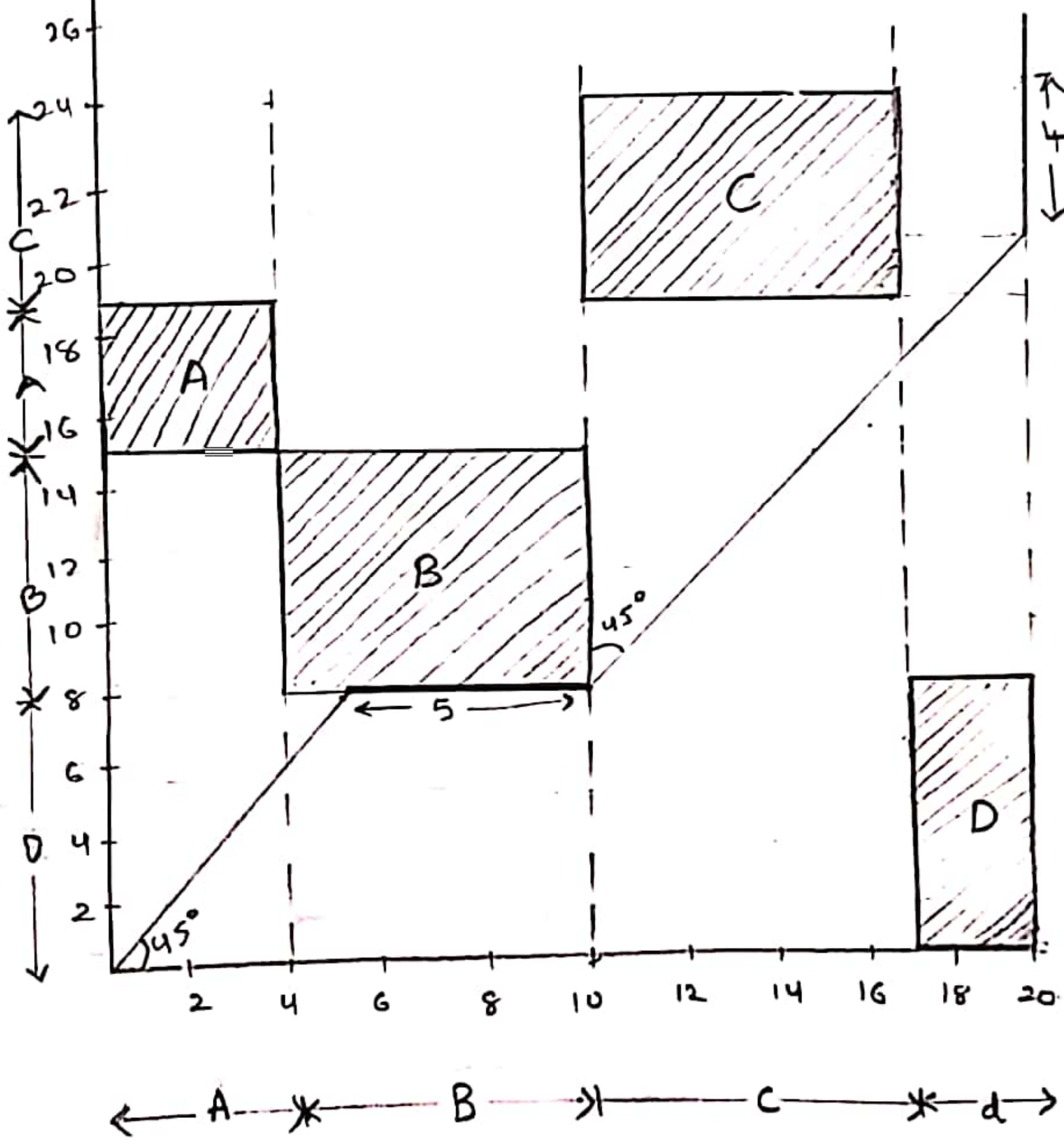
Q) Using graphical method, calculate the min time needed to process the following jobs.

machines :	A	B	C	D
Job 1 :	4	6	7	3
Job 2 :	4	7	5	8

order job 1 :	A	B	C	D
" job 2 :	D	B	A	C

Idle time on job 1 : 4 hrs

Idle time on job 2 : 5 hrs.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B
C
d

Q; A company has 6 jobs which go through 3 machines x, y and z in the order x-y-z. The processing time in mins for each job on each machine is as follows:

Job:	machine x	y	z
1	18	7	19
2	12	12	12
3	29	11	23
4	36	2	47
5	43	6	28
6	37	12	36

what should be the sequence of the job.

Sol,

Job:	1	2	3	4	5	6
x	18	12	29	36	43	37
y	7	12	11	2	6	12
z	19	12	23	47	28	36

Job:	1	2	3	4	5	6
m_p:	25	24	40	38	49	49
m_q:	26	24	34	49	34	48

Sequencing Table:

2	1	4	6	5	3

elapsed table:

Jobs	machine x Input	output	y Input	output	m--- z Input	output
2	0 + 12	12	12+12	24	24+12	36
1	12+18	30	30+7	37	37+19	56
4	30+36	66	66+2	68	68+47	115
6	66+37	103	103+12	115	115+36	151
5	103+43	146	146+6	152	152+28	180
3	146+29	175	175+11	186	186+23	209

A — B — C — D — E

elapsed time: 209 mins.

