General Physics

(103Phy)

For 1st year students (**Biology group**)

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Contents

PROPERTIES OF MATTER

Chapter (1)

UNITS AND DIMENSIONS

1.1 Introduction

To define any physical quantity in physics one must have a definite set of rules for calculating the quantity in terms of other well defined quantities that can be measured. Thus, when we speak of kinetic energy of a body, we define it as half the product of mass and the square of the velocity. If mass and velocity can be measured accurately, the kinetic energy is known accurately.

Physics is that branch of science that provides answers to questions largely depending on the behavior of non-living things in nature. Any branch of knowledge, physics, chemistry, botany, zoology or geology always helps us to know the answers for "how and why" of things. What is an atom? How do the stars and the sun produce intense heat and light? Is it possible to create or destroy matter? Many such questions are answered by the knowledge of physics.

The distance between the earth and the stars is measured in "light years". One light year is the distance traveled by light in one year(9.46×10^{15}m). The life of the universe is estimated to be millions of years. On the other hand, the size of the atom nucleus is of the order of (10^{-15}m). This shows that man lives between two infinities, viz, very small quantities and very large quantities. In comparison to the astronomical scale of time and distance, the size of man and the span of his life are insignificant. In physics, attempts are constantly made to measure these quantities (too small or too large) with the best possible accuracy.

1.2 System of units

To measure various physical quantities three fundamental units are commonly used in physics. They are (i) Length (ii) Mass and (iii) Time. All other units are derived from these fundamental units. In mechanics, derived units can be represented in terms of the fundamental units. For example, units in which the area is expressed is area of a square whose side is the unit of length. Similarly, unit of velocity is expressed by dividing the unit of length by unit of time. Such units, which depend on the powers of one or more of the fundamental units, are termed as derived units. There are two systems of units, (i) MKS and (ii) CGS.

MKS system. Here the unit of length is ***meter***, the unit of mass is ***kilogram*** and the unit of time is ***second***.

CGS system. Here the unit of length is ***centimeter***, the unit of mass is ***gram*** and the unit of time is ***second***.

1.3 S I System

International system of units (SI): is a coherent system consisting of six basic units. They are:

(1) kilogram (mass) (2) meter (length) (3) second (time) (4) ampere(current) (5) degree Kelvin (temperature) and (6) candela (luminous).

Supplementary SI units: These are plane angle and solid angle. The unit of plane angle is radian and its symbol is rad. The unit of solid angle is steradiaon.

Derived.(SI) units: Many physical quantities such as force, power, energy etc. are expressed in derived SI units. Most of the derived SI units have been named after renowned scientists.

Basic SI units

Symbol	**Units**	**Quantity**
Kg	Kilogram	mass
M	Meter	length
S	Second	time
A	Ampere	current
K	Kelvin	temperature
Cd	Candela	luminous

Basic units of mass

Unit	Symbol	Relation
kilogram	kg	1 kg
gram	g	10^{-3} kg
milligram	mg	10^{-6} kg
tone	T	10^{3} kg

Symbols for multiples and sub-multiples of units

Name	Symbol	Equivalent
Tera	T	10^{12}
Gige	G	10^{9}
Mega	M	10^{6}
Kilo	K	10^{3}
Hecta	H	10^{2}
Deca	D	10^{1}
Deci	d	10^{-1}
Centi	c	10^{-2}
Milli	m	10^{-3}
Micro	µ	10^{-6}
Nano	n	10^{-9}
Pico	p	10^{-12}
Femto	f	10^{-15}

Basic units of time

Unit	Symbol	Relation
Second	s	1 s
Minute	min	60 s
Hour	hr	3600 s
Day	d	86400 s

Basic units of length

Unit	Symbol	Relation
Meter	m	1 m
Decimeter	dm	10^{-1} m
Centimeter	cm	10^{-2} m
Millimeter	mm	10^{-3} m
Micrometer	µm	10^{-6} m
Kilometer	km	10^{3} m

1.4 Dimensions

The power of the fundamental units in terms of which a physical quantity can be represented are known as ***dimensions***. For example, in mechanics, area has two dimensions in length and is represented as:

- Area: $A = L \times L \Longrightarrow A = L^2$
- Velocity: $\upsilon = \frac{d}{t} = \frac{L}{T} \Longrightarrow \upsilon = LT^{-1}$
- Acceleration: $a = \frac{\upsilon}{t} = \frac{LT^{-1}}{T} \Longrightarrow a = LT^{-2}$
- Force: $F = m \times a = M.LT^{-2} \Longrightarrow F = MLT^{-2}$
- Work: $W = F \times d = MLT^{-2}.\, L \Longrightarrow W = ML^2T^{-2}$
- Pressure: $P = \frac{F}{A} = \frac{M\,LT^{-2}}{L^2} \Longrightarrow P = ML^{-1}T^{-2}$
- Power: $\text{power} = \frac{Work}{t} = \frac{M\,L^2T^{-2}}{T} \Longrightarrow \text{power} = ML^2T^{-3}$
- Energy: $E = \text{power} . t \Longrightarrow \text{energy} = ML^2T^{-2}$

Problems: By using the method of dimensions find the dimensions of

1. The gravitational constant.
2. The coefficient of viscosity.
3. (**hw**) The coefficient of surface tension:

1.5 Main uses of dimensional theory

1.5.1. To test the correction of equations

By substituting dimensional formulae on both sides of an equation, any physical equation can be checked. Any equation connecting physical quantities will be in accordance with the principle of homogeneity of dimensions. According to this principle, the dimensions on the left hand side of an equation are the same as the dimensions on the right hand side of the equation.

Example: Check dimensionally the following equations:

(i) $\mathbf{v^2=v_0^2+2ad}$ (ii) $\mathbf{t=2\pi\sqrt{\frac{\ell}{g}}}$

Solution

(i) $\mathbf{v^2=v_0^2}+2ad$ by using the dimensional method

$$(LT^{-1})^2 = (LT^{-1})^2 + (LT^{-2}X\ L) = (LT^{-1})^2$$

$\rightarrow$ LHS = RHS

(ii) $\mathbf{t=2\pi\sqrt{\frac{\ell}{g}}}$ by using the dimensional method

$$T = \sqrt{\frac{L}{LT^{-2}}} = \mathbf{T}$$

$\rightarrow$ LHS = RHS

1.5.2. Drive the equations

The dimensions of various physical quantities are useful in deriving the equations. Many equations have been derived in this way.

Example: Using the dimensional analysis, derive an expression for the time period of oscillation of a simple pendulum. Assume that the time period depends on (i) mass, (ii) length and (iii) acceleration due to gravity.

Solution

Assume that t, m, *l* and g are related to the equation:

$t \propto m^x \ell^y g^z$

$t = K m^x \ell^y g^z$

by using the dimensional method

$T = M^x L^y (LT^{-2})^z$

$M^0 L^0 T^1 = M^x L^{y+z} T^{-2z}$

Comparison the powers of M, L and T on both sides

$x = 0, \; y + z = 0, \quad -2z = 1$

Solving the three equations,

$x = 0, \quad y = 1/2, \quad z = -1/2$

$\therefore t = k\sqrt{\frac{l}{g}}$

Example: Show by the method of dimensions that the excess of pressure inside a soap bubble is ($k\frac{\gamma}{R}$) where γ is the surface tension and R is the radius of the bubble. Here, k is dimensionless constant.

Solution

Assume that P, γ and R are related through the equation:

$P \propto \gamma^x R^y$

$P = k\gamma^x R^y$

By using the dimensional method:

$$ML^{-1}T^{-2} = (MT^{-2})^x L^y$$

$$ML^{-1}T^{-2} = M^x L^y T^{-2x}$$

Comparison the powers of M, L, and T on both sides

x = 1, y = -1

so P= $K\gamma/$R

Example: Assuming that the factional force F between the surface of a small ball and a liquid while it falling through it depends on (i) the liquid viscosity n, (ii) the bail velocity u and (iii) the ball radius R. Show ,; dimensionally that Fα η *v* R.

Solution

Assume that F,η, *v* and R are related through the equation:

$$F \propto \eta^x v^y R^z$$

$$F = k\eta^x v^y R^z$$

By using the dimensional method

$$MLT^{-2} = (ML^{-1}T^{-1})^x (LT^{-1})^y L^z$$

$$MLT^{-2} = M^x L^{-x+y+z} T^{-x-y}$$

Comparison the powers of M, L, and T on both sides

x = 1, $-x + y + z = 1$, $-x - y = -2$

Solving the three equations,

x = 1, $y = 1$, $z = 1$

so $F = k\eta v R$

Problem: Derive by the method of dimensions, an expression for the maximum velocity u for fluid flow through a circular tube. Assume that u depends on (i) the radius of the tube R, (ii) viscosity of the fluid η, (iii) pressure difference P and (v) the length of the tube ℓ.

Problem : Test by the method of dimensions the accuracy of the relation: $\eta = \frac{\pi P R^4}{8 v \ell}$

where η is a viscosity coefficient, P is a pressure, R is a radius, v is a volume and ℓ is a length.

Chapter (2)

MECHANICAL PROPERTIES OF MATTER

2.1 Introduction

Elasticity is the property by virtue of which material bodies regain their original shape and size after the external deforming forces are removed. When an external force acts on a body, there is change in its length, shape and volume. The body is said to be strained. When this external force is removed, the body regains its original shape and size. Such bodies are called elastic bodies. Therefore, elasticity is defined as ***the property by which a body regains its original position when the forces are removed***. Steel, glass, quartz etc. are plastic bodies. The bodies which do not regain their original shape and size are called plastic bodies. Nobody is either completely elastic or completely plastic. The property of elasticity is different in different substances. Steel is less elastic than rubber. Liquids and gases are highly elastic.

2.2 Stress

When a force F is applied on a body, there will be relative displacement of the particles and due to the property of elasticity the particles tend to regain their original position. **Stress** is defined as **the acting force per unit area**. Figure (2-la) shows a bar of uniform cross sectional area A subject to equal and opposite pulls F at its ends. Consider a section through the bar, the pulls are uniformly distributed over the cross-sectional area A, as indicated in Fig. (2-lb). We define the stress at the section as the ratio of the force F to the area A:

$$\text{Stress} = F/A$$

The SI units of stress is Newton per square meter ($N \cdot m^{-2}$). This unit is also given the special name, the Pascal (Pa).

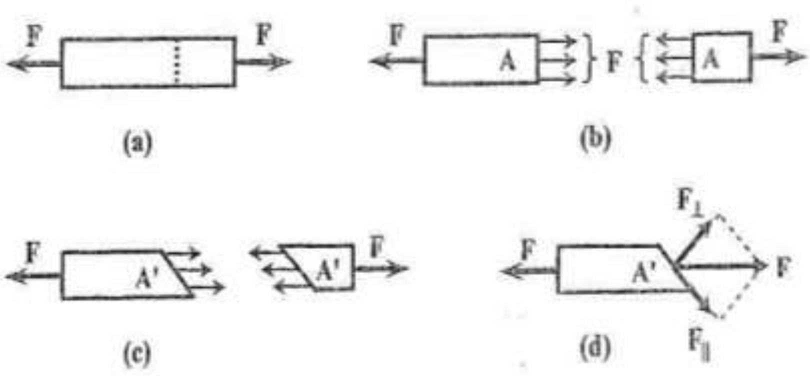


Fig. (2-1)

Consider another section through the bar at some arbitrary angle, as in Fig. (2-lc), the resultant force vector, Fig. (2-1d), can be resolved into a component F_x normal to the area A, and a component F⊥, tangent to the area. Therefore, we can define two types of stresses.

(i) Normal stress: is the perpendicular force per unit area.

(ii) Tangential stress: is the parallel force per unit area.

Normal stress = $\frac{F_{\perp}}{A}$,   Tangential stress = $\frac{F_{\parallel}}{A}$

2.3 Strain

The term strain refers to the relative change in dimensions or shape of a body that is subjected to stress. There are three types of strain:

(i) Longitudinal strain:

Figure (2-2) shows a bar of length ℓ that elongates to a length ℓ+Δℓ when equal and opposite forces F are exerted at its ends. The longitudinal strain is defined as the ratio of change in length to original length;

Longitudinal strain $=\frac{\Delta \ell}{l}$

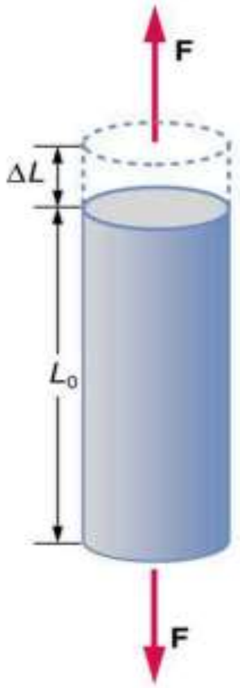


Fig. (2-2)

(ii) Volume strain:

Figure (2-3) shows a block of volume V that decrease to a volume V-ΔV when a hydrostatic pressure acts on the faces of a block the volume strain defined as **the ratio of change in volume to original volume**;

Volume strain = $-\frac{\Delta V}{v}$

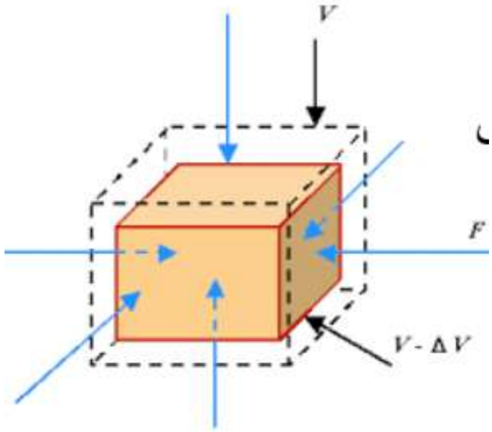


Fig. (2-3)

(iii) Shearing strain:

Shearing strain is defined as the angle of shear measured in radians. In Fig. (2-4), the surface AB is fixed and a force is applied parallel to the surface CD, so that the body is shearing by an angle θ. The angle θ measured in radians is called the shearing strain = θ

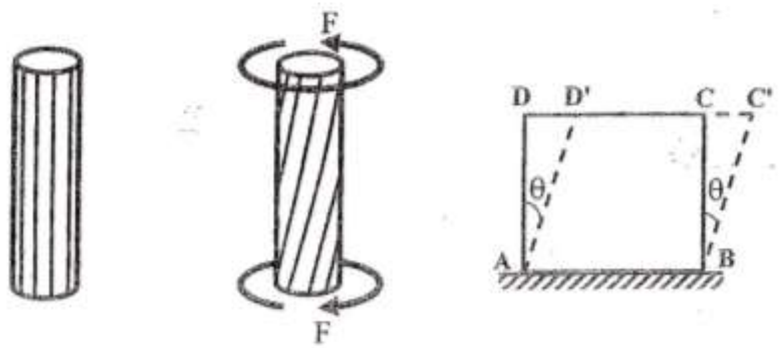


Fig. (2-4)

Problem(1): Calculate the longest length of a steel wire that can hang vertically without breaking. Breaking stress for steel $=8\times10^8 N/m^2$ and density of steel $=8\times10^3 kg/m^3$.

Problem (2): Define shearing stress and shearing strain. Calculate the torque required to twist a cylinder of length *t* and radius   through an angle θ.

2.4 Elasticity and plasticity

We may now consider the relation between each of the three kinds of stress and its corresponding strain. When any stress is plotted against the appropriate strain, the resulting stress strain diagram is found to have a shape as given in Fig. (2-5).

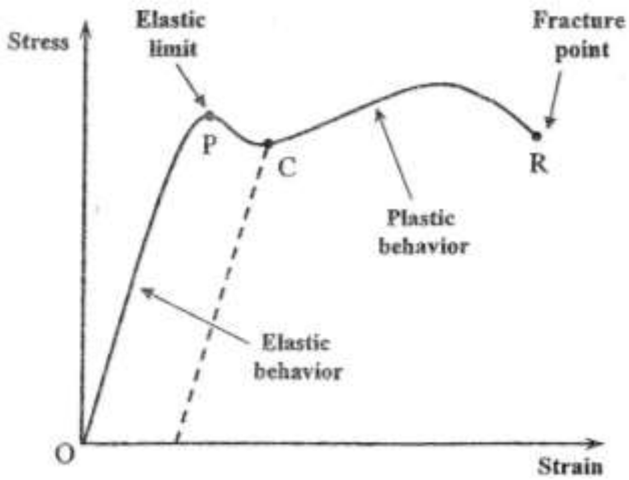


Fig. (2-5)

Elastic region (OP): During the first portion of the curve (OP), the stress and strain are proportional until, the point P, the proportional limit, is reached. The fact that there is a region in which stress and strain are proportional is called ***Hook's law***. If the load is removed at any point between O and P, the material is return to its original length. In the region OP, the material is said to be elastic and the point P is called the elastic limit. Up to this point, the force exerted by the material are conservative; when the material returns to its original shape, work done in producing the deformation is recovered.

Plastic region PR: If the material is loaded, the strain increases rapidly, and when the load is removed at some point, say C, the material does not come back to its original length but traverses the dashed line in Fig. (3-5). From P to R, the material is said to undergo plastic deformation. Increase of load beyond C produces a large increase in strain until a point R is reached **(Fracture point)** at which fracture take place.

Elasticity of living cell materials:

In many types of cells, the protoplasm has been pulled apart with micro needles until strands of it extended to great lengths. On release of these strands of protoplasm, they snapped back to their original lengths, indicating that they are elastic. Nuclei of cells have been stretched between micro needles and the indication is that nuclear material is elastic as long as it remains alive. Experiments have been performed on individual chromosomes, it was found that, the chromosomes could be extended to five times their original length while they were in the nucleus, when removed from it, they could be stretched to twenty – five times their original length.

The red blood cell has also been stretched and has been observed to snap back into its original position. In this case, the elasticity is primarily that of the red cell membrane.

There are three types of elasticity constant:

(i)Young's modulus of elasticity (Y): It is defined as the ratio of normal stress to longitudinal strain

$$Y = \frac{normal\ stress}{longitudinal\ strain} = \frac{\frac{F_{\perp}}{A}}{\frac{\Delta\ell}{\ell}} = \frac{F_{\perp}\ell}{A\Delta\ell}$$

(ii)Shear modulus (S): It is defined as the ratio of tangential stress to shearing strain.

$$S = \frac{tangential\ stress}{shearing\ strain} = \frac{\frac{F_{\parallel}}{A}}{\theta} = \frac{F_{\parallel}}{A\theta}$$

(iii)Bulk modulus of elasticity (B): It is defined as the ratio of normal stress to volume strain.

$$B = \frac{normal\ stress}{volume\ strain} = \frac{\frac{F}{A}}{-\frac{\Delta v}{v}} = -\frac{PV}{\Delta V}$$

Where P is the change in pressure, the minus sign is included in the definition of B because an increase of pressure always causes a decrease in volume.

Material	Young Modulus Y 10^{11} N m^{-2}	Shear Modulus η 10^{11} N m^{-2}	Bulk Modulus B 10^{11} N m^{-2}
Aluminium	0·70	0·30	0·70
Brass	0·91	0·36	0·61
Copper	1·1	0·42	1·4
Iron	1·9	0·70	1·0
Steel	2·0	0·84	1·6
Tungsten	3·6	1·5	2·0

Problem (3): A metallic rod of diameter 6mm. Fin d the force which must be exerted on a rod to expand it by 20% from its original length. Taking Young's modulus for the rod material is $Y = 9x10^{11}$ Nm^{-2}.

2.5 Hook's law

Consider a spring as shown in Fig (3-6), the stress required to produce a given strain depends on the nature of the material under stress. Hook's law states that within the elastic limit, stress is proportional to strain.

Stress α Strain

Stress = constant × Strain

Where the constant is called the modulus of elasticity.

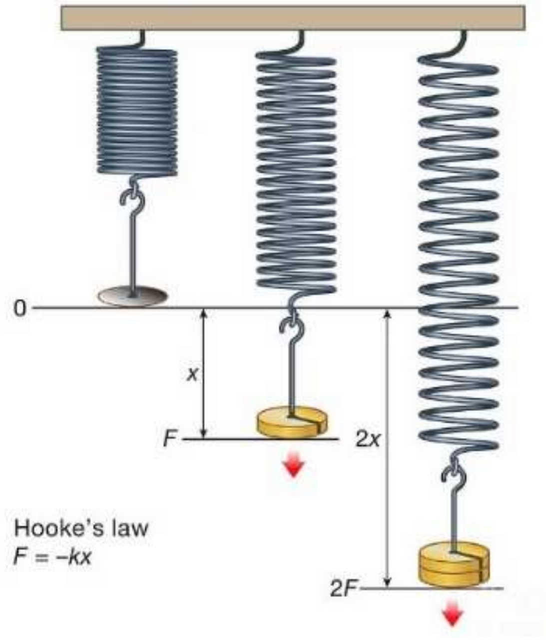


Fig. (3-6)

2.6 The force constant:

In the particular case of longitudinal stress and strain, we find that:

$$F_{\perp} = \frac{YA}{\ell}\Delta\ell$$

Then the quantity $\frac{YA}{\ell}$ is represented by a single letter k, and the elongation $\Delta\ell$ is renamed x, we have:

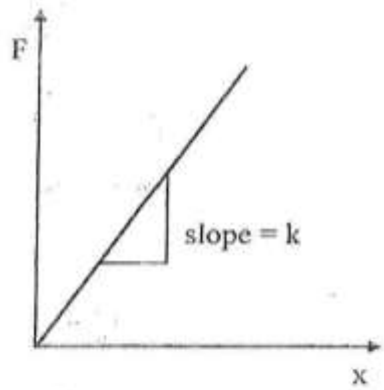


Fig. (3-7)

$$\boldsymbol{F_{\perp} = k\,\mathrm{x}}$$

The elongation of a body is directly proportional to the stretching force. **Hook's law** was first stated in this form, and later reformulated in terms of stress and strain.

When a helical spring (coil spring) is stretched, the stress and strain in the wire are nearly pure shear, but the elongation still proportional to the stretching force. That is, an equation of the form (F⊥= kx) is still valid. In this case the constant k, representing the ratio of force to elongation, is called the force constant.

2.7 work done in deforming a body

When a body is deformed by the application of external forces, the body gets strained. The work done is stored in the body in the form of energy and is called the energy of strain. Consider a wire of length ℓ, area cross section A and Young's modulus of elasticity Y, see Fig. (3-8).

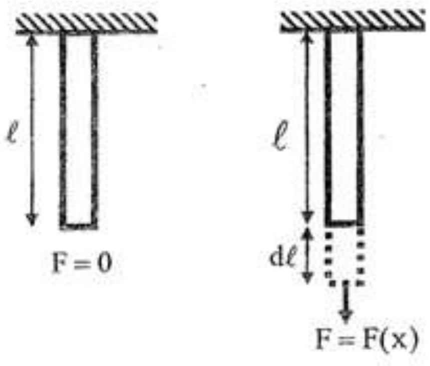


Fig. (3-8)

Let *dl* be the increase in *length* when a stretching force F is applied. Therefore, work done W:

$$W = \int dW = \int_0^x F(x)dx$$

But F = k.x

$$W = \int_0^x k.x.dx = \frac{1}{2}k.x^2\Big|_0^{dl} \qquad = \frac{1}{2}k(dl)^2 = \frac{1}{2}F.dl$$

Work done per unit volume, $W' = \frac{W}{V} = \frac{W}{Al}$

$$W' = \frac{F\,dl}{2Al} = \frac{1}{2} \text{ x } \frac{F}{A} \text{ x } \frac{dl}{l} \qquad \text{so that} \qquad W' = \frac{1}{2} \text{ x stress x strain}$$

Example: calculate the work done in stretching a uniform metallic wire of area of cross-section 10^{-6} $m^2$and length 1.5 m through 4 x 10^{-3} m. given Y = 2 x 10^{11} N/m^2.

Solution

ℓ = 1.5 m, A = 10^{-6} m^2, dℓ=4 x 10^{-3} m, Y = 2 x 10^{11} N/m^2

$$\text{strain} = \frac{d\ell}{\ell} = \frac{4 \text{ x } 10-3}{1.5}$$

$$\text{stress} = Y \text{ . strain} = \frac{2 \text{ x } 10^{11} \text{ x}4 \text{ x } 10^{-3}}{1.5} \text{ N/m}^2$$

volume of the wire = Aℓ = 10^{-6} x 1.5 m^3

$$\text{work done per unit volume} = \frac{1}{2} \text{ . stress . strain}$$

$$\text{total work done } W = \frac{1}{2} \text{ . stress . strain . volume}$$

$$W = \left(\frac{2 \text{ x } 10^{11} \text{ x}4 \text{ x } 10^{-3}}{1.5}\right) . \left(\frac{4 \text{ x } 10-3}{1.5}\right) . (1.5.\ 10^{-6}) . \qquad = 1.066 \text{ J}$$

2.8 Poisson's ratio:

Whenever a body is subjected to a force in a particular direction, there is change in dimensions of the body in the other two perpendicular directions (secondary strain). ***Poisson's ratio*** is defined as ***the ratio of secondary strain per unit stress to the longitudinal strain per unit stress.***

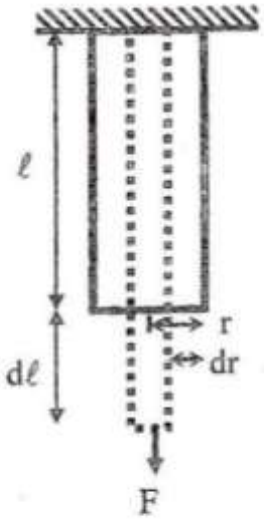


Fig. (3-9)

Consider a wire of length and radius r. The wire is fixed at one end and a force is applied at the other end. Consequently, the length of the wire increases its radius decreases, see Fig. (3-9). If the increase in length is dℓ, and the decrease in radius is dr, then

Longitudinal strain = $\frac{d\ell}{l}$ and Secondary strain = $\frac{dr}{r}$

Since Poisson's ratio σ is the ratio of secondary strain to the longitudinal strain, then

$$\sigma = \frac{-dr/r}{d\ell/\ell}$$

The Poisson's ratio σ has no units as it is a ratio of two numbers. For most of substances, the value of σ is 1/ 2.

Prove that (σ = 1/2)

The initial volume of the wire is

$$V = \pi r^2 \ell$$

If the volume of the wire remains unchanged (dV = 0) after the force has been applied, then

$$dV = 0$$

$$0 = \pi(r^2 d\ell + 2r\, dr\ell)$$

$$r d\ell = -2\,\ell dr$$

$$\therefore \sigma = \frac{-dr/r}{d\ell/\ell} = \frac{1}{2}$$

This is the maximum possible value of Poisson's ratio.

Solved Problems

1. A load of 4.0 kg is suspended from a ceiling through a steel wire of radius 2.0 mm. Find the tensile stress developed in the wire when equilibrium is achieved.

Solution:

Tension in the wire is

F = mg = 4.0 × 9.8 N.

