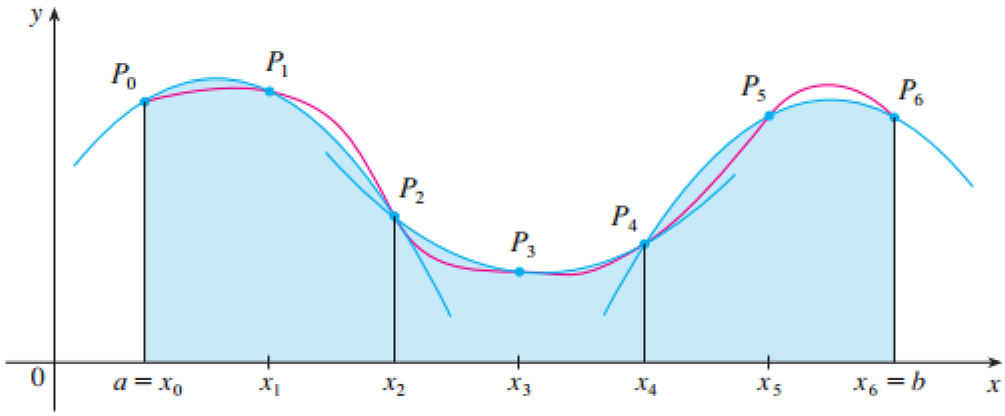
Simpson ⅜ rule

The Simpson's ⅜ integration method is a pivotal technique in numerical analysis, particularly for achieving more accurate approximations of definite integrals. It extends from the Simpson's 1/3 rule, employing cubic interpolations to enhance precision when estimating the integral of a function across a specified interval. This method addresses limitations of earlier techniques by dividing the integration interval into subintervals and utilizing cubic polynomials to approximate function behavior within these segments.



The core of Simpson's 3/8 rule lies in its strategic division of the interval into sets of three subintervals. This division enables the construction of quadratic approximations for each group of three intervals, integrating these approximations with carefully determined weights derived from function evaluations at both endpoints and midpoints. To apply Simpson's ⅜ rule effectively, a systematic approach must be followed. The interval of interest, denoted as [a, b], a being the lower bound, and b being the upper bound; this is divided into n subintervals, where n must be a multiple of 3 for accurate implementation.

The width of each subinterval *h* is calculated as $\frac{b-a}{n}$, ensuring uniformity across all divisions.

The pivotal formula, in general, for multiple intervals for Simpson's ⅜ rule is:

$$\boldsymbol{I = \int_a^b f(x)dx}$$

$$\boldsymbol{I_{Simp\frac{3}{8}} = \frac{3h}{8}\left(f(a) + 3\sum_{i=1,4,7}^{n-1} f(x_i) + 3\sum_{i=2,5,8}^{n-2} f(x_i) + 2\sum_{i=3,6,9,}^{n-3} f(x_i) + f(b)\right)}$$

And to calculate the error of this calculation the rule is:

$$\frac{(b-a)^5}{6480} f^{(4)}(\xi)$$