Idle time for machine x: $209 - 175$
$= 34$ mins

y: $209 - 50$
$= 159$ mins

z: $209 - 165$
$= 44$ mins

Q. Using graphical method to minimize the required to process the following job which should be done first

Job 1	sequence:	A	B	C	D	E
	time (hr):	6	8	4	12	4
Job 2	sequence:	B	C	A	D	E
	time (hr):	10	8	6	4	18

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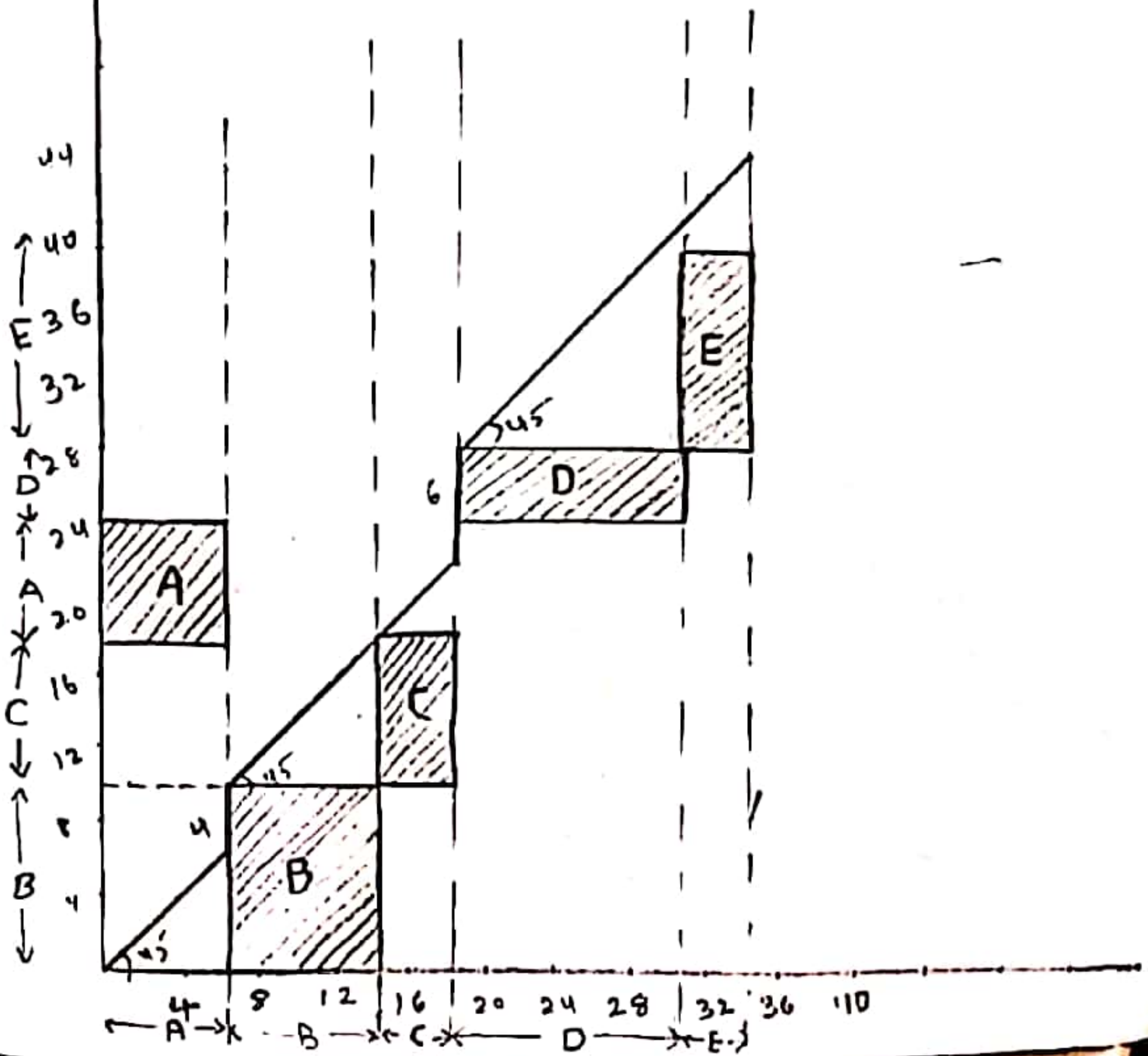


5) 6 jobs go first over machine 1 and then over machine 2. The order of the completion of jobs has no significance. The following table gives machine time in hrs, for 6 jobs and 2 machines.

Job :	1	2	3	4	5	6
machine 1 :	5	9	4	7	8	6
machine 2 :	7	4	8	3	9	5
	12	13	12 min			11

Find the sequence of jobs that minimize the total elapsed time to complete the jobs

mac A → ← mac B

sequencing table :

3	1	5	6	2	4

Elapsed Table :

Job	machine A		machine B	
	Input	Output	I/P	O/P
3	0+4	4	4+8	12
1	4+5	9	12+7	19
5	9+8	17	19+9	28
6	17+6	23	28+5	33
2	23+9	32	33+4	37
4	32+7	39	39+3	42

Total Elapsed time = 42 hrs

Idle time on mac A = 42 − 39 = 3 hrs

mac B = 42 − 36 = 6 hrs

Queuing theory: A group of items waiting to recieve service are known as Queue. Queues or waiting lines are in every day life. some of the situations where queues may be seen or patients waiting for doctors at hospitals outdoors costumers waiting at barber shop for haircuts, passengers waiting at railway booking counter, students waiting at college fee counter.