The area of cross section is

$A = \pi r^2 = \pi\times(2.0 \times 10^{-3}\ m)^2 = 4.0\ \pi \times 10^{-6}\ m^2$.

Thus, the tensile stress developed=

$$\frac{F}{A} = \frac{4\times 9.8}{4\pi\times 10^{-6}} = 3.1 \times 10^{6}\ N.m^{2}.$$

==

2. One end of a wire 2 m long and 0.2 cm^2 in cross section is fixed in a ceiling and a load of 4.8 kg is attached to the free end. Find the extension of the wire. Young modulus of steel = $2\times 10^{11}\ Nm^{-2}$. Take g = 10 m s^{-2}.

Solution:

We have $\quad Y = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\Delta\ell/\ell}$

So that, the extension is = $\Delta\ell = \frac{F\,\ell}{AY}$

As the load is in equilibrium after the extension, the tension in the wire is equal to the weight of the load = 4.8 kg × 10 ms^{-2} = 47 N

$$\Delta\ell = \frac{(47\ N)(2m)}{(0.2\times 10^{-4} m^2)\times(2\times 10^{11}\ Nm^{-2})} = 2.35\times 10^{-5}\ m.$$

==

3. Define Young's modulus, modulus of rigidity and Poisson's ratio.

4. Show that the work done per unit volume in straining a body is equal to $\frac{1}{2}$ (Stress x strain).

Chapter (3)

SURFACE TENSION

3.1 Introduction

If a sewing needle is placed on a water surface, it makes a small depression on a water surface; it makes a depression on the surface and rests there without sinking. The needle does not sink though its density is higher than that of water. In a glass tube with a small bore (capillary tube) is dipped in water, the water rises in the capillary tube. The level of water inside the capillary tube is higher than the level of water outside it. Some insects can walk on the surface of water, their feet making indentations in the surface but not penetrating it. All similar phenomena are due to the existence of a boundary surface between the liquid and other substances.

3.2. Explanation of surface tension

Consider a molecule A of a liquid lying well inside the free surface of a liquid. This molecule is attracted by all the molecules lying within the sphere of influence, Fig. (3-1). The resultant force acting on the molecule A due to all the molecules is zero.

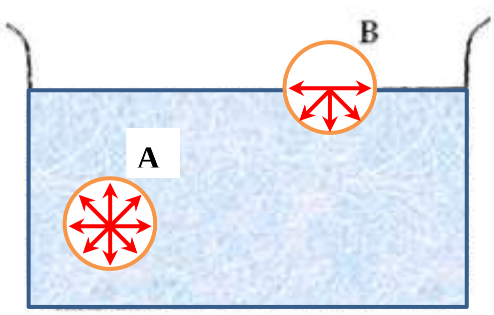


Fig. (3-1)

Consider a molecule B on the free surface of the liquid. This molecule experiences forces of attraction only due to the molecules lying in the lower half of it. The resultant of all these forces is in the downward direction. Due to this, the net inward force on the molecules lying on the surface of the liquid, the1 surface is stretched. Therefore, the free surface of a liquid at rest behaves like a stretched membrane.

For any line on the surface, the portions of surface on the two sides of the line exert pulls on it. The situation of Fig. (3-2) demonstrates this effect; a wire ring has a loop of thread attached to it as shown. When the ring and tired are dipped in a soap solution and removed, a thin film of liquid is formed in which the thread "floats" freely, as shown in part (3-2a). If the film inside the loop of tread is punctured, the thread springs out into a circular shape as in part (3-2b).

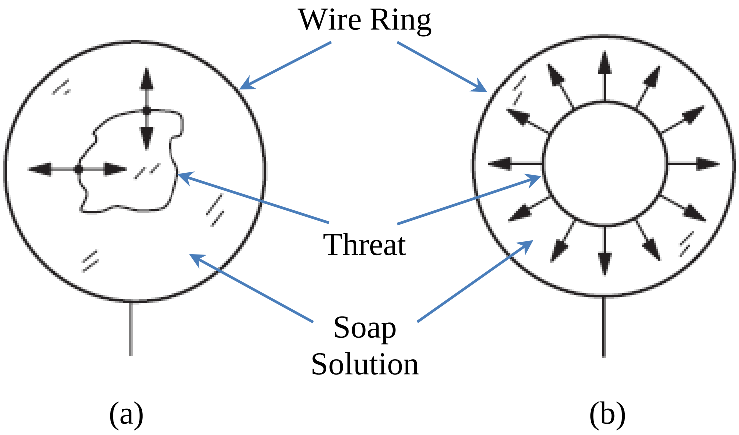


Fig. (3-2)

3.3 Surface tension and surface energy

Another simple apparatus for demonstrating surface tension is shown in Fig. (3-3). A piece of wire is bent into the shape of a U and a second piece of wire is used as a slider. When the apparatus is dipped in a soap solution and removed; the slider (if it its weight w_1 is not. too great) is quickly up to the top of the U. It may be held in equilibrium by adding a second weight w_2. Surprisingly, the same total force $F = w_1 + w_2$ will hold the slider at rest in any position.

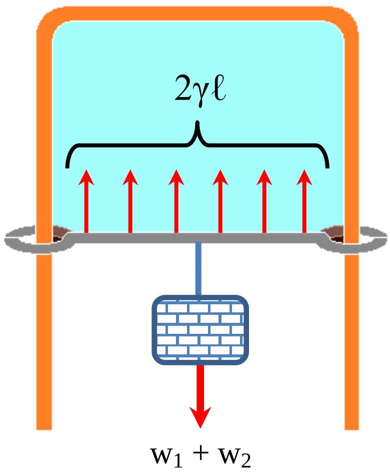


Fig. (3-3)

Let ℓ be the length of the wire slider. Since the film has two surfaces, the total length along which the surface force acts on the slider is *2l*. The *surface tension* γ in the film is defined as ***the ratio of the surface force to the length along which the force acts***(or the work done to increase the liquid surface area by a unity). Hence, in this case

$$\gamma = \frac{F}{2\ell} \qquad\qquad \gamma = \frac{Work}{Area}$$

3.4 Excess of pressure inside a spherical bubble

Consider a spherical bubble of radius r. Surface tension causes the surface of the bubble to tend to contact. Therefore, the pressure inside the bubble must be greater than the pressure outside. This excess of pressure can be calculated and it depends upon the surface tension of the liquid and the radius of the bubble.

Let the pressure inside and outside be P_1 and P_2. Then the thrust due to surface tension + the thrust due to external pressure P_2 = the thrust due to internal pressure P_1.

The thrust due to internal pressure (P_1) $= P_1 \cdot \pi r^2$

The thrust due to external pressure (P_2) $= P_2 \cdot \pi r^2$

The thrust due to surface tension $= \gamma \cdot 2\pi r$

For equilibrium,

$$\pi r^2 P_1 = 2\pi r\, \gamma + \pi r^2 P_2$$

$$(P_1 - P_2) = \frac{2\gamma}{r}$$

Let the excess of pressure given by: $P = (P_1 - P_2)$

So $P = \frac{2\gamma}{r}$

Problem (1): calculate the excess pressure inside a drop of mercury of diameter 4 mm at 20^oC (for mercury $\gamma = 465 \times 10^{-3}\ Nm^{-1}$).

3.5 Angle of contact

Figure (3-4) shows the curvature shape when two drops of water and mercury are placed on a glass substrate. When a glass plate is dipped in *water*, the water molecules cling to the surface of glass and the water molecules rise along the plate. The shape of the water surface is as shown in Fig. (3-4c). When the glass plate is dipped in *mercury*, the mercury molecules cling to the surface of glass and the liquid is depressed along the plate. The shape of the mercury surface is as shown in Fig. (3-4d).

The angle of contact is θ. Hence, the ***angle of contact*** is defined as ***the angle made by the tangent at the point of contact of the liquid surface with tangent plane to the solid surface inside the liquid.*** The angle of contact in the case of water is acute less than 90° as shown in Fig. (3-4e). In the case of mercury the angle of contact is more than 90° as shown in Fig. (3-4f).

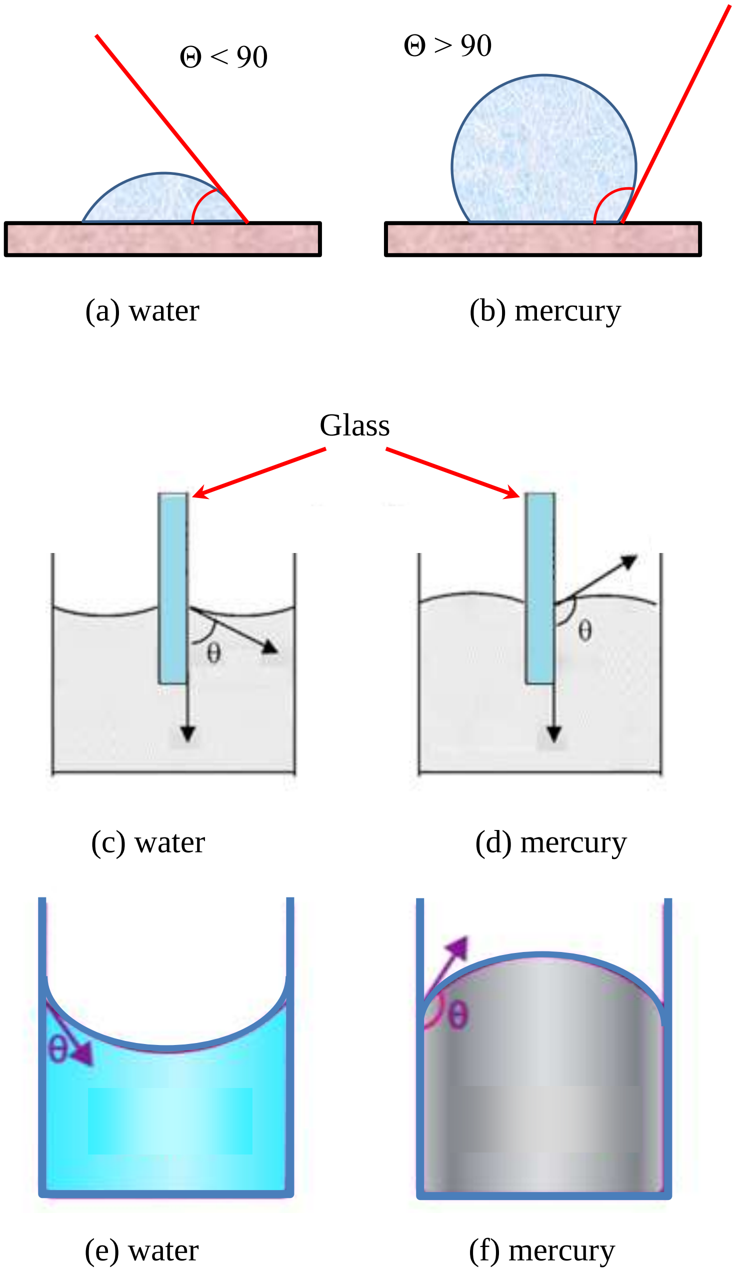


Fig. (3-4)

3.6 Capillarity

When a capillary tube of fine bore is dipped in water, water rises in tube, Fig. (3-5a). On the other hand if the tube is dipped in mercury, there is depression of mercury level in the tube, Fig. (3-5b).The property of rise or depression of a liquid inside a capillary tube is called *capillarity*. *It* is one of the most important effects of surface tension. In general, the liquids that wet the glass rise inside the capillary tube while those which do not wet the glass show a depression inside the capillary tube.

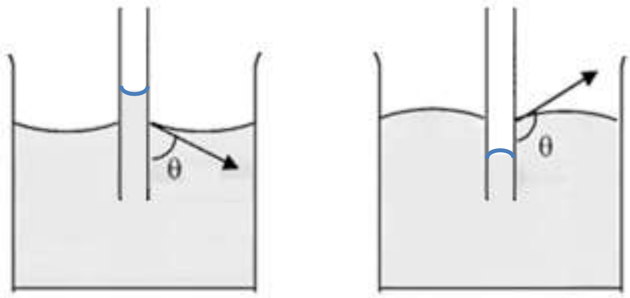


Fig. (3-5)

Capillarity is very important in a variety of life processes. A familiar example is the rising of water (actually a dilute aqueous solution) from the roots of a plant to its foliage, due partly to capillarity and osmotic pressure developed in the roots. In the higher animals, including man, blood is pumped through the arteries and veins, but capillary is still important in the smallest blood vessels, which indeed are called capillaries.

3.6 Expression for surface tension

Consider a capillary tube of radius r dipped in water as shown in Fig. (3-6). Due to surface tension, the water rise inside the capillary tube. Let h be the height of water in the tube above its free surface in the vessel. The force of surface tension γ acts tangential to the surface at the points of contact with the wall of the capillary tube. The horizontal components (γ sin θ) cancel each other, while only the vertical components (γ cos θ) are effective. Since the tube is a cylinder of radius r, the liquid makes contact with the tube along a line of length 2πr. So, the total upward force due to surface tension is 2πr γcos θ. This upward force balances the weight of the liquid column in the capillary tube $\pi r^2 h\rho g$. Therefore;

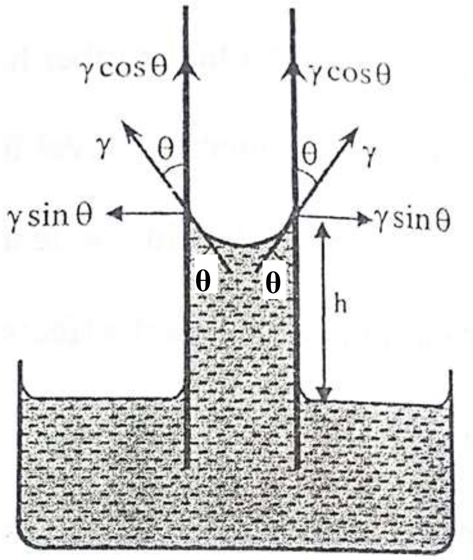


Fig. (3-6)

$$2\pi r\, \gamma \cos\theta = \pi r^2 h\rho g$$

$$2\, \gamma \cos\theta = r\, h\rho g, \qquad P = h\rho g$$

$$\gamma = \frac{r\, h\rho g}{2\cos\theta} = \frac{P\, r}{2\cos\theta}$$

The same equation holds for capillary depression.

3.7 Determination of surface tension of a liquid by Jaeger's method

Jaeger's method's is based on the principle that the excess of pressure inside an air bubble in a liquid is $\frac{2\gamma}{r}$. Here γ is the surface tension of the liquid and r is the radius of the bubble.

The apparatus consists of a Woulf's bottle D. On one side, it is fitted with a thistle funnel and on the other side it is connected to a tube BC. The tube BC is fitted with a manometer M as shown in Fig. (5-7). The end of the tube BC is joined to a capillary tube. The other end of the capillary tube is inside the liquid of which the surface tension is to be determined.

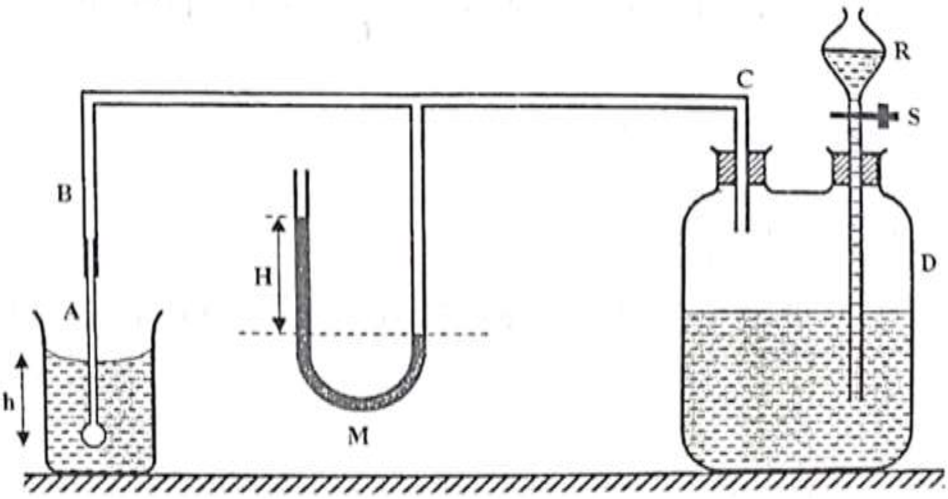


Fig. (5-7)

To start with, the stop cock S is gradually opened. The pressure of air inside the bottle increases due to flow of water into it. The liquid in the capillary tube is forced down and an air bubble appears at the end of the capillary tube. The liquid in the open end of the manometer rises. The stop cock is adjusted such that the bubble forms slowly and bursts after sometime. The process continues. If the atmospheric pressure p_a the excess of pressure inside the bubble is $\frac{2\gamma}{r}$ and the pressure of liquid column is $\rho_1 gh$, then the pressure inside the bubble is

$$P_a + \rho_1 gh + \frac{2\gamma}{r} \qquad (1)$$

Where $\rho_1$is the density of the liquid. The air pressure in the manometer

$$P_a + \rho_2 gh \qquad (2)$$

where H is the difference level in the two limbs of the manometer and ρ_2 is the density of the liquid in the manometer. Equating eqs. (1) and (2)

$$P_a + \rho_1 gh + \frac{2\gamma}{r} = P_a + \rho_2 gh$$

$$\frac{2\gamma}{r} = (\rho_2 h - \rho_1 h)g$$

$$\gamma = \frac{1}{2} gr\,(\rho_2 h - \rho_1 h)$$

The radius r of the capillary tube is measured with the help of a travelling microscope. As all the other values are known, the surface tension γ of the liquid can be determined.

The blastomere method (kinetic flow method):

The surface tension of the cell may be measured by assuming the flow of the cells through a tube dividing two containers, blastomere, as shown in Fig. (3-8). By assuming a viscosity for the cell Sichel and Burton, (1936) were able to apply Poiseuille's equation which is:

$$Q = \frac{\pi\, r6\, \Delta P}{8\mu L} \qquad \Delta P = \frac{8\mu LQ}{\pi r^{\,6}}$$

$$\gamma = \frac{\Delta P\, r}{2} \qquad \gamma = \frac{6\mu LQ}{\pi r^3}$$

Where Q is the rate of flow, r & L is the radius and length of the tube respectively, ΔP is the pressure inside the intact portion of the cell. So, we can get to the value of surface tension.

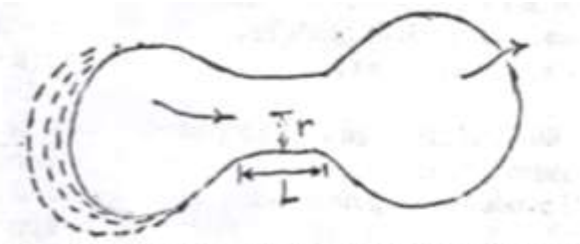


Fig. (5-8)

Insect Locomotion on Water

About 3% of all insects are to some extent aquatic. In one way or another their lives are associated with water. Many of these insects are adapted to utilize the surface tension of water for locomotion. The surface tension of water makes it possible for some insects to stand on water and remain dry.

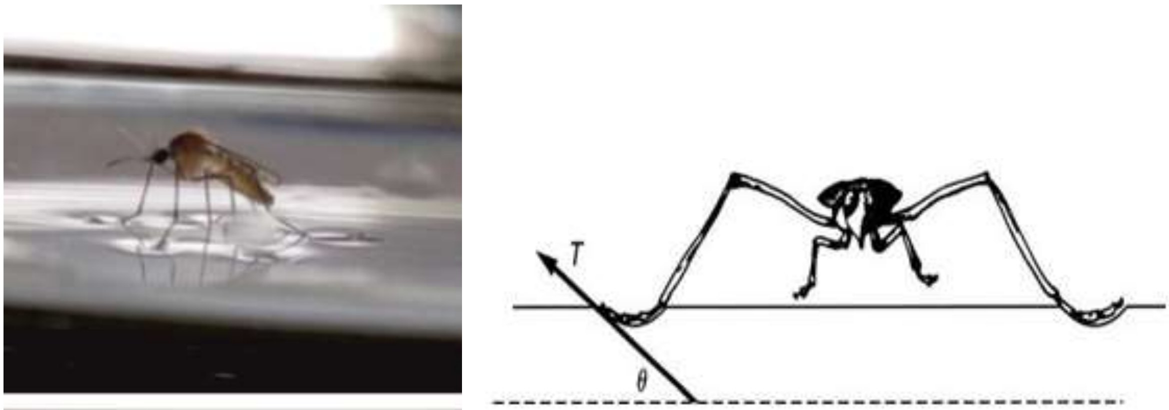


Fig. (5-9): Insect standing on water.

Let us now estimate the maximum weight of an insect that can be supported by surface tension. When the insect lands on water, the surface is depressed as shown in Figure. The legs of such an insect, however, must not be wetted by water. A waxlike coating can provide the necessary water-repulsive property.

The weight W of the insect is supported by the upward component of the surface tension; that is,

$$W= LT \sin\theta \qquad (1)$$

where L is the combined circumference of all the insect legs in contact with the water.

We assume that the insect is in the shape of a cube with side dimensions. The weight of the insect of density ρ is then

$$W= l\ 3\rho g \qquad (2)$$

Let us further assume that the circumference of the legs in contact with water is approximately equal to the dimension of the cube; that is, from Eq.2,

$$L = l = \left(\frac{W}{\rho g}\right)^{1/3}$$

The greatest supporting force provided by surface tension occurs at the angle θ 90◦ (see Fig. 7.8). (At this point the insect is on the verge of sinking.) The maximum weight W_m that can be supported by surface tension is obtained from Eq.7.22; that is,

$$W_m = L\gamma = \left(\frac{m}{\rho g}\right)^{1/3} \gamma$$

$$W_m^{2/3} = \frac{\gamma}{(\rho g)^{1/3}}$$

Solved Problems

1- Water is kept in a beaker of radius 5 cm. Consider a diameter of the beaker on the surface of the water. Find the force by which the surface on one side of the diameter pulls the surface on the other side. Surface tension of water = 0.075 Nm^{-1}.

Solution:

The length of the diameter is

$l = 2r = 2\times 5cm = 0.1$ m.

The surface tension is $S = F/l$. Thus,

$F = Sl = (0.075 \text{ N m}^{-1}) \times (0.1 \text{ m}) = 7.5 \times 10^{-3}$ N.

==

2- Find the excess pressure inside a mercury drop of radius 2.0 mm. The surface tension of mercury = 0.464 N m^{-1}.

Solution:

The excess pressure inside the drop is $P_2 - P_1 = 2\gamma/R$

$$= \frac{2 \times 0{\cdot}464 \text{ N m}^{-1}}{2{\cdot}0 \times 10^{-3} \text{ m}} = 464 \text{ N m}^{-2}$$

==

3- A 0.02 cm liquid column balances the excess pressure inside a soap bubble of radius 7.5mm. Determine the density of the liquid. Surface tension of soap solution = 0.03 N m^{-1}.

Solution:

The excess pressure inside a soap bubble is $\Delta P = 4\gamma/R$

here, the soap bubble has two surfaces, innere and outer so that the relation is given in 4γ instead of 2γ.

$$= \frac{4 \times 0{\cdot}03 \text{ N m}^{-1}}{7{\cdot}5 \times 10^{-3} \text{ m}} = 16 \text{ N m}^{-2}.$$

The pressure due to 0.02 cm of the liquid column is

$\Delta P = h\rho g = (0.02 \times 10^{-2} \text{ m})\, \boldsymbol{\rho}\, (9.8 \text{ m s}^{-2})$.

Thus, $16 \text{ N m}^{-2} = (0.02 \times 10 - 2 \text{ m})\, \boldsymbol{\rho}\, (9.8 \text{ m s}-2)$

or, $\boldsymbol{\rho} = 8.2 \times 10^{3} \text{ kg m}^{-3}$.

4- A capillary tube of radius 0.20 mm is dipped vertically in water. Find the height of the water column raised in the tube. Surface tension of water = 0.075 N m^{-1} and density of water is 1000 kg m^{-3}.

Solution:

We have, $h = \frac{2\gamma\cos\theta}{r\rho g}$

$$= \frac{2 \times 0{\cdot}075 \text{ N m}^{-1} \times 1}{(0{\cdot}20 \times 10^{-3}\text{ m}) \times (1000 \text{ kg m}^{-3}) \times (9.8 \text{ m s}^{-2})}$$

= 0.0765 m = 7.65 cm.

===

5- The lower end of a capillary tube is dipped into water and it is seen that the water rises through 7.5 cm in the capillary. Find the radius of the capillary. Surface tension of water = 7.5×10^{-2} Nm^{-1}. Contact angle between water and glass = 0°. Take g = 10 ms^{-2}.

Solution:

We have, $h = \frac{2\gamma\cos\theta}{r\rho g}$

or, $r = \frac{2\gamma\cos\theta}{h\rho g}$

$$= \frac{2 \times (7{\cdot}5 \times 10^{-2}\text{ N m}^{-1}) \times 1}{(0{\cdot}075 \text{ m}) \times (1000 \text{ kg m}^{-3}) \times (9.8 \text{ m s}^{-2})}$$

= 2.04×10^{-4} m = 0.204 mm

===

6- A liquid drop of radius R breaks up into 64 small drops. Calculate the change in energy.

7- Calculate the work done in spraying a spherical drop of mercury of radius 10^{-3} m into a million drops of equal size. Surface tension of mercury is 550×10^{-3} N/m.

8-.In Jaeger's experiment, a capillary tube of internal diameter 5×10^{-4} m dips 3×10^{-2}m inside water contained in a beaker. The difference in level of water manometer when the bubble is released is 0.09m. Calculate the surface tension of water.

Chapter (4)

FLUID DYNAMICS

4.1 Introduction

Fluid dynamics is the study of fluids in motion. It is one of the most complex branches of mechanics, as illustrated by such familiar examples of fluid flow as a river in flood. While each drop of water or each smoke particle is governed by Newton's laws of motion, the resulting equations can be exceedingly complex. So, many situations of practical importance can be represented by idealized models that are simple enough to permit detailed analysis.

When fluid is in motion, its flow can be characterized as being one of two main types. The flow is said to be **steady**, or laminar, if each particle of the fluid follows a smooth path, such that paths of different particles never cross each other. In steady flow, ***the velocity of fluid particles passing any point remains constant in time.***

Because the motion of real fluids is very complex and not fully understood, we make some simplifying assumptions in our approach. In our model of ***ideal fluid*** flow, we make the following assumptions:

1.***The fluid is nonviscous***. Internal friction is neglected.

2. ***The flow is steady***. The velocity at each point remains constant.

3. ***The fluid is incompressible***. The density of the fluid is constant.

