Recursion

CSCI 1030U - Intro to Computer Science
@IntroCS

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Outline

- Recursion
  - Recursive calls and backtracking
  - Recursive function calls and the calling stack
  - Tail recursion and optimization

Recursion

Recursion - A Motivating Example

- Say you are in line and want to know what position you have in that line
    - You could count all of the people yourself, but you are lazy and don't want to do that
    - Any ideas?

Recursion - A Motivating Example

- You could ask the person in front of you the same question
  - "What position in line are you?"

Recursion - A Motivating Example

- You could ask the person in front of you the same question
  - "What position in line are you?"
  - Assuming that they are also too lazy to count, they may ask the same question of the person in front of them

Recursion - A Motivating Example

- … and so on

Recursion - A Motivating Example

- The person at the front of the line knows that they are first, so they pass that information back to the person behind them

Recursion - A Motivating Example

- The second person adds one (for themselves) and passes back the count

Recursion - A Motivating Example

- … and so on
- Finally, you know that there are 6 people in front of you, so you must be seventh in line

Recursion

- A recursive function is one that is defined in terms of itself
  - i.e. The function calls itself, directly or indirectly
- Example:

Python:

```python
def forever():
    print("hello")
    forever()
```

C++:

```cpp
void forever() {
    cout << "hello";
    forever();
}
```

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Recursion

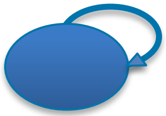
- The example below is called *direct recursion*, since the function `forever` calls itself directly

Python:

```
def forever():
    print("hello")
    forever()
```

C++:

```
void forever() {
    cout << "hello" << endl;
    forever();
}
```

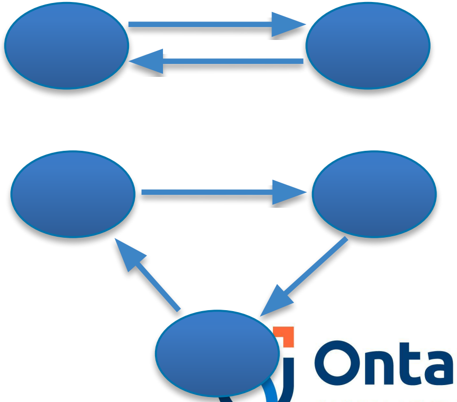


Recursion

- The example below is called *indirect recursion*, since the function `forever1` calls itself indirectly
  - This example has two functions calling each other in a cycle, but any number of functions can be involved

```
def forever1():
    print("hello")
    forever2()

def forever2():
    forever1()
```



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Recursion - Exit Conditions

- The most important thing to remember with recursion is to include an *exit condition*
  - An exit condition is a way to stop repetition
  - Similar to how a loop must always exit, so must recursion
- The example below has no exit condition
  - The result is *infinite recursion*

```
def forever(message):
    print(message)
    forever(message)
```

pythontutor.com/visualize.html

Infinite Recursion - Video Example

```
def infinite(x):
    y = 3
    print(x + y)
    infinite(x + 1)

infinite(1)
```

Recursion - Exit Conditions

- Let's define a function which has an exit condition:

```
def repeat_n_times(n, message):
    if n < 1:
        return
    print(message)
    repeat_n_times(n - 1, message)

repeat_n_times(10, "hello")
```

pythontutor.com/visualize.html

Recursion - Video Example

```python
def sum_list(elems):
    if len(elems) == 0:
        return 0
    sum_rest = sum_list(elems[1:])
    return sum_rest + elems[0]

total = sum_list([1,2,3])
```

0

Coding Exercise 05b.1

- Write a recursive implementation to calculate the Fibonacci numbers
- Recall the definition of the Fibonacci numbers is:

$$\text{Fib } n = \begin{cases} n, & \text{if } n == 0 \text{ or } n == 1 \\ \text{Fib}(n-1) + \text{Fib}(n-2), & \text{if } n > 1 \end{cases}$$

0, 1, 1, 2, 3, 5, 8, 13, 21, …

Coding Challenge 05b.1

- Create a recursive version of the factorial function
- Don't forget an exit condition!

$$n! = \begin{cases} 1, \text{ if } n == 0 \text{ or } n == 1 \\ n * (n - 1)!, \text{ if } n > 1 \end{cases}$$

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Coding Exercise 05b.2

- Write your own *recursive* version of the `filter` function, called `myfilter`, which:
  - Takes a unary function (`check`) and a list (`values`) as arguments
  - Applies the function `check` to successive each value from the list, and if the result is `True`, adds the value to the output list
- For example:

```
marks = [64.5, 87.0, 55.5, 94.0, 71.5, 46.0, 100.0]
a_grades = myfilter(lambda mark: mark >= 80.0, marks)
# a_grades should be [87.0, 94.0, 100.0]
```

