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Electron Diffraction

# 1 Abstract

This paper examines DeBroglie wavelength using electron diffraction. The data measured is then used to derive Planck's constant to be $h = 7.635 \times 10^{-34} \pm 1.792 \times 10^{-34}$ Js.. This experiment follows other experiments conducted in the 1900s in an attempt to reproduce the results found.

# 2 Introduction

The dawn of a new era within Physics was sparked with new characteristics of light that were revealed through scientific inquiry. In 1801, Thomas Young showed that light moved as a wave with his famous double slit experiment. De Broglie (1924), supposed that light has not only the property of a wave, but rather it has a particle wave duality. Davidson and Germer (1927), in their quest to make a better TV, discovered the interference effect whilst filtering electrons through a crystal of nickel. This discovery, although accidental at first, helped to prove De Broglie's hypothesis on light's behavior. Later in that same year De Broglie was able to measure the wavelength of light. In present day we use these findings to further the study of electron diffraction and crystalline structure.

# 3 Theory

Refer to Figure 1 for a schematic of an apparatus that could be use. The photon is emitted from 1.B at 20kV which is then accelerated by 1.A. It is them focused by 1.E using 1.D. ”D” is the distance between 1.E and the screen of the diffracting tube.

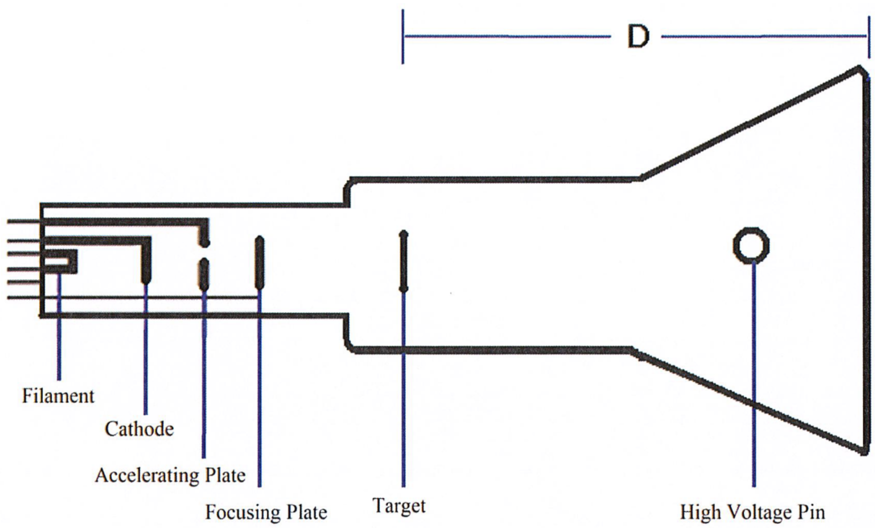


Figure 1 Schematic of electron diffraction tube

This causes the electron’s diffraction patterns to form when it comes into contact with 1E. At speed $v$ the electron passing through the crystal film has DeBrogile Wavelength of:

$$\lambda = \frac{h}{p} = \frac{h}{mv} \tag{1}$$

Where m is the mass of an electron and h is Planck’s constant. Applying Kinetic Energy of an electron which is:

$$\frac{1}{2}mv^2 = eV \tag{2}$$

Therefore since e is the charge of an electron and V is the accelerating voltage, equation 1. is now:

$$\lambda = \frac{h}{\sqrt{2meV}} \tag{3}$$

Since m, e, and h are known it follows that:

$$\lambda \approx (\frac{1.50}{V})^{\frac{1}{2}} \tag{4}$$

$\lambda$ is in nanometer, V is Volts(V). Atoms are aligned in a series of parallel planes in various directions indicated by Miller Indices (h,k,l). Referring to the angles created in Figure 2, the equation is now:

$$n\lambda = 2d\sin\theta \tag{5}$$

Where n is an integer, $\theta$ is the angle between the diffracted direction and incident direction of the electron beam, and where d is the distance between two adjacent parallel planes.