4.2 Equation of continuity

Consider an ideal fluid flowing through a pipe of non uniform size, as illustrated in Fig, (4-1), The fluid is assumed to be in stream line motion. As there is no accumulation of the fluid at any point, the amount of fluid flowing per second is the same at all cross sections of the tube. Mass of fluid that crosses the area A_1 at time interval Δt is

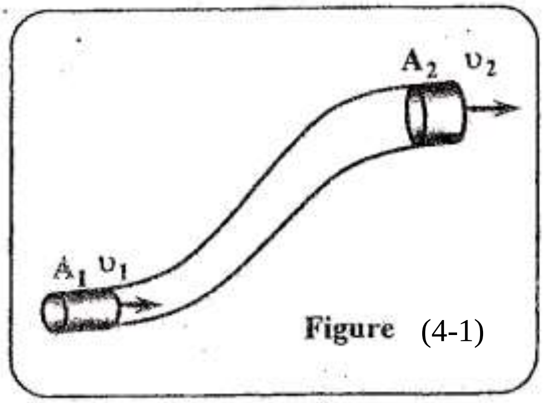


Figure (4-1)

$$m_1 = \rho V_1 = \rho A_1 d_1, \qquad d_1 = \upsilon_1 \Delta t$$

$$\Delta m_1 = \rho A_1 \upsilon_1 \Delta t$$

The mass of fluid that crosses the area A_2 at the same time interval is

$$\Delta m_2 = \rho A_2 \upsilon_2 \Delta t$$

Because the fluid is incompressible (ρ doesn't change with pressure) which is an excellent approximation for liquids under most circumstances (and sometimes for gases as well), and because the flow is steady, the mass crosses A_1 in a time interval Δt must equal the mass that crosses A_2 in the same time interval. That is

$$\Delta m_1/\Delta t = \Delta m_2/\Delta t$$

$$\rho A_1 \upsilon_1 \Delta t = \rho A_2 \upsilon_2 \Delta t$$

$$A_1 \upsilon_1 = A_2 \upsilon_2$$

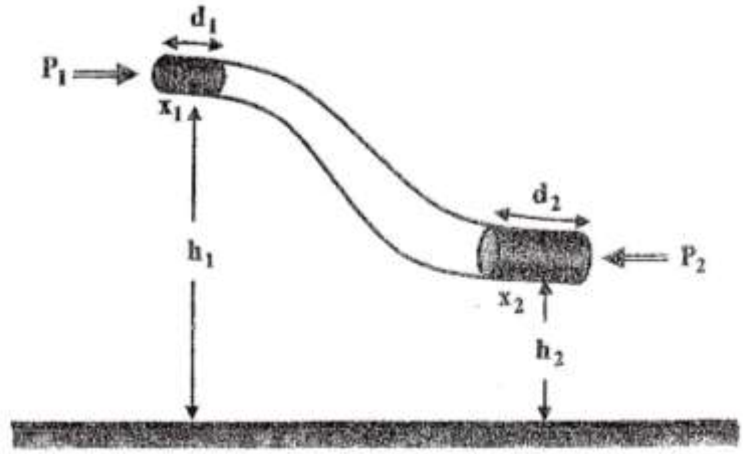
$$A\upsilon = const.$$

This equation is called ***<u>the equation of continuity</u>***

4.3 Bernoulli's equation

Bernoulli's principle states that where the velocity of a fluid is high, the pressure is low, and where the velocity is low, the pressure is high.

According to Bernoulli's equation the sum of the energies possessed by a flowing liquid at any point is constant. Consider an incompressible liquid in a pipe line, Fig. (4-2). The motion of the liquid is stream line. Let the pressure, area and velocity of the liquid at the point x_1 be A_1, P_1, and υ_1 and at x_2 be A_2, P_2, and υ_2.



(4-2)

Work done in moving a mass $m_1$of liquid a distance d_1 is

$W_1 = F_1 d_1 = P_1 A_1 \upsilon_1 \Delta t$

And the work done in moving a mass m_2 of liquid a distance d_2 is

$W_2 = -F_2 d_2 = -P_2 A_2 \upsilon_2 \Delta t$

But $A_1 \upsilon_1 = A_2 \upsilon_2$, so

$W_2 = P_2 A_1 \upsilon_1 \Delta t$

The net work done on the liquid is

$W = -(W_2 - W_1) = A_1 \upsilon_1 \Delta t (P_1 - P_2)$ (4-1)

This work done on the liquid contributes for the changes in kinetic energy and potential energy. Change in kinetic energy is

$$KE = \frac{1}{2} m_2 \upsilon_2^2 - \frac{1}{2} m_1 \upsilon_1^2$$

From the continuity equation $\Delta m_1 = \Delta m_2 = \rho A_1 \upsilon_1 \Delta t$ so

$$KE = \frac{1}{2} \rho A_1 \upsilon_1 \Delta t \, (\upsilon_2^2 - \upsilon_1^2) \qquad (4\text{-}2)$$

But change in potential energy is

$W_3 = m_2 g h_2 - m_1 g h_1$, $\Delta m_1 = \Delta m_2$

$= m_1 g \, (h_2 - h_1)$

$$W_3 = \rho A_1 \upsilon_1 \Delta t\, g\, (h_2 - h_1) \qquad (4\text{-}3)$$

From the conservation law of energy, the net work done equal to the sum of the change in kinetic energy and potential energies.

$$W = KE + W_3$$

From Eqs.(4-1), (4-2) and (4-3)

$$A_1 \upsilon_1 \Delta t\,(P_1 - P_2) = \frac{1}{2}\rho A_1 \upsilon_1 \Delta t\,(\upsilon_2^2 - \upsilon_1^2) + \rho A_1 \upsilon_1 \Delta t g\,(h_2 - h_1)$$

$$(P_1 - P_2) = \frac{1}{2}\rho(\upsilon_2^2 - \upsilon_1^2) + \rho g\,(h_2 - h_1)$$

$$P_1 + \frac{1}{2}\rho\upsilon_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho\upsilon_2^2 + \rho g\, h_2$$

Which means that

$$P + \frac{1}{2}\rho\upsilon^2 + \rho g h = const.$$

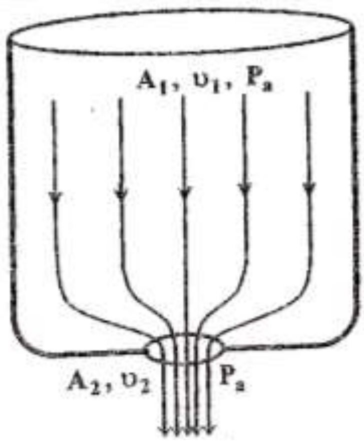
at every point in the fluid, where h is the height of the center of the tube above a fixed reference level.

This equation represents ***Bernoulli equation***.

4.4. Applications of Bernoulli's equation

4.4.1 Torricelli's law

Figure (4-3) represents an open tank of cross sectional area A,, filled to a depth h with a liquid of density ρ. The liquid flows out of an orifice of area A,. Let υ_1 and υ_2 be the speeds at points (1) and (2). The quantity υ_2 is called the speed of efflux. The pressure at point (2) is atmospheric, pressure, P_a. Applying Pernoulli's equation to point (1) and (2), and take the bottom of the tank as our reference level, we get:



(4-3)

$$P_1 + \tfrac{1}{2}\rho\upsilon_1^2 + \rho g h_1 = P_2 + \tfrac{1}{2}\rho\upsilon_2^2 + \rho g\, h_2$$

$$P_a + \frac{1}{2}\rho\upsilon_1^2 + \rho g h_1 = P_a + \frac{1}{2}\rho\upsilon_2^2 + 0$$

Suppose also that $A_1 = A_2$. Because υ_1^2 is very much less than υ_2^2 and can be neglected. So

$$\upsilon_2^2 = 2gh \qquad (4\text{-}3)$$

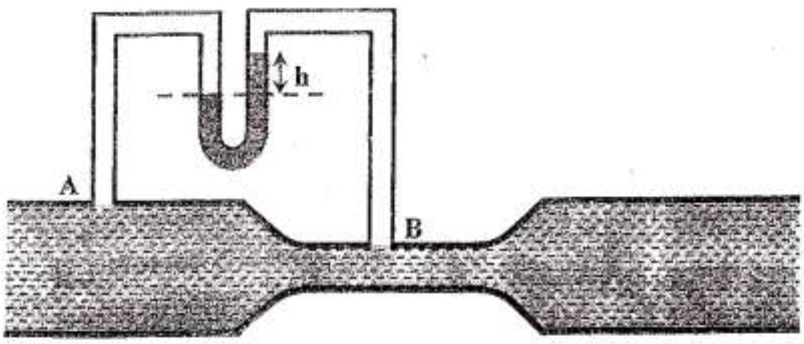
$$\upsilon = \sqrt{2gh}$$

That is, the speed of efflux is the same as the speed of a body acquire in falling freely through a height h. This is ***Torricelli's law***.

4.4.2.Venturi tube

A venturi tube is essentially a pipe with a narrow constriction (the throat). The flowing fluid speeds up as it passes through this constriction, so the pressure is lower in the throat.

A venturi tube consists of a horizontal tube as shown in Fig. (6-4). It has different cross-sections at A and B. It is used to find the rate of flow of a liquid when the motion of the liquid is steady and non turbulent. The rate of flow of the liquid crossing through any cross-section of the pipe is constant. It-means that the amount of liquid crossing per second at A is equal to the amount of liquid crossing per second at B. but the area of cross section at A is larger and at B is small. Therefore, the velocity of flow of the liquid at A is less and at B it is more.



(4-4)

According to Bernoulli equation

$$P_1 + \frac{1}{2}\rho\upsilon_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho\upsilon_2^2 + \rho g\, h_2$$

Here $h_1 = h_2$ so,

$$\frac{1}{2}\rho(\upsilon_2^2 - \upsilon_1^2) = (P_1 - P_2)$$

From the continuity equation $A_1 \upsilon_1 = A_2 \upsilon_2$ so,

$$\upsilon_2 = \frac{A_1}{A_2} \upsilon_1$$

$$\frac{1}{2} \rho \upsilon_1^2 \left(\frac{A_1^2}{A_2^2} - 1 \right) = (P_1 - P_2)$$

$$\upsilon_1^2 = \frac{2(P_1 - P_2)}{\rho (A_1^2 - A_2^2)} A_2^2$$

$$\upsilon_1 = A_2 \sqrt{\frac{2(P_1 - P_2)}{\rho (A_1^2 - A_2^2)}}$$

The pressure difference at A and B is indicated by the manometer limbs.

Circulation of the Blood

The circulation of blood through the body is often compared to a plumbing system with the heart as the pump and the veins, arteries, and capillaries as the pipes through which the blood flows. This analogy is not entirely correct. Blood is not a simple fluid; it contains cells that complicate the flow, especially when the passages become narrow. Furthermore, the veins and arteries are not rigid pipes but are elastic and alter their shape in response to the forces applied by the fluid. Still, it is possible to analyze the circulatory system with reasonable accuracy using the concepts developed for simple fluids flowing in rigid pipes.

Figure 8.4 is a drawing of the human circulatory system. The blood in the circulatory system brings oxygen, nutrients, and various other vital substances to the cells and removes the metabolic waste products from the cells. The blood is pumped through the circulatory system by the heart, and it leaves the heart through vessels called *arteries* and returns to it through *veins*.

The mammalian heart consists of two independent pumps, each made of two chambers called the *atrium* and the *ventricle*. The entrances to and exits from these chambers are controlled by valves that are arranged to maintain the flow of blood in the proper direction. Blood from all parts of the body except the lungs enters the right atrium, which contracts and forces the blood into the right ventricle. The ventricle then contracts and drives the blood through the pulmonary artery into the lungs. In its passage through the lungs, the blood releases carbon dioxide and absorbs oxygen. The blood then flows into the left atrium via the pulmonary vein. The contraction of the left atrium forces the blood into the left ventricle, which on contraction drives the oxygen-rich blood through the aorta into the arteries that lead to all parts of the body except the lungs. Thus, the right side of the heart pumps the blood through the lungs, and the left side pumps it through the rest of the body.

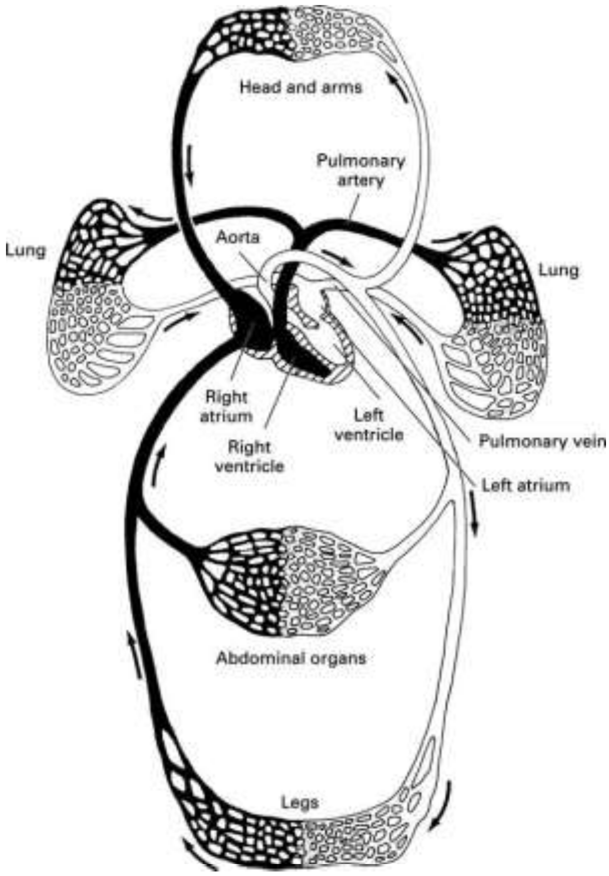


Fig. (4-5): Schematic diagram showing various routes of the circulation.

The large artery, called the *aorta*, which carries the oxygenated blood away from the left chamber of the heart, branches into smaller arteries, which lead to the various parts of the body. These in turn branch into still smaller arteries, the smallest of which are called ***arterioles*.**

Measurement of Blood Pressure

The arterial blood pressure is an important indicator of the health of an individual. Both abnormally high and abnormally low blood pressures indicate some disorders in the body that require medical attention. High blood pressure, which may be caused by constrictions in the circulatory system, certainly implies that the heart is working harder than usual and that it may be endangered by the excess load. Blood pressure can be measured most directly by inserting a vertical glass tube into an artery and observing the height to which the blood rises (see Fig. 8.5). This was, in fact, the way blood pressure was first measured in 1733 by Reverend Stephen Hales, who connected a long vertical glass tube to an artery of a horse. Although sophisticated modifications of this technique are still used in special cases, this method is obviously not satisfactory for routine clinical examinations. Routine measurements of blood pressure are now most commonly performed by the cut-off method. Although this method is not as accurate as direct measurements, it is simple and in most cases adequate. In this technique, a cuff containing an inflatable balloon is placed tightly around the upper arm. The balloon is inflated with a bulb, and the pressure in the balloon is monitored by a pressure gauge. The initial pressure in the balloon is greater than the systolic pressure, and the flow of blood through the artery is therefore cut off. The observer then allows the pressure in the balloon to fall slowly by releasing some of the air. As the pressure drops, she listens with a stethoscope placed over the artery downstream from the cuff. No sound is heard until the pressure in the balloon decreases to the systolic pressure. Just below this point the blood begins to flow through the artery; however, since the artery is still partially constricted, the flow is turbulent and is accompanied by a characteristic sound. The pressure recorded at the onset of sound is the systolic blood pressure. As the pressure in the balloon drops further, the artery expands to its normal size, the flow becomes laminar, and the noise disappears. The pressure at which the sound begins to fade is taken as the diastolic pressure.

In clinical measurements, the variation of the blood pressure along the body must be considered. The cut-off blood pressure measurement is taken with the cuff placed on the arm approximately at heart level.

Chapter (5)

VISCOSITY

5.1 Introduction

A FLUID is normally defined as a substance which is incapable of sustaining a shearing stress. However, this definition is only applicable when the fluid is at rest. If relative motion takes place a measurable resistance is encountered and the fluid is said to exhibit viscosity. Consider a fluid flowing without turbulence over a fixed surface AB (Figure 5.1). Experimentally, it is found that a layer of the fluid at D, at a distance x + dx from AB, moves with a velocity u + du greater than that of a layer of the fluid at C, at a distance x from AB, which moves with velocity u. The velocity gradient between the layers C and D is thus du/dx.

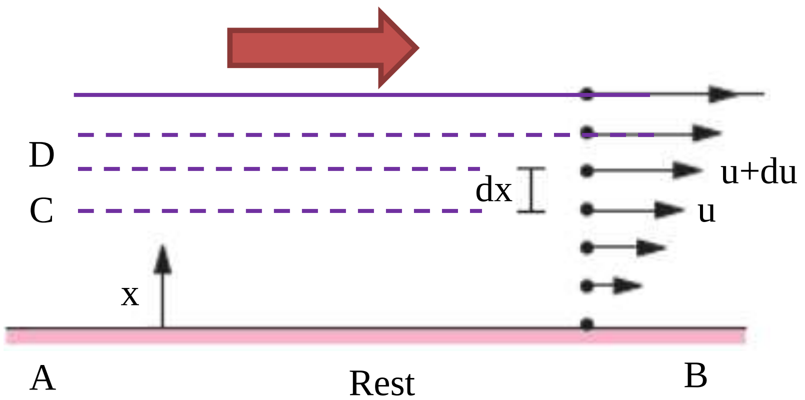


Fig. (5-1)

Newton assumed that for a fluid moving in parallel layers, the shearing stress at any point is directly proportional to the velocity gradient and thus formulated his law of viscous flow

$$\frac{F}{A} \propto \frac{du}{dx}$$

or

$$\frac{F}{A} = \eta \frac{du}{dx} \qquad (5.1)$$

where F is the tangential viscous force between two layers of area A, at a distance dx apart, moving with relative velocity du. η is called the coefficient of viscosity of the fluid. This law is valid only when the fluid is moving with streamline motion. It is not applicable when turbulence occurs. From equation

(5.1) it can be seen that $\eta = (F/A)/du/dx$ and the dimensions of η are $ML^{-1}T^{-1}$. In the SI system the unit is thus Nsm^{-2} Alternatively the centipoise, equal to 10^{-3} Nsm^{-2}, may be used. An alternative expression to viscosity is the so-called kinematic viscosity of a fluid. This is equal to η/ρ where ρ is the density of the fluid concerned and the unit in the SI system is the centistoke equal to $10^{-6}\ m^2s^{-1}$.

5.2 Poiseuille's Equation

Consider a liquid flowing through a narrow horizontal tube of radius a and length I, under a pressure difference P between the ends of the tube. Assume the liquid moves with streamline motion parallel to the axis of the tube and that the liquid in contact with the walls of the tube is at rest. When steady conditions are attained let the velocity of the liquid at a distance r from the axis of the tube be u and let the velocity gradient be du/dr. This viscous drag per unit area is thus η du/dr and this acts over all the surface area of the inner cylinder of liquid in a direction opposed to the pressure gradient down the tube. The total viscous drag on this cylinder of liquid is thus η du/dr. $2\pi r\, l$. The force tending to accelerate the liquid cylinder is P. πr^2 and thus, when steady conditions exist, this force must be balanced by the viscous drag, i.e.

$$P\pi r^2 = -\eta \frac{du}{dr} . 2\pi r l$$

$$-r\,dr = \frac{2\eta l}{P} . du \qquad (5.2)$$

At the walls of the tube $r = a$ and $u = 0$. Integrating equation **(5.2)** between the limits $r = a$ to $r = r$ gives

$$a^2 - r^2 = \frac{4\eta l}{P} . u$$

i.e.

$$u = \frac{P}{4\eta l}(a^2 - r^2) \qquad (5.3)$$

The volume of liquid which flows through the tube per second between the radii r and $r + dr$ is given by

$$dV = 2\pi r\, dr \,.\, u = \frac{P\pi}{2\eta l}(a^2 - r^2)\, r\, dr$$

and hence, the total volume of liquid flowing through the tube per second may be obtained by integration and is given by

$$V = \int_0^a \frac{P\pi}{2\eta l}(a^2 - r^2)\, r\, dr = \frac{P\pi a^4}{8\eta l} \qquad (5.4)$$

This expression relating the volume of liquid flowing through a tube per second, V, to the pressure difference, P, the coefficient of viscosity η, and the length l and radius, a, of the tube, is known as Poiseuille's formula.

7.3 Stoke's falling body viscometer

Stoke's law, which was derived from hydrodynamical considerations for a perfectly homogeneous continuous fluid of infinite extent, states that

$$F = 6\pi\eta a u \qquad (5.5)$$

Where F is the viscous retarding force exerted on a sphere, of radius a, moving with uniform velocity u, through a fluid whose coefficient of viscosity is η. It was shown by Newton that when a body is acted on by a constant force and is also subjected to a resistance proportional to its velocity, then ultimately, when the force resisting its motion is equal and opposite to the constant force causing the motion, the body attains a constant or terminal velocity. Thus, when a sphere is allowed to fall under gravity in a viscous fluid, when it has attained its terminal velocity, the viscous retarding force is equal to the force causing the motion of the sphere, i.e. its weight. Hence;

$$6\pi\eta a u = \tfrac{4}{3}\pi a^3(\rho - \sigma)\, g \quad \ldots\ldots \quad (5.6)$$

here ρ is the density of the material of the sphere and σ is the density of the liquid. Equation (5.6) affords a convenient method for the determination of the coefficient of viscosity of a liquid where the liquid is available in an appreciable quantity.

Basically, the experiment involves allowing metal spheres of known radius and density to fall in a vertical glass tube filled with the liquid under investigation. Measurement of the transit time between two fixed marks on the side of the tube gives the terminal velocity and thus the only unknown in equation (5.6) is η, the coefficient of viscosity. The apparatus is shown in Figure 5.2.

The experiment consists of a large glass cylinder, with a length of the order of 1 m and a diameter of 10 cm, filled with the liquid whose coefficient of viscosity is required. Two fixed marks are made by winding black cotton round the cylinder at distances roughly one third and two-thirds of the distance from the top of the vessel respectively. It is assumed that the sphere has attained its terminal velocity in the first third of the liquid and the subsequent terminal velocity is determined by measurement of the transit time between the fixed points, a known distance apart. The spheres are steel ball bearings of various diameters, of the order of 1-4 mm. Their diameter is carefully measured with a micrometer screw gauge and they are thoroughly wetted with the liquid under investigation. Each sphere is subsequently fed, in turn, through a short vertical glass tube placed centrally as shown, thereby ensuring that they fall axially through the liquid. The transit time is carefully noted for each sphere, several different measurements being made for each different diameter. Now equation (5.6) can be rearranged in the following form:

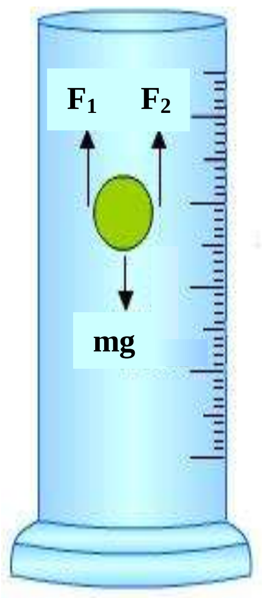


Fig. (5-2)

$$\eta = \frac{2}{9} a^2 g(\rho - \sigma)\frac{t}{s} \qquad \cdots \quad (5.7)$$

where s is the distance between the fixed marks and t is the transit time. Hence, for a given liquid at constant temperature, a^2t is constant and the graph of t against l/a^2, should be a straight line. The value of η, for the liquid being investigated, may then be obtained from the slope of the line. The temperature of the liquid must be maintained constant since viscosity varies with temperature. An improvement on the experiment described above is to place the cylinder inside a constant temperature bath. Equation (5.5) was derived by Stokes for a sphere falling in a continuous fluid of infinite extent. In the experiment described, therefore, corrections must be made for the boundary conditions appertaining at the walls and base of the cylinder containing the liquid. Ladenburg showed that to correct for the wall effect, the true velocity of the sphere is given by:

$$u_{\infty} = u\left(1 + 2{\cdot}4\frac{a}{R}\right) \quad \ldots\ldots \quad (5.8)$$

where u is the observed velocity and R is the radius of cross-section of the cylinder containing the fluid. Similarly, to correct for the effect of the base of the cylinder, the following formula is used

$$u_{\infty} = u\left(1 + 3{\cdot}3\frac{a}{h}\right) \quad \ldots\ldots \quad (5.9)$$

where h is the total height of the liquid. Hence, incorporating these two corrections into equatioı (7.7) gives

$$\eta = \frac{2}{9}\frac{(\rho - \sigma)\,ga^2}{u[1 + 2{\cdot}4(a/R)]\,[1 + 3{\cdot}3(a/h)]} \quad \ldots \quad (5.10)$$

Medical Applications of Viscosity

1- **Erythrocytes sedimentation rate:** By considerindg that the liquid is the blood plasma and the inclusions are the erythrocytes, then, when the densities of the plasma and erythrocytes are known, and also the radius of the blood cell and gravitation then the sedimentation rate v can be determined from

$$\eta = \frac{2}{9}\frac{r^2 g(\rho_p - \rho_f)}{V_s}$$

2- **Rheumatic fever:** This can be diagnosed from that the rheumatic fever causes collection of RBCS which results in an increase in the radius of red blood cell r causing v_s to be increased and so the rate of sedimentation.

3- **Anemia:** Hematocrit or packed cell volume PCV: the percentage of red blood cell in the blood in case of normal conditions equal to 45%.

In case of anemia, this ratio is decreased and taken as a good indicator.

Solved Problems

1- A large wooden plate of area 10 m^2 floating on the surface of a river is made to move horizontally with a speed of 2 m/s by applying a tangential force. If the river is 1 m deep and the water in contact with the bed is stationary, find the tangential force needed to keep the plate moving. Coefficient of viscosity of water at the temperature of the river = 0.01 poise.

(1 poise = 0.1 N s m^{-2})

Solution:

The velocity decreases from 2 m s–1 to zero in1 m of perpendicular length. Hence, velocity gradient

$$= dv/dx = 2\ \text{s}^{-1}.$$

Now, $$\eta = \left|\frac{F/A}{dv/dx}\right|$$

or, $$10^{-3}\ \frac{\text{N–s}}{\text{m}^2} = \frac{F}{(10\ \text{m}^2)\,(2\ \text{s}^{-1})}$$

or, F = 0.02 N.