Arrival rate: The rate at which costumers arive to be serviced is known as arrival rate and is randomly distributed according to the poison distribution. It is denoted by λ

Service rate: The rate at which one service channel can perform the required costumer is called the service rate and is randomly distributed according to the poison distribution. It is denoted by μ.

Traffic intensity (or) utilization factor: It is defined as the ratio between arrival rate to service rate. It is denoted by $\rho = \frac{\lambda}{\mu}$ always < 1

Average no. of costumers in system $= \frac{\rho}{1-\rho}$

Average no. of costumers in Queue $= \frac{\rho^2}{1-\rho}$

average time of costumers in the queue $= \frac{\lambda}{\mu(\mu-\lambda)}$

average waiting time that a costumer spends in the system : $\frac{1}{\mu-\lambda}$

probability that there are n costumers in the system $= \rho^n (1-\rho)$

probability of empty or ideal system : $1-\rho$

probability of queue size be $\geq m$: ρ^m

Q) At a cycle repair shop, on an average ~~Early~~ every 5 mins a costumer arrive. The service time is 4 min/ on an average. The inter arrival costumer. Suppose that the inter arrival time follows poison distribution & service time are exponentially distributed.

Find : 1) ~~Traffic~~ intensity

2) Averge time of costumers in the queue.

3) Avge no. of costumers in queue.

4) Probability of an ideal system.

arrival rate $\lambda = 5$ mins $= 1/5$ mins

service rate $\mu = 4$ mins $= 1/4$ mins.

i) ultimate factor $\rho = \frac{\lambda}{\mu} = \frac{1/5}{1/4} = \frac{4}{5} = 0.8$.

ii) Avg. time of costumers in queue $= \cancel{\frac{\rho^2}{1-\rho^2}} = \frac{\lambda}{\mu(\mu-\lambda)}$

$$= \frac{1/5}{1/4(1/4-1/5)} = \frac{1/5}{\frac{1}{4}\left(\frac{1}{20}\right)} = 16 \text{ mins}$$

iii) Avg no. of Customers in queue $= \frac{(0.8)^2}{1-(0.8)} = \frac{0.64}{0.2} = 3.2$.

iv) Probability of an ideal system $= 1-\rho = 1-0.8 = 0.2$

Simplex method / Linear programming LPP: is a mathematical technique. This technique is applied for choosing the best alternatives from a set of feasible ~~of~~ alternatives. This LPP technique is designed to help managers in planning, decision making to allocate the resource.

types of LPP:

i. Graphical method.

ii. Formulation method.

iii. Non-graphical method (simplex)

i. Graphical method: Find the max. value of $z = x_1 + 3x_2$. subject to constraints $3x_1 + 6x_2 \le 8$, $5x_1 + 2x_2 \le 10$, $(x_1, x \ge 0)$

Prof → max

cost → min.

Sol: given,

$3x_1 + 6x_2 = 8$

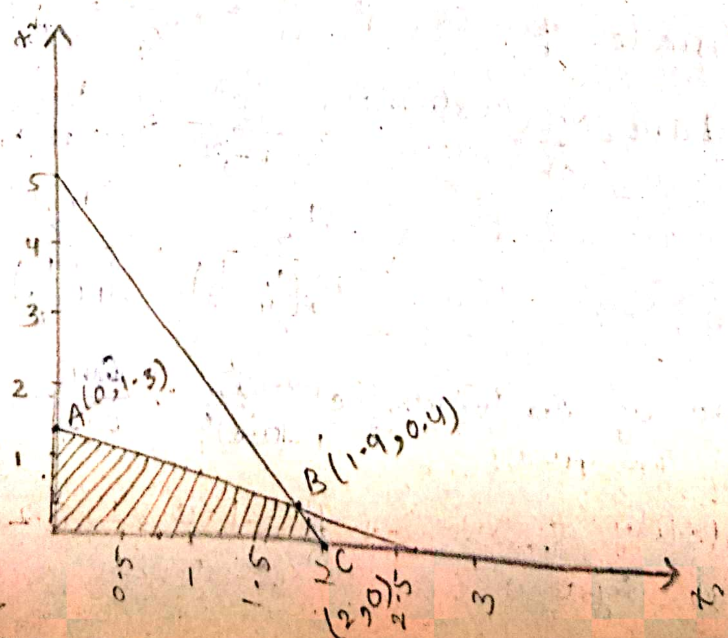
let $x_1 = 0$ — $6x_2 = 8$ — $\boxed{x_2 = 1.3}$

let $x_2 = 0$ — $3x_1 = 8$ — $\boxed{x_1 = 2.6}$

$5x_1 + 2x_2 = 10$

let $x_1 = 0$ — $\boxed{x_2 = 5}$

let $x_2 = 0$ — $\boxed{x_1 = 2}$



A(0, 1.3) ; B = ...

max: $z = x_1 + 3...$

at A ⇒ z = 0 + 3...

at B ⇒ z = 1.9...

at C ⇒ z = 2 + ...