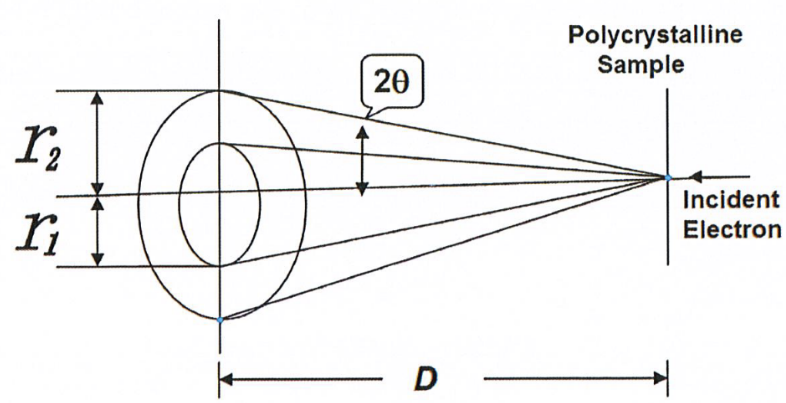


Figure 2 Schematic of electron beam diffraction from a crystal film

For small $\theta$, $\sin\theta$ can be approximated as $\theta = \frac{r}{2D}$, where r is the same as $r_1$ and $r_2$ in Figure 2, and D is the D in both of the figures above. The distance between two parallel planes of Miller indices is:

$$d = \frac{a}{(h^2 + k^2 + l^2)^{\frac{1}{2}}} \tag{6}$$

Plugging Equation 6 into Equation 5 yields:

$$\lambda = \frac{2a \sin\theta}{n(h^2 + k^2 + l^2)^{\frac{1}{2}}} \tag{7}$$

Let $H = nh$, $K = nk$, and $L = nl$, thus:

$$\lambda = \frac{2a \sin\theta}{(H^2 + K^2 + L^2)^{\frac{1}{2}}} = \frac{r}{D} \frac{a}{(H^2 + K^2 + L^2)^{\frac{1}{2}}} \tag{8}$$

Thus for $n^(th)$ order Brags diffraction for any Miller indices plane can be considered $1^(st)$ order Bragg diffraction of plane. Another form of Equation 8 without using the small angle approximation is:

$$\sin\theta = \sin(\frac{1}{2}\arctan(\frac{r}{D})) \tag{9}$$

Electron Wavelength can be found with Equation 3 or Equation 8, from which crystal lattice constant or Miller indices for a specific diffraction ring can be determined using the table below:

Table 1.1

| $h \quad k \quad l$ | $h^2 + k^2 + l^2$ | $(h^2 + k^2 + l^2)^{1/2}$ |
|---|---|---|
| 111 | 3 | 1.732 |
| 200 | 4 | 2.000 |
| 220 | 8 | 2.828 |
| 311 | 11 | 3.316 |
| 222 | 12 | 3.464 |
| 400 | 16 | 4.000 |
| 311 | 19 | 4.358 |

# 4 Experimental Apparatus and Procedures

## 4.1 Specifications for Apparatus

The high voltage is between 0-20kV, DC current, and is adjustable. The filament Voltage is 6.3V, with a current of 0.8mA. For this setup the diffraction sample is Gold (Au). The apparatus has a screen diameter of 130mm, and the overall dimensions are 360mmx200mmx500mm.