==

2- The velocity of water in a river is 18 km h^{-1} near the surface. If the river is 5 m deep, find the shearing stress between the horizontal layers of water. The coefficient of viscosity of water = 0.01 poise. (1 poise = 0.1 N s m^{-2})

Solution:

The velocity gradient in vertical direction is:

$$\frac{dv}{dx} = \frac{18\ \text{km h}^{-1}}{5\ \text{m}} = 1{\cdot}0\ \text{s}^{-1}$$

The magnitude of the force of viscosity is:

$$F = \eta A\,\frac{dv}{dx}$$

The shearing stress is:

$$F/A = \eta\,\frac{dv}{dx} = (10^{-2}\,\text{poise})\ (1{\cdot}0\ \text{s}^{-1}) = 10^{-3}\ \text{N m}^{-2}.$$

Part (2) Heat

I. Effect of heat on Matter

The earliest evidence of human use of fire for warmth and light comes from caves occupied by Peking man about half a million years ago. In spite of this use, it has been only in relatively recent times that people have understood that heat is energy and that temperature is a measure of the amount of that energy present in a body. As a form of energy, heat can be converted into work. Although it is customary to speak of sensible heat and latent heat, since the word heat is restricted to energy being transferred. When heat energy is given to a body one or more of the following effects are observed:

i. Change in color.

ii. Increase in size (expansion).

iii. Change in electric properties.

iv. Change in chemical composition.

v. Body gets hotter (increase in temperature).

vi. Change in state (solid to liquid or liquid to vapour).

All these effects can be observed experimentally and some of them are used to measure the change in temperature.

Zeroth Law of the Thermodynamics

Two bodies are said to be in **thermal equilibrium** if no transfer of heat takes place when they are placed in contact. We can now state the Zeroth law of thermodynamics as follow:

"If two bodies A and B are in thermal equilibrium and A and C are also in thermal equilibrium , then B and C are also in thermal equilibrium."

The Zeroth law allows us to introduce the concept of temperature to measure the hotness or coldness of a body. All bodies in thermal equilibrium are assigned equal temperature. A hotter body is assigned higher temperature than a colder body. Thus, the temperature of two bodies decide the direction of heat flow when the two bodies are put in contact. Heat flows from the body at higher temperature to the body at lower temperature.

1. Heating and Cooling Curves

Substances in our environment are usually classified as solids, liquids, or gases. These forms are called **phase of matter**. When heat energy is given to a substance an increase of temperature is observed and a change of phase may take place. The reverse changes take place on cooling. For example, when water is heated sufficiently, it changes to steam; or when enough heat is removed, water changed to ice. Such changes can be studied by the what are called heating or cooling curves. If the temperature of 1 gm of ice is recorded at regular intervals as it heats, and the readings are plotted on a temperature-time graph, a heating curve is obtained as in Fig. (1). At some particular temperature (0°C and 100°C) the graph becomes

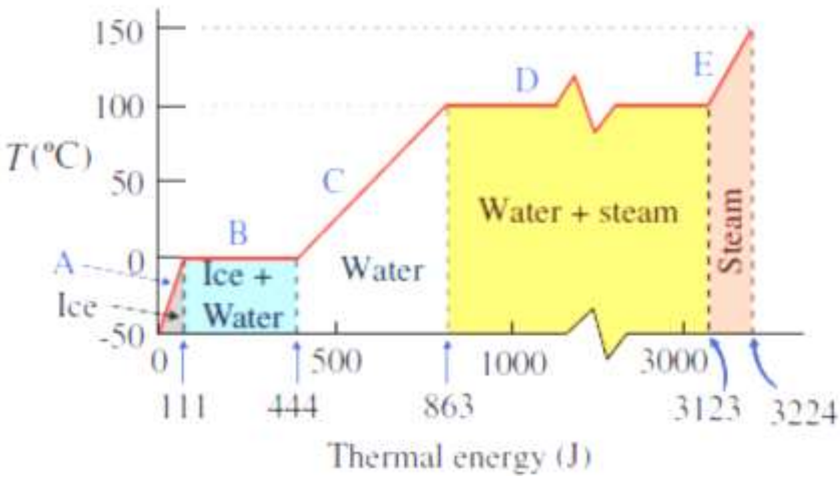


Figure (1)

horizontal. Although heat energy is continually given, the temperature does not rise for a certain time. At these temperatures ice melts and water boils, respectively. If, on the other hand, hot vapour cools and temperature-time is plotted a cooling curve will be obtained with horizontal parts at the same temperatures (100°C and 0°C). It is at those temperatures the substance during heating is melted (melting point), boiled (boiling point). On cooling, at this temperature the substance is condensed at the temperature of the boiling point and is solidified or freezed at constant temperature called freezing point which equals to the temperature of the melting point. At these constant temperatures the state is changed and two phases are existed in equilibrium. The melting points and the boiling points of various substances vary considerably, as Table (1) shows.

Table (1): Melting and boiling points of various substances

Substance	Melting point (°C)	Boiling point (°C)
Helium	-269.65	-268.93
Nitrogen	-209.97	-195.81
Oxygen	-218.79	-182.97
Water	0.00	100.00
Sultur	119	444.60
Lead	327.3	1750
Aluminum	660	2450
Silver	960.80	2193
Gold	1063.00	2660
Copper	1083	1187

2. Thermal Expansion

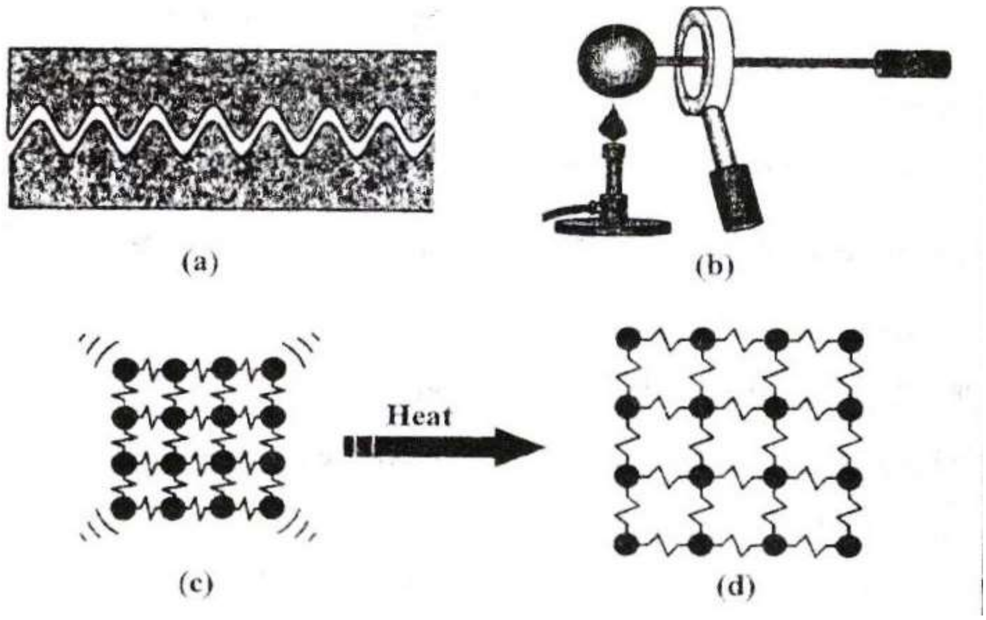


Figure 2

When a material is heated or cooled, it changes its dimensions. Most materials expand as their temperature increases. This phenomenon, known as thermal expansion, plays an important, role in engineering applications. The sagging of telephone wires in summer is due to the liner expansion in summer, Fig. (2a). When iron rods are used in the construction of a building, gaps are left between the ends of the beam and the walls, to provide for the expansion in a summer. The ball just pass through the ring at room temperature but it does not pass through the same ring when it is heated to a higher temperature, Fig. (2b).

The thermal expansion of a body is a consequence of the change in the average separation between its atoms or molecules. Figure (2c) shows a diagram of a crystal lattice in two dimensions. Heating causes the molecules to vibrate with greater amplitude in the lattice, thus increasing the volume of the solid as in the Fig. (2d).

2.1 Coefficient of Linear Expansion

The expansion of a solid can be in length, area or volume. In case of a body in the form of a wire or a rode, only the change in length with temperature is usually considered. The change in length ΔL over a temperature range from T_1 to T_2 is proportional to both the original length L_0 and to the temperature range ΔT. Thus,

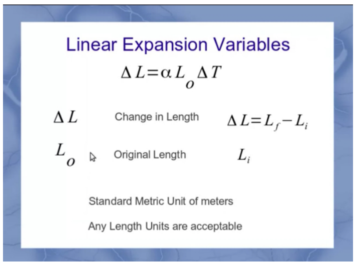
$$\Delta L \propto L_0 \Delta L$$

$$\Delta L = \alpha L_0 \Delta L$$

or

$$L - L_0 = \alpha L_0 (T_2 - T_1)$$

$$L = L_0 [1 + \alpha \Delta T] \qquad (2)$$



Where L is the final length and the proportionality constant α is called **the coefficient of linear expansion.** It is defined in a deferential form **as the ratio between the frictional change in length: dL/L_o to the change in temperature dT**, thus

$$\alpha = 1/L_o \, . \, dL/dT$$

2.2 Coefficient of Area and Volume Expansion

$$A = A_0 (1 + 2\alpha \Delta T) \qquad A = A_0 (1 + \beta \Delta T) \qquad (3)$$

Where β is the area expansion coefficient

$$\beta = 2\alpha$$

Thus, the coefficient of area expansion of a substance is twice its coefficient of linear expansion.

$$V = V_0 (1 + 3\alpha \Delta T) \qquad V = V_0 (1 + \gamma \Delta T) \qquad (4)$$

where γ is the volume expansion coefficient and, thus,

$$\gamma = 3\,\alpha$$

Thus, the coefficient of volume expansion of a substance is three times its coefficient of linear expansion. The ratio between the three coefficients is

$$\alpha : \beta : \gamma = 1 : 2 : 3 \qquad (5)$$

As it is somewhat difficult to determine β and γ directly, for solids, the values of α is determined and the values of β and γ are taken approximately equal to 2α and 3α respectively. Values of expansion coefficients of some materials are given in Table (2).

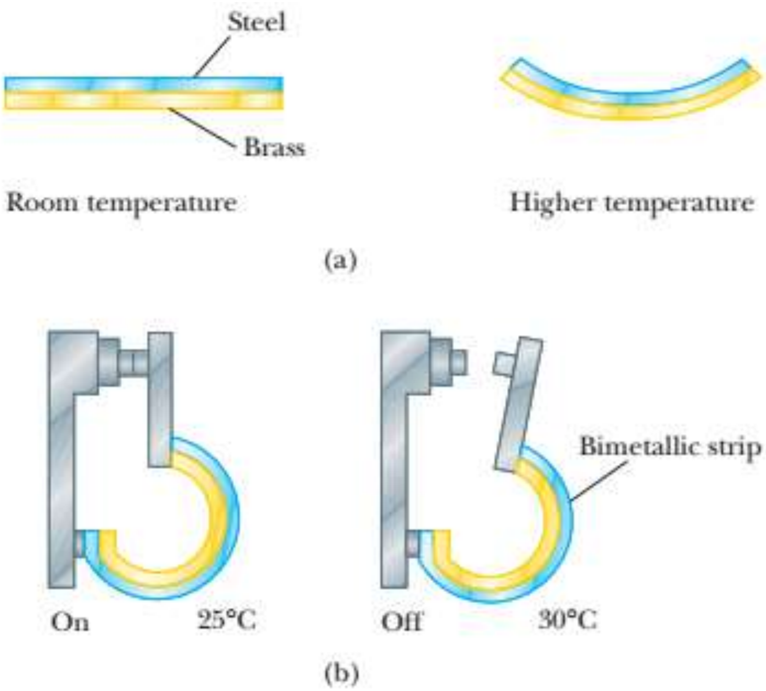
Table (2): Expansion coefficients for some material near room temperature

Material	**$\alpha \times 10^{-6}$ (^{o}C)$^{-1}$**	**Material**	**$\gamma \times 10^{-4}$ (^{o}C)$^{-1}$**
Pyrex	3.2	Air	0.36
Glass	9.0	Helium	0.36
Steel	11	Alcohol	1.12
Copper	17	Benzene	1.24
Bronze	19	Acetone	1.50
Brass	19	Mercury	1.82
Aluminum	24	Glycerine	4.85
Lead	29	Gasoline	9.60

As Table (2) indicates, each substance has its own characteristic coefficients of expansion. For example, when the temperature of a brass rod and a steel rod of equal length are raised by the same amount from some common initial value, the brass rod expands more than the steel rod because brass has a larger coefficient of expansion than steel. A simple device called a bimetallic strip that utilizes the principle is found in practical devices such as thermostats.

Applications

Thermometers and Thermostats: Many devices for the automatic control of temperature depend upon expansion. Such devices are called "thermostats". A bimetallic is made of pieces of different metals bonded together, and the strip bends toward the metal with the smaller thermal expansion. As illustrated in Fig. (5b), the strip can be calibrated with s scale to measure temperature. Some electric thermostats depend upon the bending of a bimetal strip when heated or cooled, Fig. (5c), to switch the current on or off. When heated, strip bends and makes contact with screw, thus completing circuit, and vice versa.



Thermometers and Temperature Scales

Temperature is a measure of the amount of heat energy possessed by and object. Thus the temperature defines the thermal state of a body i.e., the degree of hotness or coldness of the body Temperature provides an indicator of the direction of internal energy flow. When two bodies are in contact, internal energy goes from the one at the higher temperature to the one at the lower temperature.

Thermometers are devices used to define and measure the temperature of a system. All thermometers make use of the change in properties are (1) the change in volume of a liquid, (2) the change in length of a solid, (3) the change in pressure of a gas at constant volume, electric resistance of a conductor, and (6) the change in color of some object. For a given substance and a given temperature range, a temperature scale can be established based on any one of these physical quantities.

1. Temperature Scales

There are three main scales commonly used in the world today to measure temperature: the Celsius (°C) scale, the Fahrenheit (°F) scale, and the Kelvin (K) scale, see Fig. (1). Each of these scales uses a different set of divisions based on different reference points.

(i) **Celsius scale:** Anders Celsius was a Swedish astronomer credited with the invention of the Celsius scale. Celsius chose the melting point of ice (0°C) and the boiling point of water (100°C) as his two reference temperatures to provide for a simple method of thermometer calibration. Celsius divided the difference in temperature between the freezing and boiling points of water into 100 degrees.

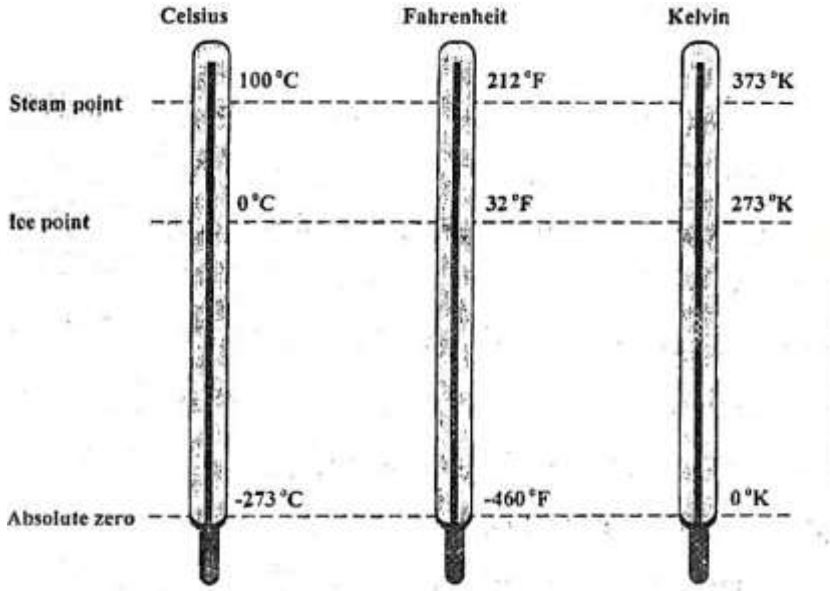


Fig. (1) Temperature scales

(ii) **Fahrenheit scale:** Daniel Gabriel Fahrenheit was a German physicist who credited with the invention of the alcohol and mercury thermometers. At this scale, the melting point of ice was set at 32°F and temperature between the freezing and boiling points of water into degrees. The fahrenheit scale is still commonly used in the United states.

(iii) **Kelvin scale:** Lord William Kelvin was a Scottish physicist who devised the Kelvin (K) scale. This scale is based on the idea of absolute zwro, the theoretical temperature at which all molecular motion stops. The exists in the universe (-273.15°C). The Kelvin scale uses the same unit of division as the Celsius scale; however, it resets the zero point to absolute zero (-273.15°C). The freezing point of water is (273.15K) and (373.15K) is the boiling point of water.

The relation between changes in temperature on the Celsius, Fahrenheit and Kelvin scales is

$$\frac{T_c}{100} = \frac{T_f - 32}{180} = \frac{T_k - 137.15}{100}$$

Or

$$T_c = \frac{5}{9}(T_f - 32)$$

$$T_f = \frac{5}{9}(T_k - 273.15) + 32$$

$$T_k = T_c + 273.15$$

2. Thermometry

Thermometry involves measuring certain physical property of a substance that is sensitive to temperature change. Some physical properties which change with temperature are the length of a rod, the volume of a liquid, the electric resistance of a wire or the color of a lamp filament etc. On the basis of such temperature dependent various kinds of thermometers have been developed. These are such as (a**) liquid thermometers** based on the thermal expansion of liquids (mercury or alcohol), (b**) gas thermometers** working on the thermal expansion of gases, (c) **resistance thermometers** depending on the variation of electrical resistance of metal with temperature, (d) **thermoelectric thermometers** of the junction of two metals and, (e) **radiation thermometers** which determine the temperature of a body by measuring the thermal radiation emitted by it.

The substance whose physical property varying with temperature is used in thermometer is called thermometric substance and the corresponding physical property is termed as thermometric property. Suppose that X is a thermometric property to be used in a temperature scale. Also suppose that the temperature T has the linear relationship with X as:

$$X = \delta T$$

Where δ is constant.

The last equation can give as many temperature scales as the values of X are. To calibrate the thermometer, a standard reference temperature is taken. Thus, at 0°C, 100°C and T°C

$$X_0 = \delta\ (0),$$

$$X_{100} = \delta(100)$$

and

$$X_T = \delta T$$

Hence

$$X_{100} - X_0 = 100\delta,$$

$$X_T - X_0 = \delta T$$

So

$$\frac{T}{100} = \frac{X_T - X_O}{X_{100} - X_O}$$

or

$$T = 100\,\frac{X_T - X_O}{X_{100} - X_O} \qquad (2)$$

Equation (2) defines the temperature T on the scale in which the property X of the substance has been employed. Thus for mercury scale, the above equation would give the unknown temperature T in the form

$$T_{mercury} = 100\,\frac{l_T - l_O}{l_{100} - l_O} \quad {}^{o}C$$

Where ℓ's are lengths of mercury column at various temperatures.

Similarly for a constant volume gas temperature scale

Medical thermometer

Normal human body-temperature

It is the typical *temperature* range found in *humans*. Its range is typically stated as 36.5–37 °C (97.7–98.6 °F).

Human body temperature varies. It depends on gender, age, time of day, exertion level, health status (such as illness and menstruation), what part of the *body* the measurement is taken at, state of consciousness (waking, sleeping, sedated), and emotions. Body temperature is kept in the normal range by *thermoregulation*, in which adjustment of temperature is triggered by the central nervous system.

A medical thermometer (clinical thermometer):

It is used for *measuring* human or animal *body temperature*. The tip of the thermometer is inserted into the *mouth* under the *tongue* (oral or sub-lingual temperature), under the *armpit* (axillary temperature), into the *rectum* via the *anus* (rectal temperature), into the *ear* (tympanic temperature), or on the *forehead* (temporal temperature).

Fig(1-4b.)A medical/clinical mercury thermometer showing the temperature of 37.7 °C (99.9 °F)

Fig(1-4c) Electronic clinical thermometer.

5. Solved Examples

Example (1):

An aluminum tube is 3m long at 20°C. What is its length at (a) 100 °C and (b) 0.0 °C?

---------------------------------------solution---------------------------------------

$L_0 = 3m$ $T_1 = 20°C$ $\alpha = 24 \times 10^{-6}\ (°C)^{-1}$

$L = ?$ (a) $T_2 = 100\ °C$ (b) $T_2 = 0.0\ °C$

$L = L_0\,[1 + \alpha(t_2 - T_1)]$

(a) at $T_2 = 100°C$

$L = 3\,[1 + 24 \times 10^{-6}\,(100 - 20)] = 3\,[1 + 0.001920] = 3.00576m$

(b) at $T_2 = 0.0\ °C$

$L = 3\,[1 + 24 \times 10^{-6}\,(0.0 - 20] = 3\,[1 - 0.00048] = 2.99856\ m$

Example (2):

A steel railroad track has a length of 30 m at 0.0°C. (1) What is its length on a hot day when the temperature is 40°C ?

---------------------------------------solution---------------------------------------

$L0 = 30m,\ T = 40\ °C,$ $\alpha = 11 \times 10^{-6}\ (°C)^{-1},\ Y = 20 \times 10^{10}\ N/m^2$

(a) $\Delta L = \alpha L \Delta T = 11 \times 10^{-6} \times 30 \times 40 = 0.013m$

So the length at 40°C = $L0 + \Delta L = 30 + 0.013m$

Quiz

1- *If you quickly plunge a room-temperature mercury thermometer into very hot water, the mercury level will (a) go up briefly before reaching a final*

Example:

The resistance of a platinum resistance thermometer is 2Ω at 100°C. At what temperature will the resistance become 2.3 Ω

---------------------------------------**solution**---------------------------------------

Since

$$T = 100 \times \frac{R_T - R_o}{R_{100} - R_o}$$

Here R_T= 2.3Ω, R_o= 2Ω, R_{100}= 2.5Ω

$T = 100 \times \frac{2.3-2}{2.5-2} = 60^{o}C$

5. Questions and Problems

1. The resistance of platinum resistance thermometer at 19°C is 3.5Ω and at 99°C is 3.66 Ω. At what temperature will its resistance be 4.3 Ω?

2. A substance is heated from -12°F to 150°F. What is its change in temperature on (a) the Celsius scale and (b) the Kelvin scale?

III. Specific Heat

In the preceding chapters, the concept of "temperature" and its measurement were discussed. The study of the interaction that takes place during the approach to thermal equilibrium leads to the concept of heat. Heat is a form of energy that is transferred as a consequence of heat. Heat is a form of energy that is transferred as a consequence of a difference in temperature between a system and its transferred as a consequence of a difference in temperature between a system and its surroundings. The transference of heat from a hot body to a cold body is analogous to the flow of water from a high level to a low level.

The increase in temperature of a body when heated depends on its mass and the material of which it is composed. If 100g of copper and 100g of water are heated by similar burners for the same time, the rise in temperature is not the same in the two cases. The rise in temperature depends on the quantities of heat given to the body and the nature of its material.

1. Quantity of Heat

The quantity of heat Q gained or lost by a substance is proportional to its mass m and the change in temperature ΔT. Thus,

$$Q \propto m\,\Delta T$$

Then, the relation expresses Q is

$$Q = mc\,\Delta T \qquad (1)$$

Where c is constant depending on the nature of the material. The unit for measuring the quantity of heat is the **calorie** which it is the **amount of heat required to raise the temperature of one gram of water through one degree centigrade**. The standard unit of **calorie** recommended by the International Union of pure and Applied Physics: The amount of heat necessary to raise the temperature if 1 gram of water from 14.5°C to 15.5°C.

2. Heat Capacity

Since different substance have different capacities for heat, as indicated above, they can be distinguished by a quantity known as heat capacity of a body. The heat capacity of a body is the quantity of heat required to raise the temperature of the whole of the body through one degree. If m is the mass of the body and c its specific heat, its heat capacity is,

$$C = mc \qquad (2)$$

3. Specific Heat

Specific heat is defined as the quantity of heat required to raise the temperature of one gram of substance through 1°C. The specific heat of a substance is not constant and it is different at different temperatures. Ordinary, the specific heat determined is the mean specific heat. From Eq. (1), the mean specific heat c is given by.

$$c = \frac{Q}{m\Delta T}$$

For qualitative work, if dQ heat is given to raise the temperature of m grams of a substance through dT then.

$$dQ = mc\,dT$$

and

$$c = \frac{1}{m}\frac{dQ}{dT}$$

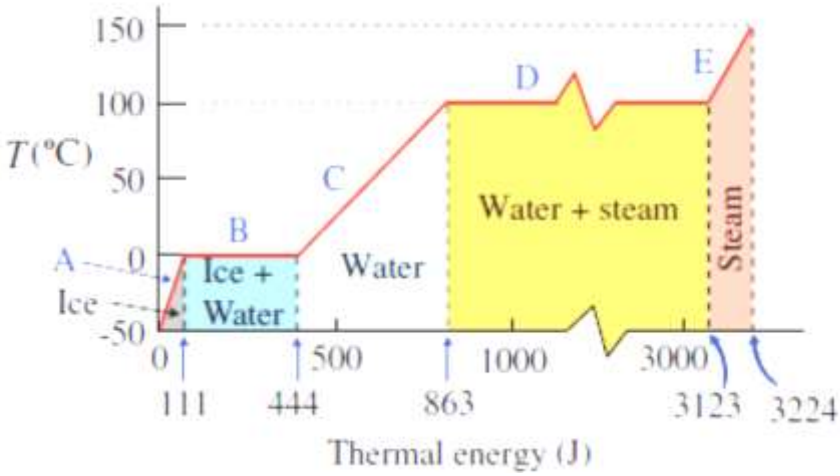
4. Latent Heat

We can add heat to a substance without raising its temperature if the substance is undergoing a change of state. If, for example, we add heat slowly to a an amount of ice, we will observe that the temperature does not rise until all the ice is meted as in Fig. (1). The heat supplied is utilized in changing the ice to water. Thos heat is stored as thermal energy in the water, and is given out when the water changes back to ice. As this heat cannot be detected by a thermometer, it is usually referred to as latent (word latent means invisible).

The formal definition of latent heat is the energy given out or absorbed without a change in temperature and is given by

$$Q = mL$$

Q is the heat, m is the mass, L is a constant for a certain material and is called the latent heat of fusion (for melting) or vaporization (for boiling).



4.1. Latent Heat of Fusion

When a solid substance changes from the solid phase to the liquid phase, energy mist be supplied in order to overcome the molecular attractions between the particle of the solid. This energy does not bring about a change in temperature, see Fig. (1). We call this energy latent heat. With this in mind, we define the latent heat of fusion:

"The latent heat of fusion of a substance is the amount of heat required to convert unit mass of the solid into without a change in temperature".

$$Q = mL_f$$

4.2. Latent Heat of Vaporization

In the last section, we stated that while a solid is changing to a liquid there is no change in temperature. Similarly, when a liquid is changing to a vapor ther is no change in temperature, see Fig. (1). To change a given mass of water at the boiling point into vapor or steam, a definite amount of heat must be supplied; when an equal mass of steam condenses to water, an equal amount of heat is given out. The heat supplied is called the latent of vaporization. The latent heat of vaporization is the energy required to convert the molecular forced of attraction between the particles of a liquid, and bring them to the vapor state, where the attractions are minimal. The definition of the latent heat of vaporization is:

"The latent heat of vaporization of a substance is the amount of heat required to convert unit mass of the liquid into the vapor without a change in temperature".

$$Q = mL_v$$

5. Conservation of Energy

When two bodies at different temperatures are placed in contact with each other then heat will pass from the body at higher temperature to the body at lower temperature until both arrive at the same temperature.