∴ maximum ...

Q) Find the ...

$7x_2$ subject ...

$3x_1 + 8x_2 \le$ ...

Sol: given

$x_1 + x_2$ ...

let $x_1 = 0$

let $x_2 = 0$

$10x_1 + 7$...

let x_1 ...

let x_2 ...

$A = (0, 1.3)$; $B = (1.9, 0.4)$; $C = (2, 0)$.

max: $z = x_1 + 3x_2$.

at $A \Rightarrow z = 0 + 3(1.3) = 3.9$

at $B \Rightarrow z = 1.9 + 3(0.4) = 1.9 + 1.2$
$= 3.1$

at $C \Rightarrow z = 2 + 0 = 2$.

$\therefore$ maximum value at $A = 3.9$

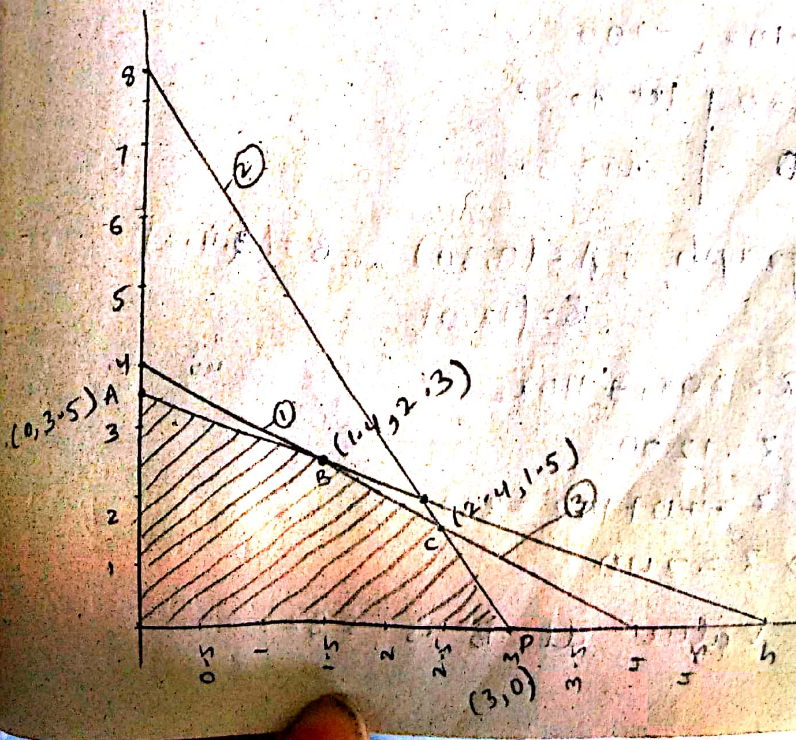
Q) Find the maximum value of $z = 5x_1 + 7x_2$ subject to constrain $x_1 + x_2 \le 4$; $3x_1 + 8x_2 \le 24$; $10x_1 + 7x_2 \le 35$, $x_1, x_2 \ge 0$.

Sol: given

$x_1 + x_2 = 4$
let $x_1 = 0 \Rightarrow x_2 = 4$
let $x_2 = 0 \Rightarrow x_1 = 4$

$3x_1 + 8x_2 = 24$
let $x_1 = 0 \Rightarrow 8x_2 = 24$
$x_2 = 3$
let $x_2 = 0 \Rightarrow 3x_1 = 24$
$x_1 = 8$.

$10x_1 + 7x_2 = 35$
let $x_1 = 0 \Rightarrow x_2 = 5$
let $x_2 = 0 \Rightarrow x_1 = 3.5$.



$A=(0,3.5)$ $B=(1.4,2.3)$ $C=(2.4,1.5)$
$D=(3,0)$.

max i $z = 5x_1 + 7x_2$.

At A $\Rightarrow z = 7(3.5) = 24.5$

At B $\Rightarrow z = 5(1.4) + 7(2.3)$
$= 7.0 + 16.1$
$= 23.1$

At C $\Rightarrow z = 5(2.4) + 7(1.5)$
$= 10.0 + 10.5$
$= 20.5$.

At D $\Rightarrow z = 15$.

$\therefore$ max value at A is 24.5.

Q) Find the min value of $z = 20x_1 + 40x_2$ subject to constrain $36x_1 + 6x_2 \geq 180$; $3x_1 + 12x_2 \geq 36$; $20x_1 + 10x_2 \geq 100$; $x_1, x_2 \geq 0$.

Sol) consider,

$36x_1 + 6x_2 = 180$

let $x_1 = 0$, $x_2 = 30$. | let $x_2 = 0$, $x_1 = 5$

$3x_1 + 12x_2 = 36$

let $x_1 = 0$, $x_2 = 3$ | $x_2 = 0$, $x_2 = 12$.

$20x_1 + 10x_2 = 100$

let $x_1 = 0$, $x_2 = 10$ | let $x_2 = 0$, $x_1 = 5$.

From graph; $A = (0,30)$ $B = (2.4, 8)$
$C = (12,0)$.

4.8 3
40 192.

