Analysis of Average Travel Time for Stateless Opportunistic Routing Techniques

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Abstract—**Wireless network applications, such as, searching, routing, self stabilization and query processing can be modeled as random walks on graphs. Stateless Opportunistic routing technique is a robust distributed routing technique based on random walk approach , where nodes transfer the packets to one of their direct neighbors uniformly, until the packets reach their destinations. Simplicity in execution, fault tolerance, low overhead and robustness to topology changes made it more suitable to wireless sensor networks scenarios. But the main disadvantage of stateless opportunistic routing is estimating and studying the effect of network parameters on the packet latency. In this work, we derived the analytical expressions for mean latency or average packet travel time for r-nearest neighbor cycle, r-nearest neighbor torus networks. Further, we derived the generalized expression for mean latency for m-dimensional r-nearest neighbor torus networks and studied the effect of number of nodes, nearest neighbors and network dimension on average packet travel time.**

Index Terms—**Wireless sensor networks, Delay tolerant networks, Random walks, Opportunistic forwarding, Spectral graph theory**

I. INTRODUCTION

In Stateless opportunistic routing, packets are forwarded to the next available neighbors in a random walk fashion until they reach the destinations. Estimation of access time, commute time, cover time and mixing time for random walks are discussed in [1]. Opportunistic forwarding provides significant performance gains and increases the throughput in the wireless networks [2], [3]. The problem of searching for a node or a piece of data and the hitting time of the node has been studied in [8]. Analytic formulas for maximum expected latency has been derived for regular wireless networks [7]. But this work does not studied the Mean Latency metric which is very important metric to study the packet delay and it also does not provide the upper and lower bounds for latency. In our work we derived the mean latency expressions for r-nearest neighbor networks. The motivation behind the using finite sized networks is most of the practical WSN/adhoc networks are finite sized, such as applications in health, military and security in buildings. The r-nearest neighbor networks [7] with varying number of nodes represents the notion of geographical proximity in the wireless sensor networks/ adhoc networks, where, nearest neighbors r captures the overhead or nodes' transmission radius. The advantage of this kind of analysis and

theoretical results is they will play a critical role in the design of wireless sensor networks before the network operations, and also easier to perform than real experiments and thousands of simulation trails. This work provides the understanding of mean latency in terms of number of nodes, nearest neighbors and network dimension and gives the important insights for estimating latency in wireless networks. Further, we also studied the effect of wireless network parameters on packet delay in flat fading environments. For that, we used the system model proposed in [12] for designing the topology coefficients.

The rest of the paper is organized as follows. In Section II, we have given a brief overview about Mean Latency or average travel time. In Section III, we have given the generalized expressions for eigen values of the Laplacian matrix for r-nearest neighbor networks. In Section IV, Mean Latency expressions and bounds are derived for r-nearest neighbor cycle, r-nearest neighbor torus and m-dimensional r-nearest neighbor networks have been derived. In Section V, we have studied the network parameters effect on latency for arbitrary network model. In Section VI, we compared the simulation results with analytical results obtained in the Section IV.

II. MEAN LATENCY OF RANDOM WALKS

Given an undirected graph $G = (V, E)$, where V is the set of nodes and E is the set of edges. Let A be a adjacency matrix of G and d_i be the degree of i where each node $i \in V$. Let $D = diag\,(d_i)$ be a diagonal matrix of node degrees, then $P = D^{-1}A$ is a symmetric transition matrix associated with a random walk on G. Let $\pi = [\pi_i]_{1 \le i \le n}$ is a stationary distribution probability vector. In this case random walk on G is reversible,.i.e. $\pi_i p_{ij} = \pi_j p_{ji}$ and distribution can be expressed as

$$\pi_i = \frac{d_i}{\sum_k d_k} = \frac{d_i}{d} \qquad (1)$$

Definition 1: Normalized Laplacian matrix for undirected graph G is defined as

$$N = D^{-\frac{1}{2}}(D - A)D^{-\frac{1}{2}} \qquad (2)$$

Where N is symmetric and positive semi-definite. Let λ_k, v_k be the eigen values and the corresponding eigen vectors of