In this process

Heat lost by one body = Heat gained by the other

The formula for heat lost or heat gained during the process is given by

$$Q = mc\,\Delta T$$

$$Q = m \times c \times (T_f - T_i)$$

Where

m = mass of the body,

c = specific heat of the body,

$T_f - T_i$ = raise or fall in temperature

The above principle of heat (conservation of heat law) has been used for the measurement of heat and also the specific heat of substance.

5.1. Specific Heat of Solids (Mixture Method)

Mixture method is the one most commonly used in the laboratory for determining the specific heat of solids. The apparatus consists of two parts, the heater and the calorimeter, see Fig. (3). The heater consists of two coaxial cylinders, the annular space between them being supplied with a steady flow of steam. The top of the air chamber (inner one) is closed with a cork while the bottom by a trap door through which the solid may be dropped into the calorimeter. The calorimeter is a copper vessel placed in a wooden box packed with wood to reduce loss of heat by conduction. The calorimeter is provided with a copper stirrer and a sensitive thermometer.

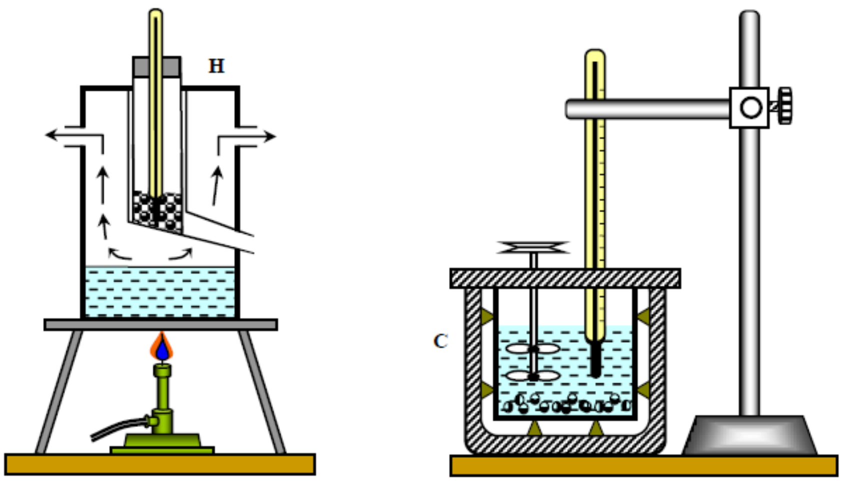


Figure 3: Determination of specific heat of a solid by mixture method

The solid, in the form of a small pieces, is weighted and suspended inside the heater by a thread passing through the cork. Steam is passed through the heater from a boiler, so that the solid is heated, the empty dry calorimeter, with the stirrer is weighted. Water is taken in the calorimeter and the calorimeter and contents are weighted again. The mass of water taken is then readily found. The calorimeter is placed back inside the wooden box and the temperature of the water is noted.

When the solid has attained the steady maximum temperature, the calorimeter is pushed under the trap door of the heater and the solid dropped into the calorimeter. The contents of the calorimeter are well stirred and the highest temperature reached is noted.

Suppose c1 is specific heat of a given solid. M_1 grams of the solid at T_1°C is added to m_3 of water at T_2°C in s calorimeter whose mass is m_2 and its specific heat is c_2. Let T3°C be the final temperature of the mixture, see Fig. (4). By equating the heat lost by the solid Q_s to the heat gained by the calorimeter and water ($Q_c + Q_w$) we get

Heat lost by solid = Heat gained by (calorimeter + water)

$$Q_s = Q_c + Q_w$$

$$M_1 c_1 (T_3 - T_1) = (m_2 c_2 + m_3 c_3)(T_3 - T_2) \qquad (8)$$

From which c_1 can be determined.

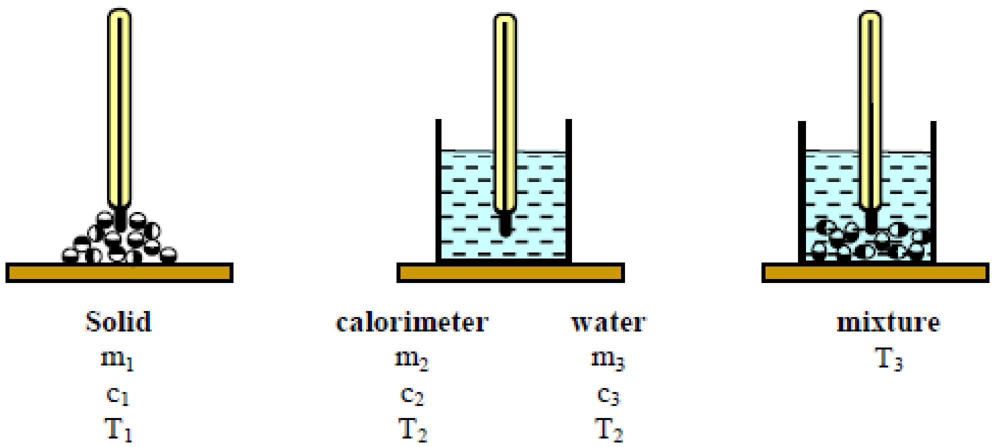


Figure 4: Data taken in the mixture methode

Where

m_1, c_1	are the mass and the specific heat of the liquid
m_2, c_2	are the mass and the specific heat of the calorimeter.
T_1	is the initial temperature of the liquid
T_2	is the final temperature of the liquid
I	is the current flowing
V	is the potential difference across the resistance R
t	is the time in second

6. Solved Examples

Example (1):

Two hundred thousand joules of heat is removed from a 25kg block of ice initially at -5°C. What is its final temperature?

(c_{ice} = 2110J/kg °C)

--**Solution**--

$Q = 20000 = 2 \times 10^5 J$ $\quad\quad$ $m = 25kg$

$T_1 = -5°C$ $\quad\quad$ $T_2 = ?$

$Q = mc\Delta T = mc(T_2 - T_1)$

$-2 \times 10^5 = 25 \times 2110 \times [T_2 - (-5)]$

$T2 = 8.8°C$

Example (2):

One kg of water at 50°C is added to 3kg of water at 6°C. What is the final temperature of the mixture?

--**Solution**--

$m1 = 1kg$ $\quad\quad$ $T1 = 50°C$ $\quad\quad$ $m2 = 3kg$

$T2 = 6°C$ $\quad\quad$ $T3 = ?$

Heat lost = Heat gained

$m_1c_1(T - T_1) = m_2c_2 (T - T_2)$

$1 \times c \times (50 - T_3) = 3 \times c \times (T_3 - 6)$

$50 - T_3 = 3T_3 - 18$

$4T_3 = 68 \quad\quad$ or $T_3 = 17°C$

Heat Transfer

Thermal energy is related to the temperature of matter. For a given material and mass, the higher the temperature, the greater its thermal energy. Heat transfer of t he exchange of thermal energy through a body or between bodies which occurs when there is a temperature difference. The temperature distribution and the heat flow are of interest in many scientific and engineering applications, such as the design of heat exchangers, nuclear – reactors cores, heating and air conditioning systems, and solar energy system.

There are three way in which heat is transferred from one place to another. The three ways are by conduction, convection, and radiation. Conduction is the transfer of heat in which thermal energy is transferred from molecule in a material with no perceptible motion of the material. Convection is the transfer of heat by mass motion of the heated material. Radiation is the transfer of thermal energy by electromagnetic waves.

Heat is transferred by conduction a solid or fluid at rest. Conduction needs a medium in which to take place, whereas radiation can take place in a vacuum with no material carrier. Heat is transferred by convection in fluids in motion. In fact, conduction and radiation are the two basic modes of heat flow; convection can be regarded as conduction with fluid in motion.

1. Transfer of Heat by Conduction

Heat conduction can be visualized as the result of molecular collisions. As one end of the object is heated, the molecules there move faster and faster. As they collide with their slower neighbor, they transfer some of their energy to these molecules whose speeds increase.

Consider the conduction of heat through a slab of material of face area A and thickness Δx, with a difference of temperature between the face of ΔT. The rate of heat flow through the slab is found to be proportional to the temperature gradient ΔT/Δx and the area A. Then;

$$\frac{\Delta Q}{\Delta t} \propto A\frac{\Delta T}{\Delta x} \qquad \text{or} \qquad H \propto A\frac{\Delta T}{\Delta x}$$

For infinitesimal thickness dx, across which there is a temperature difference dT, we obtain the fundamental law of heat conduction.

$$H = \frac{dQ}{dT} = -KA\frac{dT}{dX}$$
(1)

where dT/dx is called the temperature gradient, and k is a constant of proportionality called the thermal conductivity of the material. It is defined as, the rate of heat flow by conduction per unit area per unit temperature gradient. The thermal conductivities, k, for various substances are given in Table (1). The minus sign in Eq. (1) denotes the fact that thermal energy flows in the direction of decreasing temperature.

Table (1): Thermal conductivities of some substances

Metals	k(W/m.°C)	Gases	k(J/s.m.K)
Silver	427	Air	0.0234
Copper	397	Hydrogen	0.1720
Aluminum	238	Oxygen	0.0238
Gold	314	Nitrogen	0.0234
Iron	79.5	Helium	0.1380

1.1. Linear Flow of Heat

Consider the heat flow through a uniform rod of length L and constant cross sectional area A as illustrated in Fig. (1). The rod is insulated so that thermal energy cannot escape from its surface except at the ends, which are in thermal contact with heat reservoirs having temperatures T_1 and T_2.

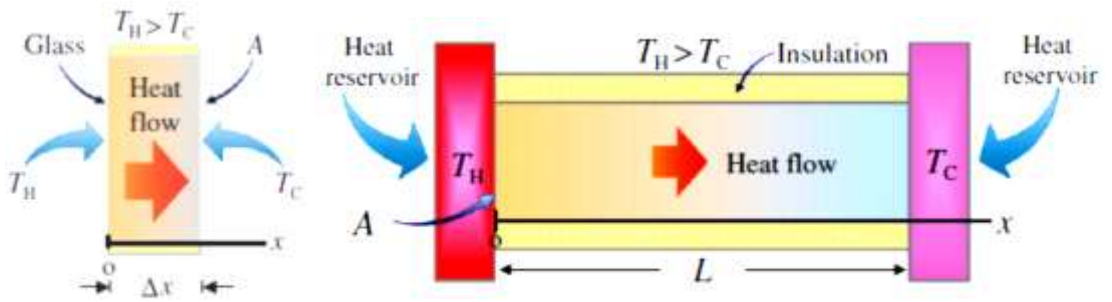


Figure 1 : Conduction of heat through an insulating material

In a steady state the temperature at each point along the rod is constant in time. In this case, the temperature at each point along the rod is constant in time. In this case, the temperature gradient dT/dx is the same at all cross sections and is

$$\frac{dT}{dx} = \frac{T_2 - T_1}{L}$$

Therefore, the rate of heat flow through the rod is given from Eq. (1) by

$$H = \frac{dQ}{dt} = -KA\frac{T_2 - T_1}{L} \qquad (2)$$

3. Transfer of Heat by Convection

Although liquids and gases are not very good conductors of heat, they can transfer heat quite rapidly by **convection**. Convection is the process in which heat is transferred by the mass movement of molecules from one place to another. For instance, the air above a radiator (or other type of heater) expands as it is heated and hence its density decrease; because its density less, it rises.

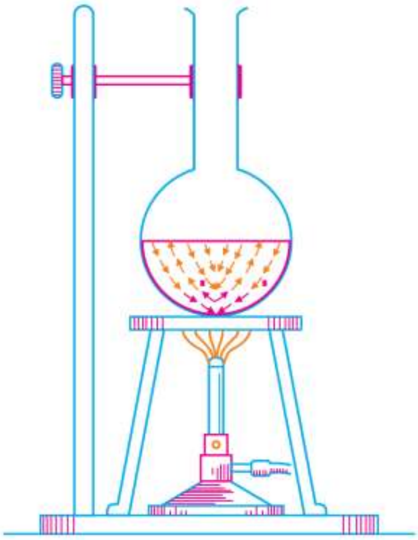


Figure 8: Convection of heat

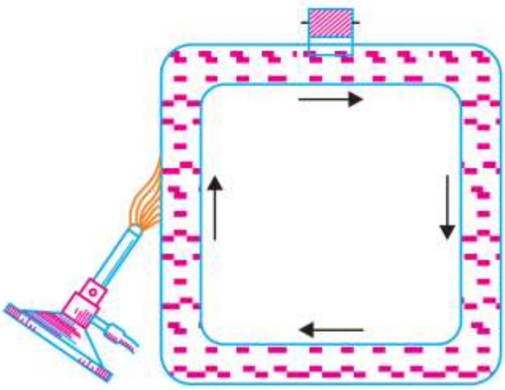


Figure 9: Convection of current

Take a flask containing water. Add a large crystal of $KMnO_4$. Heat the flask, as in Fig. (8). Colored streaks of water rise up and move to the sides. Water rise up and move to the sides. Water at the bottom heated and move up. Water from the sides reaches the bottom, gets heated and rises up. The process continues. This phenomenon is called convection. Each molecule of water comes to the hot point, takes heat and moves up.

Take a rectangular glass tube and fill it with colored water. Heat the side tube gently. The convection currents are set up due to the movement of the heated water molecules. The direction of movement of water molecules is indicated by the movement of the colored water.

Near the sea, water becomes less warm than the land during day time. The heated air on the land surface moves up. Air from the surface of sea moves towards the sea shore. This is called see breeze. Convection currents are set up. During night when land becomes colder than water, the air over the surface of water is warmer and moves upward. Air from the land moves towards the sea. This is called land breeze, see Fig. (10).

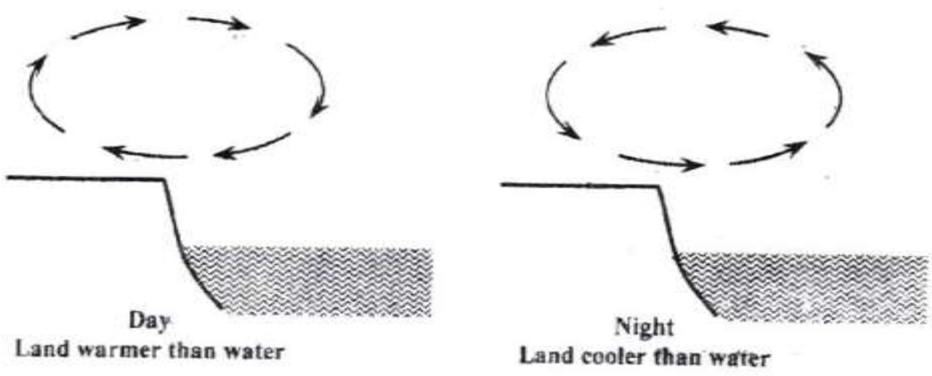


Fig. (10): Illustration of sea and land breezes

The mathematical theory of convection quite complex. The rate of heat transfer by convection to or from a surface can be calculated from the following equation:

$$H = \frac{dQ}{dt} = hAK\Delta T \quad (8)$$

Where h is the convection coefficient, A is the area of the surface, and ΔT is the temperature difference between the surface and the main body of the fluid.

4. Transfer of Heat by Radiation

Convection and conduction require the presence of matter. Radiative heat transfer does not require a medium to pass through. Thus, it is the only form of heat transfer present in vacuum. It uses electromagnetic radiation (photons), which travels at the speed of light and is emitted by any matter with temperature above 0K (-273°C). Radiative heat transfer occurs when the emitted radiation strikes another body and is absorbed. An example of this is the transfer of heat from the Sun to the earth. It travels through ninety million miles of space in which three is no material substance in about 8 minutes.

4.1. Blackbody Radiation

A blackbody refers to an opaque object that emits thermal radiation. A perfect blackbody is one that absorbs all incoming light. If heated to a high temperature, a blackbody will begin to glow with thermal radiation. An approximation to a perfect blackbody may be obtained by a hollow sphere having a small opening with the inside walls having a rough, dull surface as shown in Fig. (11).The radiation enters or leaves the cavity through a small hole. Part of the radiation entering the cavity will be absorbed by its walls and part reflected radiations escape through the hole, so that after many internal reflections nearly all the radiation is absorbed and the body approximates a blackbody.

At the beginning of the 20^{th} century, scientist Lord Rayleigh, an Max Plank studied the black body radiation using such a device. After much work, Planck was able to describe the intensity of light emitted by a blackbody as a function of wavelength. Planck's work on blackbody is one the areas of physics that led to the foundation of the wonderful science of Quantum Mechanics, but that is unfortunately beyond the scope of this course.

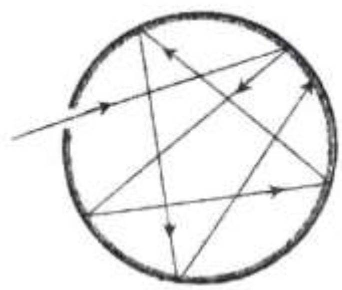


Fig. (11): An approximation of a perfect absorber (Blackbody)

Plank was found that the temperature of a blackbody increases, the total amount of light emitted per second increases, and the wavelength of the spectrum's peak shifts to the bluer colors, see Fig. (12). For example, an iron bar becomes orange-red when heated to high temperatures and its color shifts toward blue and white as it is heated further.

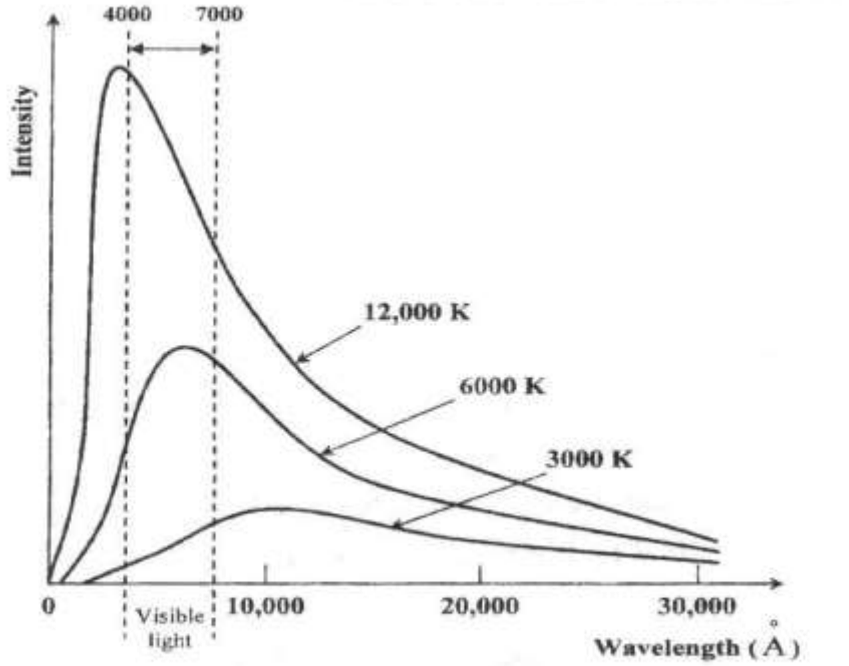


Fig. (12): The wavelength of the spectrum's peak shifts to the bluer colors

4.2. Stefan-Botzmann's Law

It was mentioned at the beginning that the quantity of radiation emitted by a body depends on its temperature. In fact, the total radiation emitted by a body increases very rapidly as the temperature is raised.

According to Stefan-Boltzmann, **the rate of emission of radiation from a black body is directly proportional to the fourth power of its absolute temperature**. So the rate at which energy leaves the black body is

$$R = \frac{dQ}{dt} = \sigma T^4 \quad (9)$$

where R is the radiation power per unit area and, σ is a universal constant called the Stefan-Boltzmann constant which has the value

$$\sigma = 5.67 \times 10^{-8} \text{ watt/m}^2\text{K}^4$$

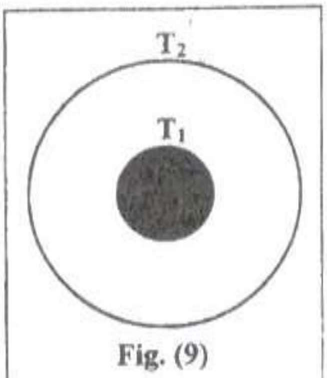


Fig. (9)

If the body is not perfect black and its emissivity is ,then ε, then

$$R = \frac{dQ}{dt} = \varepsilon\sigma T^4 \quad (10)$$

where ε varies between zero and one, depending on the nature of the surface. For a perfectly black body ε = 1. Whereas shiny surfaces have ε close to zero and thus emit less radiation. Not only do shiny surfaces emit less radiation, but they absorb little of the radiation that falls upon them (most is reflected). Black and very dark object absorb nearly all the radiation that falls on them. Thus, a good absorber is also a good emitter.

Any body not only emits energy by radiation, but it also absorbs energy radiated by other bodies. If a body is at a temperature T_1 as in Fig. (9), it radiates energy by a rate $R_1 = \varepsilon\sigma T^4{}_1$. If the body is surrounded by an environment at temperature T_2, the surroundings radiate energy by a rate

$R_2 = \varepsilon\sigma T^4{}_2$. The net rate of radiation heat flow from the body per unit area is given by R_1-R_2, So

$$R = \frac{dQ}{dt} = \varepsilon\sigma(T_1^4 - T_2^4) \qquad (11)$$

Example (1): Two black concentric spheres are temperatures of 200k and 300k. The space in between the two spheres is evacuated. Calculate the net rate of energy transfer between the two spheres is evacuated. Calculate the net rate of energy transfer between the two spheres. ($\sigma = 5.67 \times 10^{-8}$ M.K.S. units)

Solution:

$T_1 = 300$k, $T_2 = 200$K, $\varepsilon = 1$, $\sigma = 5.672 \times 10^{-3}$

$$R = \sigma(T_1^4 - T_2^4)$$

$$R = 5.67 \times 10^{-8} \times [(300)^4 - (200)^4]$$

$$R = 386.68 \text{ Watts/m}^2$$

Part (3) Geometrical Optics

1-1 The Nature of Light

Before the beginning of the nineteenth century, light was considered to be a stream of particles that either was emitted by the object being viewed or emanated from the eyes of the viewer. Newton, the chief architect of the particle theory of light, held that particles were emitted from a light source and that these particles stimulated the sense of sight upon entering the eye. Using this idea, he was able to explain reflection and refraction.

Most scientists accepted Newton's particle theory. During his lifetime, however, another theory was proposed—one that argued that light might be some sort of wave motion. In 1678, the Dutch physicist and astronomer Christian Huygens showed that a wave theory of light could also explain reflection and refraction.

In 1801, Thomas Young (1773–1829) provided the first clear demonstration of the wave nature of light. Young showed that, under appropriate conditions, light rays interfere with each other. Such behavior could not be explained at that time by a particle theory because there was no conceivable way in which two or more particles could come together and cancel one another.

Additional developments during the nineteenth century led to the general acceptance of the wave theory of light, the most important resulting from the work of Maxwell, who in 1873 asserted that light was a form of high-frequency electromagnetic wave. Hertz provided experimental confirmation of Maxwell's theory in 1887 by producing and detecting electromagnetic waves.

Although the wave model and the classical theory of electricity and magnetism were able to explain most known properties of light, they could not explain some subsequent experiments. The most striking of these is the photoelectric effect, also discovered by Hertz: when light strikes a metal surface, electrons are sometimes ejected from the surface.

As one example of the difficulties that arose, experiments showed **that the** kinetic energy of an ejected electron is independent of the light intensity. This finding contradicted the wave theory, which held that a more intense beam of light should add more energy to the electron. An explanation of the photoelectric effect was proposed by Einstein in 1905 in a theory that used the concept of quantization developed by Max- Planck (1858–1947) in 1900. The quantization model assumes that the energy of a light wave is present in particles called photons; hence, the energy is said to be quantized. According to Einstein's theory, the energy of a photon is proportional to the frequency of the electromagnetic wave:

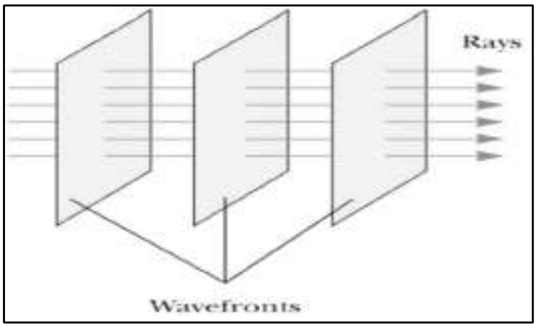
$$E = hf$$

where the constant of proportionality $h = 6.63 \times 10^{-34}$ J . s is Planck's constant . In view of these developments, light must be regarded as having a dual nature.

Light exhibits the characteristics of a wave in some situations and the characteristics of a particle in other situations. Light is light, to be sure. However, the question "Is light a wave or a particle?" is inappropriate. Sometimes light acts like a wave, and at other times it acts like a particle**.**

1.1 Characteristics of light:

1. The light passes through transparent materials.
2. The light travels through the vacuum.
3. The light is reflected when incident on a highly polished smooth surface.
4. The light will be refracted when incident on a boundary between two different transparent surfaces.



5. The speed of light decreases in a medium in which its density is high.
6. The speed of light in vacuum is 3×10^{8} m/sec.
7. The interference is taken place from the light emerges from two nearest coherent sources.
8. The light is diffracted (change in its path) when incident on a narrow or an edge.
9. The light is polarized when passes through some transparent crystalline materials.

1.2 THE RAY APPROXIMATION IN GEOMETRIC OPTICS

In studying geometric optics, we shall use of an important property of light that can be understood based on common experience.

Light travels in a straight-line path in a homogeneous medium until it encounters a boundary between two different materials. As shown in figure (1), a ray of light is an imaginary line drawn along the direction of travel of the light beam. A wave front is a surface passing through the points of a

Figure -1 A plane wave travelling to the right. Note that the rays, corresponding to the direction of wave motion, are straight lines perpendicular to the wave fronts.

wave that have the same phase and amplitude. For instance, the wave fronts in figure (1) could be surfaces passing through the crests of waves.

1.3 THE REFRACTIVE INDEX

The ***index of refraction***, or ***refractive index***, of any optical medium is defined as the ratio between the speed of light in a vacuum and the speed of light in the medium:

$$Refractive\ index = \frac{speed\ in\ vacuum}{speed\ in\ medium}$$

$$n = \frac{c}{v}$$

The letter n is customarily used to represent this ratio. Using the speed of light in different mediums, we obtain the following values for the refractive indices:

For glass: n = 1.520

For water: n = 1.333

For air: n = 1.000

Accurate determination of the refractive index of air at standard temperature (0°C) and pressure (76 mmHg) gives:

$$n = 1.000292 \quad \text{for air}$$

Different kinds of glass and plastics have different refractive indices. The most commonly used optical glasses range from 1.52-1.72.