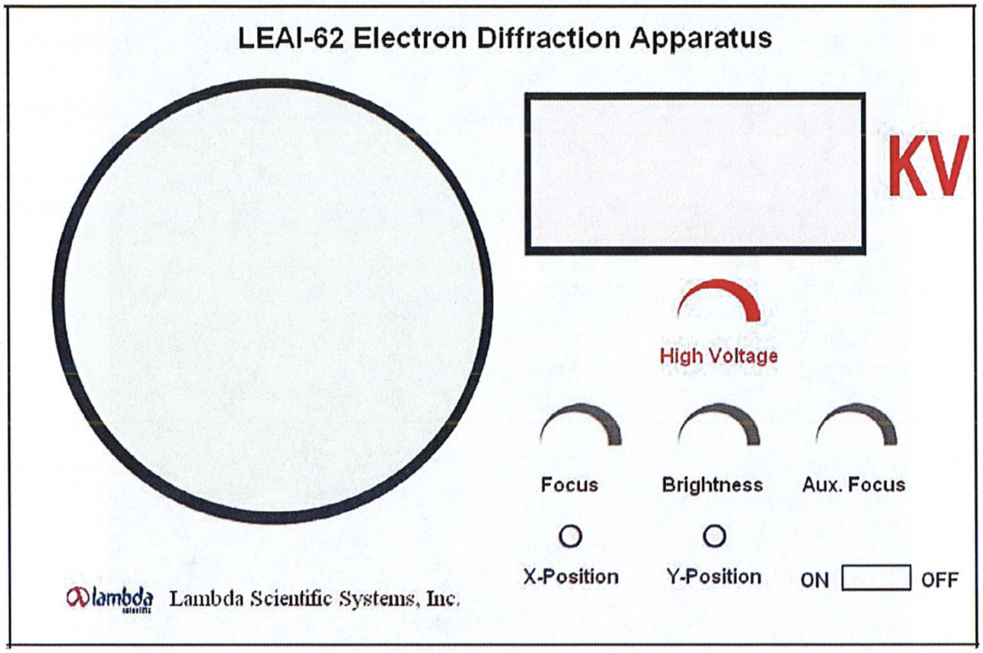


Figure 3 Schematic of front panel

## 4.2 Procedure

Before the power is on, turn high voltage knob to minimum. Next, turn on power to the device. Allow for it to warm up for 10 minutes. Set Voltage to 60kV, adjust the brightness and the focus until a light is observed on the screen. Adjust the X-Position and the Y-Position knobs to bring the center of the pattern as close to the center of the screen as possible. Do not leave the bright spot of the pattern in one place for an excessive amount of time, it will burn the screen. Allow for the system to stabilize, then begin to collect data (refer to data collection for more details). Once done collecting data turn the high voltage to minimum then turn off the whole apparatus.

## 4.3 Data Collection

For this experiment the data was collected using pictures of the screen with a ruler placed on top of the screen. This was done to prevent burning and/or any other damage that could be done to the apparatus. At 5 different voltages a photo was taken with a ruler placed on top of the screen. This will allow for the rings distance from center to be measured and then compared to the Miller Indices in Table 1. This data will later be used in comparison to DeBrogile's Wavelength. These results will also allow for the derivation of Planck's constant, which is $\lambda^2 = \frac{1}{V}$.

# 5 Data analysis, results, and uncertainties.

The results of this experiment are shown in Fig. 4. The error in diffraction radius, $5 \times 10^{-4}$m was determined by the precision of the measuring device along with the uncertainty in the voltage, which was determined to be $50$V. The uncertainty in the length D was determined by the apparatus to be $3 \times 10^{-3}$m.

The general shape of this graph is linear, which fits the linearized dependency described in the square of Eq. 8, and the solid line represents the trendline for the graph, from which Planck's Constant was obtained using the following equation

$$k = \frac{h}{\sqrt{2me}} \tag{10}$$

where k is the slope of the trendline, m is the mass of an electron in kg, and e is the charge of an electron in C. For further analysis of our data, we used a combined Eq. 3 and Eq. 8 to get

$$h = \frac{r\alpha\sqrt{2meV}}{D(H^2 + K^2 + L^2)^{1/2}} \tag{11}$$

which was used to calculate h for each data point. Then, averaging h for each voltage group, we obtained Fig. 5.

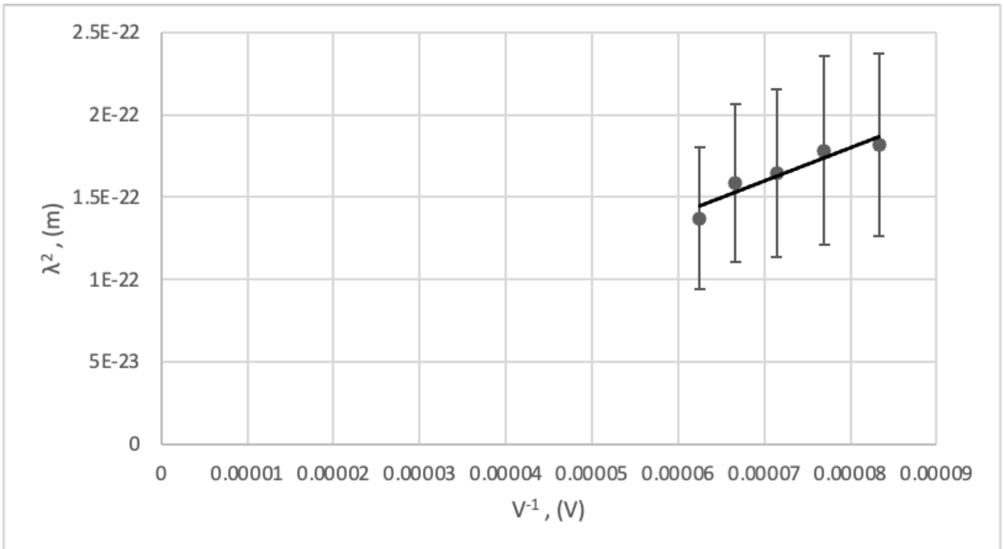


Figure 4 Dependence of wavelength squared on the reciprocal of voltage. The solid line represents the trendline of the graph.