N, then the hitting time of random walk [1] from node s to node t as H_{st}, which can be expressed as

$$H_{st} = 2m \sum_{k=2}^{n} \frac{1}{\lambda_k} \left(\frac{v_{kt}^2}{d(t)} - \frac{v_{ks}v_{kt}}{\sqrt{d(t)d(s)}} \right) \quad (3)$$

Lemma 1: The mean latency or average random walk travel time T [10] between every arbitrary pair of nodes is equal to

$$T = \frac{2}{n-1} Tr(L^+) \quad (4)$$

Where $Tr(L^+)$ represents trace of the Moore-Penrose inverse of Laplacian matrix and n denotes the number of nodes.

III. r-NEAREST NEIGHBOR NETWORKS

A. r-nearest neighbor cycle

The r-nearest neighbor cycle C_n^r can be represented by a circulant matrix [4]. A circulant matrix is defined as

$$\begin{bmatrix} a_1 & a_2 & ....a_{n-1} & a_n \\ a_n & a_1 & ....a_{n-2} & a_{n-1} \\ . & . & & . & . \\ . & . & & . & . \\ a_3 & a_4 & ....a_1 & a_2 \\ a_2 & a_3 & ....a_n & a_1 \end{bmatrix} \quad (5)$$

and j-th eigen value of a circulant matrix can be expressed as

$$\lambda_j = a_1 + a_2\omega^j + ... + a_n\omega^{(n-1)j} \quad (6)$$

where ω be the n-th root of 1. Then ω is the complex number:

$$\omega = \cos\left(\frac{2\pi}{n}\right) + i\sin\left(\frac{2\pi}{n}\right) = e^{\frac{i2\pi}{n}} \quad (7)$$

The 1-nearest cycle and 2-nearest cycle are shown in Fig.1 and Fig.2 respectively. Let the adjacency matrix A and the degree matrix D of 1-nearest cycle, then they can be written as

$$A = \begin{bmatrix} 0 & 1 & 0 & ....0 & 1 \\ 1 & 0 & 1 & ....0 & 0 \\ . & . & . & & . & . \\ . & . & . & & . & . \\ 0 & 0 & 0 & ....0 & 1 \\ 1 & 0 & 0 & ....1 & 0 \end{bmatrix} \quad (8)$$

$$D = \begin{bmatrix} 2 & 0 & 0 & ....0 & 0 \\ 0 & 2 & 0 & ....0 & 0 \\ . & . & & . & . \\ . & . & & . & . \\ 0 & 0 & ....2 & 0 \\ 0 & 0 & ....0 & 2 \end{bmatrix} \quad (9)$$

Theorem 1: The generalized expression for eigenvalues of Laplacian matrix L for r-nearest neighbor cycle C_n^r can be expressed as,

$$\lambda_j(L(C_n^r)) = 2r + 1 - \frac{\sin\frac{(2r+1)\pi j}{n}}{\sin\frac{\pi j}{n}} \quad (10)$$

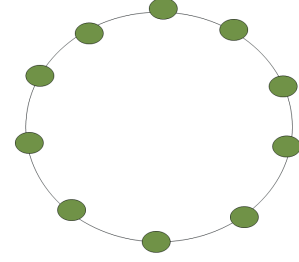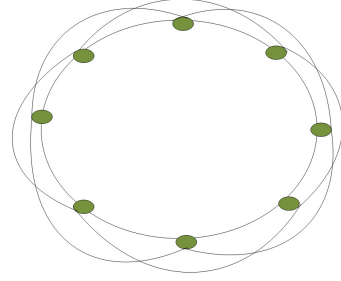
Fig. 1.   1-nearest neighbor cycle

Fig. 2.   2-nearest neighbor cycle

where $j = 0, 1, ...(n-1)$.

Proof: From (7), we can observe that, the first row is enough to obtain the eigen values of any circulant matrix.