The ***optical density*** of any transparent medium is a measure of its refractive index. A medium with a relatively high refractive index is said to have a high optical density, while one with a low refractive index is said to have a low optical density.

1.4 OPTICAL PATH

To derive one of the most important fundamental principles in geometric optics, it is appropriate to define a quantity called the *optical path*. The path d of a ray of light in any medium is given by the product *velocity* times *time*:

$$d = vt$$

Since by definition $n = c/v$, which gives $v = c/n$, we can write

$$d = \frac{c}{n}t \quad or \quad nd = ct$$

The product nd is called the optical path Δ:

$$\Delta = nd$$

The optical path represents the distance light travels in a vacuum (Δ), in the same time, it travels a distance d in the medium. If a light ray travels through a series of optical media of thickness $d, d', d'',\ldots$ and refractive indices $n, n', n'',\ldots$, the total optical path is just the sum of the separate values:

$$\Delta = nd + n'd' + n''d'' + \ldots.$$

Figure (2) is a diagram illustrating the meaning of optical path is shown in. Three media of length d, d', and d'', with refractive indices n, n', and n'', respectively, are shown touching each other. Line AB shows the length of the actual light path through these media, while the line CD shows the distance Δ, the distance light would travel in a vacuum in the same amount of time t.

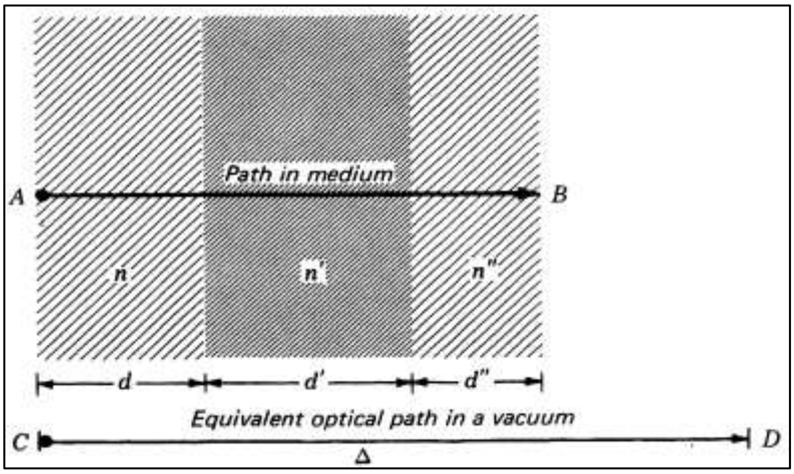


Fig. (2) The optical path through a series of optical media

1.5 REFLECTION AND REFRACTION

Reflection of light

When a light ray traveling in a transparent medium encounters a boundary, leading into a second medium, part of incident ray is reflected back into the first medium. Fig (3a) shows several rays of a beam of light incident on a smooth, mirror-like, reflecting surface. The reflected rays are parallel to each other, as indicated in the figure. Reflection of light from such a smooth surface is called **specular reflection**. On the other hand, if the reflecting surface is rough, as in figure (3b), the surface reflects the rays in a variety of directions. Reflection from any rough surface is known as **diffuse reflection**.

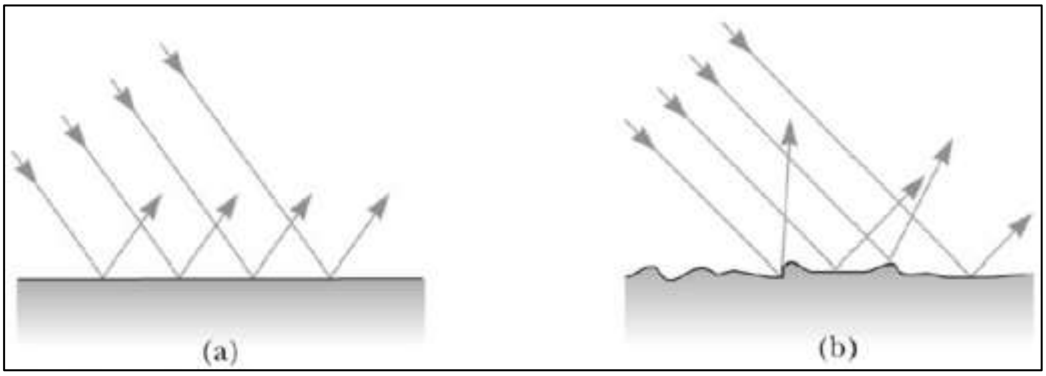


Figure (3) A schematic representation of (a) specular reflection, where the reflected rays are all parallel to each other, and (b) diffuse reflection, the reflected rays travel in random directions.

Consider a light ray traveling in air and incident at some angle on a flat, smooth surface, as in figure (4). The incident and reflected rays make angles θ_1 and θ'_1 respectively, with a line perpendicular to the surface at the point where the incident ray strikes the surface. We call this line the *normal* to the surface.

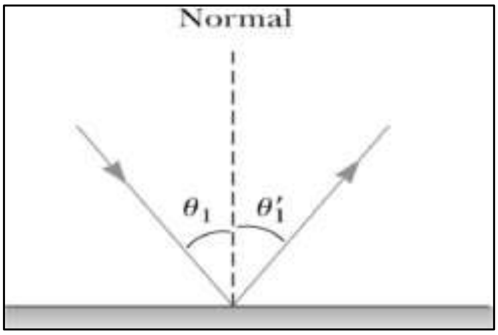


Figure (4) According to the law of reflection, $\theta_1 = \theta'_1$

There are two important laws of reflection

The <u>*first states*</u> that;

The angle of incidence = the angle of reflection

$$\theta_1 = \theta'_1$$

The <u>*second states*</u> that; the incident ray, the reflected ray and the normal all lie in the same plane, which is perpendicular to the interface separating the two media.

Refraction of Light

When a ray of light traveling through a transparent medium encounters a boundary leading into another transparent medium, as in figure 5, part of the ray is reflected and part enters the second medium. The ray that enters the second medium is bent at the boundary and is said to be *refracted*.

The angle of refraction, θ_2, in figure 5 depends on the properties of the two media and on the angle of incidence, through the relationship

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1} = constant$$

where v_1 is the speed of light in medium 1, and v_2 is the speed of light in medium 2, note that the angle of refraction is also measured with respect to the normal.

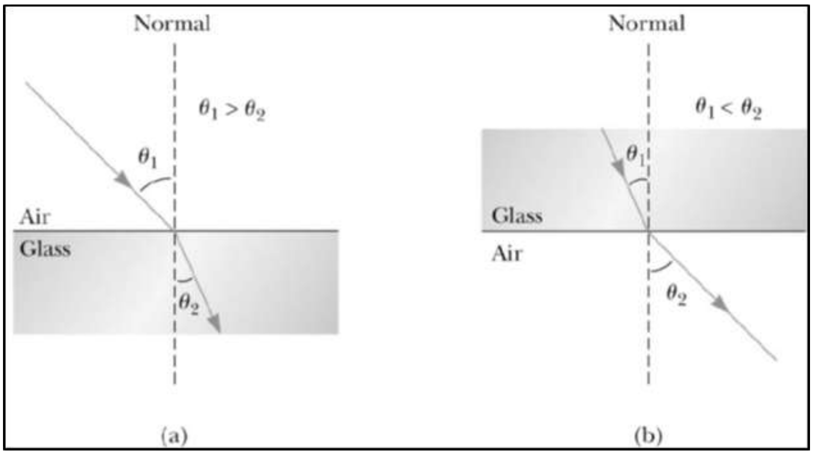


Figure (5) A ray obliquely incident on an air-glass interface. The refracted ray is bent toward the normal because $v_2 < v_1$.

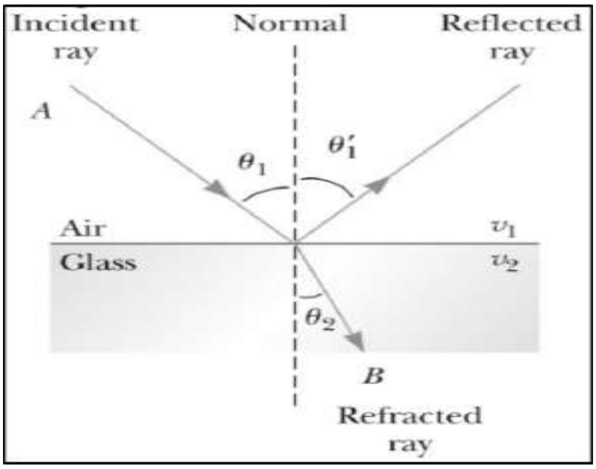


Figure (6). (a)When the light beam moves from air into glass, its path is bent toward the normal. (b)When the beam moves from glass into air, its path is bent away from the normal.

When light moves from a material in which its speed is high to a material in which its speed is lower, the angle of refraction θ_2 is less than the angle of incidence θ_1. The refracted ray therefore bends toward the normal, as shown in figure 6a .If the ray moves from a material in which it travels slowly to a material in which its speed is high , the angle of refraction θ_2 is greater than θ_1 as shown in figure 6b.

There are two important laws of refraction;

The first states that ; the sin of the angle of incidence and the sin of angle of refraction bear a constant ratio one to the other

$$\frac{\sin\theta_1}{\sin\theta_2} = constant$$

In addition the constant is found to have exactly the ratio of the refractive indices of the two media *n*, *n'*.

Hence, we can write

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n\prime}{n}$$

which can be written as

$$n \sin\theta_1 = n' \sin\theta_2$$

which is known as **Snell's law.**

The second states tha**t;** the incident, refracted rays and the normal, all lie in the same plane.

1.6 *Derivation of the laws of reflection and refraction from Fermat's Principle*.

Fermat's principle states that;

The path taken by a ray of light in passing from one point to the other is the path of minimum time.

(i) Derivation the Law of reflection. Let *PA* in figur 7a be the incident ray and *AQ* the reflected ray; *i* and *r* the angles of incidence and reflection respectively. We have to show that the first law of reflection, which given by**,**

$$\angle i = \angle r$$

is compatible with principle of least time.

Let the perpendicular distances *PL* and *QM* be denoted by h_1 and h_2 and the total length on *x*-axis intercepted by these perpendiculars, i.e., *LM* is equal *p*.

The optical path PAQ is given by

$$S = PA + AQ$$

$$=[h_1^2 + (p - x)^2]^{\frac{1}{2}} + [h_2^2 + x^2]^{\frac{1}{2}}$$

Differentiating and equating the first derivative to zero, we have

$$\frac{ds}{dx} = \frac{1}{2}\,\frac{2(p-x)\,(-1)}{[h_1^2+(p-x)^2]^{\frac{1}{2}}} + \frac{1}{2}\,\frac{2x}{[h_2^2+x^2]^{\frac{1}{2}}} = 0,$$

$$\therefore \qquad \frac{(p-x)}{[h_1^2+(p-x)^2]^{\frac{1}{2}}} = \frac{x}{[h_2^2+x^2]^{\frac{1}{2}}}$$

i.e. $\sin i = \sin r$ or $i = r$

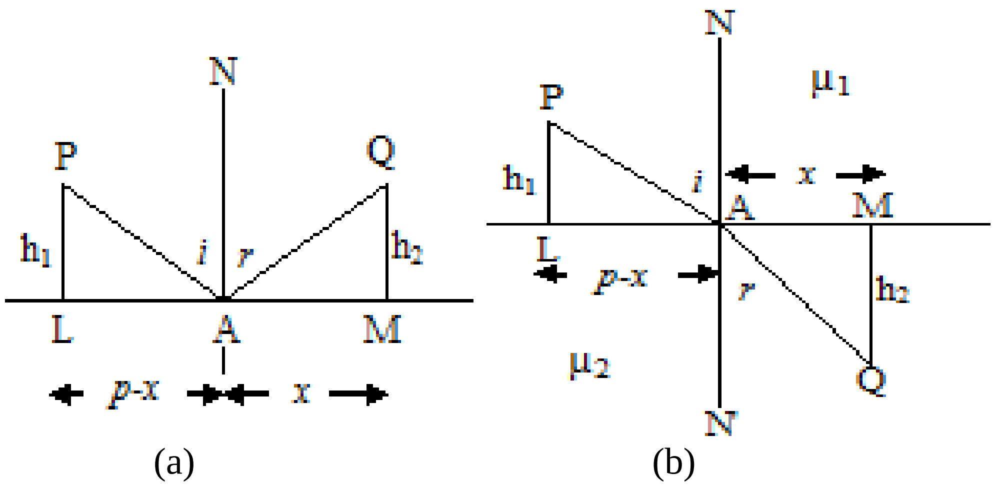


Fig.7 Laws of optics on Fermat's principle

(ii) Law of refraction. In Figure 7b PAQ is the path of a ray from the medium of refractive index μ_1 into a medium of refractive index μ_2 passing *via* a point A at the interface, then the optical path,

$$S = \mu_1 PA + \mu_2 AQ$$

Taking A as origin, $AM = x$ and $AL = (p-x)$, then

$$S = \mu_1[h_1^2 + (p - x)^2]^{\frac{1}{2}} + \mu_2[h_2^2 + x^2]^{\frac{1}{2}}$$

According to Fermat's principle the first derivative of the expression above must vanish, that is

$$\frac{ds}{dx} = 0$$

Now, $$\frac{ds}{dx} = \frac{\frac{1}{2}\mu_1}{[h_1^2+(p-x)^2]^{\frac{1}{2}}} \times (-2p + 2x) + \frac{\frac{1}{2}\mu_2}{[h_2^2+x^2]^{\frac{1}{2}}} \times 2x = 0$$

$$\therefore \qquad \mu_1 \frac{(p-x)}{[h_1^2+(p-x)^2]^{\frac{1}{2}}} = \mu_2 \frac{x}{[h_2^2+x^2]^{\frac{1}{2}}}$$

Now, $\frac{(p-x)}{[h_1^2+(p-x)^2]^{\frac{1}{2}}} = \sin i$ *and* $\frac{x}{[h_2^2+x^2]^{\frac{1}{2}}} = \sin r$

$\mu_1 \sin i = \mu_2 \sin r$ The Snell's law is, therefore, proved.

Example :-

A light ray of wavelength 589 nm (produced by a sodium lamp) traveling through air is incident on a smooth, flat slab of crown glass at an angle of 30.0^o to the normal, as sketched in figure 8 .Find the angle of refraction, θ_2.

Solution Snell's law can be rearranged as

$$\text{Sin } \theta_2 = \frac{n1}{n2} \sin \theta_1$$

From the table n_1 = 1.00 for air and n_2 = 1.52 for crown glass. Therefore, the unknown refraction angle is determined by

$$\sin \theta_2 = \left(\frac{1.00}{1.52}\right) (\sin 30.0^o) = 0.329$$

$$\theta_2 = \sin^{-1}(0.329) = 19.2^0$$

We see that the ray is bent toward the normal, as expected.

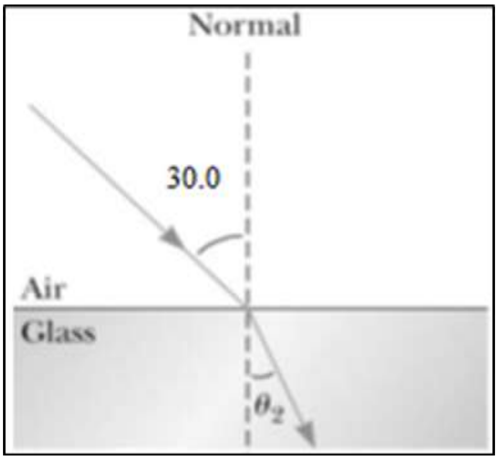
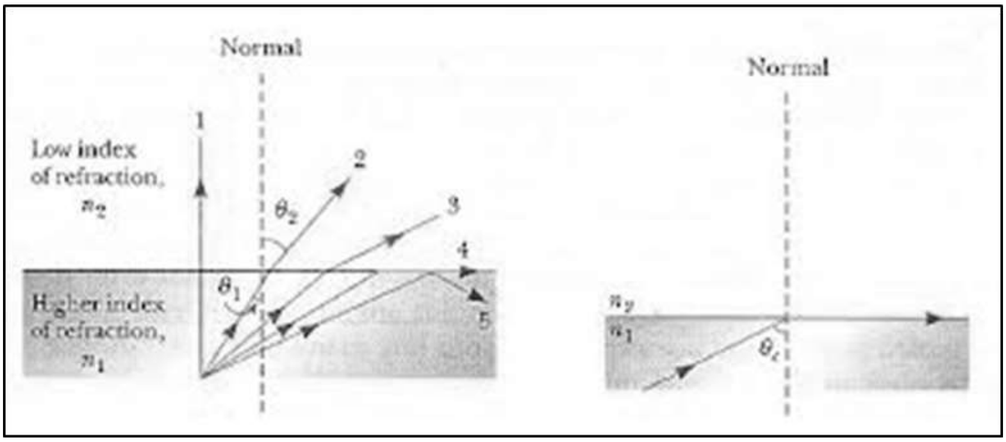


Figure 8: Refraction of light by glass.

1.7 TOTAL INTERNAL REFLECTION

An interesting effect called ***total internal reflection*** can occur when light attempts to move from a medium with a high index of refraction to one with a lower index of refraction. Consider a light beam traveling in medium 1 and meeting the boundary between medium 1 and medium 2, where n_1 is greater than n_2 (Fig.9a). The possible directions of the beam are indicated by rays 1 through 5. Note that the refracted rays are bent away from the normal because n_1 is greater than n_2.At some particular angle of incidence θ_c, called the ***critical angle***, the refracted light ray moves parallel to the boundary so that $\theta_2 = 90^0$ (fig.9b). For angles of incidence greater than θ_c, the beam is entirely reflected at the boundary, as is the ray 5 in figure 9a. This ray is reflected at the boundary as though it had struck a perfectly reflecting surface. It and all rays like it obey the law of reflection; that is, the angle of incidence equals the angle of reflection.



(a) (b)

Figure 9 (a) Rays from a medium with index of refraction n_1 travel to a medium with index of refraction n_2, where $n_1 > n_2$.As the angle of incidence increases, the angle of refraction θ_2 increases until θ_2 is 90° (ray4). For even larger angles of incidence, total internal reflection occurs (ray5). (b)The angle of incidence producing a 90° angle of refraction is often called ***the critical angle*** $\boldsymbol{\theta_c}$.

We can use Snell's law to find the critical angle. When $\theta_1 = \theta_c$, $\theta_2 = 90^o$ and Snell's law gives

$$n_1 \sin \theta_c = n_2 \sin 90^o = n_2$$

$$\text{Sin } \theta_c = \frac{n2}{n1} \quad \text{for } n_1 > n_2 \qquad \text{(i)}$$

Note that this equation can be used only when n_1 is greater than n_2.That is

total internal reflection occurs only when light attempts to move from a medium of high index of refraction to a medium of lower index of refraction.

If n_1 were less than n_2, Equation i would give sin θ_c>1, which is an absurd result because the sine of an angle can never be greater than unity.

Chapter 2

Mirrors

This chapter is concerned with the images formed when spherical waves fall on flat and spherical surfaces. We find that images can be formed by reflection or by refraction and that mirrors and lenses work because of this reflection and refraction. Such tools, commonly used in optical instruments and systems, are described in detail. In this chapter we use the ray approximation and assume that light travels in straight lines, both steps are valid because here we are studying the field called ***geometrical optics***. In this chapter we discuss the manner in which optical tools such as mirrors form images.

2. Image formed by spherical mirrors

2.1.Concav mirrors

A spherical mirror has the shape of a segment of sphere.

Figure (10a) shows the cross section of spherical mirror with it's surface represented by the solid curved black line. Such mirror, in which light is reflected from the inner is called a ***concave mirror***. The mirror has a radius of curvature R, and its center of curvature is located at point C.

Point V is the center of spherical segment, and a line drawn from C to V is called the principal axis of the optical system.

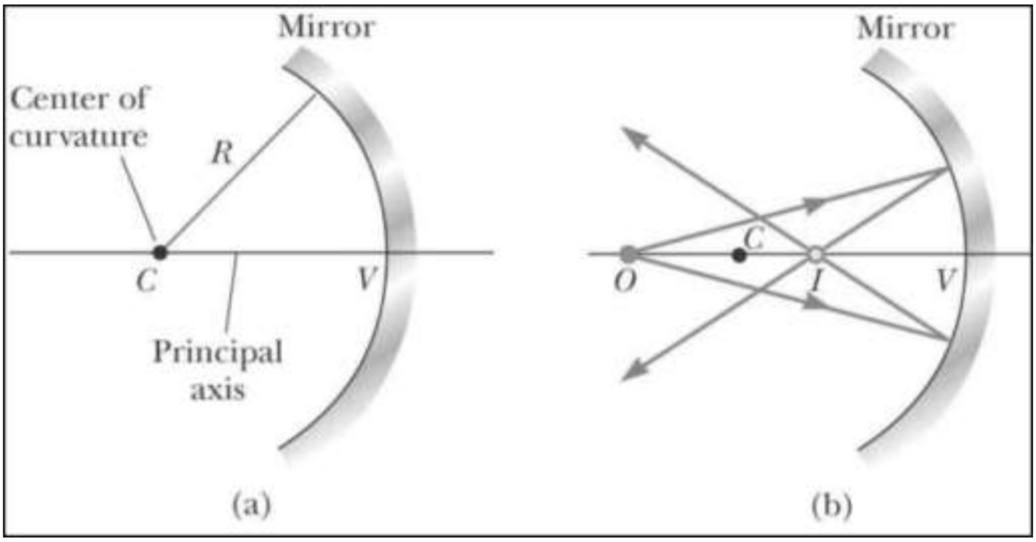


Fig. (10): (a) A concave mirror of radius R whose center of curvature is at C. (b) A point object placed at O forms a real image at I.

The radius of curvature (R) is the radius of the sphere which the mirror is a portion of it.

The center of curvature (C) is the center of the sphere which the mirror is a portion of it.

The pole of the mirror (V) is the point in the center of curvature of the mirror .

The focus (I) is the point at which the parallel rays are collected after reflection from the mirror.

The focal length (IV) is the distance between the focus and the pole of the mirror.

Now consider a point source of light placed at point O in Fig, (10b), located on the principal axis to the left of mirror.

Several diverging rays originating at O are shown. After reflecting from the mirror, these rays converge (come together) at the image point I. As a result, a real image is formed.

In which follows, we assume that all rays that diverge from the object make a small angle with the principal axis. All such rays reflect through the image point, as in Fig. (10b). Rays that are far from the principal axis, as in Fig. (11) converge to other points on the principal axis, producing a blurred image. This effect, called ***spherical aberration***, which is presented to some extent for any spherical mirror.

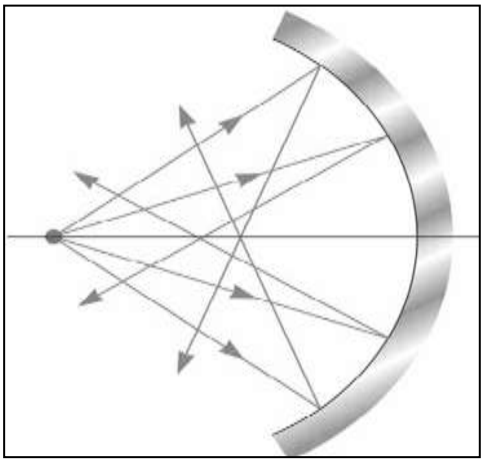


Fig (11) Rays at large angles from the principal axis reflect from the mirror to intersect the principal axis at different points, resulting in a blurred image. This is called ***spherical aberration***.

2.2. Ray diagram for concave mirrors

The position and size of images formed by mirrors can be determined by using the ray diagram. In order to locate the image, two rays are constructed, as shown in Figs (12a) and (12b). These rays start from the top of the object and are drawn as follows:

- Ray 1 is drawn from the top of the object parallel to the principal axis and is reflected back through the focal point, F.
- Ray 2 is drawn from the top of the object through the focal point, thus, it reflected parallel to the principal axis.

The intersection of these rays locates the image.

Images formed by spherical surfaces;

(1) Image formed by concave mirror:

(a) The object is very far (∞) the image formed is real, inverted and very small and formed at the principal focus.

(b) The object is in a distance larger than twice the focal length (far than the center of curvature) Fig.(12b). The formed image is real, inverted and small at distance larger than the focal length and smaller than twice the focal length (between the focus and center of curvature).

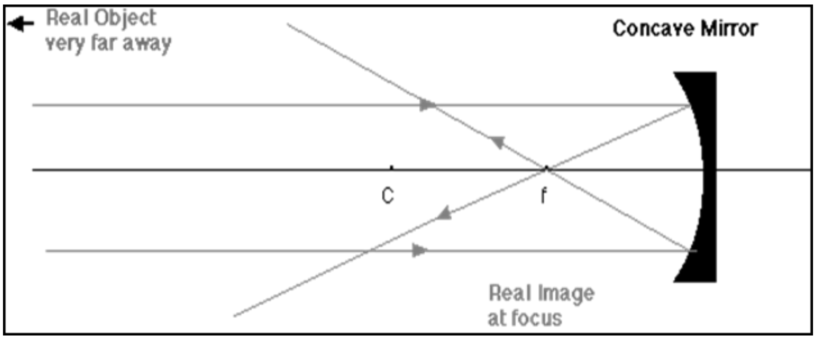
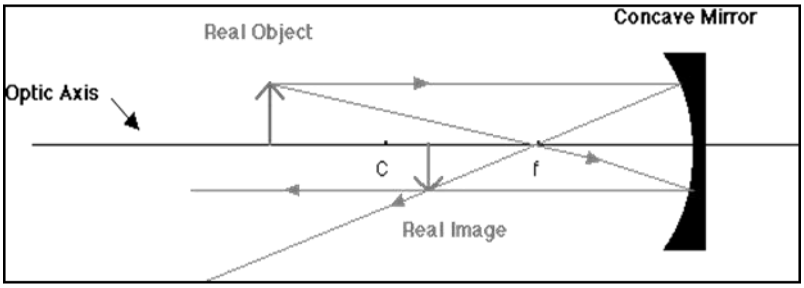


Fig.(12a)



Fig(12b)

(c) The object is in a distance equal to twice the focal length (at the center of curvature) Fig(12c). The formed image is real, inverted, and equal to the object and at distance equal to the object distance at the center of curvature.

(d) The object is at a distance larger than the focal length and shorter than its twice (between the focus and center of curvature). The formed image is real, inverted, and large and at distance larger than twice the focal length (far than center of curvature).