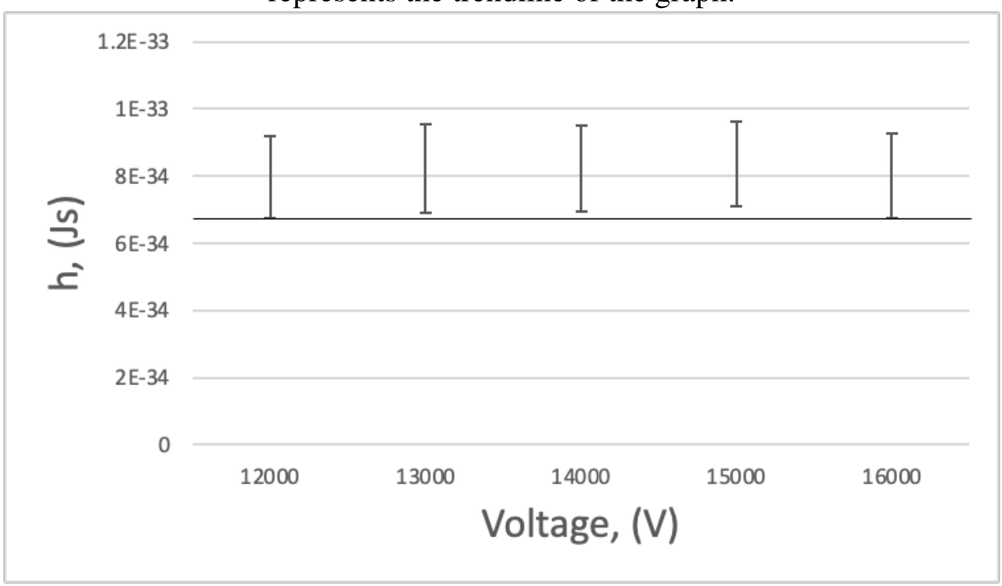


Figure 5 The average calculated value of h for each voltage. The solid line represents the accepted value of h. The error bars represent the standard deviation within each voltage.

To calculate the error term for each data point, we used the following formula

$$\frac{\Delta h}{h} = \frac{\Delta r}{r} + \frac{1}{2} \cdot \frac{\Delta V}{V} + \frac{\Delta D}{D} \tag{12}$$

where the uncertainty terms are those described above and h is calculated using Eq. 11.

With this, we calculated the overall error terms using the sum of the squares of each individual error term

$$\Delta h_{tot} = \sqrt{\sum \Delta h_i^2} \tag{13}$$

this gives us our calculated value of $h = 7.635 \times 10^{-34} \pm 1.792 \times 10^{-34}$ Js

## 5.1 Discussion

While the data obtained in these experiments is described by Eq. 11, there are some deviations for larger radii as can be seen in Fig. 4. This is likely due to human error when measuring as any error in measurement is magnified when working on larger scales. This is validated by the fact that the first data point seems to be off the trendline in Fig. 4. On the other hand, for increasingly smaller radii, there could be more error due to the scale of the measurement device. Thus, we can conclude the measurement ranges used in this experiment were optimal measurement ranges.

Other qualitative measurement uncertainties include the “fuzziness” of the image, resulting in difficulty measuring the data, and a lack of data points. These could be improved with a better image-capturing device and more data samples respectively.

## 5.2 Conclusion

Using the electron diffraction apparatus, we were able to measure Planck’s constant h. The method we used resulted in $h = 7.635 \times 10^{-34} \pm 1.792 \times 10^{-34}$ Js. The measured values are in agreement with the commonly accepted value of $h = 6.62607015 \times 10^{-34}$.

# 6 References

1. *LEAI-62 Electron Diffraction Apparatus*, PHYS 340 manual, Purdue University, W. Lafayette