Tail Recursion

Tail Recursion

- Tail recursion is a special case of recursion where the recursive call is the last thing to happen before the function returns
  - It is noteworthy since this kind of recursion can be easily optimized:
    1. Converting to an iterative equivalent
    2. Simplifying the calling stack

```
def print_n_times(n, message):
    if n == 0:
        return
    print(message)
    print_n_times(n - 1, message)

print_n_times(5, ´Hello´)
```

Tail Recursion - Discussion

- Is the following Fibonacci function tail-recursive?

```
def fibonacci(n):
    if n <= 1:
        return n
    return fibonacci(n-1) + fibonacci(n-2)
```

pythontutor.com/visualize.html

Tail Recursion - Calling Stack

- Each stack frame on the calling stack remembers the *return address*, which is the address of the instruction immediately following the function call
  - Since the very next action after the recursive call is a return, we can simplify the stack significantly
  - Instead of each recursive call waiting for other recursive calls to exit, only to return, we can just have the innermost recursive call return to the original return address

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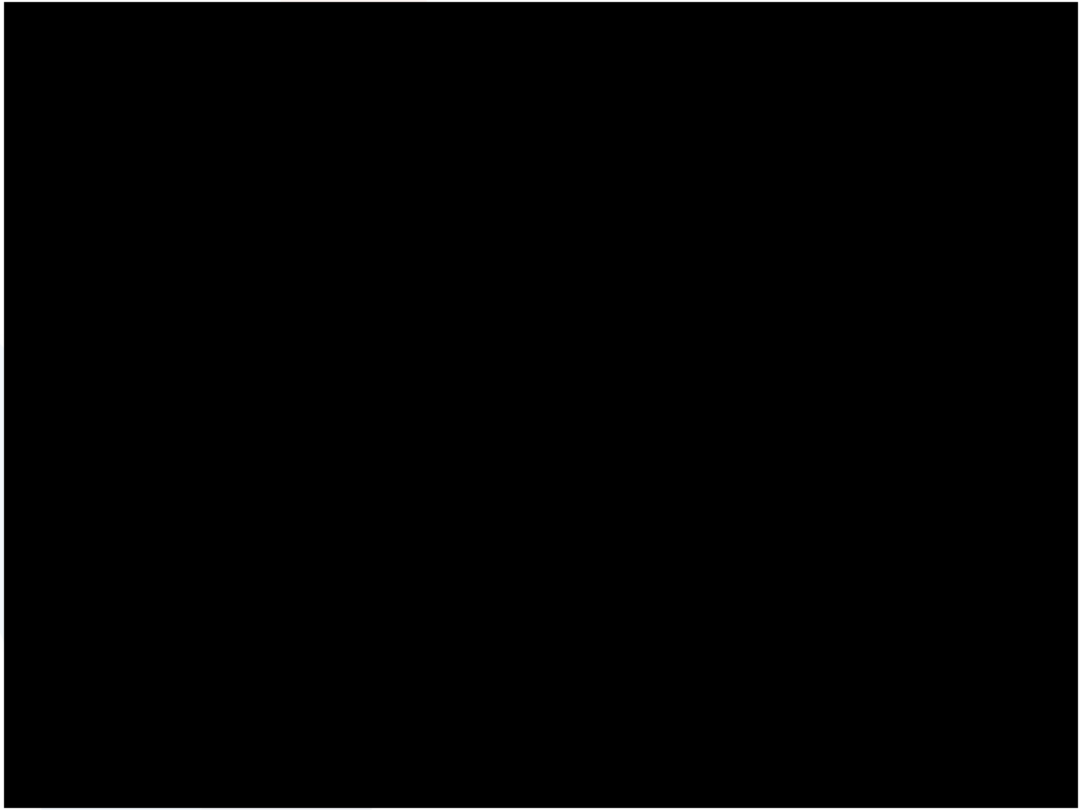
Tail Recursion - Video Example



Tail Recursion - Video Example

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```
def sum_list_tail(elems, base = 0):
    if len(elems) == 0:
        return base
    return sum_list_tail(elems[1:], base + elems[0])

total = sum_list_tail([1,2,3])
```

Hacker's Corner: Iterative Conversion

- Converting a tail-recursive function to iteration is usually straightforward:

```
def print_n_times(n, message):
    if n == 0:
        return
    print(message)
    print_n_times(n - 1, message)
```

```
def print_n_times(n, message):
    for i in range(n):
        print(message)
```

Wrap-up

- Recursion
    - Recursive calls and backtracking
    - Recursive function calls and the calling stack
    - Tail recursion and optimization

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Coming Up

- Motivation
  - What are classes?
  - What are objects?
  - Objects and classes in real life
  - Member variables and functions
- Writing object-oriented programs
  - Python syntax