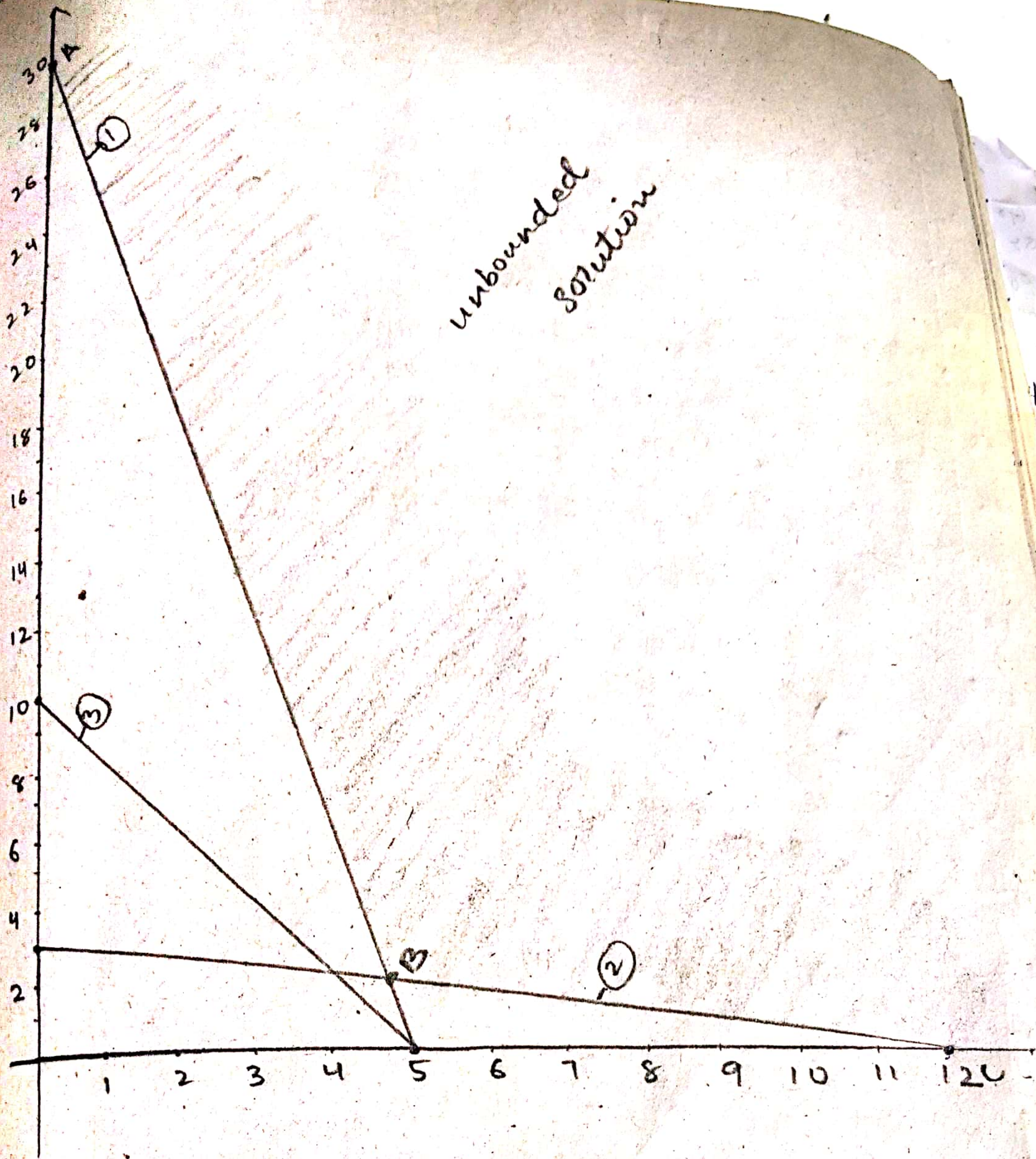
min i $z = 20x_1 + 40x_2$.

At A $\Rightarrow z = 1200$

At B $\Rightarrow z = 40 + 192 = 232$.

At C $\Rightarrow z = 240$.

$\therefore$ min value at B is 232.