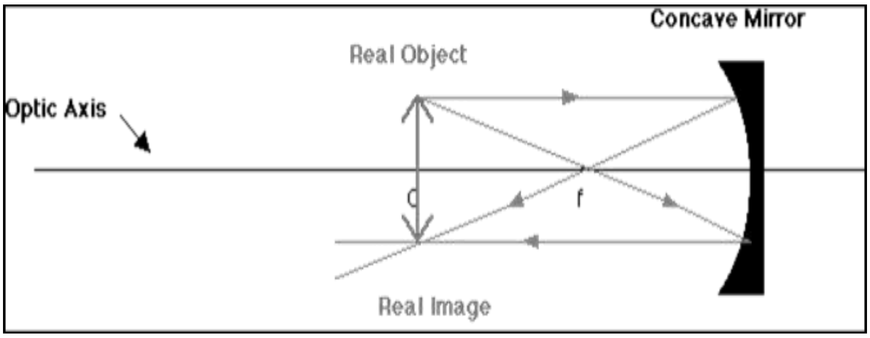


Fig.(12c)

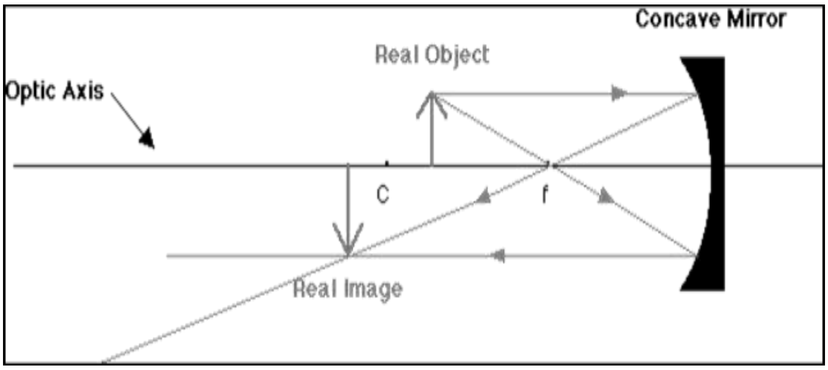


Fig.(12d)

(e) The object is at the focal length ***the principal focus***; The rays reflect parallel and the image formed in ∞

(f) The object is at distance shorter than the focal length. The image formed is virtual, upright, large and behind the mirror.

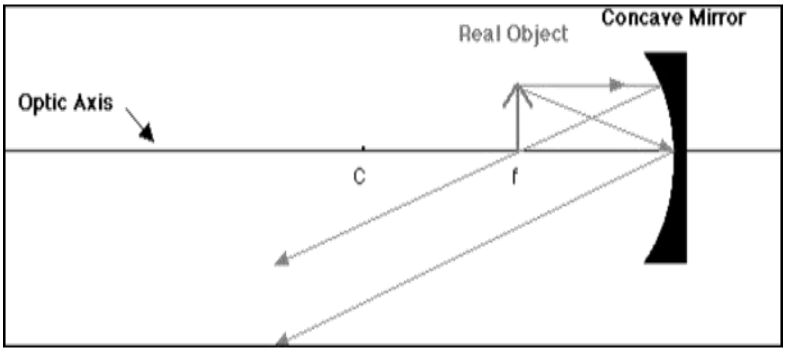


Fig.(12e)

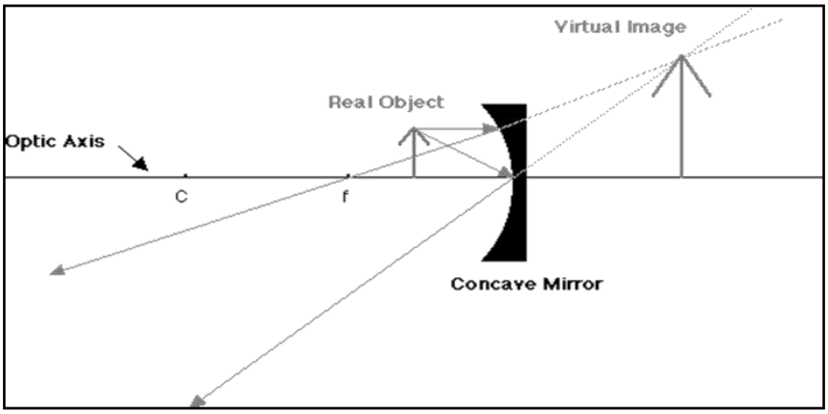


Fig.(12f)

2.3 The sign convention in mirror and also in lenses

The distance of the object or the image from the refracting surface is a vector quantity and these distances must be represented with proper signs. The convention of signs used is in accordance with the conventions of
coordinate geometry as shown in the following table.

Sign Convention for Spherical Mirrors and Thin Lenses

Applies to: Mirror and Thin Lens Equation: $1/d_0+1/d_i=1/f$

Magnification Equation: ***Image height/Object height***$=h_i/h_0= - d_i/d_0$

	Spherical mirrors	Lenses
Focal length (f)	+ for concave mirrors	+ for convex lens
	- for convex mirrors	- for concave lens
Object distance (d_0)	+ if the object is in front of the mirror (real object)	+ if the object is to the left of the lens (real object)
	- if the object is behind of the mirror (virtual object)	- if the object is to the right of the lens (virtual object)
Image distance (d_i)	+ if the image is in front of the mirror(real image)	+ for an image (real) formed to the right of the lens by a real object
	- if the image is behind the mirror(virtual image)	- for an image (virtual) formed to the left of the lens by a real object
Magnification (m)	+ if an image that is upright with respect to the object	+ if an image that is upright with respect to the object
	- if an image that is inverted with respect to the object	- if an image that is inverted with respect to the object

Mirror equation

2.4 Concave Mirror

Consider a point object O on the principal axis of a concave mirror (Fig 13). A ray OA is reflected along AI and another ray OP is reflected back from P along PO. I is the image of the object O.

Here, OP = u, CP = R, PI = F, and AM = x

In the Δ ACO, $\gamma = \alpha + \theta$ or $\theta = \gamma - \alpha$.

In the Δ AIO, $\beta = \alpha + \theta + \theta = \alpha + 2\theta$

$\therefore$ $\beta = \alpha + 2(\gamma - \alpha) = 2\gamma - \alpha$

$$2\gamma = \alpha + \beta \qquad (1)$$

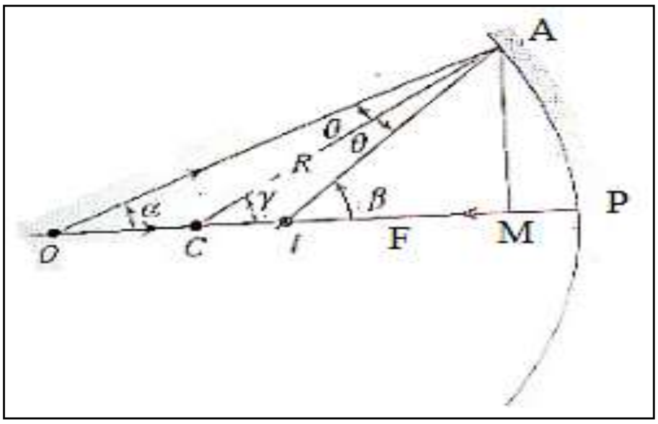


Fig.13

Now, calculating the values of α, β and γ,

(For a small angle, tan α = α and for small curvature take M coincident with P).

$$\alpha = \frac{AM}{OM} \cong \frac{x}{OP} = \frac{x}{u}$$

$$\beta = \frac{AM}{IM} \cong \frac{x}{IP} = \frac{x}{v}$$

$$\gamma = \frac{AM}{CM} \cong \frac{x}{CP} = \frac{x}{R}$$

Substituting these values in equation (1) one gets:-

$$\frac{2x}{R} = \frac{x}{u} + \frac{x}{v}, \text{ Further, } R = 2f$$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \qquad (2)$$

Special case, When R = ∞, the surface becomes plane and it acts as a plane mirror.

$$\frac{1}{v} + \frac{1}{u} = \frac{2}{R} \qquad \text{or} \qquad \frac{1}{v} + \frac{1}{u} = \frac{2}{\infty},$$

or $\frac{1}{v} + \frac{1}{u} = 0$ or $\frac{u+v}{uv} = 0$

$\therefore$ $u + v = 0$ or $v = -u$

Therefore, the image is virtual and formed behind the mirror at the same distance as the object is in front of the mirror.

Moreover, the size of the image is equal to the size of the object.

Example:-

Assume that a certain concave spherical mirror has a focal length 12 cm, find the location of the image for object at a distance of 27 cm and 12 cm from the mirror and describe the image in each case.

The solution

Using the mirror equation

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

$$\frac{1}{27}+\frac{1}{v}=\frac{1}{12}$$

v = 21.6 cm

the image is inverted and real.

When the object distance is 12 cm, the object is located at the focal point:

$\frac{1}{12}+\frac{1}{v}=\frac{1}{12}$ v = ∞

2.5 Convex mirror

Figure (14) shows the formation of an image by a convex mirror, that is, the rays of light are reflected from the outer, convex surface. This is sometimes called a diverging mirror because the rays from any point on a real object diverge after reflection as though they were coming from some point behind the mirror. The image in Fig (14) is virtual because the reflected rays only appear to originate at the image point. Furthermore, the image is always upright and smaller than the object and it is formed between f and the pole, as shown in the figure.

We do not derive any equations for convex spherical mirrors because we can use Eq.(2) for either concave or convex mirrors.

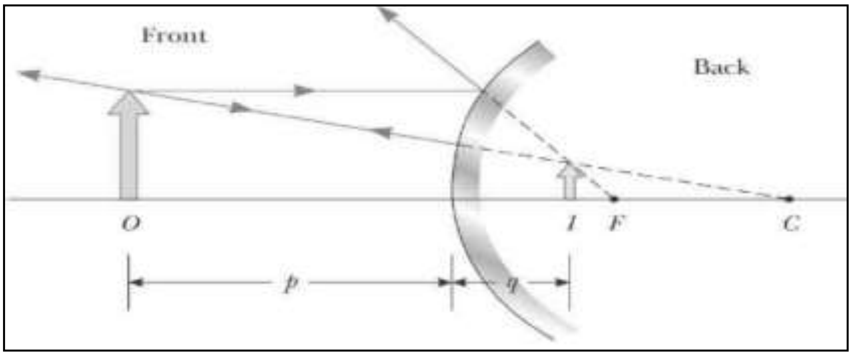


Fig (14) *Formation of an image by a convex mirror.*

2-6-Image formed by refraction

In this section, we describe how images are formed by the refraction of rays at a spherical surface of a transparent material.

Consider two transparent media with indices of refraction n_1 and n_2, where the boundary between the two media is a spherical surface of radius R, Fig (15). We assume that the object at point O is in the medium whose index of refraction is n_1. Furthermore, of all rays leaving O, let us consider only those that make a small angle with the axis and with each other. As we shall see, all such rays originating from the object point are refracted at the spherical surface and focus at a single point I, the image point.

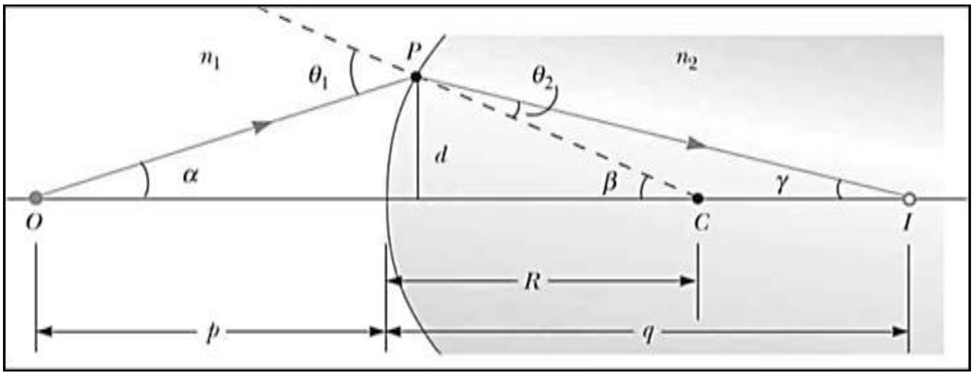


Fig. (15)

Let us proceed by considering the geometric construction in Fig (15), which shows a single ray leaving point O and focusing at point I, Snell's law applied to this refracted ray gives

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Because the angles θ_1 and θ_2 are assumed small, we can use the small angle approximation $\sin\theta \approx \theta$ (angles in radians).

Therefore, Snell's law becomes

$$n_1 \theta_1 = n_2 \theta_2$$

Now we use the fact that an exterior angle of any triangle equals the sum of the two opposite interior angles. Applying this to the triangles OPC and PIC in Fig. (15) gives

$$\theta_1 = \alpha + \beta$$

$$\beta = \theta_2 + \gamma$$

If we combine the last three expressions, and eliminate θ_1 and θ_2, we find

$$n_1\alpha + n_2\gamma = (n_2 - n_1)\,\beta \qquad (3)$$

Again, in the small angle approximation, tan θ≈ θ, and so we can write the approximate relationships

$$\alpha \cong \frac{d}{p} \qquad\qquad \beta \cong \frac{d}{R} \quad ,\gamma = \frac{d}{q}$$

If we substitute these expressions into Eq. (3) and divide by d one gets:-

$$\frac{n_2}{q} - \frac{n_1}{p} = \frac{n_2 - n_1}{R} \tag{4}$$

For a fixed object distance p, the image distance q ,from the transparent medium, is independent of the angle that the ray makes with the axis. This result tells us that all rays focus at the same point I.

2-7.Plane refracting surface

If the refracting surface is plane, then R approaches infinity and Eq.(4) reduces to

$$\frac{n_1}{p} = \frac{-n_2}{q}$$

$$q = \frac{-n_2}{n_1}\, p \tag{5}$$

The ratio n_2/n_1 represents the index of refraction of medium 2 relative to the medium 1.

From Eq.(5) the sign of q is opposite to that of p.

This means that, the image formed by a plane refracting surface is on the same side of the surface as the object.

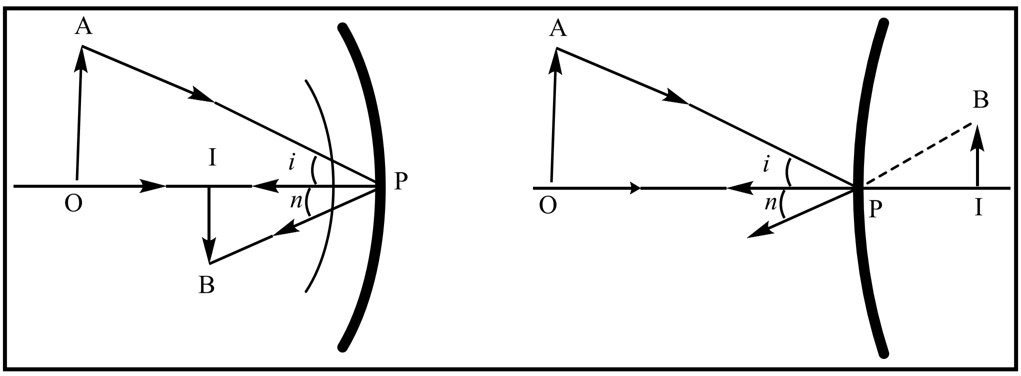
2.8 Magnification of the image

(1) Lateral or transverse magnification.

It is defined as the ratio of the height of the image to the height of the object.

$$m = \frac{height\ of\ the\ image}{height\ of\ the\ object}$$

In Fig. 16, IB is the image of the object OA.



CONCAVE CONVEX

Fig.(16)

$$\angle APO = \angle BPI$$

$$i = r$$

$$\tan i = \frac{OA}{OP}, \qquad \tan r = \frac{IB}{IP}$$

$$\frac{OA}{OP} = \frac{IB}{IP} \qquad Or \qquad \frac{IB}{OA} = \frac{IP}{OP}$$

Here, $IP = v$, $OP = u$

$$m = \frac{IB}{OA} = \frac{IP}{OP} = \frac{v}{u} = -$$

Example (1):

An object 4 cm high is placed at 20 cm from a convex mirror having a focal length of 8 cm. Find the position of the image, the magnification of the mirror and the height of the image.

Solution

Because of the mirror is convex, the focal length is negative. By using the second law

$$\frac{1}{20} + \frac{1}{q} = \frac{-1}{8} \qquad q = -5.71\ cm$$

The negative sign indicates that the image is virtual and formed behind the mirror.

The magnification of the mirror is given by

$$M = \frac{-q}{p} = 0.286 \qquad i.e. \quad the\ image\ is\ erect.$$

The height of the image is given by

$$M = \frac{h'}{h} = \frac{h'}{4} = 0.286$$

h' = 1.144 cm i.e. Small image.

Example(2):

A small fish is swimming at a depth d below the surface of a pond. What is the apparent depth of the fish as viewed from directly overhead ?

Solution

Because of the refracting surface is plane,

$R = \infty$

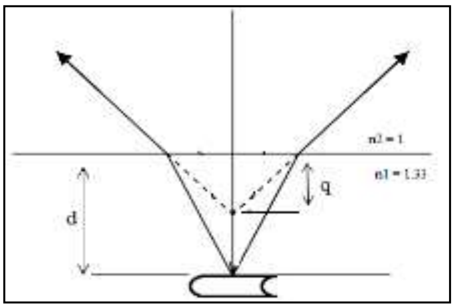


Fig.(17)

So we can use Eq.(5)

where $n_1 = 1.33$, $n_2 = 1$, $p = d$

$$\therefore \qquad q = \frac{-n2}{n1} p = \frac{-1}{1.33} d = -0.75\ d$$

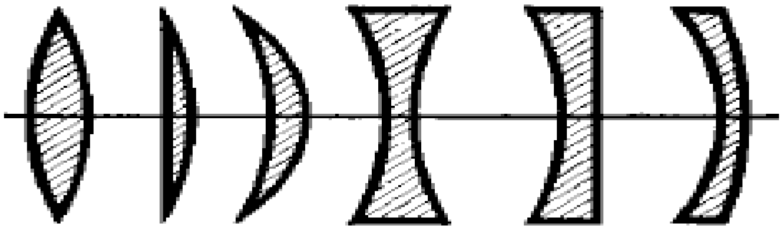
Since q is negative, the image is virtual, as in the figure 17.

Chapter 3 **Lenses**

3. Refraction through Lenses

A lens is a portion of a transparent refracting medium bounded by two spherical surfaces or by one spherical surface and a plane surface. Lenses are usually made of glass. The line joining the centers of curvature of the two spherical surfaces is known as the principal axis. If one of the surfaces is plane, the axis is a straight line normal to the surface drawn through the center of curvature of the other surface. A plane through the axis is called the principal section of the lens. Optical center of a lens is a point on the principal axis, when the lens is thin the incident ray passing through the optical center is considered to go in a straight without any deviation.

The following types of lenses are in common use (Fig. 3.1).The first three are convergent lenses and the last three are divergent lenses.



1 2 3 4 5 6

1. Double convex or biconvex lens 2. Plano-convex lens
3. Concavo-convex lens 4. Double concave or bi-concave lens 5. Plano-concave lens 6. Convexo-concave lens

3. 1.Ray diagram for thin lenses

Ray diagram are very convenient for locating the image formed by a thin lens or a system of lenses**.** Figure (3-2) illustrates this method for three single-lens situations. To locate the image of a converging lens, Figs. (3-2a) and (3-2b), the following two rays are drawn from the top of the object:

- Ray 1 is drawn parallel to the optical axis. After being refracted by the lens, this ray passes through one of the focal points.
- Ray 2 drawn through the center of the lens. This ray continues in a straight line.

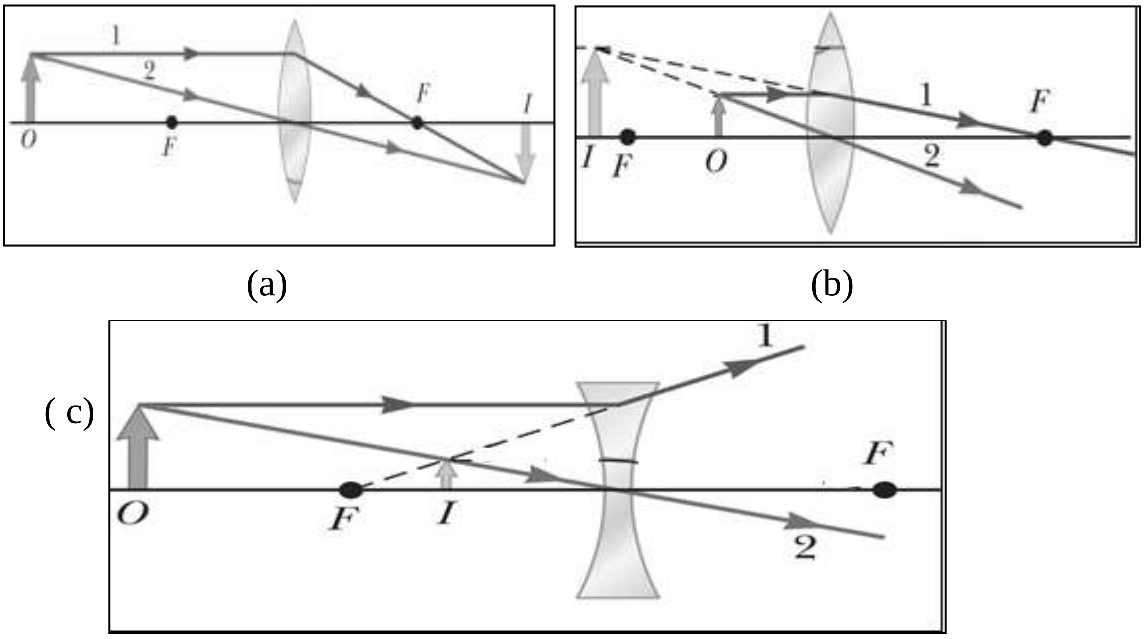


Fig. (3-2): Ray diagrams for locating the image formed by a thin lens

A similar construction is used to locate the image of diverging lens, as shown in Fig. (3-2c).

3.2. Refraction Through a thin lens

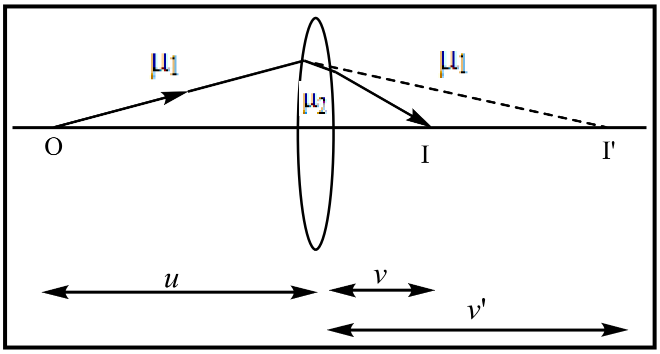


Fig. 3.3

Consider a thin lens enclosing a medium of refractive index μ_2 and separating it from a medium of refractive index μ_1 on its two sides. Let R_1 and R_2 be the radii of curvature of the two co-axial spherical surface and O is a point object situated on the principal axis.

An image I' is formed by refraction at the first surface and let its distance from the pole of the first surface be equal to v' .

Then, $$\frac{\mu_2}{v\prime} + \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} \qquad (3\text{-}1)$$

Because the rays are refracted from the second surface of the lens, the virtual image I' may be regarded as the object for the second surface and the final image is formed at I which lies in the medium of refractive index μ_1. If the distance of the final image from the pole of the second surface is equal to v, (also, $u = v'$) " at very small values of t".

Then, $$\frac{\mu_1}{v} - \frac{\mu_2}{v'} = \frac{\mu_1 - \mu_2}{R_2} \qquad \textbf{(3-2)}$$

In this case the rays are passing from the medium of refractive index μ_2 (*i.e.*, lens) to the medium of refractive index μ_1.

Adding (3-1) to (3-2) one gets:

$$\frac{\mu_1}{v} + \frac{\mu_1}{u} = (\mu_2 - \mu_1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Dividing by μ_1

$$\frac{1}{v} + \frac{1}{u} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

If the lens is placed in air $\mu_1 = 1$ and $\frac{\mu 2}{\mu 1} = \mu$, where μ, is the refractive index of the material of the lens.

Then,

$$\frac{1}{v} + \frac{1}{u} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Note: (1) It is to be remembered that these equations will hold true only for paraxial rays and for a thin lens where the thickness of the lens can be taken negligibly small as compared to u, v, R_1 and R_2.

(2) While solving numerical problems, proper signs for μ, v, R_1 and R_2 are to be used.

3.3 Principal Foci

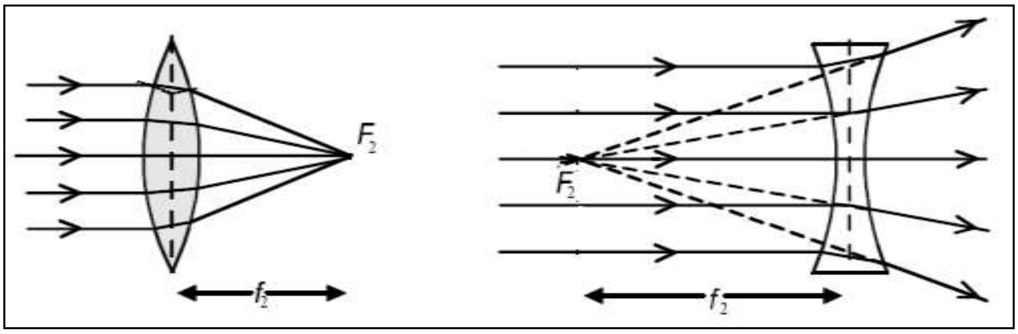
In the formula,

$$\frac{1}{v} + \frac{1}{u} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

if $u = \infty$, $$\frac{1}{u} = \frac{1}{\infty} = 0$$

$$\frac{1}{v} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \qquad (3\text{-}3)$$

This value of v is known as second Principal focal length and the position of the image corresponding to the axial point object lying at infinity is termed second Principal focus of the lens. (Fig 3.4)



(i) (ii)

Fig. 3.4

In a concave lens, f_2 is –ve and in a convex lens it is +ve, according to the sign convention.

In the case of a concave lens F_2 is virtual and in case of a convex lens F_2 is real.

$$\therefore \frac{1}{f_2} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \qquad (3\text{-}4)$$

Considering the case when the image is formed at infinity

$$v = \infty$$

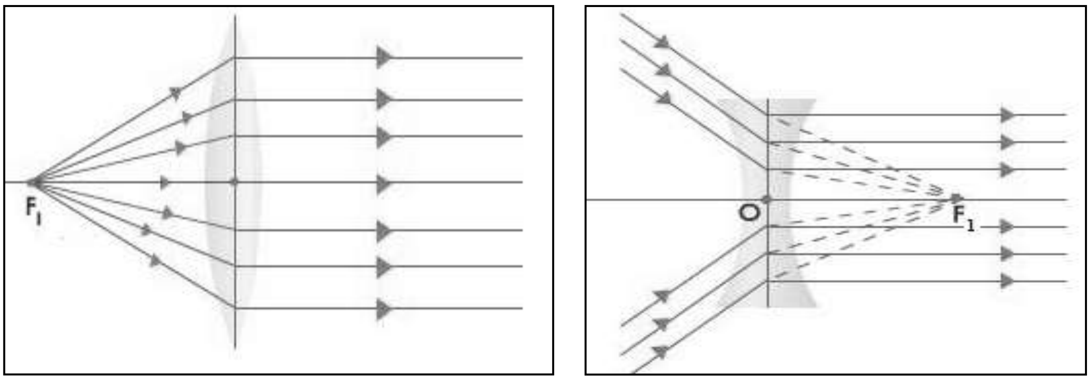
$$\frac{1}{v} = \frac{1}{\infty} = 0$$

$\therefore$ From the equation, $\frac{1}{v} + \frac{1}{u} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$$+\frac{1}{u} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

This value of u is known as First principal focal length of the lens and is denoted by f_1 (Fig. 3.4)

$$\frac{1}{f_1} = +(\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \qquad (3\text{-}5)$$



(i) (ii)

Fig. 3.5

If the focal length of the lens is *f* then from equations (3-3,4,5).