The first row of adjacency matrix (A), degree matrix (D) and Laplacian matrix (L) can be written as follows,

$$A_{1n} = \begin{bmatrix} 0 & \underbrace{1\ 1\ 1........0..........1\ 1\ 1}_{2r\ times} \end{bmatrix} \quad (11)$$

$$D_{1n} = [2r\ 0\ 0\ 0 .....0\ 0\ 0\ 0] \quad (12)$$

$$L_{1n} = \begin{bmatrix} 2r & \underbrace{-1\ -1\ -1 .....0......\ -1\ -1\ -1}_{2r\ times} \end{bmatrix} \quad (13)$$

By using (13) and (6), we can write the

$$\lambda_j(L(C_n^r)) = 2r - 2\sum_{i=1}^{r} \cos\left(\frac{2\pi ji}{n}\right) \quad (14)$$

Lemma 2: Trigonometric identity of Dirichlet kernel [5]

$$1 + 2\sum_{j=1}^{r} \cos(jx) = \frac{\sin\left(r + \frac{1}{2}\right)x}{\sin\left(\frac{x}{2}\right)} \quad (15)$$

Hence, from the Lemma 2, (14) can be rewritten as,

$$\lambda_j(L(C_n^r)) = 2r + 1 - \frac{\sin\frac{(2r+1)\pi j}{n}}{\sin\frac{\pi j}{n}} \quad (16)$$

B. *r-nearest neighbor torus*

A torus can be seen in Fig. 3 and it can be represented by the $n \times n$ block circulant matrix A as

$$A = \begin{bmatrix} A_0 & A_1 & .... A_{n_1-2} & A_{n_1-1} \\ A_{n_1-1} & A_0 & .... A_{n_1-3} & A_{n_1-2} \\ . & . & . & . \\ . & . & . & . \\ A_1 & A_2 & .... A_{n_1-1} & A_0 \end{bmatrix} \quad (17)$$

where the number of nodes $n = n_1^2$, then each block A_i, for $i = 0, 1...(n_1 - 1)$ represents $n_1 \times n_1$ circulant matrices.

Lemma 2: Let G be the cartesian product of two graphs G' and G'' with vertex sets V' and V'' and edge sets E' and E''. Let the eigen values of G' are $\lambda_1\left(G'\right).........\lambda_p\left(G'\right)$ and G'' are $\lambda_1\left(G''\right).........\lambda_q\left(G''\right)$, where $p = \left|V'\right|$ and $q = \left|V''\right|$. Let the vertex set of G is $r = |V|$, which can be expressed as $V = \left|V'\right| \times \left|V''\right|$ [9]. Then, the eigen values of G can be expressed as

$$\lambda_k\left(G\right) = \lambda_i\left(G'\right) + \lambda_j\left(G''\right) \quad (18)$$

,where $i \in \{1, 2, ...p\}$, $j \in \{1, 2, ...q\}$ and $k \in \{1, 2, ...r\}$.

Remark 2: (18) also holds for eigen values of the Laplacians L' and L'' of graphs of G' and G'' respectively[6].

Theorem 2: The generalized expression for eigenvalues of Laplacian matrix L for r-nearest neighbor torus T_n^r can be expressed as

$$\lambda_{j_1,j_2}\left(L(T_{k_1,k_2}^r)\right) = 4r + 2 - \frac{\sin\frac{(2r+1)\pi j_1}{k_1}}{\sin\frac{\pi j_1}{k_1}} - \frac{\sin\frac{(2r+1)\pi j_2}{k_2}}{\sin\frac{\pi j_2}{k_2}} \quad (19)$$

where $j_1 = 0, 1, 2, ...(k_1 - 1)$, $j_2 = 0, 1, 2, ...(k_2 - 1)$.