4) Find the min value of $z = 20x_1 + 10x_2$

s/c $x_1 + 2x_2 \le 40$

$3x_1 + x_2 \geq 30$

$4x_1 + 3x_2 \geq 60$; $x_1, x_2 \geq 0$

sol) $x_1 + 2x_2 = 40$

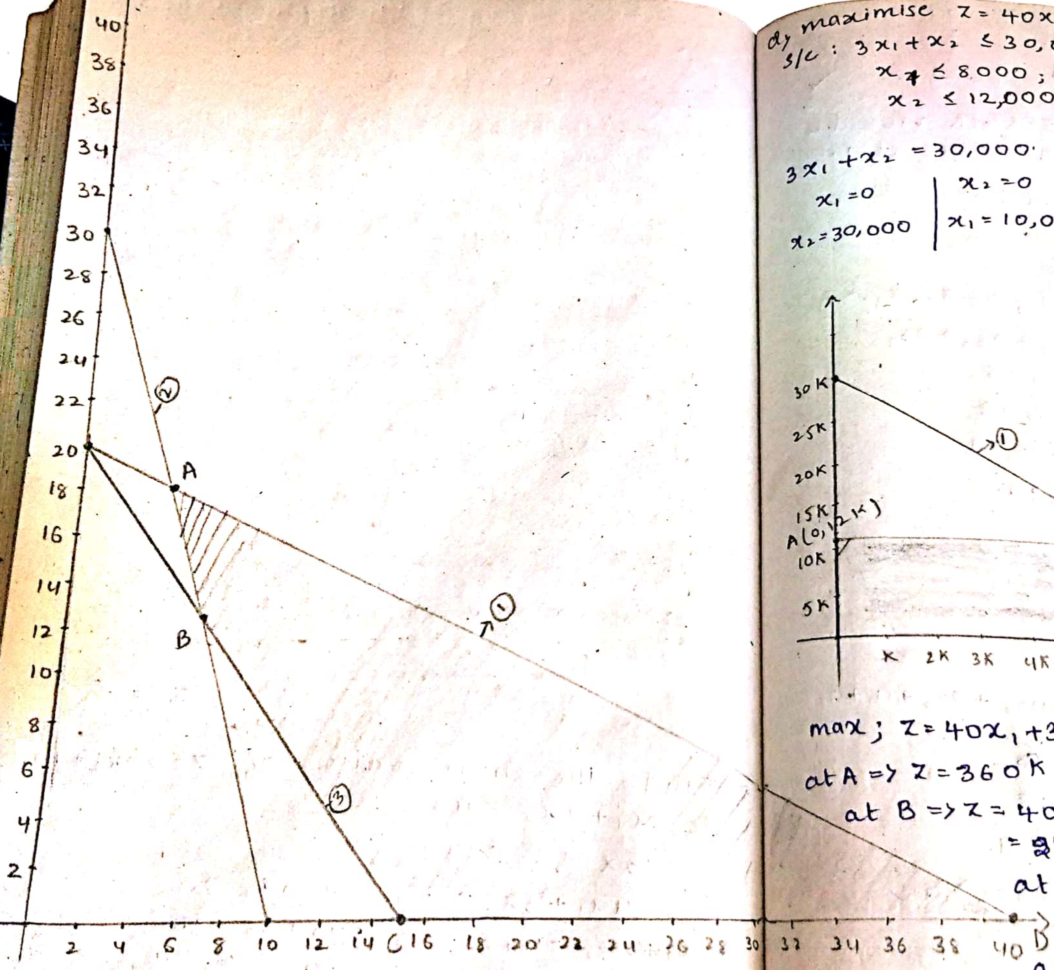
let $x_1 = 0$, $x_2 = 20$	$x_2 = 0$, $x_1 = 40$

$3x_1 + x_2 = 30$

$x_1 = 0$, $x_2 = 30$	$x_2 = 0$, $x_1 = 10$.

$4x_1 + 3x_2 = 60$

$x_1 = 0$, $x_2 = 20$	$x_2 = 0$, $x_1 = 15$



d) maximise $z = 40x_1 + 30x_2$

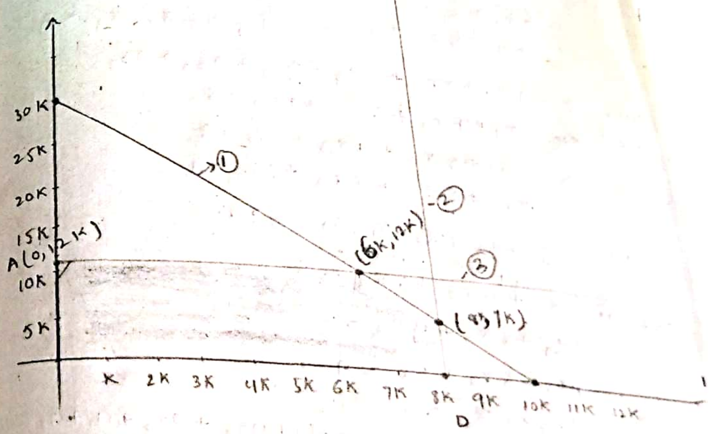
s/c : $3x_1 + x_2 \leq 30{,}000$

$x_1 \leq 8{,}000$;

$x_2 \leq 12{,}000$; $x_1, x_2 \geq 0$

$3x_1 + x_2 = 30{,}000$

$x_1 = 0$	$x_2 = 0$	$x_1 = 8000$	$x_2 = 12{,}000$
$x_2 = 30{,}000$	$x_1 = 10{,}000$	$x_2 = 0$	$x_1 = 0$



max ; $z = 40x_1 + 30x_2$

at A => $z = 360K$

at B => $z = 40(6K) + 30(12K)$

$= 240K + 360K = 600K$

at C => $z = 320K + 210K$

$= 530K$

at D => $z = 320K$

max at B i.e. 600K.

Simplex method:

$\leq$ (s) $\quad$ $\geq$ (a, s) $\quad$ $=$ ($\leq$, $\geq$)

s = slack variable
a = artificial variable

Q) maximize $Z = 2x_1 + 3x_2 + 4x_3$

S/C : $x_1 + 4x_2 + 6x_3 \leq 6$

$2x_1 + 8x_2 + 4x_3 \leq 8$

$3x_1 + 6x_2 + 7x_2 \leq 9$; $x_1, x_2, x_3 \geq 0$.