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

This is the lens equation and is applicable to a concave or a convex lens.

3.4 Power of a lens

The power of a lens is the measure of its ability to produce convergence of a parallel beam of light. A convex lens of large focal length produces a small converging effect on the rays of light and a convex lens of small focal length produces a large converging effect. Due to this reason, the power of a convex lens is taken as + ve and a convex lens of large focal length has low power and a convex lens of small focal length has high power. On the other hand a concave lens produces divergence. Therefore, its power is taken as negative.

The unit in which power of a lens is measured is called diopter (D).

A convex lens of focal length one meter has a power = +1 diopter, and

a convex lens of focal length 2 meters has a power = + $\frac{1}{2}$ diopter.

Mathematically,

$$\text{Power} = \frac{1}{\text{focal length in meters}}$$

If two lenses of focal lengths f_1 and f_2 are in contact then

$$\frac{1}{F} = \frac{1}{f1} + \frac{1}{f2}$$

$$P = P_1 + P_2$$

Where P_1 and P_2 are the powers of the two lenses and P is the equivalent power.

Example:-

A diverging lens has a focal length of -20 cm. An object 2 cm in height is placed 30 cm in front of the lens. Locate the position of the image and find the magnification and the height of the image.

The answer

p = 30 cm, f = -20 cm, Using a lens equation one gets:

1- $\frac{1}{30} + \frac{1}{q} = \frac{-1}{20}$ q = -12 cm

$\therefore$ The image is virtual

2- The magnification M = $\frac{-q}{p} = \frac{-(-12)}{30} = 0.4$

3- 0.4 = $\frac{h'}{h} = \frac{h'}{2}$ $\therefore$ h' = 0.8 cm

Example:-

A converging glass lens (n = 1.52) has a focal lens of 40 cm in air. Find its focal length when it is immersed in water, which has an index of refraction of 1.33.

The answer

We can use the lens maker's formula in air and water where R_1,R_2 are constants in the two media

$$\frac{1}{fa} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Where n = 1.52

In water we set

$$\frac{1}{f_w} = (n'-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Where n' is the index of refraction of glass relative to water. *i.e.* n' = 1.52/1.33 = 1.14.

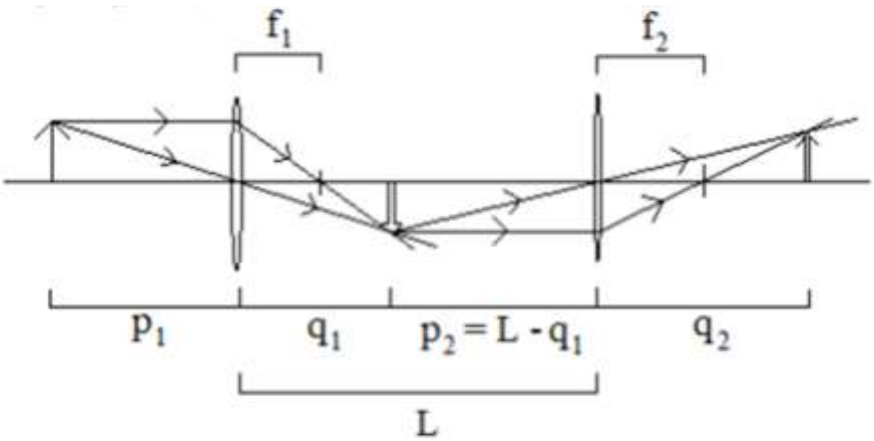
Dividing the two equations gives

$$\frac{f_w}{f_a} = \frac{n-1}{n'-1} = \frac{1.52-1}{1.14-1} = 3.71$$

$\therefore$ f_w =40 x 3.71 = 148.4 cm

Multiple-lens systems

If we have one lens behind another, we can simply treat the image formed by the first lens as an object for the second lens. For example, suppose we have two convergent lenses, with focal lengths f1 and f2, separated by a distance L. The object is located at a distance p1 in front of the first lens. We want to locate the image and find the magnification. This is a typical ray diagram, to show our distances and sign conventions:



For this particular diagram, all the quantities are positive. If one of the lenses was divergent, its focal length would be negative; if one or both of the images was on the same side as its corresponding object, the value of q for that image would be negative. Let us calculate q2 for given p1, L, f1 and f2. The equations for the two lenses are

$$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f_1} \qquad \frac{1}{L - q_1} + \frac{1}{q_2} = \frac{1}{f_2}$$

We can simply use the first equation to calculate q1 and plug into the second. For example, suppose that the object is 50cm away, the lenses are 120cm apart, the first lens has a focal length of 30cm and the second lens has a focal length of 100cm. Then, our equation will gives

$$q_1 = \frac{1}{1/f_1 - 1/p_1} = \frac{1}{1/30cm - 1/50cm} = 75cm$$
$$q_2 = \frac{1}{1/f_2 - 1/(L - q_1)} = \frac{1}{1/100cm - 1/(120 - 75)cm} = -82cm$$

This means that the final image will be virtual, and will be 82 centimeters in front of the second lens (in the direction of the object). This is unlike the ray diagram above, where the final image is real and behind the second lens. We can calculate the magnification of this two-lens contraption. Since the magnification of the first lens is M1 = -h1 / h, where h is the height of the object and h1 is the height of the first image, and M2 = -h2 / h1, since the first image is now the object and h2 is the height of the second image, M1M2 = h2 / h = M is the total magnification from the object to the final image (the sign cancels because a sequence of two inverted real images give an upright real image, so the magnification for a real image is positive.) Since M1 = -q1 / p1 and M2 = -q2 / p2 = q2 / (L - q1), the magnification is

$$M = M_1 M_2 = \frac{q_1 q_2}{p_1(L - q_1)}$$

For our example, this gives

$$M = \frac{75cm(-82cm)}{50cm(120cm - 75cm)} = -2.7$$

The final image is therefore inverted, and 2.7 times larger than the original.

Chapter 4

Optical instruments

4- THE HUMAN EYE

As human beings our sense of vision is one of our most prized possessions. For those of us that enjoy normal vision this marvelous gift of nature is the most useful of all recording instruments, yet in a few instances it should not be relied upon to tell the truth. As an illustration of how unreliable vision can be, mention should be made of a whole group of phenomena known as *optical illusions.*

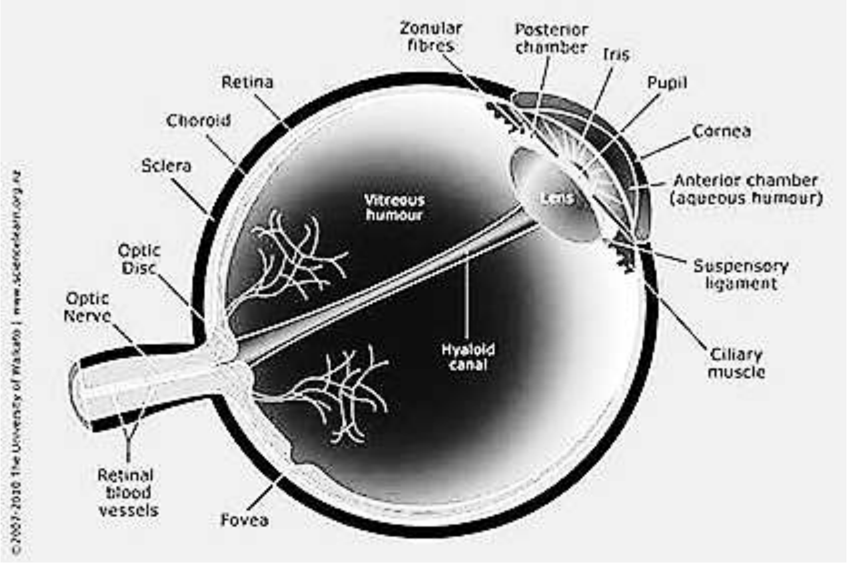


Fig.(4-1)

In spite of these imperfections in our vision, most of us are able to enjoy the beauties of color, form, and motion, all made possible by illumination with visible white light. The eye is like a fine camera, with a shutter, iris, and lens system on one side and a sensitive film called the retina on the other (Fig.4-1). The function of the lens system is to focus on the retina an image of objects to be seen. Like a camera, the iris diaphragm opens wider for faint light and closes down for bright sunlight. The pigment determining the color of the eye is in the iris.

The retina of the eye contains hundreds of cones and rods, whose function it is to receive light pulses and change them into electric currents. How these electric currents are produced by the cones and rods, and how they are translated by the brain into what we call vision is only partially understood by scientists working in the field.

It is known that the cons respond to bright light only and are responsible for our distinction of colors. Rods, on the other hand, are sensitive to faint light, to motion and to slight variations in intensity.

At the very center of the retina is a slightly yellowish indentation called the *fovea*. This small area contains a large number of cons and no rods. It is on this spot in each eye that one focuses the image of objects one wishes to see in minute detail. Note, for example, that when one looks at any single word on this page, words close by are quite blurred.

We divide the subject of light perception into two parts: (1) the optical components leading to the formation of sharp images on the retina and (2) the property of the nerve canal and brain to interpret the electrical impulses produced. When light from any object enters the eye, the lens system forms *a real but inverted image on the retina*. While all the images are inverted, it is of course a most amazing fact that we interpret them by the brain as being erect.

4-1. MAGNIFIERS

The magnifier is a positive lens whose function it is to increase the size of the retinal image over and above that which is formed with the unaided eye. The apparent size of any object as seen with the unaided eye depends on the angle subtended by the object (see Fig.4-2). As the object is brought closer to the eye, from A to B to C in the diagram, accommodation permits the eye to change its power and to form a larger and larger retinal image. There is a limit to how close an object may come to the eye if the latter is still to have sufficient accommodation to produce a sharp image.
Although the nearest point varies widely with various individuals, 25.0 cm is taken to be the standard *near point*, sometimes called *the distance of most distinct vision*. At this distance, indicated in Fig. 4-3(a), the angle subtended by object or image will be called θ.

If a positive lens is now placed before the eye in the same position, as in diagram (4-3b), the object y can be brought much closer to the eye and an image subtending a larger angle θ' will be formed on the retina. What the positive lens has done? is to form a virtual image y' of the object y and the eye is able to focus upon this virtual image.

Any lens used in this manner is called a *magnifier* **or** *simple microscope*. If the object y is located at F, the focal point of the magnifier, the virtual image y' will be located at infinity and the eye will accommodated for distant vision as illustrated in fig. 4-3 (c). If the object is properly located a short distance inside of F as in diagram (b), the virtual image may be formed at the distance of most distinct vision and a slightly greater magnification obtained, as will now be shown.

The angular magnification M is defined as the ratio of the angle θ' subtended by the image to the angle θ subtended by the object.

$$M = \frac{\theta'}{\theta} \qquad (4\text{-}1)$$

From diagram (b) the object distance s is obtained by the regular thin-lens formula as

$$\frac{1}{s} + \frac{1}{-25} = \frac{1}{f} \qquad or \qquad \frac{1}{s} = \frac{25+f}{25f}$$

From the right triangles, the angle θ and θ' are given by

$$\tan\theta = \frac{y}{25} \qquad \text{and} \qquad \tan\theta' = \frac{y}{s} = y\,\frac{25+f}{25f}$$

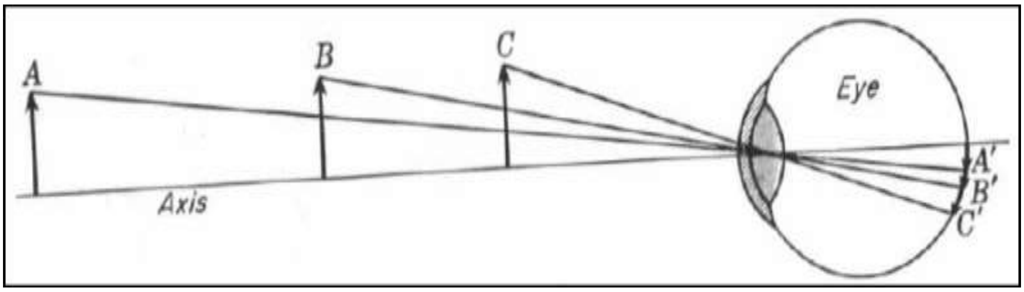


Figure 4-2 The angle subtended by the object determines the size of the retinal image.

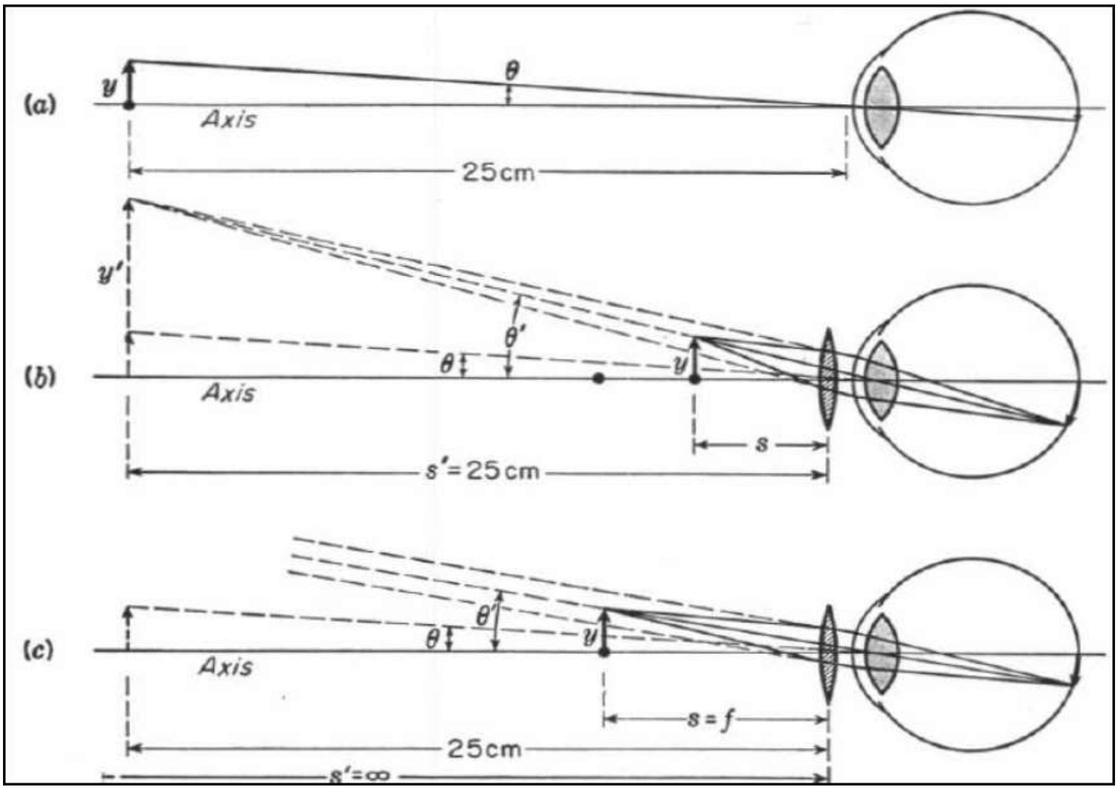


Figure 4-3. The angle subtended by (a) an object at the near point to the naked eye, (b) the virtual image of an object inside the focal point, (c) the virtual image of an object at the focal point

.For small angles the tangents can be replaced by the angles

themselves to give the following approximate relations,

$$\theta = \frac{y}{25} \qquad and \qquad \theta' = y\,\frac{25+f}{25f}$$

Giving for the magnification, from eq. (4-1),

$$M = \frac{\theta'}{\theta} = \frac{25}{f} + 1 \tag{4-2}$$

In diagram (4-3b), the object distance s is equal to the focal length, and the small angles θ and θ' are given by

$$\theta = \frac{y}{25} \qquad and \qquad \theta' = \frac{y}{f}$$

Giving for the magnification

$$M = \frac{\theta\prime}{\theta} = \frac{25}{f} \tag{4-3}$$

The angular magnification is therefore larger if the image is formed at the distance of most distinct vision. For example, let the focal length of a magnifier be 1 in. or 2.5 cm. for these two extreme cases, Eqs. (4-2) and (4-3) give

$$M = \frac{25}{2.5} + 1 = 11 \quad and \quad M = \frac{25}{2.5} = 10$$

Because magnifiers usually have short focal lengths and therefore give approximately the same magnifying power for object distances between 25.0 cm and infinity, the simpler expression 25/f is commonly used in labeling the power of magnifiers. Hence a magnifier with a focal length of 2.5 cm will be marked 10 and another with a focal length of 5.0 cm will be marked 5, etc.

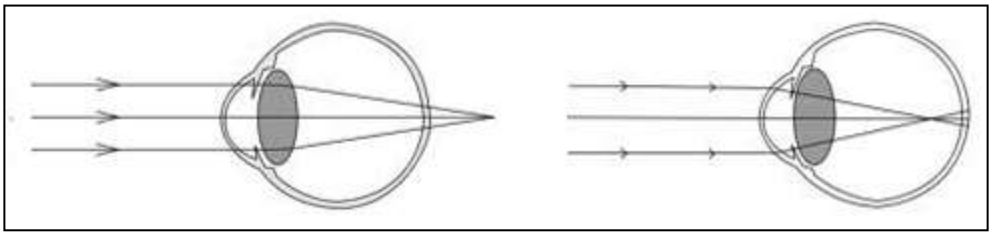
4.2 Spectacle lenses

The ability of the human eye to focus on nearby and distant objects, attributed to the crystalline lens, is most prominent in children. Changing the shape of the lens is accomplished by a rather complicated system of ligaments and muscles. Due to tension in the lens capsule, the crystalline lens, if completely free, would tend to become spherical in shape. Surrounding the edge of the lens is an annular ring called the *sciliary muscle*, which on contracting squeezes the lens, causing it to bulge. In effect this reduces the focal length of the lens, bringing nearby objects to a sharp focus on the retina.

If the *sciliary muscle* relaxes, *the suspensory ligaments* pull outward on the lens periphery, causing it to flatten. This increases the focal length, bringing distant objects to focus on the retina. This ability is part of the process of vision called *accommodation*.

As a person grows older, the crystalline lens becomes harder and harder, and the muscles that control its shape grow weaker and weaker, thus making accommodation more and more difficult. This condition is referred to as *presbyopia*. When the length of the eyeball is such that incident parallel light rays converge to a point behind the retina, the person is far-sighted and is said to have *hypermetropia* [see fig. 4-4(a)]. When parallel rays come to a focus in front of the retina, as in diagram (4-4b), the person is near sighted and is said to have *myopia*.

In order to correct these defects in one's vision, a converging lens of the appropriate focal length is placed in front of the *hypermetropic eye*, and a diverging lens is placed in front of the *myopic eye*. A positive lens adds some convergence to the rays just before they reach the cornea, thereby enabling the person to see distant objects in sharp focus .A diverging lens in front of the myopic eye can bring distant objects to a sharp focus.



(a) Hypermetropia, farsighted (b) Myopia, nearsighted

Figure 4-4. Typical eye defects, largely present in the adult population

4-3.Microscopes

In the seventeenth century convex lenses were used as magnifying glasses. Afterwards two or more lenses were used to form powerful microscopes.

In 1648, Hooke made use of microscopes in the study of existence of cells in animal and vegetable tissues.

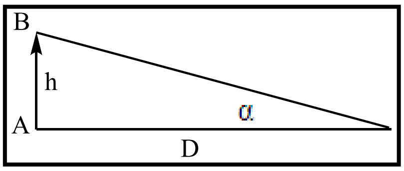
The angular magnification of a microscope in normal use is

$$M = \frac{Angle\ subtended\ at\ the\ eye\ by\ the\ image}{Angle\ subtended\ at\ the\ unaided\ eye\ by\ the\ object\ placed\ at\ the\ near\ point}$$

$$M = \frac{\beta}{\alpha}$$

Simple Microscope (Magnifying Glass)

Suppose an object of length h is placed at the near point A and viewed by the eye [fig. 4-4]. Then the visual angle is α, where



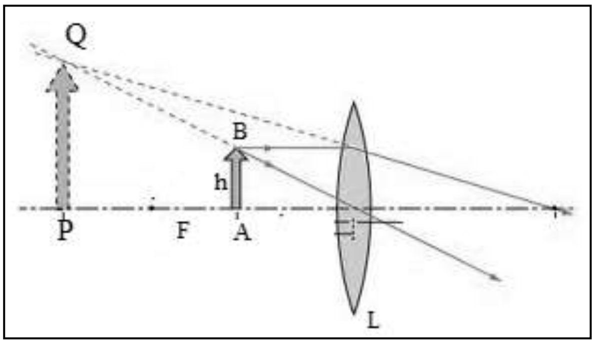


Fig.4-4

$$\tan \alpha = \frac{h}{D} \qquad (4\text{-}4)$$

Now, suppose that a convex lens L of focal length f is used as a magnifying glass and the object AB is placed between O and F, such that the magnified erect image PQ is formed. If the observer's eye is close to the lens then the distance OP is equal to the least distance of distinct vision. Here the visual angle β is given by

$$\tan \beta = \frac{PQ}{D} \qquad (4\text{-}5)$$

Dividing 4-5 by 4-4,

$$\frac{\tan \beta}{\tan \alpha} = \left[\frac{PQ}{h}\right]$$

$$\frac{\beta}{\alpha} = \frac{PQ}{h} \qquad \text{(For small angles)}$$

But, $$\frac{PQ}{h} = \frac{v}{u} = \frac{D}{u}$$

IF M is the magnifying power

$$M = \frac{\beta}{\alpha} = \frac{D}{u}$$

$\therefore \quad M = \frac{D}{u}$

From the general low $\quad \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

Here, v = - D, so

$$\therefore \quad -\frac{1}{D} + \frac{1}{u} = \frac{1}{f}$$

$$-1 + \frac{D}{u} = \frac{D}{f}$$

$$\frac{D}{u} = 1 + \frac{D}{f}$$

$$M = 1 + \frac{D}{f}$$

Example:-

A convex lens of focal length 10 cm is used as a magnifying glass. Find the magnifying power when (i) the image is formed at infinity (ii) the image is formed at the least distance of distinct vision (25 cm from the lens).

(i) When the image is formed at infinity

$$M = \frac{D}{u} = \frac{25}{10} = 2.5$$

(ii) When the image is formed at the near point

$$M = 1 + \frac{D}{f} = 1 + \frac{25}{10}$$

$$M = 3.5$$

4-4.Compound Microscope

The magnifying power of a simple microscope can be increased by decreasing the focal length of the lens as $M = 1 + \frac{D}{f}$

But due to constructional difficulties, the focal length of a lens cannot be decreased beyond a certain limit. Moreover the lens of small focal length has a small diameter because the curvature of the surface is large and the field of view is small.

Therefore, to increase the magnifying power, two separate lenses are used. The lens near the object is called the objective and the other which is nearer the eye is known as the eyepiece. The objective and the eyepiece are both convex lenses. The objective is of small diameter and small focal length (high power) whereas the eyepiece is of large focal length than the objective.

The object AB is placed at a distance slightly greater than the focal length of the objective (fig. 4-5). An inverted image A'B' is formed at A'. The eyepiece is adjusted so that the distance of A' from it is less than its focal length. As the eyepiece acts as a simple magnifying glass, the final image PQ is formed at P which is magnified and virtual.

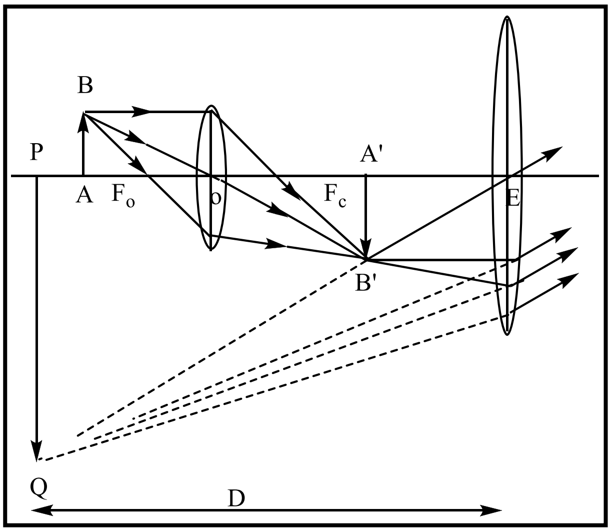


Fig.4-5

Magnifying Power. When the microscope is in normal use the final image PQ is formed at the near point at a distance D from the eye.

$$M = \frac{Angle\ subtended\ at\ the\ eye\ by\ the\ final\ image\ at\ D}{Angle\ subtended\ at\ the\ eye\ by\ the\ object\ at\ the\ same\ distance\ D}$$

$$\therefore \qquad M = \frac{\frac{PQ}{D}}{\frac{AB}{D}} = \frac{PQ}{AB}$$

$$M = \frac{PQ}{A'B'} \ x\ \frac{A'B'}{AB}$$

As $\frac{PQ}{A'B'} = M_e = Magnification\ of\ the\ eypiece$

and $\frac{A'B'}{AB} = M_o = Magnification\ of\ the\ objective$

$$\therefore \qquad M = M_e \text{ x } M_o$$

As the eyepiece acts as a simple magnifying glass, its magnifying power

$$= 1 + \frac{D}{f}$$

$$M_e = 1 + \frac{D}{f_e}$$

Where f_e is the focal length of the eyepiece.

If the distance of the image A'B' from the objective = v and the distance of the object AB from the objective = u

then, $M_o = \frac{v}{u}$

Also $\frac{1}{v} + \frac{1}{u} = \frac{1}{f_0}$

$\therefore \quad \frac{1}{v} + \frac{1}{u} = \frac{1}{f_o}, \ \frac{v}{u} = \left(\frac{v}{f_o} - 1\right)$

$\therefore \quad M_o = -\frac{v}{u} = -\left(\frac{v}{f_o} - 1\right)$

$\therefore \quad M = M_e \text{ x } M_o = -\frac{v}{u}\left(1 + \frac{D}{f_e}\right)$

or $M = -\left(\frac{v}{f_o} - 1\right)\left(1 + \frac{D}{f_e}\right)$

In the case of most of the microscopes the distance between the objective and the eyepiece is fixed. The lenses are fixed at the two ends of a tube. Then the microscope is focused by moving it bodily either towards the object or away from the object.

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