Proof : T_n^r can be represented by Cartesian product of two r-nearest neighbor cycles. So from the *Lemma* 2, we can write the $\lambda_{j_1,j_2}\left(L(T_{k_1,k_2}^r)\right)$ as,

$$\lambda_{j_1,j_2}\left(L(T_{k_1,k_2}^r)\right) = \lambda_{j_1}\left(L(C_{k_1}^r)\right) + \lambda_{j_2}\left(L(C_{k_2}^r)\right) \quad (20)$$

From *Remark* 2, we can write the expressions for $\lambda_{j_1}\left(L(C_{k_1}^r)\right)$ and $\lambda_{j_2}\left(L(C_{k_2}^r)\right)$, substituting them in (20) proves the theorem.

Theorem 3: The generalized expression for eigenvalues of Laplacian matrix L for m-dimensional r-nearest neighbor torus can be expressed as

$$\lambda_{j_1,j_2,...j_m}\left(L(T_{k_1,k_2....k_m}^r)\right) = (2r + 1)m - \sum_{i=1}^{m}\left(\frac{\sin\frac{(2r+1)\pi j_i}{k_i}}{\sin\frac{\pi j_i}{k_i}}\right) \quad (21)$$

Proof: r-nearest neighbor m-dimensional torus can be represented by Cartesian product of m number of r-nearest
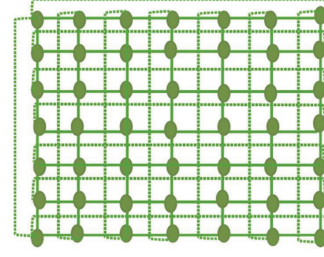

Fig. 3. Two dimensional torus

neighbor cycles. So from the *Lemma* 2, we can write the $\lambda_{j_1,j_2....j_m}\left(L(T_{k_1,k_2....k_m}^r)\right)$ as,

$$\lambda_{j_1,j_2,...j_m}\left(L(T_{k_1,k_2....k_m}^r)\right) = \lambda_{j_1}\left(L(C_{k_1}^r)\right) + \lambda_{j_2}\left(L(C_{k_1}^r)\right) .... + \lambda_{j_m}\left(L(C_{k_m}^r)\right) \quad (22)$$

From *Remark* 2, we can substitute the expressions for $\lambda_{j_1}\left(L(C_{k_1}^r)\right)$, $\lambda_{j_2}\left(L(C_{k_2}^r)\right)$ and $\lambda_{j_m}\left(L(C_{k_m}^r)\right)$ in (22), which proves the theorem.

IV. MEAN LATENCY ANALYSIS FOR r-NEAREST NEIGHBOR NETWORKS

Theorem 5: The mean latency T of r-nearest neighbor cycle C_n^r between every arbitrary pair of nodes is

$$T\left(C_n^r\right) = \sum_{j=2}^{n}\left(\frac{2}{(n-1)\left((2r+1) - \frac{\sin\frac{(2r+1)\pi j}{n}}{\sin\frac{\pi j}{n}}\right)}\right) \quad (23)$$

Proof: From (10), we can write

$$Tr(L^+) = \sum_{j=2}^{n}\frac{1}{\lambda_j(L)}$$

$$= \sum_{j=2}^{n}\left(\frac{1}{\left((2r+1) - \frac{\sin\frac{(2r+1)\pi j}{n}}{\sin\frac{\pi j}{n}}\right)}\right) \quad (24)$$

Substituting the (24) in (4), proves the theorem.

Theorem 6: The mean latency of r-nearest neighbor torus T_{k_1,k_2}^r between every arbitrary pair of nodes is

$$T(T_{k_1,k_2}^r) = \sum_{j_1=1}^{k_1-1}\sum_{j_2=0}^{k_2-1}\left(\frac{2}{(k_1+k_2-1)\left((4r+2) - \frac{\sin\frac{(2r+1)\pi j_1}{k_1}}{\sin\frac{\pi j_1}{k_1}} - \frac{\sin\frac{(2r+1)\pi j_2}{k_2}}{\sin\frac{\pi j_2}{k_2}}\right)}\right) \quad (25)$$

Proof: From (19), we can write

$$Tr(L^+) = \sum_{j_1=1}^{k_1-1} \sum_{j_2=0}^{k_2-1} \frac{1}{\lambda_{j_1,j_2}(L)}$$

$$= \sum_{j_1=1}^{k_1-1} \sum_{j_2=0}^{k_2-1} \left(\frac{2}{\left((4r+2) - \frac{\sin\frac{(2r+1)\pi j_1}{k_1}}{\sin\frac{\pi j_1}{k_1}} - \frac{\sin\frac{(2r+1)\pi j_2}{k_2}}{\sin\frac{\pi j_2}{k_2}} \right)} \right) \quad (26)$$

Substituting the (26) in (4), proves the theorem.