Ans) $Z = 2x_1 + 3x_2 + 4x_3 + 0S_1 + 0S_2 + 0S_3$.

case I : S/C : $x_1 + 4x_2 + 6x_3 + 1S_1 = 6$

$2x_1 + 8x_2 + 4x_3 + 1S_2 = 8$

$3x_1 + 6x_2 + 7x_3 + 1S_3 = 9$

$x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$.

$Z = 2x_1 + 3x_2 + 4x_3 = 0$

case II : $x_1 + 4x_2 + 6x_3 \geq 6$

$2x_1 + 8x_2 + 4x_3 \geq 8$

$3x_1 + 6x_2 + 7x_3 \geq 9$.

$x_1, x_2, x_3 \geq 0$.

$\Rightarrow Z = 2x_1 + 3x_2 + 4x_3 + 0S_1 + 0S_2 + 0S_3 + a_1 + a_2 + a_3$.

$x_1 + 4x_2 + 6x_3 + a_1 - 1S_1 = 6$

$2x_1 + 8x_2 + 4x_3 + a_2 - 1S_2 = 8$

$3x_1 + 6x_2 + 7x_3 + a_3 - 1S_3 = 9$.

$x_1, x_2, x_3, a_1, a_2, a_3, S_1, S_2, S_3 \geq 0$

Case III : $Z = 2x_1 + 3x_2 + 4x_3$

S/C $x_1 + 4x_2 + 6x_3 \leq 6$

$8x_2 + 4x_3 \geq 8$

$3x_1 + 6x_2 + 7x_3 \geq 9$

$3x_1 + 6x_2 + 7x_3 \leq 9$

$\Rightarrow Z = 2x_1 + 3x_2 + 4x_3 + 0s_1 + 0s_2 + 0s_3 + 0s_1 + a_2 + a_3$

$x_1 + 4x_2 + 6x_3 + 1s_1 = 6$

$8x_2 + 4x_3 + a_2 - s_2 = 8$

$3x_1 + 6x_2 + 7x_3 + a_3 - S_3 = 9$

$3x_1 + 6x_2 + 7x_3 + S_4 = 9.$

$x_1, x_2, x_3, S_1, S_2, S_3, a_2, a_3 \geq 0$

Graphical method of solving LPP:

i) Formulate the linear programming problem.

ii) Plot the constraint lines considering them as equations.

iii) Identify the feasible solution region.

iv) Locate the corner points of the feasible solution.

v) calculate the value of the objective function on the corner points.

vi) Choose the point where the objective function has optimal value.

method (maximization):

Steps for Non-graphical describing the problem

i) Setup the inequalities constraints

ii) Introduce slack variable and convert inequalities into equations.

iii) Enter the Equalities into the simplex table.

iv) calculate Z_j and $C_j - Z_j$ value for the Solution

v) Determine the Entreping variable by choosing the highest $C_j - Z_j$ coloum.

vi) Compute the value of the Key row and key coloum.

vii) If there is no non negative $C_j - Z_j$ values the final solution has been obtained.

Q) Use simplex method, maximise $z = 5x_1 + 3x_2$

S/C : $x_1 + x_2 \le 2$

$5x_1 + 2x_2 \le 10$

$3x_1 + 8x_2 \le 12$

$x_1, x_2 \ge 0$.

Sol) $x_1 + x_2 + S_1 = 2$

$5x_1 + 2x_2 + S_2 = 10$

$3x_1 + 8x_2 + S_3 = 12$

; $z = 5x_1 + 3x_2 + 0S_1 + 0S_2 + 0S_3$.

; $x_1, x_2, S_1, S_2, S_3 \ge 0$

$1 - \frac{0}{1}$

$2 - \frac{1 \times 0}{1}$

$12 - \frac{36}{8}$ 6 8.

6

$8 - \frac{6}{5}$

$\frac{6}{5}$

$\frac{34}{5}$

Basic variables	Solu V
0 S_1	
0 S_2	
0 S_3	

z

$5x_1$

$0S_2$

$0S_3$

∴ In

are

Sin

Q) Sol

me

$Z = 5x_1 + 3x_2 + 0S_1 + 0S_2 + 0S_3$

Basic variables	Solution values	x_1 (C_j = 5)	x_2 (C_j = 3)	S_1 (C_j = 0)	S_2 (C_j = 0)	S_3 (C_j = 0)	min ratio $\frac{S.V}{KC}$
0 S_1	2	1	1	1	0	0	2
0 S_2	10	5	2	0	1	0	2
0 S_3	12	3	8	0	0	1	4 select min +ve value
	$Z_j = \Sigma C_i x_i$	0	0	0	0	0	
	$C_j - Z_j$	5	3	0	0	0	
		select max +ve value					
$5x_1$	2	1	1	1	0	0	div with Key Element
$0S_2$	0	0	-3	-5	1	0	
$0S_3$	6	0	5	-3	0	1	
	Z_j	5	5	5	0	0	stop the prob when $C_j - Z_j$'s values are 0 or -ve
	$C_j - Z_j$	0	-2	-5	0	0	

$a - \frac{b \times c}{d} = 10 - \frac{5 \times 2}{1}$

$\therefore$ In maxisation case the values of $C_j - Z_j$ all are 0 or -ves

since, $x_1 = 2$; $x_2 = 0$

$z = 5x_1 + 3x_2$

$= 5(2) + 3(0) = 10$.

