Rotation matricies

yaw pitch roll

$$R = R_z(\alpha)\, R_y(\beta)\, R_x(\gamma) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix}$$

rotation matrix

$$R = \begin{bmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma & \cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma & \sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma \\ -\sin\beta & \cos\beta\sin\gamma & \cos\beta\cos\gamma \end{bmatrix}$$

inverse of rotation matricies => euler angles

$$\phi_x = \operatorname{atan2}(r_{32}, r_{33})$$

$$\phi_y = \operatorname{atan2}\left(-r_{31}, \sqrt{r_{31}^2 + r_{33}^2}\right)$$

$$\phi_z = \operatorname{atan2}(r_{21}, r_{11})$$

Homogeneous transform matrix

$$T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$d = \{x, y, z\}$ positional translations

$R = \{r_{11}, \ldots r_{33}\}$ rotation matrix

* transform matricies can be generated / combined for each joint
  => forwards kinematics

DH parameters (Denavit Hartenberg)

- Rules for assigning coordinate frames

1. z axis must be axis of rotation
2. x axis must be perpendicular to z axis in immediate surrounding frames
3. y axis must follow right hand rule
4. x axis must intersect z axis from previous joint

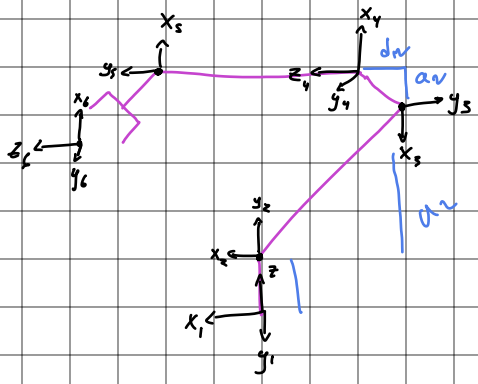
joint	ϕ	α	d	a
1				
2				
3				
4				
5				
6				

ϕ = rotation on previous z from old to new x
α = angle of common normal between z axis
d = offset along z from old to new normal
a = length of common normal
↳ length offset along x (length of arm)

=> using 4 parameters per joint results in the transform matrix

$$\begin{bmatrix} \cos\phi_n & -\sin\phi_n \cos\alpha_n & \sin\phi_n \sin\alpha_n & a_n \cos\phi_n \\ \sin\phi_n & \cos\phi_n \cos\alpha_n & -\cos\phi_n \sin\alpha_n & a_n \sin\phi_n \\ 0 & \sin\alpha_n & \cos\alpha_n & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C\phi_n & -S\phi_n C\alpha_n & S\phi_n S\alpha_n & a_n C\phi_n \\ S\phi_n & C\phi_n C\alpha_n & -C\phi_n S\alpha_n & a_n S\phi_n \\ 0 & S\alpha_n & C\alpha_n & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$


x_5
y_5
x_6
z_6
y_6
x_4
z_4
y_4
d_N
a_N
y_5
x_5
y^2
y_2
x_2
z
x_1
y_1