Theorem 7: The mean latency of r-nearest neighbor torus $T_{k_1,k_2,\ldots k_m}^r$ between every arbitrary pair of nodes is

$$T(T_{k_1,k_2,\ldots k_m}^r) = \sum_{j_1=1}^{k_1-1} \sum_{j_2=0}^{k_2-1} \cdots\cdots \sum_{j_m=0}^{k_m-1} \left(\frac{2}{\left((2r+1)m - \sum_{i=1}^{m} \frac{\sin\frac{(2r+1)\pi j_i}{k_i}}{\sin\frac{\pi j_i}{k_i}} \right)} \right) \quad (27)$$

Proof: From (21), we can write

$$Tr(L^+) = \sum_{j_1=1}^{k_1-1} \sum_{j_2=0}^{k_2-1} \cdots\cdots \sum_{j_m=0}^{k_m-1} \frac{1}{\lambda_{j_1,j_2,\ldots,j_m}(L)}$$

$$= \sum_{j_1=1}^{k_1-1} \sum_{j_2=0}^{k_2-1} \cdots\cdots \sum_{j_m=0}^{k_m-1} \left(\frac{2}{\left(\sum_{i=1}^{m} k_i - 1 \right)\left((2r+1)m - \sum_{i=1}^{m} \frac{\sin\frac{(2r+1)\pi j_i}{k_i}}{\sin\frac{\pi j_i}{k_i}} \right)} \right) \quad (28)$$

Substituting the (28) in (4), proves the theorem.

Lemma 3: The mean latency T between every arbitrary pair of nodes satisfies the following bound [11]:

$$\frac{2}{(n-1)\lambda_1(L)} \leq T \leq \frac{2}{\lambda_1(L)} \quad (29)$$

Where $\lambda_1(L)$ is the second smallest eigen value of Laplacian matrix and n represents number of nodes.

Theorem 8: The bounds for mean latency T for r-nearest neighbor cycle can be expressed as,

$$\frac{2\sin\frac{\pi}{n}}{(n-1)((2r+1)\sin\frac{\pi}{n} - \sin\frac{(2r+1)\pi}{n})} \leq T \leq \frac{2\sin\frac{\pi}{n}}{(2r+1)\sin\frac{\pi}{n} - \sin\frac{(2r+1)\pi}{n}} \quad (30)$$

Proof: Substituting the n=1 in (4) gives $\lambda_1(L)$, which can be substituted in the (29) proves the theorem.

Similarly we can prove the bounds for mean latency for m-dimensional torus network as

$$\frac{2\sin\frac{\pi}{k_1}}{(n-1)((2r+1)\sin\frac{\pi}{k_1} - \sin\frac{(2r+1)\pi}{k_1})} \leq T \leq \frac{2\sin\frac{\pi}{k_1}}{(2r+1)\sin\frac{\pi}{k_1} - \sin\frac{(2r+1)\pi}{k_1}} \quad (31)$$

V. OPPORTUNISTIC FORWARDING FOR ARBITRARY NETWORKS

We consider the arbitrary network model as shown in Fig. 9, where the nodes are distributed arbitrarily, but their positions are known. The following propagation model has been considered

$$P_{R_j} = \frac{p_{ij}}{1 + (r_{ij}/r_0)^\eta} \quad (32)$$

where P_{R_j} is the power received by node j when node i transmits, r_{ij} is the distance between nodes i and j, η is the

path loss exponent and r_0 is a reference distance. If $r_{ij} \gg r_0$, the receiver is in the transmit antenna far-field, where the received power is inversely proportional to r_{ij}^η. Conversely, if $r_{ij} \ll r_0$, then the received power is approximately equal to the transmitted power.