Q) Solve the following LP problem using simplex method. maxi $z = 10x_1 + 15x_2 + 20x_3$

s/c : $2x_1 + 4x_2 + 6x_3 \leq 24$

$3x_1 + 9x_2 + 6x_3 \leq 30$

$x_1, x_2, x_3 \geq 0$.

Sol) $2x_1 + 4x_2 + 6x_3 + 1S_1 = 24$

$3x_1 + 9x_2 + 6x_3 + 2S_2 = 30$

$x_1, x_2, x_3, S_1 + S_2 \geq 0$

$z = 10x_1 + 15x_2 + 20x_3 + 0S_1 + 0S_2$

Basic variables	Solution values	c_j: 10 x_1	15 x_2	20 x_3	0 s_1	0 s_2	min ratio $\frac{S.V}{K.C}$
$0\,s_1$	24	2	4	$\boxed{6}$	1	0	4
$0\,s_2$	30	3	9	6	0	1	5
	Z_j =	0	0	0	0	0	
	$C_j - Z_j$ =	10	15	$\boxed{20}$	0	0	
$20\,x_3$	$\frac{24}{6} = 4$	$\frac{2}{6} = \frac{1}{3}$	$\frac{4}{6} = \frac{2}{3}$	1	$\frac{1}{6}$	0	12
$0\,s_2$	6	$\boxed{1}$	5	0	−1	1	6
	Z_j	$\frac{20}{3}$	$\frac{40}{3}$	20	$\frac{20}{6}$	0	
	$C_j - Z_j$	$\boxed{\frac{10}{3}}$	$\frac{5}{3}$	0	$-\frac{20}{6}$	0	
$20\,x_3$	2	0	−1	1	$\frac{1}{2}$	$-\frac{1}{3}$	
$10\,x_1$	6	1	5	0	−1	1	
	Z_j	10	$-20+50$ = 30	$20+0$ = 20	$10-10$ = 0	$-\frac{20}{3}+10 = \frac{10}{3}$	
	$C_j - Z_j$	0	−15	0	0	$-\frac{10}{3}$	

$a - \frac{b \times c}{d}$

$3 - \frac{12}{6}$ $\quad 0 - \frac{1}{3}$ $\quad \frac{45-40}{3}$ $\quad \frac{2}{3} \times \frac{\frac{5}{3}}{1}$ $\quad 15$ $\quad 4 - \frac{\frac{2}{3}}{1}$ $\quad \frac{1}{6} - \frac{-\frac{1}{3}}{1}$ $\quad \frac{1}{6} + \frac{1}{3}$ $\quad \frac{1}{6} - \frac{-\frac{1}{3}}{1}$ $\quad \frac{1}{6} + \frac{1}{3} = \frac{3}{6}$ $\quad \frac{-20+30}{3}$

we get: $x_1 = 6$; $x_2 = 0$; $x_3 = 2$

$Z = 10x_1 + 15x_2 + 20x_3$

$= 60 + 40 = 100$

If a ques is given in min, change it to max by changing signs

min $Z: x_1 + 2x_2 + 4x_3$

s/c: $2x_1 + 4x_2 + 5x_3 \geq 2$

$x_1 + 2x_2 + 7x_3 \leq 4$

$2x_1 + 3x_2 + 6x_3 \geq 3$

max: $Z = -x_1 - 2x_2 - 4x_3$

s/c $-2x_1 - 4x_2 - 5x_3 \leq -2$

$x_1 + 2x_2 + 7x_3 \leq 4$

$-2x_1 - 3x_2 - 6x_3 \leq -3$

Formulation meth...

Q) A Furniture c... chairs. data ... consumed an... wood and la... consumed in ... of company w... and chairs ... the total pr...

wood

Labo...

Profi...

sol) let x_1 b...

maxi: Z...

s/c...

Q) The r... on the... which... run...

The ... 200 ... -ment... and S...

Formulation method.

Q) A Furniture company manufacture table and chairs. data given below shows resources consumed and unit profit. Here it is assumed wood and labour are 2 resources which are consumed in manufacturing furniture. management of company wishes to determine how many tables and chairs should be made to maximise the total profits and formulate.

	T	C	Availability
wood	30	20	300
Labour	5	10	110
Profit	6	8	

Sol: let x_1 be the tables, x_2 be the chairs

prof - maxi
cost - min

maxi : $Z = 6x_1 + 8x_2$

s/c: $30x_1 + 20x_2 \leq 300$

$5x_1 + 10x_2 \leq 110$.

$x_1, x_2 \geq 0$.

Q) The manager of an oil company must decide on the optimal mix of 2 possible processes of which the inputs and output for production run on as follows:

	Input		Output	
Process	Crude A	Crude B	Gasoline x	Gasoline y
1	5	3	5	8
2	4	5	4	4

The max amount available for crudes A and B is 200 units and 150 units respectively. market requirements shows that atleast 100 units of gasoline x and 80 units of gasoline y must be produce. The profit / production run from process 1 and 2 are Rs. 300 and 400 respectively. Solve LPP by graphical method.

maxi : $Z = 300x_1 + 400x_2$.

$s/c : 5x_1 + 4x_2 \leq 200$

$3x_1 + 5x_2 \leq 150$

$5x_1 + 4x_2 \geq 100$

$8x_1 + 4x_2 \geq 80$.

$x_1, x_2 \geq 0$.