The relationship between the coefficients a_{ij} and the distances r_{ij} can be expressed as

$$a_{ij} = \frac{1}{1 + \left(r_{ij}/r_{c_{ij}} \right)^\alpha} \quad (33)$$

where α is a positive coefficient and $r_{c_{ij}}$ is the coverage radius, which depends on the transmit power.

From the (33), the relationship between the coverage radius $r_{c_{ij}}$, power coefficients p_{ij} and the minimum required power for communication p_{min} can be expressed as

$$r_{c_{ij}} = r_0 \left(\frac{p_{ij}}{p_{\min}} - 1 \right)^{1/\eta} \quad (34)$$

where

$$r_0 = \sqrt{\frac{\log(n) + c_n}{\pi n}} \quad (35)$$

using the (34) and (32) the topology coefficients can be written in terms of the power coefficients p_{ij}, denoting the power used by node i to transmit to node j as

$$a_{ij} = \frac{r_0^\alpha \left(p_{ij} - p_{\min} \right)^{\alpha/\eta}}{r_0^\alpha \left(p_{ij} - p_{\min} \right)^{\alpha/\eta} + r_{ij}^\alpha p_{\min}^{\alpha/\eta}} \quad (36)$$

Remark 1 : From the (37), it is evident that the topology co-efficients depends on power coefficients p_{ij} and other wireless network parameters α, p_{min}, r_0, r_{ij}, η.

Remark 2: From the (37) it is also evident that symmetric wireless links present in the network when $p_{ij} = p_{ji}$ and asymmetric wireless links exits when $p_{ij} \neq p_{ji}$. Here, we studied the symmetrical wireless networks.

VI. SIMULATION RESULTS

To validate the derived Mean latency analytical expressions, we compared with the simulation results and observed that both agree with each other. As shown in the Fig. 4, we plotted mean latency T against nearest neighbors for n=300, and observed that mean latency decreases with r. From Fig.5, we can see the mean latency versus number of nodes n for $r = 1$ for r-nearest neighbor cycle. Similarly to observe the Mean latency variation for two dimensional finite networks, we plotted mean latency versus k_1 and k_2. We have taken r=1 to plot Fig. 7. and $k_1=k_2$=1000 to plot Fig.6. To study the effect of network dimension on mean latency, we have taken the k_1=16, k_2=18, k_3=20 and k_4=22 and r varied from 1 to 4. We have observed that, Mean latency is decreased with the network dimension and it further decreases with increase in r.

To study the effect of few more wireless network parameters on expected packet delay (EPD), we have used the (3) and to
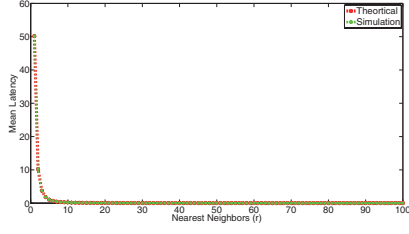
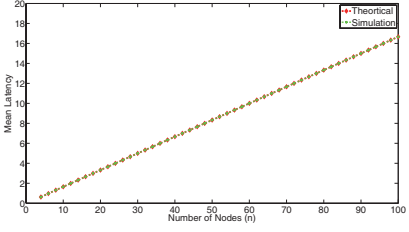Fig. 4.   Mean latency versus Nearest neighbors for r-nearest neighbor cycle

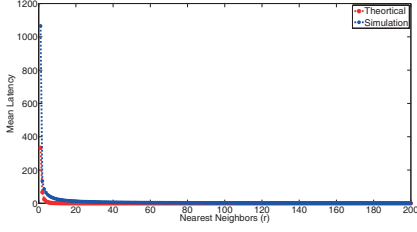Fig. 7.   Mean latency versus k_1 and k_2 for r-nearest neighbor torus

Fig. 5.   Mean latency versus number of nodes for r-nearest neighbor cycle
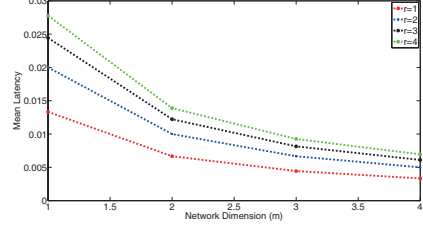
Fig. 8.   Mean latency versus Network Dimension for r-nearest neighbor torus

Fig. 6.   Mean latency versus Nearest neighbors for r-nearest neighbor torus

Fig. 9.   Wireless Network Topology

generate the topologies as shown in Fig.9 we have used the (37).

$$EPD = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} H_{ij}}{n(n-1)} \qquad (37)$$

where H_{ij} denotes the delay between node i and node j and n represents number of nodes.

After estimating the EPD, we try to understand the effect of wireless network parameters on EPD.To evaluate how the network density affects the EPD, topologies has been generated with size, $A = 1 \times 1 \, m^2$. From Fig. 10, we can see that, packet delay increases with path loss exponent, here we have taken $p_{min} = 0.1$ and $n = 30$. Fig. 11 shows the impact of Minimum Received Power on EPD for $n = 30$ in freespace ($\alpha = 2$) and multipath ($\alpha = 4$) communication. From the simulation results, EPD increases with Minimum Received Power (p_{min}) and it increases further in multipath environments. From the proposed analytic modeling, topological coefficients has been derived and the connectivity threshold has been introduced to get the binary adjacency matrix which defines the wireless network topology. Fig. 12 shows the impact of connectivity threshold on EPD for $p_{min} = 0.1$ in freespace ($\alpha = 2$) and multipath ($\alpha = 4$) communication.
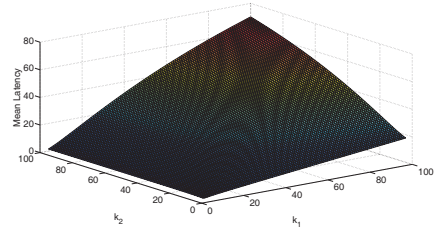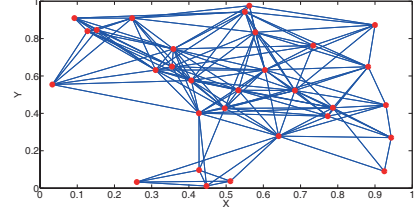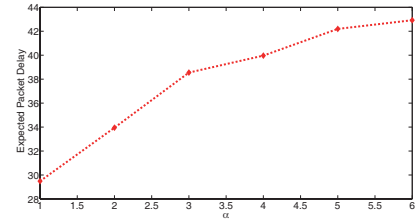

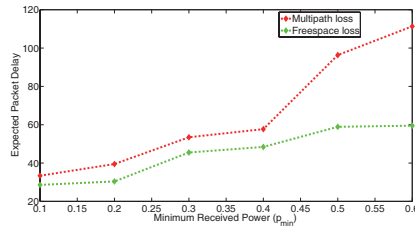Fig. 10.   Expected Packet Delay versus path loss exponent α

Fig. 11.   Expected Packet Delay versus P_{min} for random topology

VII. CONCLUSIONS

In this work, analytical expressions for Mean latency or average travel time has been derived for r-nearest neighbor cycle, r-nearest neighbor torus and m dimensional r nearest
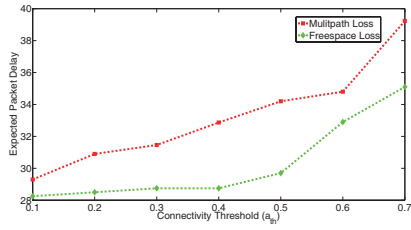
Fig. 12. Expected Packet Delay versus Connectivity Threshold for random topology

neighbor torus networks. We have given the theoretical bounds for mean latency in terms of number of nodes and nearest neighbors. Further, we also studied the latency for arbitrary wireless networks in flat fading environments.

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